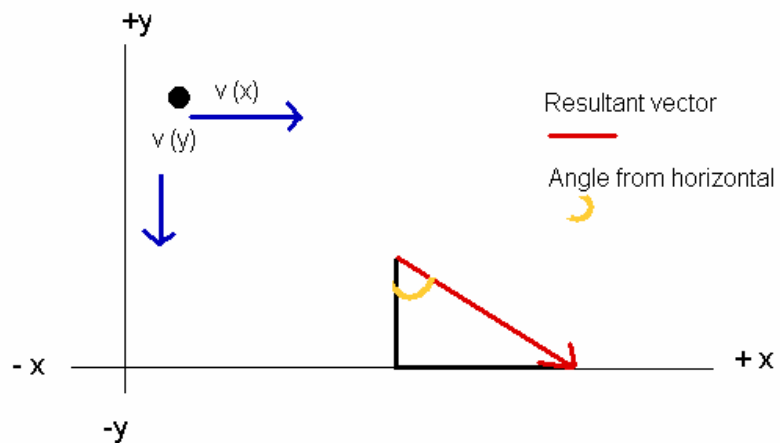


Physics 590 Homework, Week 6

Week 6, Homework 1

Prob. 6.1.1

A descent vehicle landing on the moon has a vertical velocity toward the surface of the moon of 35 m/s. At the same time it has a horizontal velocity of 55 m/s. (a) At what speed does the vehicle move along its descent path? (b) At what angle with the vertical is this path?



- a) To find the speed the vehicle descends with we need to combine the speed along the x coordinate with the speed along the y coordinate using the Pythagorean Theorem.

b)

$$\text{Speed of descent} = \sqrt{(v_x)^2 + (v_y)^2}$$

$$\text{Speed of descent} = \sqrt{(55\text{m/s})^2 + (35\text{m/s})^2}$$

$$\text{Speed of descent} = 65.192 \text{ m/s}$$

$$\text{Speed of descent} = 65.\text{m/s}$$

- c) To find the angle to the vertical we need to use the magnitude of the x coordinate (speed in the x direction) as the opposite side of the angle to the vertical, divided by the magnitude of the resultant (speed of descent) as the

hypotenuse of the of a right triangle made between the angles. The sin of the needed angle equals that ratio.

$$\sin\theta = (55\text{m/s})/(65\text{m/s}) = .8461538462$$

$$\sin^{-1}(.8461538462) = \theta$$

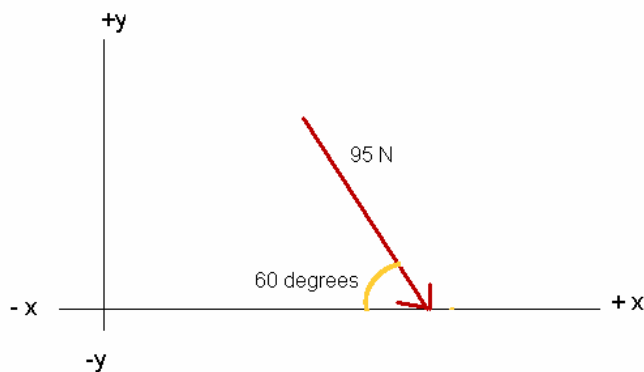
$$\Theta = 57.79577^\circ$$

$$\Theta = 57^\circ$$

Prob. 6.1.2

A lawnmower is pushed across a lawn by applying a force of 95 N along the handle of the mower. The handle makes an angle of 60.0° with the horizontal. (a) What are the horizontal and vertical components of the force? (b) The handle is lowered so that it makes an angle of 30.0° with the horizontal. What are the horizontal and vertical components of the force?

a) Trigonometry is used to find the force applied in the vertical and horizontal directions that provides for the resultant force and angle given.



$$\text{Horizontal force} = F(x) = 95\text{N} * \cos 60^\circ$$

$$\text{Horizontal force} = F(x) = 47.5 \text{ N}$$

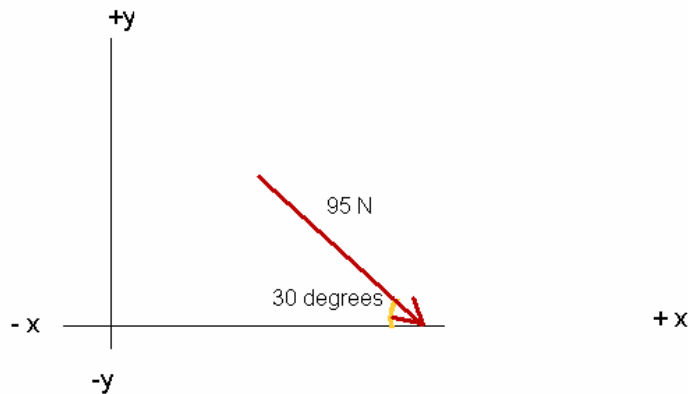
$$\text{Vertical force} = F(y) = 95\text{N} * \sin 60^\circ$$

$$\text{Vertical force} = 82.3 \text{ N}$$

$$\text{Horizontal force} = F(x) = 48\text{N}$$

$$\text{Vertical force} = 82\text{N}$$

b) Trigonometry is used to find the force applied in the vertical and horizontal directions that provides for the resultant force and angle given.



$$\text{Horizontal force} = F(x) = 95\text{N} * \cos 30^\circ$$

$$\text{Horizontal force} = F(x) = 82.3 \text{ N}$$

$$\text{Vertical force} = F(y) = 95\text{N} * \sin 30^\circ$$

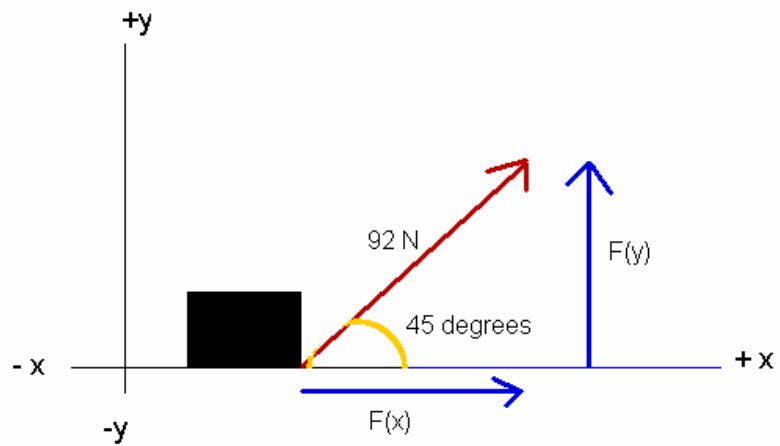
$$\text{Vertical force} = 47.5 \text{ N}$$

$$\begin{aligned} \text{Horizontal force} &= F(x) = 82\text{N} \\ \text{Vertical force} &= 48\text{N} \end{aligned}$$

Prob. 6.1.3

A force of 92 N is exerted on a heavy box by means of a rope. The rope is held at an angle of 45° with the horizontal. What are the vertical and horizontal components of the 92 N force?

Trigonometry is used to find the force applied in the vertical and horizontal directions that provides for the resultant force and angle given.



Horizontal force = $F(x) = 92\text{N} * \cos 45^\circ$

Horizontal force = $F(x) = 65.05\text{ N}$

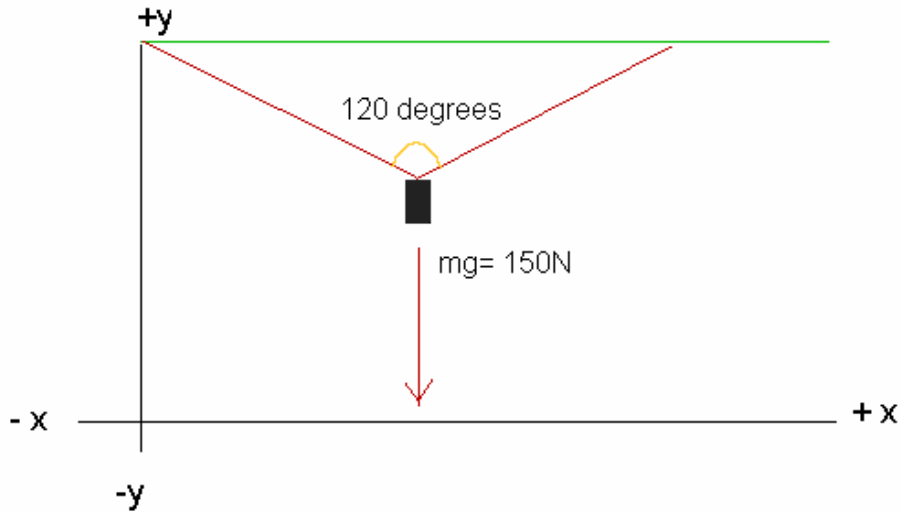
Vertical force = $F(y) = 92\text{N} * \sin 45^\circ$

Vertical force = 65.05 N

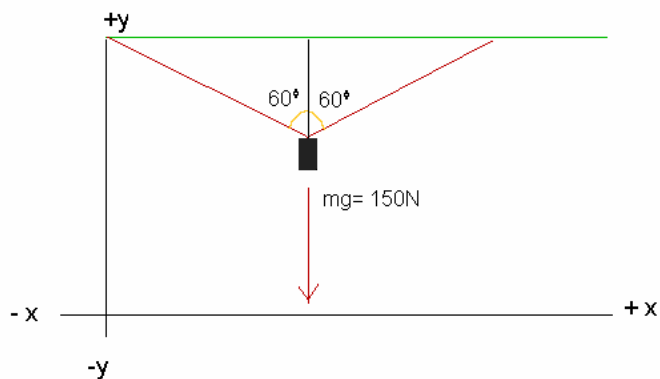
Horizontal force = $F(x) = 65\text{N}$ Vertical force = 65N

Prob. 6.1.4

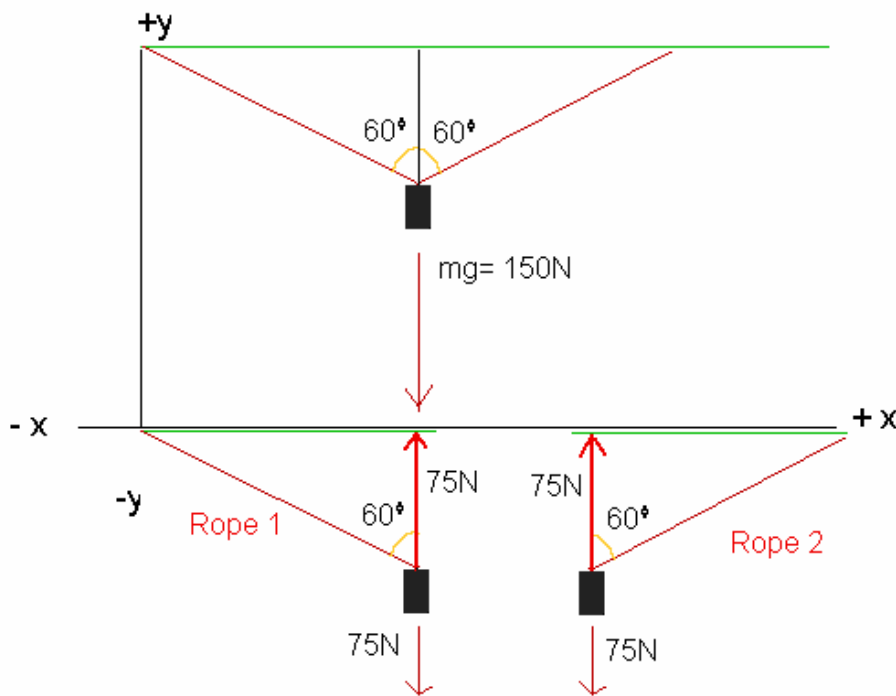
A street lamp weighs 150 N. It is supported equally by two wires that form an angle of 120° with each other. What is the tension in these wires?



In order to stop the traffic light from accelerating down, an equal but opposite resultant force needs to be applied. Each wire supports the light's weight equally. The resultant force splits the 120° angle in half. Two 90° triangles are formed between the resultant force and the horizontal. Giving two 30° - 60° - 90° triangles.



The triangles need to have a combined force along the resultant that is equal and opposite to the weight of the light.



The tension in the ropes is found dividing the adjacent side of the angle (75 N) by the $\cos 60^\circ$

$$T = (75\text{N})/(\cos 60^\circ)$$

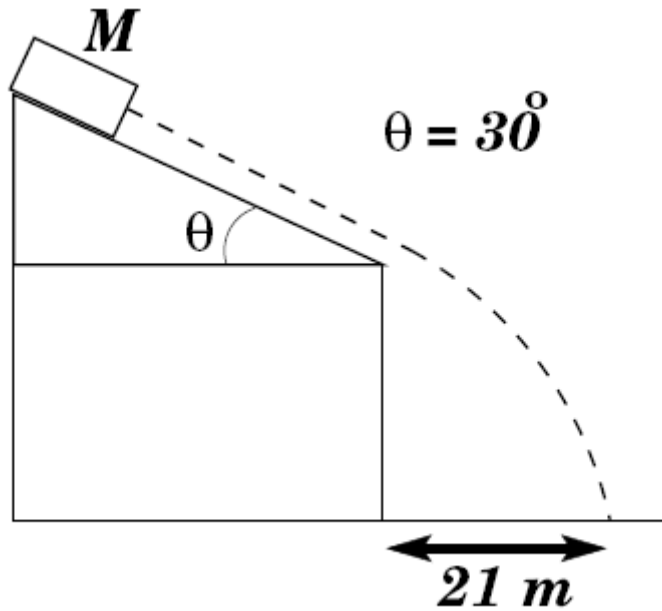
$$T = 150\text{N}$$

The tension is the same magnitude for each rope, but their x components are opposite each other, giving rope 1 a negative vector, and rope 2 a positive vector.

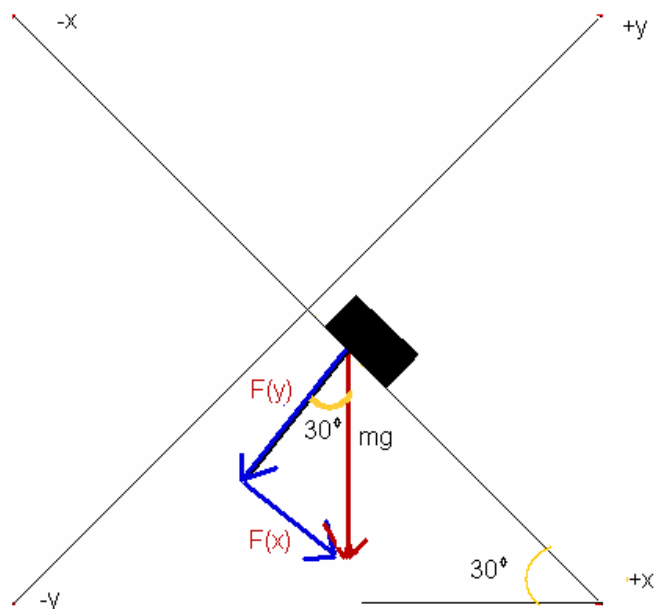
$T(\text{rope 1}) = -150\text{N}$ $T(\text{rope 2}) = 150\text{N}$
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Prob. 6.1.5

A mass, M , starts from rest and slides down a frictionless incline. As it leaves the incline, its speed is 24 m/s (see the figure below). (a) What is the acceleration of the mass while on the incline? (b) What is the length of the incline? (c) How long does it take the mass to reach the floor after it leaves the top of the incline?



- a) Acceleration of the mass on the incline: I will arrange my x axis along the path of the ramp, and the y axis will be perpendicular to the ramp. Motion can only be along the x axis, and the greatest force (mg) will become the hypotenuse of any subsequent triangle drawn from the figure. With the hypotenuse and a given angle I can find the component sides, one of which will be the force along the path of motion. Dividing that force by the mass will yield the acceleration of the ramp.



$$F(x) = mg * \sin 30^\circ$$

$$F(x) = ma(x) = m(9,8 \text{ m/s/s}) * \sin 30^\circ$$

$$ma(x) = m(4.9 \text{ m/s/s})$$

$$\frac{(\text{mass})a(x)}{\text{mass}} = \frac{(\text{mass})(4.9 \text{ m/s/s})}{\text{mass}}$$

$$a(x) = 4.9 \text{ m/s/s}$$

The acceleration of the mass on the incline = 4.9 m/s/s

b) The distance is found using the formula $(v)^2 = (v_0)^2 + 2ad$

$$v_0 = 0 \text{ m/s}$$

$$v = 24 \text{ m/s}$$

$$a = 4.9 \text{ m/s/s}$$

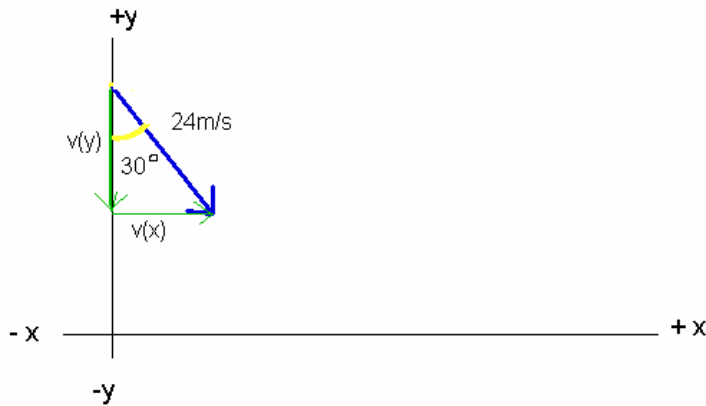
$$(24 \text{ m/s})^2 = (0 \text{ m/s})^2 + 2(4.9 \text{ m/s/s})d$$

$$\frac{578 \text{ m}^2/\text{S}^2}{9.8 \text{ m/s/s}} = d$$

$$d = 58.7755$$

The length of the incline = 59 m

c) For this part of the problem I will redraw my x axis along the horizontal, and my y axis along the vertical. It is coming down at an initial angle of 30 degrees to the horizontal with a speed of 24 m/s. When that speed is broken down into the x component vector speed, dividing the distance of 21 m by the x component speed will give the time it took to fall. (It must fall for that time: longer time and it would yield a longer distance, shorter time would yield a shorter distance)



$$v(x) = (24\text{m/s}) * \cos 30^\circ$$

$$v(x) = 20.78 \text{ m/s}$$

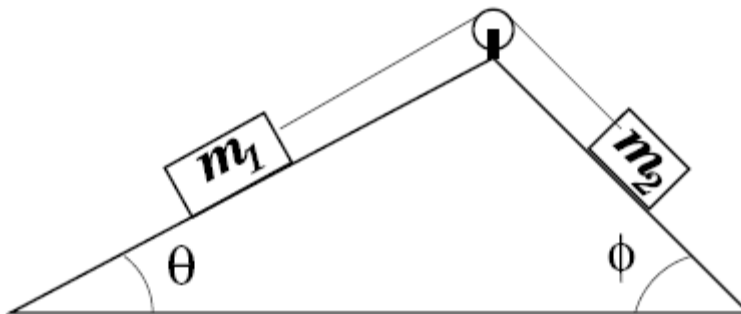
Time for mass to fall from edge of ramp:

$$(21\text{m}) / (20.78 \text{ m/s}) = 1.01 \text{ s}$$

Time for mass to fall from edge of ramp: 1.0s

Prob. 6.1.6

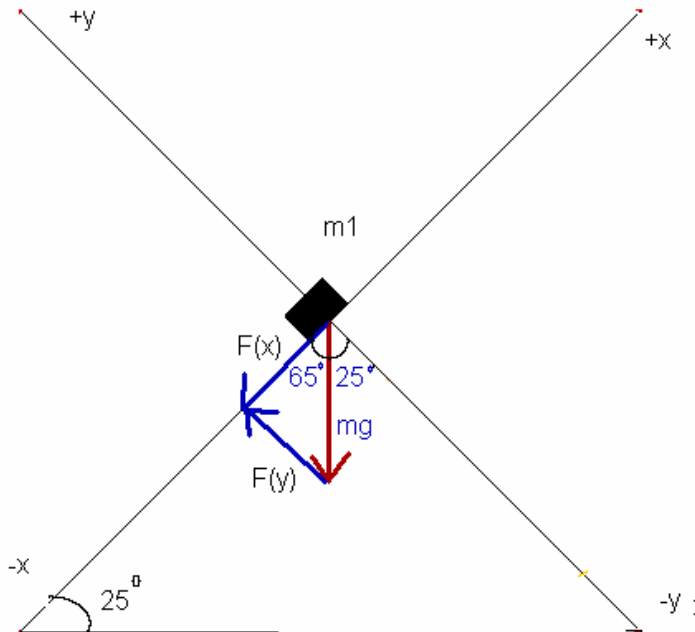
Two masses are attached to each other by a rope that passes over a frictionless, massless pulley as shown in the figure below. Find the direction and magnitude of the acceleration of mass $m_2 = 2.00 \text{ kg}$ if $m_1 = 3.00 \text{ kg}$, $\theta = 25^\circ$ and $\phi = 45^\circ$.



In order to find the acceleration of m_2 we must first find the net force acting on each mass. Without friction the only force acting on the masses is their weights, but the forces are directed along

angles and the normal force of the ramp will cancel a portion of their weight. I will set up my x axis along the path of motion for both masses, and set up my y axis perpendicular to the x axis. From there I will use mg for each mass as the hypotenuse of a right triangle to solve for their component vector forces. The y vector force and the normal force will cancel, leaving the net force to equal the x component force. The 2 forces will be opposite in direction, so the force with the greater magnitude x component force will indicate the direction the masses will move in. The combined forces then need to be divided by the total mass ($m_1 + m_2$) to get the acceleration.

For m_1 :

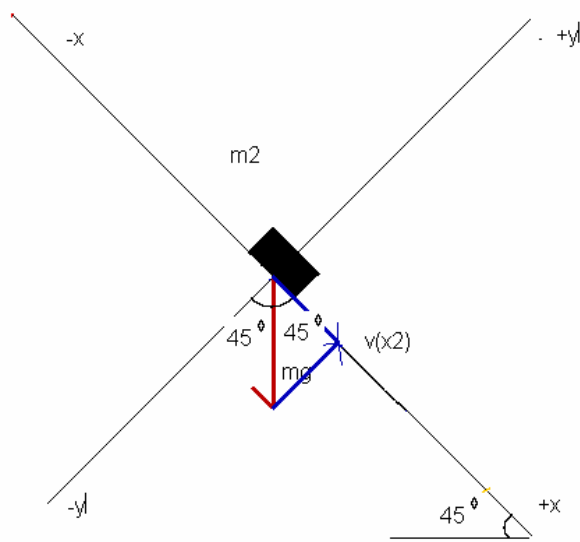


$$F(m_1x) = mg * \cos 65^\circ$$

$$F(m_1x) = -(3.00\text{kg})(9.8\text{m/s/s}) * \cos 65^\circ$$

$$F(m_1x) = -12.42\text{N}$$

For m2

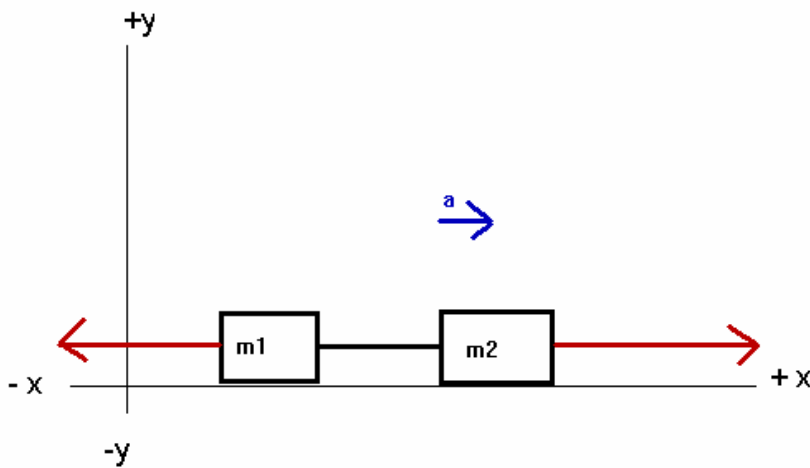


$$F(m_2x) = mg \cdot \cos 45^\circ$$

$$F(m_2x) = (2.00\text{kg})(9.8\text{m/s}^2) \cdot \cos 45^\circ$$

$$F(m_2x) = 13.86\text{N}$$

$$F(m_2x) + F(m_1x) = \Sigma F$$



$$\Sigma F = 13.86\text{N} - 12.42\text{N}$$

$$\Sigma F = 1.44\text{N to the right}$$

$$a = \Sigma F / \Sigma m$$

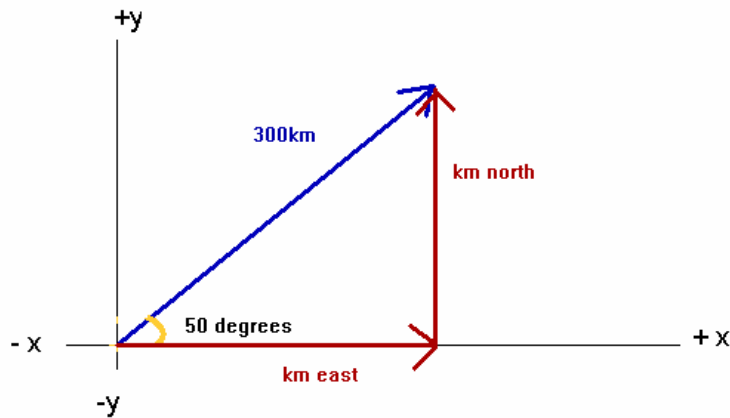
$$a = (1.44\text{N}) / (2.00\text{kg} + 3.00\text{kg})$$

$$a = .288 \text{ m/s/s}$$

Mass 2 will accelerate down its incline with an acceleration of .288 m/s/s

Prob. 6.1.7

A plane travels on a heading of 50.0° north of east for a distance of 3.00×10^2 km. How far north and how far east, in kilometers, does the plane travel?



$$\text{Distance north} = 300\text{km} * \sin 50^\circ$$

$$\text{Distance north} = 229.813\text{km}$$

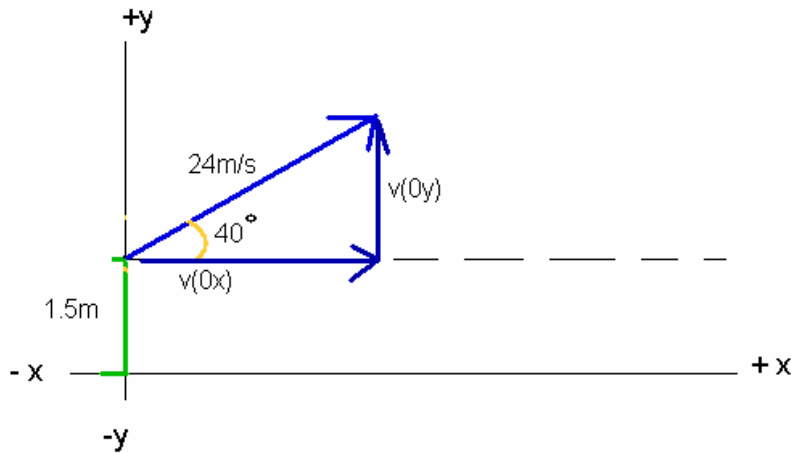
$$\text{Distance east} = 300\text{km} * \cos 50^\circ$$

$$\text{Distance east} = 192.836\text{km}$$

Distance north = 230km
Distance east = 193km

Prob. 6.1.8

An archer shoots an arrow into the air with an initial velocity of 24 m/s at an angle $\theta = 40^\circ$. The arrow leaves the bow at a height of 1.5 m above the ground. Find (a) the horizontal distance at which the arrow hits the ground (b) the maximum height the arrow reaches (c) the horizontal distance the arrow travels before hitting the ground assuming the archer adjusts the angle to 50° .



Of the three things that are asked to be solved for the maximum height is most convenient, after the vector has been broken down into its component parts. It will give us the time to reach the maximum height, then using the same formula we can get the time to reach the ground. Combining those two times will give us the total time the arrow is in the air. That time multiplied by the velocity in the x direction will give us the range. In order to solve for section c, follow the same steps substituting 50 degrees for the angle.

$$v_{0y} = \sin 40^\circ * 24 \text{ m/s}$$

$$v_{0y} = 15.4269 \text{ m/s}$$

$$v_{0x} = \cos 40^\circ * 24 \text{ m/s}$$

$$v_{0x} = 18.3851 \text{ m/s}$$

Solve for the time to maximum height

$$v_y = v_{0y} + at$$

$$0 \text{ m/s} = 15.4 \text{ m/s} + (-9.8 \text{ m/s/s})t$$

$$\frac{-15.4\text{m/s}}{(-9.8\text{m/s/s})} = t$$

$$t = 1.57\text{s to reach max height}$$

To get the distance traveled in the y direction

$$d_y = v_{0y} * t + \frac{1}{2} at^2$$

$$y = (15.4 \text{ m/s})(1.57\text{s}) + \frac{1}{2}(-9.8\text{m/s/s})(1.57\text{s})^2$$

$$y = 24.178 - 12.078 = 12.09\text{m}$$

Total height from the ground

$$12.09\text{m} + 1.5\text{m} = 13.59\text{m}$$

b) maximum height above the ground = 13.6m

Solve for the time it takes to fall 13.6m

$$d_y = v_{0y} * t + \frac{1}{2} at^2$$

$$-13.6\text{m} = 0\text{m/s} * t + \frac{1}{2}(-9.8\text{m/s/s})t^2$$

$$\frac{-27.2\text{m}}{-9.8\text{m/s/s}} = t^2$$

$$t = 1.67\text{s}$$

The total time the arrow is in the air = t(going up) + t(going down)

$$T = 1.67\text{s} + 1.57\text{s} = 3.24\text{s}$$

The range = $v_{x0} * T$

$$18.3851\text{m/s} * 3.24\text{s} = 59.57\text{m}$$

a) The horizontal distance covered = 60m

c)

$$v_{0y} = \sin 50^\circ * 24 \text{ m/s}$$

$$v_{0y} = 18.3851 \text{ m/s}$$

$$v_{0x} = \cos 50^\circ * 24 \text{ m/s}$$

$$v_{0x} = 15.4269 \text{ m/s}$$

Solve for the time to maximum height

$$v_y = v_{0y} + at$$

$$0 \text{ m/s} = 18.4 \text{ m/s} + (-9.8 \text{ m/s/s})t$$

$$\frac{-18.4 \text{ m/s}}{-9.8 \text{ m/s/s}} = t$$

$$t = 1.88 \text{ s}$$

t = 1.88s to reach max height

To get the distance traveled in the y direction

$$d_y = v_{0y} * t + \frac{1}{2} at^2$$

$$y = (18.4 \text{ m/s})(1.88 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s/s})(1.88 \text{ s})^2$$

$$y = 34.592 - 17.319 = 17.201 \text{ m}$$

Total height from the ground

$$17.201 \text{ m} + 1.5 \text{ m} = 18.701 \text{ m}$$

c_b) maximum height above the ground = 18.7m

Solve for the time it takes to fall 18.7m

$$d_y = v_{0y} * t + \frac{1}{2} at^2$$

$$-18.7\text{m} = 0\text{m/s} * t + \frac{1}{2}(-9.8\text{m/s/s})t^2$$

$$\frac{-37.4\text{m}}{-9.8\text{m/s/s}} = t^2$$

$$t = 1.95\text{s}$$

The total time the arrow is in the air = t(going up) + t(going down)

$$T = 1.95\text{s} + 1.88\text{s} = 3.83\text{s}$$

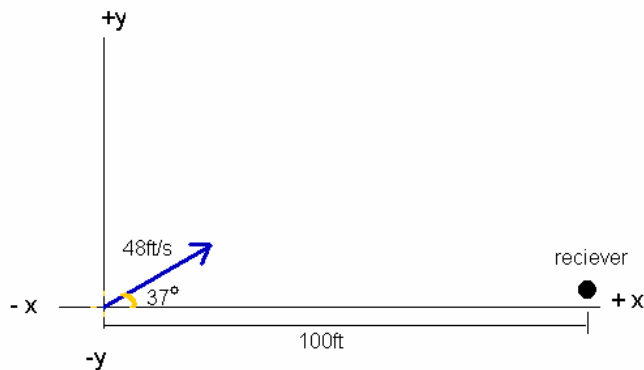
The range = $v_{x0} * T$

$$15.4269\text{m/s} * 3.83\text{s} = 59.08\text{m}$$

c_a) The horizontal distance covered = 59m

Prob. 6.1.9

A player kicks a football at an angle of 37° with the horizontal and with an initial speed of 48 ft/s. A second player standing at a distance of 100 ft from the first in the direction of the kick starts running to meet the ball at the instant it is kicked. How fast must he run in order to catch the ball just before it hits the ground?



First I will break the motion into its component vectors, then find the time to the maximum height. That time doubled will equal the time the ball is in the air. That time multiplied by the velocity in the x direction will give me the distance the punt will go. That distance subtracted from 100 feet will give me the distance the receiver must travel. That distance divided by the time the ball is in the air will yield the minimum speed the player needs to field the ball without it bouncing.

$$v_{0y} = \sin 37^\circ * 48 \text{ ft/s}$$

$$v_{0y} = 28.89 \text{ ft/s}$$

$$v_{0x} = \cos 37^\circ * 48 \text{ ft/s}$$

$$v_{0x} = 38.33 \text{ ft/s}$$

Solve for the time to maximum height

$$v_y = v_{0y} + at$$

$$0 \text{ m/s} = 28.89 \text{ ft/s} + (-32 \text{ ft/s/s})t$$

$$\frac{-28.89 \text{ ft/s}}{-32 \text{ ft/s/s}} = t$$

$$t = 0.90 \text{ s}$$

$$t = 0.90 \text{ s to reach max height}$$

$$0.90 \text{ s} * 2 = \text{total time in the air}$$

$$t = 1.8 \text{ s}$$

$$\text{Distance punt covers} = v(x) * t$$

$$38.33 \text{ ft/s} * 1.8 \text{ s} = 68.994 \text{ ft}$$

Distance receiver needs to cover

$$100 \text{ ft} - d(\text{punt})$$

$$100 \text{ ft} - 69 \text{ ft} = 31 \text{ ft}$$

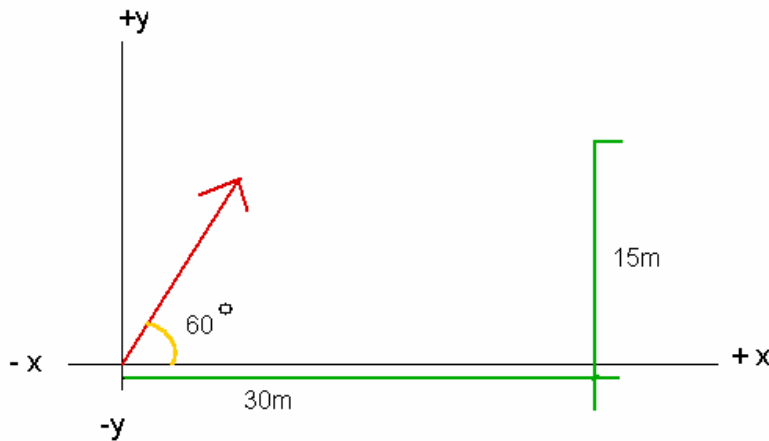
Speed receiver need to have to just catch the ball = d/t

$$31\text{ft}/1.8\text{s} = 17\text{ ft/s}$$

Speed receiver need to have to just catch the ball = 17 ft/s

Prob. 6.1.10

A projectile shot at an angle of 60° above the horizontal strikes a building 30 m away at a point 15 m above the point of projection. (a) Find the initial speed of projection. (b) Find the magnitude and direction of the velocity of the projectile when it strikes the building.



The method I devised to solve this problem is to use the height of the target to get an initial velocity in the y direction. The problem I have is that time is not given.

- 1) I determined that by plugging in values for t , I could find v_{0y} .
- 2) By multiplying that velocity by the tangent of the opposite angle (30°), I would get the velocity in the x direction.
- 3) By multiplying that velocity by the time I plugged in, I would get the distance it would travel in the x direction.

- 4) When the x distance equals 30m and the y distance equals 15m, I will have the components to calculate the resultant velocity.
- 5) From there I calculate the velocity in the y direction for the time solved for.
- 6) Combine that with the velocity in the x direction to get the resultant magnitude and the angle it strikes the building with.

First I will try 2.0s

$$1) \quad d_y = v_{0y} * t + \frac{1}{2} at^2$$

$$15m = v_{0y} * 2.0s + \frac{1}{2}(-9.8m/s/s)(2.0s)^2$$

$$15m = v_{0y} * 2.0s - 9.8m$$

$$24.8m = v_{0y} * 2.0s$$

$$\frac{24.8m}{2.0s} = v_{0y}$$

$$v_{0y} = 12.4m/s$$

$$2) \quad \tan 30^\circ * v_{0y} = v_{0x}$$

$$\tan 30^\circ * 12.4m/s = 7.159m/s$$

$$v_{0x} = 7.159m/s$$

$$3) \quad d_x = v_{0x} * t$$

$$d_x = 7.159m/s * 2.0s = 14.318m$$

That time is not long enough for the projectile to travel 30 meters, even though its height is 15 meters.

I will try 3.0s

$$1) d_y = v_{0y} * t + \frac{1}{2} at^2$$

$$15m = v_{0y} * 3.0s + \frac{1}{2}(-9.8m/s/s)(3.0s)^2$$

$$15m = v_{0y} * 3.0s - 44.1m$$

$$59.1m = v_{0y} * 3.0s$$

$$\frac{59.1m}{3.0s} = v_{0y}$$

$$v_{0y} = 19.7m/s$$

$$2) \tan 30^\circ * v_{0y} = v_{0x}$$

$$\tan 30^\circ * 19.7m/s = 11.37m/s$$

$$v_{0x} = 11.37m/s$$

$$3) d_x = v_{0x} * t$$

$$d_x = 11.37 * 3.0s = 34.11m$$

That time is a little too long. The projectile will land further down range than is required.

The time must be between 2.0 and 3.0 seconds

I will try 2.74s

$$1) d_y = v_{0y} * t + \frac{1}{2} at^2$$

$$15m = v_{0y} * 2.74s + \frac{1}{2}(-9.8m/s/s)(2.74s)^2$$

$$15m = v_{0y} * 2.74s - 36.787m$$

$$51.787m = v_{0y} * 2.74s$$

$$\frac{51.787\text{m}}{2.74\text{s}} = v_{0y}$$

$$v_{0y} = 18.90\text{m/s}$$

$$2) \tan 30^\circ * v_{0y} = v_{0x}$$

$$\tan 30^\circ * 18.90\text{m/s} = 10.91\text{m/s}$$

$$v_{0x} = 10.91\text{m/s}$$

$$3) d_x = v_{0x} * t$$

$$d_x = 10.91 * 2.74\text{s} = 29.89\text{m}$$

$$d_x = 30\text{m}$$

$$4) v_0 = \sqrt{(v_{0x})^2 + (v_{0y})^2}$$

$$v_0 = \sqrt{(10.91\text{m/s})^2 + (18.90\text{m/s})^2}$$

$$v_0 = 21.8\text{m/s}$$

Initial speed of the projectile = 21.8m/s

$$5) v_{y15\text{m}} = v_{0y} + gt$$

$$v_{y15\text{m}} = 18.90\text{m/s} + (-9.8\text{m/s/s})(2.74\text{s}) = -7.95\text{m/s}$$

The negative velocity indicates that it is no longer traveling up, but is on its way down.

$$6) \tan \theta = \text{opp/adj}$$

$$\tan \theta = v_{y15\text{m}} / v_{0x}$$

$$\tan \theta = (-7.95\text{m/s}) / (10.91\text{m/s})$$

$$\tan\theta = -0.7286892759$$

$$\Theta = \tan^{-1}0.7286892759$$

$$\Theta = -36.08^\circ$$

The angle it hits the building is 36° below the horizontal with a speed of 7.95 m/s