It is useful to express this result in terms of the electric field exterior to the pair of plates.

First consider the case where the outer shell is grounded $V_b = 0$ and find the charge states of the two conductors when $\Delta V = V_a = V_o$. Then solve the general two plate problem by computing the $2 \times 2$ capacitance matrix and solve for the charge states for general values of $V_a$ and $V_b$. For this calculation it is necessary to define a reference potential $V(s_o) = 0$, and by convention take the reference point $s_o > b$. Find the charges of both surfaces when: (a) $V_a = V_o$ and $V_b = 0$ (this should reproduce your earlier result) and for $V_a = 0$ and $V_b = V_o$ (reversed bias). Find the electrostatic energies for both charge configurations.

**H6.P1 Electric field at the edge of an anisotropic conductor**

Problem 2.57 implies that the electric field at a sharp edge of a conducting disk or a conducting needle can be very large. In this problem we calculate how the magnitude of the field depends on the curvature at the edge of the conducting object for two prototypical shapes.

(a) A disk is obtained by considering the limit of the result of 2.57 when $a = b \to R$ and $c \to 0$. The edge field in the plane of the disk at its edge diverges $\propto c^p$. Find the power $p$.

(b) A needle is obtained by considering the limit $a = L/2$, $b = c \to 0$. The edge field i.e. parallel to the needle at its end diverges $\propto c^p$. Find the power $p$. To appreciate the strength of this divergence, evaluate the magnitude of the electric field at the tip if the total charge on the needle is $Q = 10^{-9}$ C and the small width parameter $c = 10 \mu m$.

**H6.P2 Concentric Spherical Capacitors**

Two concentric spherical shells have radii $a$ (inner shell) and $b$ (outer shell). The elementary result for the (scalar) spherical capacitance of this system is

$$C_s = 4\pi \varepsilon_o \frac{ab}{b-a}$$

(a) Find the full $2 \times 2$ capacitance matrix for this system of coupled conductors.

(b) If the inner shell is held at potential $V_a = V_o$ and the outer shell is grounded $V_b = 0$ find the charges on the two conductors $Q_a$ and $Q_b$, their sum $Q_{tot} = Q_a + Q_b$ and the total electrostatic energy of this system.

(c) For the complementary geometry where the outer shell is held at potential $V_b = V_o$ and the inner shell is grounded $V_a = 0$ find the charges on the two conductors $Q_a$ and $Q_b$, their sum $Q_{tot} = Q_a + Q_b$ and the total electrostatic energy of this system.