This can be solved either by carrying out a line integral, or by direct solution of the Poisson equation \( \nabla^2 V = -\rho/\epsilon_0 \). A solution by a direct integral over the source distribution is done in problem 2.28.

2. Specification of the potential is problematic for an infinite line charge since there is no far field where the potential can be assumed to vanish (i.e., you can’t get far away from an infinitely long line!). Consequently, you should define a reference point for your solution that sets \( V = 0 \) at some arbitrary separation \( s_0 \). Alternatively, you can carry out a calculation for a finite line charge of length \( L \) (where \( V \to 0 \) for \( s \gg L \)) and set \( V(s \to \infty) = 0 \).

3. Do parts (a) and (b) only.

**H4.P1 Dipole Fields**

The exterior electric field \( \mathbf{E}^e \) for a dipole shell, i.e., a surface charge density on a sphere of radius \( R \) given by

\[
\sigma(\theta) = \sigma_0 \cos \theta
\]

can be written in the form

\[
\mathbf{E}^e = \frac{p}{4\pi\epsilon_0} \left[ \frac{2 \cos \theta \mathbf{e}_r + \sin \theta \mathbf{e}_\theta}{r^3} \right]
\]

where \( p \) is a constant giving the total dipole moment of the shell. (Note that this is an exact expression for the exterior field of a dipole shell).

(a) Integrate over the charge distribution to evaluate the constant \( p \) expressing your answer in terms of the known quantities \( \sigma_0 \) and \( R \).

(b) Show (by explicit calculation) that \( \nabla \cdot \mathbf{E}^e = 0 \).

**H4.P2 Two Dimensional Dipoles**

In this problem we extend the discussion of the field from the charge configuration shown in Griffiths Fig. 2.28 to a continuous charge distribution with cylindrical symmetry. Two uniformly charged cylinders with volume charge density \( \pm \rho \) and equal radius \( R \) are aligned along the \( z \) axis with centers offset by a perpendicular distance \( d \). Define \( \vec{d} \) as the vector from the axis of the positively charged cylinder to the negatively charged cylinder. We’ll define the locations of these centers \( \pm(d/2)\mathbf{e}_x \), and you should consider the limiting case that \( R \gg d \neq 0 \).

(a) Find the electric field (magnitude and direction) in the region where the cylinders overlap (this is a region with zero net charge density).

(b) The electric field exterior to this charge distribution can be expressed \( \vec{E} = E_r \mathbf{e}_r + E_\phi \mathbf{e}_\phi \). Find explicit expressions for \( E_r \) and \( E_\phi \) in terms of \( \rho, R, r, \phi \) and fundamental constants to leading nonvanishing order in \( d/s \).

(c) Find shapes of the equipotentials for this system by deriving an expression for the parametric curve \( s(\phi) \) where \( V(s, \phi) = V_0 \).