Fighting Crises with Secrecy*

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Abstract

How does central bank lending during a crisis restore confidence? Emergency lending facilities which are opaque (in that names of borrowers are kept secret) raise the perceived average quality of bank assets in the economy, creating an information externality that prevents runs. Stigma (the cost of a bank’s participation at the lending facility becoming public) is desirable to implement opacity as an equilibrium outcome, as no bank wants to reveal its participation status. The central bank’s key policy instrument for limiting the use of lending facilities while maintaining secrecy is the haircut applied to bank assets used as collateral.

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1 Introduction

How does central bank emergency lending during a financial crisis restore confidence? During the financial crisis of 2007-2008 the Federal Reserve introduced a number of new emergency lending programs, including the Term Auction Facility, the Term Securities Lending Facility, and the Primary Dealers Credit Facility, which provide public funds to financial intermediaries in exchange for private assets. Also as part of their Emergency Economic Stabilization Act, the U.S. Treasury implemented similar programs, such as the Troubled Asset Relief Program (TARP). What these facilities had in common was their initial specific design to hide borrowers’ identities. Secrecy was also integral to the special crisis lending programs of the Bank of England and the European Central Bank. Plenderleith (2012), asked by the Bank of England to review their Emergency Lending Facilities (ELA) during the financial crisis, wrote: “Was secrecy appropriate in 2008? In light of the fragility of the markets at the time . . . ELA is likely to be more effective if provided covertly” (p. 70). Even before the Federal Reserve came into existence, private bank clearinghouses would also open an emergency lending facility during banking panics keeping the identities of borrowing banks secret. See Gorton and Tallman (2018).

The secrecy of these lending facilities is not without controversy. Hiding the identities of weak or insolvent banks has been widely criticized, resulting in fierce calls for transparency. During the recent crisis, for instance, Bloomberg and Fox News sued the Fed (under the Freedom of Information Act) to obtain the identities of borrowers. Similarly, TARP was initially designed to maintain secrecy of borrowers, but this was quickly overturned by the Senate Congressional Oversight Panel. In this paper, however, we show that lending facilities that replace “dubious private assets” with “good public bonds” in secret are indeed optimal. Then political pressures that force transparency during crises may lead to their inefficient implementation; such that more banks are bailed out than necessary.

Secrecy creates an information externality by raising the average quality of assets in the banking system without replacing all assets under suspicion, mitigating the de-

1Bernanke (2010): “. . . [because of] the competitive format of the auctions, the TAF [Term Auction Facility] has not suffered the stigma of the conventional discount window” (p.2).
sire of depositors to examine all banks’ assets, thereby avoiding a withdrawal of funds (runs hereafter) from those banks that are found to have less than average asset quality. A crisis here is an information event, as in Dang, Gorton, and Holmström (2013) and Gorton and Ordonez (2014), and as such its solution has a strong information content. Lending facilities recreate confidence and avoid inefficient examination of banks’ portfolios. Secrecy minimizes the social cost of resorting to lending facilities.

But even when the central bank may prefer secrecy to reduce intervention costs, banks could have different plans, individually disclosing their participation (or lack thereof) and making the intended secrecy not sustainable. How can opaque lending facilities be sustainable in equilibrium? We show that stigma (the negative inference made about a bank’s quality from its need to borrow from the government) is endogenous to the implementation of the intervention and essential to discourage participating banks from disclosing their participation. But wouldn’t non-participating banks have incentives to reveal their lack of participation? We show that in equilibrium those banks would face a run without any gain in perception about their quality. In short, stigma aligns the secrecy incentives of the government with those of individual banks.

In stark contrast with the literature, in our setting a run is not a coordination failure (as there is a single depositor per bank) but instead an information failure. Our interpretation of a run is not one in which the depositor suddenly requests “give me the money!” but rather says “show me the money!” We abstract from coordination failures to focus on this novel dimension of distrust in banks that leads to close examination of their portfolio before withdrawing, a behavior common during events of financial distress. Our view of stigma is also in contrast to the more standard view in the literature, usually thought of in the context of normal, non-crisis, times in which a single bank does not want to be identified as weak by borrowing from the discount window for fear of a run. Neither the bank nor the central bank want this bank-specific information revealed. But, a financial crisis is a state of the world in which the whole banking system is stigmatized. The run is already happening: holders of short-term debt doubt all banks’ backing for their debt. Addressing this with a policy of opacity is intended to create the rational belief that the system is solvent. Here, opacity is not the means to avoid stigma.

Our view captures both depositors seeking to withdraw from banks but also repo lenders wanting to withdraw via higher haircuts on the collateral or not rolling over their loans at all. Evidence that not all banks experience runs upon examination is provided by Perignon, Thesmar and Vuillemey (2017) who show that in the European market for unsecured wholesale certificates of deposit, some banks experiences funding dry-ups while other banks did not.
and a run on a single bank but instead stigma is the means to avoid transparency and a run on the whole system.

In our model households are born with endowments already deposited in a bank. They can withdraw immediately to consume or keep the deposits in the bank. A bank is an institution that also has its own funds and has the proprietary access to two productive investment opportunities. One, which we denote collateral, generates an asset that is pledgeable and the other, which we denote project, is stochastic, ex-ante efficient and not pledgeable. The first will be financed with its own resources and is then used to collateralize the deposits to finance the second.

The underlying problem in the economy is a scarcity of good collateral (“safe debt” to back repo, for example). The bank’s own collateral will be used to back deposits (e.g., the land on which a building is being built). When good collateral is scarce, an efficient substitute is ignorance about which private assets used as collateral are good and which are bad, as in the absence of information even banks with bad collateral can avoid early withdrawals and invest in the project. In that case, good and bad collateral are pooled in an informational sense. When this pooling results in a high enough perceived average value of banks’ portfolios, depositors do not examine their own bank’s portfolio (no run), maintain their deposits in the bank and banks efficiently invest in their projects.

A crisis happens when an (exogenous) event occurs (a fall in home prices, for instance) causing depositors to run, examining the bank’s portfolio and withdrawing if the bank has bad collateral. If this happens, banks can react by reducing their investment scale thus reducing the amount borrowed. This can avoid information acquisition, or they can give in to the run, be examined by depositors and hope the depositors find out that the collateral is good so it can invest at the optimal scale. In either case, absent government intervention, aggregate consumption falls during a crisis.

The government’s goal is to prevent a systemic run, which means avoiding information acquisition about all banks’ portfolios. How does a central bank end a crisis in our model? First, the central bank opens an emergency lending facility (called the discount window throughout the paper) that exchanges government bonds for private collateral at a haircut (discount throughout the paper). As banks have heterogeneous collateral (because of idiosyncratic shocks), which banks go to the discount window depends on their private information about their portfolio quality and on the discount. The central bank can adopt a policy of opacity in which the identities of banks going to the discount
window are not revealed, or a policy of transparency in which identities are revealed. We both provide conditions under which opacity is the socially preferred policy and show that individual banks do not want to deviate from this policy by revealing their participation status.

Restoring confidence during crises means raising the perceived average quality of bank assets in the economy. As, with opacity, lending government bonds to some banks increases the expected bank portfolio value for all banks, this intervention creates an information externality that also prevents runs for non-participating banks. The central bank’s choice of the discount controls how many banks participate in the discount window and by doing so determines the perceived average quality of private collateral remaining in the economy. As participating banks are those with lower quality of collateral on average, they do not want to individually reveal their participation and face a higher likelihood of an idiosyncratic run in the future. Similarly, non-participating banks cannot credibly signal a high quality of collateral in the future by revealing no participation, but face a run during the ensuing crisis. Endogenous participation and stigma is what makes the opacity policy sustainable in equilibrium.

Related Literature: A prominent branch of the literature has studied the effects of lending facilities in helping the economy during crises. Most of this work is based on the premise that a crisis is given by an exogenous tightening in credit constraints (see Gertler and Kiyotaki (2010) for a discussion) and lending facilities are set up to provide loans directly (increasing the role of governments as credit providers), open a discount window (mostly to improve inter-banking operations) and injecting equity (to increase the net worth of banks). In our model the tightening of credit constraints is endogenous and created by an informational reaction in credit markets to changes in fundamentals, and then we emphasize the role of lending facilities in relaxing the informational tightening of credit conditions.

There is also a large literature on lenders-of-last-resort summarized by Freixas, Gianinni, Hoggarth and Soussa (1999 and 2000) and by Bignon, Flandreau, and Ugolini (2009). Recent work also includes Flandreau and Ugolini (2011) and Bignon, Flandreau, and Ugolini (2009) who document the development of the Bank of England and the Bank of France as lenders-of-last-resort. Unlike the existing literature, we focus on why secrecy surrounds interventions during crises, the sustaining role of stigma, and

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how discounts are determined during a crisis.

Our paper also contributes to the recent discussion about the optimal disclosure of information about financial variables by governments and regulators. A clear example is the literature that studies stress tests and focuses on the possible impact of information disclosure about a bank’s portfolio on funding (Alvarez and Barlevy (2015) and Leitner and Williams (2017)), on runs (Bouvard, Chaigneau, and De Motta (2015), Faria-e Castro, Martinez, and Philippon (2017) and Williams (2017)) and on risk-sharing (Goldstein and Leitner (2018)). For a survey see Goldstein and Sapra (2013). Our work differs from this literature in several ways. First, stress tests are primarily about preventing crises, not fixing them. Second, the disclosure of stress test is the revelation of a state/type, while the disclosure of lending facilities participation is the revelation of a choice. Finally, the stress test literature focuses on solving a planning problem, whereas we are not only interested in the optimality of the policy but also on its sustainability as an equilibrium outcome, in particular the role of discount rates to pin down the stigma cost that discourages banks from deviating from the central bank’s intended disclosure plan.

Among the cited papers on stress tests, Alvarez and Barlevy (2015) is the closest to ours in terms of focusing on the disclosure choice of banks. They study disclosure of stress tests by interconnected banks. During crises, when contagion is strong, the planner may need to force disclosure because banks would individually prefer opacity. In our case the planner wants opacity but banks may prefer disclosure, potentially making the intended plan non-sustainable. While in principle it is easy to force disclosure of hard information, preventing it may prove more challenging. They show the former may be necessary, we show that the latter is feasible because of endogenous stigma on the design of lending facilities.

Another related paper is Philippon and Skreta (2012), who study optimal interventions in markets with adverse selection. In a different context, but as in our case, the goal is to find an intervention that maximizes investment at the lowest possible cost to taxpayers. Also as in our case, the outside option of not participating in the intervention tool is endogenous to the extent of participation. While in their paper, however, participation is observable by assumption and signals the quality of the individual, in our paper both participation and its disclosure are choices of the planner when designing the optimal intervention and choices of individual banks, which affect the sustainability of such intervention.
The paper proceeds as follows. In Section 2 we specify the model, including depositors’ choices to run or not and banks’ choices of how much to invest. In Section 3 we describe crises and the structure of interventions that we consider. Section 4 concerns the equilibrium when the economy is in a crisis and the central bank opens a lending facility. First, we solve the equilibrium for each discount and disclosure policy, and then we obtain the combination that maximizes welfare. We also consider the role of stigma in sustaining the optimal disclosure policy. Section 5 concludes. All proofs are contained in the Online Appendix.

2 Model

2.1 Environment

We study a two-period economy populated by a mass $1$ of households and a mass $1$ of banks. Households live for a single period, are risk-neutral and are indifferent between consuming at the beginning or at the end of the period in which they live. Each household is born with an endowment $D$ of consumption good (numeraire), which is already deposited in a bank (accordingly we refer to them as depositors), which is available for withdrawal at no cost at the beginning of the period.

Banks live for two periods and are also risk-neutral. Each bank is matched with a single household, has its own funds (bank equity), and two single-period investment opportunities in each period. One investment opportunity generates proceedings that are observable and verifiable in court. The other does not. This structure guarantees that the bank finances the former with own funds and use it to collateralize the depositors’ funds to finance the latter. Accordingly, we call the first, verifiable, project as the collateral and the second simply as the project.

Collateral: With probability $p_{it}$ the collateral of bank $i$ in period $t$ is of good type, in which case it delivers $C$ units of numeraire at the end of the period. With the complementary probability, the collateral is of bad type and does not deliver any numeraire at the end of the period. The collateral types are iid across periods and across banks. We refer to $p_{it}$ as the quality of the collateral held by bank $i$ in period $t$. The quality of collateral depends on an aggregate component and on an idiosyncratic component, $p_{it} = \bar{p}_t + \eta_i$. The aggregate component, $\bar{p}_t$, is time-varying and captures the average
quality of collateral in the economy. The idiosyncratic component, \( \eta_i \), is persistent and captures heterogeneity of banks’ ability to manage good collateral, where \( \eta_i \sim F[-\bar{\eta}, \bar{\eta}] \), with \( E(\eta_i) = 0 \) and \( \bar{\eta} \) such that \( \rho_i \in [0, 1] \).

Collateral Information: In terms of information, no agent (neither the bank nor the depositor) knows the collateral type at the beginning of the period but there is asymmetric information about its quality: while both banks and depositors know the average quality of collateral in the economy, \( \bar{\eta} \), each bank privately observes its own idiosyncratic component \( \eta_i \), so the bank has a better assessment of its own collateral quality, \( p_{it} \). At the end of the period, after all contracts are settled, the collateral type becomes public knowledge. Even though no agent knows the collateral type at the beginning of the period, depositors have a technology to privately investigate the collateral type at the beginning of the period at a cost \( \gamma \) in terms of numeraire. Using this technology they can determine whether the collateral will deliver \( C \) units of numeraire at the end of the period.\(^8\) We assume that acquiring information is a private activity, but the depositor can credibly disclose (certify) such private information immediately upon acquisition if it is in his best interest to do so. This information technology induces depositors to learn about the bank’s collateral type before deciding whether or not to keep their funds in the bank.

Project: The project pays \( A \min\{K, K^*\} \) with probability \( q \) and 0 otherwise, where \( K^* \) is the maximum feasible scale of production. As mentioned, the result of the project (whether it succeeds or fails) is known by the bank but it is neither observable nor verifiable by the depositors. We make the following parametric assumptions about the project’s payoffs:

Assumption 1 Project payoffs

- \( qA > 1 \). It is ex-ante optimal to finance the project up to a scale \( K^* \).
- \( D > K^* + \gamma \). It is feasible to implement the optimal investment just using deposits.
- \( C > K^* \). Good collateral is sufficient to cover the optimal loan size.\(^9\)

\(^7\)An example of collateral in our setting is the construction of a building, financed by the banker’s own funds and then sold to a property manager in charge of selling the units and dealing with tenants. The quality of the construction depends both on aggregate components (i.e., an aggregate TFP shock) and idiosyncratic components (i.e., the expertise of the banker to monitor the contractor).

\(^8\)Assuming that banks can also learn the bank’s collateral type does not modify the main insights. See Gorton and Ordonez (2018) for double-sided information acquisition in a model without banks.

\(^9\)Even though the bank could use the proceeds of the good collateral to finance the project up to \( K^* \)
2.2 Timing

The timing during a single period is as follows. At the beginning of the period, a bank offers a contract to its depositor that consists of an investment plan (the commitment to invest a certain amount $K$ in the project), an interest rate for the deposits (a promised repayment $R \geq D$ at the end of the period), and the collateral that secures the deposits (a fraction $x$ of the collateral to be delivered to the depositor in case the bank defaults). Conditional on this contract, depositors choose whether or not to examine the bank’s collateral and then decide whether or not to withdraw the deposits at the beginning of the period. At the end of the period, two sequential movements take place. First projects pay and loan contracts are fulfilled. Then, collateral types are revealed and banks sell their collateral (the fraction not handed out to lenders because of default) to households (making a take-it or leave-it offer to a randomly matched household).

Formally, we have a sequential game between a bank and its depositors in which a bank chooses a contract to maximize profits conditional on the constraints imposed by information acquisition and withdrawal choices of depositors. Modifying the announced investment the bank can (in effect) change these choices. Accordingly, in what follows we first discuss depositors’ examination and withdrawal choices and then banks’ investment choices.

2.3 Withdrawal Choices

We say there is a run on a bank when its depositor examines the bank’s collateral at the beginning of the period, and withdraws the funds upon finding out that the collateral is bad. Here we characterize the optimal contract in case of a run and in case of no run. While the bank knows its own collateral quality $p_{it}$, households just have an expectation about the probability that a particular bank’s collateral is good, which we denote as $E(p_{it}|I)$, where $I$ is the information set of depositors at the time of deciding whether to withdraw at the beginning of the period. In this section we focus on a single bank and a single period, so we dispense with the subindex $it$. this is not possible because the bank has to finance the project at the beginning of the period and the collateral proceeds happen at the end of the period.


2.3.1 Bank Run (Depositor Examines the Bank)

Assume the depositor examines the bank’s collateral at the beginning of the period, spending $\gamma$ of numeraire. When examination occurs both the depositor and the bank learn the type of collateral at the beginning of the period. If the collateral is good, it is enough to guarantee the recovery of deposits at the end of the period, even if the bank finances the project at the optimal scale $K^*$ and the project fails. This is because $C + D - K^* > D$ by assumption. If the collateral is bad, the depositor withdraws the deposits at the beginning of the period because if the bank were to finance the project it can always claim the project has failed and turn over the worthless collateral to the depositor.

What is the bank contract triplet $(R_r, x_r, K_r)$ (the promised repayment at the end of the period, the fraction of collateral that backs the loan and the investment scale) that induces the depositor to examine the collateral and allows for investment if the collateral is good? To begin to answer this first define the expected quality of collateral of a bank offering a contract that induces a run as $E^r(p) \equiv E(p|\text{run})$, which is an equilibrium object. With probability $E^r(p)$ the depositor’s expected payoffs are $qR_r + (1 - q)(D - K_r + x_rC) - \gamma$. With probability $(1 - E^r(p))$, the depositor always withdraws and loses the cost of examination, then obtaining $D - \gamma$.

Before using these elements to write the expression for the expected depositor’s payoffs, there is an additional constraint we exploit: a truth-telling condition given by $R_r = D - K_r + x_rC$. If it were the case that $R_r < D - K_r + x_rC$ the bank would always sell the collateral to another household at $C$ to repay the loan (notice that a buying household can infer the collateral is good by observing that a loan that presumes examination has been issued). In contrast, if it were the case that $R_r > D - K_r + x_rC$ the bank would always default on the depositor.

Using the truth-telling condition, the expected payoffs for a depositor are then $E^r(p)(R_r - \gamma) + (1 - E^r(p))(D - \gamma)$, independent of $K_r$. Assuming depositors do not have negotiation power and break even, $D = E^r(p)R_r + (1 - E^r(p))D - \gamma$. This participation constraint pins down the interest rate:

$$R_r = D + \frac{\gamma}{E^r(p)},$$

which is independent of $q$ (the probability the project pays off).
The contract that maximizes the bank’s payoffs under runs is then a triplet, \((R_r, x_r, K_r)\), where \(R_r\) is given by equation (1), \(x_r = \frac{R_r - D + K_r}{C}\) (from the truth telling condition) and \(K_r = K^*\) (from feasibility of optimal investment upon finding the collateral is good). Now we can compute the bank’s ex-ante expected profits given this contract. As the bank knows its collateral is good with probability \(p\), and that it will suffer a withdrawal in case its collateral is bad, its ex-ante expected profits are \(p(D + qA K^* - K^* - R_r) + pC\).

Substituting \(R_r\) in equilibrium, the period expected profits (net of the expected price of the collateral at the end of the period, \(pC\)) from inducing a run are:

\[
E_r(\pi|p, E^r(p)) = \max \left\{ pK^*(qA - 1) - \frac{p}{E^r(p)} \gamma, 0 \right\}.
\]

These expected profits depend not only on the probability that the bank’s collateral is good, \(p\), but also on the average quality of the collateral of all other banks inducing a run. This implies that there is cross-subsidization among banks that face a run: banks with \(p > E^r(p)\) end up paying more to compensate depositors for the information costs in expectation, as \(\frac{p}{E^r(p)} > 1\). The opposite happens for banks with \(p < E^r(p)\).

### 2.3.2 No Bank Run (Depositor Does Not Examine the Bank)

Another possible contract is one where the depositor does not produce information. In this case, the depositor expects to obtain \(R_{nr}\) in case of repayment. And in the case of default, a fraction \(x_{nr}\) of the collateral of expected value \(E^{nr}(p)C\) (where \(E^{nr}(p) \equiv E(p|\text{no run})\) is the expected quality of collateral among banks that offer a contract that does not trigger examination); this is \(D - K_{nr} + x_{nr} E^{nr}(p)C\). In this case the truth-telling constraint is given by \(R_{nr} = D - K_{nr} + x_{nr} E^{nr}(p)\). If \(R_{nr} < D - K_{nr} + x_{nr} E^{nr}(p)C\), the bank can sell the collateral at a price \(E^{nr}(p)C\) (recall information about the type is revealed after contracts are settled) and repay always. In contrast, if \(R_{nr} > D - K_{nr} + x_{nr} E^{nr}(p)C\), the bank would always default. Then, \(x_{nr} = \frac{R_{nr} - D + K_{nr}}{E^{nr}(p)C}\).

The depositor is indifferent between participating in the contract or not if \(D = R_{nr}\), which implies that no run is a feasible contract if and only if \(x_{nr} = \frac{K_{nr}}{E^{nr}(p)C} \leq 1\). This gives the first technological constraint on investment from this contract,

\[
K_{nr} \leq E^{nr}(p)C.
\]
The second, informational, constraint on investment comes from guaranteeing that the depositor does not have an incentive to deviate and examine the bank’s collateral privately at the beginning of the period. Depositors want to deviate if the expected gains from acquiring information, evaluated at $R_{nr}$ (and then at $x_{nr}$), are greater than the gains from not acquiring information. This is

$$(1 - E^{nr}(p))D + E^{nr}(p) [qR_{nr}^R + (1 - q)[D - K_{nr} + x_{nr}C]] - \gamma > D.$$  

Substituting in the definitions of $R_{nr}$ (and then $x_{nr}$), there is no incentive to privately deviate and examine the bank’s collateral if:

$$K_{nr} < \frac{\gamma}{(1 - q)(1 - E^{nr}(p))}. \quad (4)$$

Intuitively, the gains from examining the collateral and withdrawing if the collateral is bad are increasing in the size of the loan and in the risk of default. If the bank scales back the project (hence the loan), the depositor has less of an incentive to acquire information about the bank’s collateral.

Imposing constraints (3) and (4), the project size that is consistent with a no-run contract is

$$K_{nr}(E^{nr}(p)) = \min \left\{ K^*, \frac{\gamma}{(1 - q)(1 - E^{nr}(p))}, E^{nr}(p)C \right\}, \quad (5)$$

and the bank’s expected profits (net of the expected price of the collateral at the end of the period, $pC$) are

$$E_{nr}(\pi|E^{nr}(p)) = K_{nr}(E^{nr}(p))(qA - 1). \quad (6)$$

## 2.4 Investment Choices

The bank would like to invest at optimal scale, but that may trigger collateral examination. By choosing the investment level, a bank can then choose between facing a run or not. The optimal decision of how much to invest in the project is then isomorphic to the bank announcing an investment strategy that either finances the project at the optimal scale (triggering a run) or at a restricted scale (avoiding a run). All banks’ decisions, however, must be consistent in the aggregate. Even though banks are not fundamentally linked to each other, their choices of whether to trigger examination or not affect the inference of depositors about the quality of their collateral.
As we have shown, the expected profits of a bank suffering a run (equation 2) depends both on \( p \) and \( E^r(p) \) while the expected profits of a bank without a run (equation 6) only depends on \( E^{nr}(p) \). Since \( E_r(\pi) \) increases in \( p \) while \( E_{nr}(\pi) \) is independent of \( p \), if a bank with collateral of quality \( p \) announces an investment plan that induces a run, then all \( p' > p \) will also. Similarly, if a bank with a collateral of quality \( p \) announces an investment plan that avoids a run, then all \( p' < p \) will also. Intuitively, a bank with high collateral quality is more willing to open the collateral to examination and to face the lottery that a run represents. Hence, banks with better collateral in expectation are more inclined to announce large investments that induce examination.

This implies that the optimal investment strategy is given by a cutoff rule under which all banks with \( p < p^* \) restrict their investments to avoid runs and all banks with \( p > p^* \) invest at the optimal scale and open themselves to examination of the collateral (a run), where \( p^* \) is the collateral quality that makes a bank indifferent between a run or not.

\[
E_r(\pi|p^*, E^r(p)) = E_{nr}(\pi|E^{nr}(p)),
\]

and where \( E^r(p) = E(p|p > p^*) \) and \( E^{nr}(p) = E(p|p < p^*) \).

More formally, allowing for corner solutions in which all banks either face a run or not, the equilibrium cutoff is such that

\[
p^* = \begin{cases} 
\bar{p} + \bar{\eta} & \text{if } E_r(\pi|\bar{p} + \bar{\eta}, \bar{p} + \bar{\eta}) < E_{nr}(\pi|\bar{p}) \\
p^* & \text{s.t. } E_r(\pi|p^*, E(p|p > p^*)) = E_{nr}(\pi|E(p|p < p^*)) \\
\bar{p} - \bar{\eta} & \text{if } E_r(\pi|\bar{p} - \bar{\eta}, \bar{p}) > E_{nr}(\pi|\bar{p} - \bar{\eta})
\end{cases}
\]

As both \( E^r(\pi|p^*, E(p|p > p^*)) \) and \( E^{nr}(\pi|E(p|p < p^*)) \) increase with \( p^* \) there may be multiple \( p^* \) in equilibrium. In what follows we focus on the largest \( p^* \), this is the best equilibrium that guarantees the highest sustainable output. The next Proposition shows that in such an equilibrium the threshold \( p^* \) increases with the average quality of collateral in the economy, \( \bar{p} \). That is, the higher the average quality of collateral in the economy the fewer banks face runs and collateral examination.

**Proposition 1** In the best equilibrium, the threshold \( p^* \) is increasing in \( \bar{p} \). There exist beliefs \( p^H > p^L \) such that if \( \bar{p} > p^H \) no bank faces a run and if \( \bar{p} < p^L \) all banks face runs.
Remark on off-equilibrium beliefs: When the threshold $p^*$ is a corner (either $p^* = \bar{p} - \eta$ or $p^* = \bar{p} + \eta$), the expected quality of the collateral of a bank following a strategy that is off-equilibrium is not well-defined. If $p^* = \bar{p} + \eta$ no bank faces a run and then $E^r(p)$ is not well-defined as it is an off-equilibrium strategy. The same is the case for $E^{nr}(p)$; if $p^* = \bar{p} - \eta$, then all banks are expected to face a run. Following the Cho and Kreps (1987) criterion, we assume that if a bank follows a strategy not supposed to be followed in equilibrium, depositors believe the bank holds a collateral that maximizes its incentives to deviate from the expected strategy. I.e., if $p^* = \bar{p} + \eta$ and a depositor observes a bank investing in a large project so as to induce a run, then the depositor believes that the bank has the highest available quality collateral, $E^r(p) = \bar{p} + \eta$. Similarly, if $p^* = \bar{p} - \eta$ and a depositor observes a bank investing in a project that discourages examination of the collateral, then the household believes that the bank has the lowest available quality collateral, $E^{nr}(p) = \bar{p} - \eta$.

3 Crises and Interventions

3.1 Introducing a Crisis

In the previous section we have characterized the contracts and investment levels of all banks in the economy as a function of the aggregate $\bar{p}_t$ in each period $t$. In this section we restrict attention to a situation in which the economy suffers a crisis in the first period and gets back to normal in the second period. How the economy fares during the crisis in the first period will also determine output in the second period.

The crisis in the first period comes from a shock to the economy as follows. First, we capture the informational nature of crises by assuming that $\bar{p}_{t=1} = p_L < p_H$, as defined in Proposition 1, such that all banks face examination (runs) in the absence of inter-vention. Second, we capture the turbulent nature of crises by assuming that absent interventions, there are potential information leakages; with probability $\varepsilon$ the bank’s collateral type is revealed. In the second period the economy goes back to normal, which we capture by assuming that $\bar{p}_{t=2} = p_H > p_H^*$, as defined in Proposition 1, such that, absent a crisis in the first period there would not be a run on any bank.

When the economy is in normal times, absent a previous crisis, it achieves the maximum potential consumption: households consume $D$ and all banks invest at the op-
timal scale, producing an additional amount $K^*(qA - 1)$ of numeraire to consume. Aggregate consumption is then

$$W_N = D + p_HC + K^*(qA - 1).$$

During crises, however, absent government intervention, all banks face runs, depositors examine the banks’ collateral and only a fraction $p_L$ of banks retain their deposits, at an informational cost $\gamma$, while the remaining fraction $(1 - p_L)$ of banks face withdrawals at the beginning of the period and are not able to finance their projects.  

In this case, aggregate consumption is

$$W_C = D + p_LC + p_LK^*(qA - 1) - \gamma < W_N.$$

### 3.2 Tools to Fight the Crisis: Lending Facilities

We now introduce a central bank, a planner that manages a particular set of lending facilities that can be used to relax the incentives to acquire information in the economy and boost investment and consumption during the crisis in the first period. This is the timing of these facilities.

1. The central bank opens a discount window that exchanges $B$ government bonds (hereafter “bonds” for short) per unit of collateral, to be paid in numeraire at the end of the period. It also announces whether it will reveal the identities of banks participating at the discount window (a policy of transparency) or not (a policy of opacity). The central bank chooses both $B$ and the disclosure strategy, committing to the announced policy.

2. Conditional on the announced policies, banks choose whether to go to the discount window (participate) or not, and critically, whether to disclose such participation to its depositors, potentially in opposition to the intended government’s disclosure policy. If a bank does not participate, its collateral type is revealed with leakage probability $\varepsilon$. If a bank participates, and its participation becomes

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10 The assumption of information leakages during crises becomes irrelevant when all banks are investigated, as the types of all collateral are revealed anyway.

11 Borrowing a bond corresponds to using the Fed’s Term Securities Lending Facility; see Hrung and Seligman (2011), but bonds could also be thought of as cash or reserves.
public, there is an endogenously determined stigma cost, denoted by $\chi$. Stigma arises from learning about the bank’s idiosyncratic collateral quality that affects the future likelihood of runs and then future bank’s profits. The magnitude of the stigma cost will be determined in equilibrium depending on the aggregate use of the discount window and, as we will discuss, it is critical in the implementation of opacity.\footnote{See Hu and Zhang (2019), Anbil (2017), Armantier et al. (2015), Ennis and Weinberg (2013) and Furfine (2003) for evidence on the relevance of stigma for participation on discount windows.}

3. At the end of the first period, discount window participants with successful projects repay deposits using the proceeds from the project and repurchase their collateral back from the central bank. Failing participants lose their bonds to depositors, who redeem them at the central bank. Successful banks that did not participate repay their deposits with the proceeds from the project and retain their collateral. Failing non-participants default and hand over their collateral to their depositors.

4. The central bank liquidates the collateral left in its possession (not repurchased) but imperfectly, only extracting a fraction $\phi$ of its value at the end of the period. The numeraire generated by liquidating such collateral plus distortionary taxes (transfers at a social cost $\delta$ per unit of numeraire) are used to redeem bonds.

In addition to a new player – the central bank – we have added three additional exogenous parameters in this section: the leakage probability, $\varepsilon$, the inefficiency of public liquidation, $\phi$, and the social cost of distortionary taxation, $\delta$. The leakage probability just avoids discontinuous changes in discount window participation and better highlights the forces behind opacity. Inefficiency of public liquidation and distortionary taxation costs are simply introduced to avoid the trivial result that the central bank should always lend at no discount.

We have also added the concept of stigma cost, $\chi$, which is endogenous and plays a more fundamental role. Banks participating in lending facilities will be those with worse collateral in expectation. As banks operate in subsequent periods, revealing participation (and then revealing that they have worse collateral) exposes them to future runs, even when the economy returns to normal times.

Our focus is on the interplay between socially-optimal disclosure (optimal for a planner that maximizes aggregate consumption, present an future, net of intervention dis-
tortions) and individually-optimal disclosure (optimal for an individual bank that maximizes its own profits, present and future). We will show that in our model these two are aligned: when the planner wants transparency, banks also have incentives to privately disclose participation, and when the planner wants opacity, the threat of a run discourages banks that do not participate and stigma discourages banks that do participate to reveal their statuses, implementing the socially optimal allocation as an equilibrium outcome.

4 The Role of Opacity and Stigma in Fighting Crises

In this section we solve the central bank’s problem in two steps. First, we compute the equilibrium and welfare in the economy under opacity (Op) and then under transparency (Tr), as a function of the bonds $B$ that the central bank exchanges per unit of collateral through the discount window. Then we obtain the optimal $B^*$ that maximizes welfare in each case and allows the central bank to choose the best disclosure policy, $\{Op, Tr\}$. We will discuss policy in terms of the central bank’s discount. Define

$$B \equiv \bar{p}C,$$

such that the central bank choosing $\bar{p}$ implicitly chooses how many bonds $B$ to offer per unit of collateral. Then the discount for a bank with collateral of expected quality $E(p)$ is given by $(E(p) - \bar{p})C$. For convenience, and based on this one-to-one mapping between $\bar{p}$ and $B$, we will discuss comparative statics in terms of $\bar{p}$.\(^{13}\)

In announcing its policy the central bank cannot force banks to participate in the discount window, or depositors to not examine collateral and maintain deposits in banks. Most importantly, the central bank cannot force banks to follow its desired disclosure policy and keep in secret their own participation (or lack thereof).

As such, in computing welfare, the central bank has to take into account two equilibrium objects, the fraction of banks participating in the discount window, $y(\bar{p}|\{Op, Tr\})$ and the probability of runs, $\sigma(\bar{p}|\{Op, Tr\})$. After we obtain these equilibrium objects, we compute the incentives of banks to deviate and individually report their participation status, potentially making the announced disclosure policy unsustainable.

\(^{13}\)We can also define the haircut by the ratio $1 - \frac{B}{E(p)C} = 1 - \frac{\bar{p}}{E(p)}$. When we refer to discount, it can also be interpreted as the haircut of government bonds.
4.1 Ending a Crisis with Opacity

By intervening, the central bank cannot force depositors to maintain their funds in a bank, or banks to participate. The central bank can, however, choose the right discount to steer those decisions. Now we solve for how agents react to the discount rate when participation is maintained in secret.

4.1.1 Bank Runs with Opaque Discount Windows

When a depositor chooses whether to examine the bank’s collateral (run) or not, he takes the fraction of banks participating in the discount window, $y$, as fixed (this is, of course, an equilibrium object that we endogenize in the next section). As there is no information about participation, the expected value of (any) bank’s portfolio is $yB + (1 - y)E_{nw}(p)C$, where $E_{nw}(p)$ is the expected collateral quality of banks that do not to participate (also an equilibrium object). Given this expectation, depositors are indifferent between withdrawing or not at the beginning of a period when $D = qR_{nr}^R + (1 - q)R_{nr}^D$, where $R_{nr}^D$ is now given by the expected return in case the bank defaults, $R_{nr}^D = D - K_{nr} + x_{nr}[yB + (1 - y)E_{nw}(p)C]$.

In equilibrium any bank will promise in expectation the same in case of repayment or default (for the same truth-telling reasons as in the previous section), so $D = R_{nr}^R = R_{nr}^D$. We can then obtain the fraction of collateral in the portfolio that is promised to depositors in case of default,

$$x_{nr} = \min \left\{ \frac{K_{nr}}{yB + (1 - y)E_{nw}(p)C}, 1 \right\}.$$

Now we can compute the incentive of a depositor to privately acquire information about the portfolio of the bank. At a cost $\gamma$ the depositor can privately learn whether the bank has bonds or private collateral in its portfolio, and in the case where the bank has private collateral, its type. The benefits of acquiring information are as follows: with probability $y$ the bank has bonds and the depositor that examines the portfolio does not withdraw, getting a payoff of $qR_{nr}^R + (1 - q)[D - K_{nr} + x_{nr}B] - R_{nr}^D$. With probability $(1 - y)(1 - E_{nw}(p))$ the bank has bad private collateral and the depositor withdraws at the beginning of the period, getting a payoff of $D - \gamma$. Finally, with probability
the bank has good private collateral, and the depositor keeps his deposits in the bank, getting a payoff of \( qR_{nr}^R + (1 - q)[D - K_{nr} + x_{nr}C] - \gamma \).

Adding the previous payoffs weighted by the respective probabilities, considering that \( R_{nr}^R = D \) and \( x_{nr}(yB + (1 - y)E^{nw}(p)C) = K_{nr} \), there is no examination as long as

\[
K_{nr}(E^{nw}(p)) \leq \frac{\gamma}{(1 - q)(1 - y)(1 - E^{nw}(p))}.
\]  

(8)

and we can define \( \sigma(\tilde{p}) \) (making explicit it depends in equilibrium on the discount offered by the central bank), the probability of a run, as

\[
\sigma(\tilde{p}) = \begin{cases} 
0 & \text{if } K < \frac{\gamma}{(1-q)(1-y(\tilde{p}))(1-E^{nw}(p|\tilde{p}))} \\
[0, 1] & \text{if } K = \frac{\gamma}{(1-q)(1-y(\tilde{p}))(1-E^{nw}(p|\tilde{p}))} \\
1 & \text{if } K > \frac{\gamma}{(1-q)(1-y(\tilde{p}))(1-E^{nw}(p|\tilde{p}))}.
\end{cases}
\]  

(9)

The next Proposition is trivial and it comes from comparing the condition for no information acquisition in the absence of intervention (equation 4) and in the presence of intervention (equation 8).

**Proposition 2** Runs are less likely with intervention when there are many banks participating at the discount window (i.e., high \( y \)).

4.1.2 Participation in Opaque Discount Windows

We have solved for the depositors’ incentives to examine a bank’s portfolio conditional on the fraction of banks participating at the discount window, \( y \). Now we show, in four stages, that this fraction \( y \) is in equilibrium a function of \( \tilde{p} \). First, define

\[
L(p, \tilde{p}) \equiv pK^\star(qA - 1) - \frac{p}{E^{nw}(p|\tilde{p})} \gamma + pC
\]

as the “(L)ow” bank’s expected payoffs when the bank faces a run given that it did not participate in the discount window (as in equation 2). Second, define

\[
H(K(\tilde{p})) \equiv K(\tilde{p})(qA - 1) + pLC
\]
as the “(H)igher” bank’s expected payoffs when the bank does not face a run and is able to invest $K(\bar{p})$ (as in equation 6).\footnote{Here we focus on a situation with no bank purposefully choosing a run, and then $E(p) = p_L$.} Finally, define

$$d(\bar{p}) \equiv (p_L - \bar{p})C$$

as the bank’s discount when borrowing from the discount window.

Having defined all these elements, we can compute the gains of a bank $p$ from participating at the discount window. If the discount is determined by $\bar{p}$, the expected payoffs of a bank $p$ of no participation are

$$E_{nw}(\pi|p, \bar{p}) = \sigma(\bar{p})L(p, \bar{p}) + (1 - \sigma(\bar{p}))[1 - \varepsilon)H(K) + \varepsilon L(p, \bar{p})]$$

while the expected payoffs of participation are

$$E_{w}(\pi|p, \bar{p}) = \sigma(\bar{p})[H(K) - d(\bar{p}) - \chi(p, \bar{p})] + (1 - \sigma(\bar{p}))[H(K) - d(\bar{p})].$$

As can be seen, these payoffs depend both on the quality of the bank’s collateral, $p$, and on the discount from participating in the discount window, $\bar{p}$.

### 4.1.3 Opacity Equilibrium

Since both the probability of runs and participation depend on each other and both depend on the discount $\bar{p}$, the next four lemmas characterize the equilibrium depositors’ run (examination) strategies and banks’ participation strategies, which can be divided into four ranges of $\bar{p}$, under opacity.

**Lemma 1** Very low discount region.

There exists a cutoff $\bar{p}_h < p_L + \eta$ such that, for all $\bar{p} \in [\bar{p}_h, p_L + \eta]$ ("very low discount region"), no depositor runs (that is, $\sigma(\bar{p}) = 0$) and all banks borrow from the discount window (that is, $y(\bar{p}) = 1$). There is no stigma in equilibrium (that is, $\chi(p, \bar{p}) = 0$ for all $p$).

This lemma shows that there are always levels of discount low enough ($\bar{p}$ large enough) to induce all banks to participate at the discount window and, based on this outcome, no depositor examines any bank’s portfolio, keeping their deposits available for investment. Further, as all banks participate, there is no stigma if participation is revealed.
Lemma 2 Low discount region.

There exists a cutoff $\tilde{p}_m < \tilde{p}_h$ such that, for all $\tilde{p} \in [\tilde{p}_m, \tilde{p}_h)$ (“low discount region”), no depositor runs (that is, $\sigma(\tilde{p}) = 0$), not all banks participate (this is, $y(\tilde{p}) < 1$) and participation declines with the level of discount (that is, $y(\tilde{p})$ increases with $\tilde{p}$). Stigma is positive and increases in expectation with the level of discount (that is, $\chi(p, \tilde{p})$ increases with $\tilde{p}$).

Intuitively, when the discount is low ($\tilde{p}$ is large), many banks choose to borrow at the discount window because the cost in terms of exchanging private collateral for bonds at a low discount more than compensate for the risk of a run and information about the private collateral being revealed. Given high participation, depositors do not have incentives to run and examine any bank’s portfolio.

In this region, banks that choose not to participate are those with high collateral quality, and then participation reveals relative low collateral quality, creating a stigma cost in case participation is revealed. However, even though stigma is positive in case of participation, in equilibrium there is no examination that reveals participation, so in equilibrium no bank faces a stigma cost. This is relevant later when studying private incentives to disclose participation, as stigma is an off-equilibrium threat that does not arise in equilibrium but would arise with voluntary disclosure.  

Lemma 3 Intermediate discount region.

There exists a cutoff $\tilde{p}_l < \tilde{p}_m$ such that, for all $\tilde{p} \in [\tilde{p}_l, \tilde{p}_m)$ (“intermediate discount region”), depositors run with positive probability but not always (that is, $\sigma(\tilde{p}) \in (0, 1)$) and a constant fraction $y$ of banks go to the discount window (that is, $y(\tilde{p}) = y(\tilde{p}_m) > 0$).

In the intermediate discount range the equilibrium cannot involve pure run strategies by depositors. Since participation when depositors do not run is low, depositors have incentives to run. In contrast, if depositors run, banks have more incentives to participate, discouraging runs. Then, depositors have to be indifferent between running or not. As the discount increases in this range, banks lower incentives to participate have to be compensated for by a larger probability of runs that increases the costs of no participation. As there are runs on the equilibrium path, stigma costs are realized.

Exploiting data on the use of discount windows and TAF programs during the recent financial crisis, Hu and Zhang (2019) show that (i) banks participating in discount windows were smaller and weaker and (ii) among banks participating in TAF auctions, those with worse collateral bid the highest for funds. These findings are consistent with our implication that only banks under serious distress and bad collateral are willing to pay the discount, and then stigma is an endogenously determined cost.
Lemma 4  High discount region.

There exists a cutoff $\tilde{p}_l > 0$ such that, for all $\tilde{p} \in (0, \tilde{p}_l]$ (“high discount region”), banks do not borrow from the discount window (that is $y(\tilde{p}) = 0$) and depositors always run (that is, $\sigma(\tilde{p}) = 1$). There is no stigma in equilibrium (that is, $\chi(p, \tilde{p}) = 0$ for all $p$).

Intuitively, when the discount is high enough ($\tilde{p}$ is low), no bank chooses to borrow from the discount window, even if depositors run. Given this reaction, depositors always run. The economy generates the same consumption as in the case of a crisis without intervention. As there is no participation, stigma is based on off-equilibrium beliefs, which however cannot overturn this result. If beliefs are such that (off-equilibrium) participating banks have collateral with quality above average, then there is no gain in the second period beyond what is achievable with average quality. If, on the other hand, beliefs are such that (off-equilibrium) participating banks have collateral with quality below average, then there is a stigma cost and banks are even more discouraged to participate.

The equilibrium strategies derived in Lemmas 1-4 are illustrated in Figure 1. On the horizontal axis we show the average discount $d(p_L, \tilde{p})$, the red solid function shows the fraction of depositors who run, $\sigma(\tilde{p})$, and the black dashed function shows the fraction of banks that participate in the discount window, $y(\tilde{p})$. We use the average discount instead of $\tilde{p}$ as it is more intuitive to think of the discount as the cost of participation. The strategies in the “very low discount region” $[0, d(p_L, \tilde{p}_h)]$ are shown in Lemma 1, in the “low discount region” $[d(p_L, \tilde{p}_h), d(p_L, \tilde{p}_m)]$ are shown in Lemma 2, in the “intermediate discount region” $[d(p_L, \tilde{p}_m), d(p_L, \tilde{p}_l)]$ in Lemma 3 and in the “high discount region” $[d(p_L, \tilde{p}_l), C]$ in Lemma 4.

4.2  The Role of Stigma in Sustaining Opacity

The previous analysis is based on the presumption that the central bank announces and commits to a policy of not disclosing participation. But, this policy of opacity unravels if banks choose to individually reveal their participation status. There are two possible deviations. Banks that participated hold bonds and may want to disclose it to avoid runs. Banks that did not participate may want to announce that they did not need to participate. An opaque policy cannot be implemented if these any deviations happens. Once banks disclose participation, no disclosure is a signal of no participation, and vice
versa. Considering these deviations is particularly relevant in our framework, since the gains from opacity come from sustaining the functioning of credit for all banks, but without lending to all banks, just a fraction of them. Here we study the incentives of banks to deviate from public opacity, and then the implementability of secrecy as a way to combat crises.

What prevents participants from disclosing their participation? The answer is stigma: as participants in equilibrium are those with the lowest collateral quality, and the idiosyncratic component is persistent, disclosure would put those banks in a weak position to prevent runs in the second, normal, period. But then what prevents non-participants (with higher collateral quality) from disclosing their participation? The answer is a run: non-participants are subject to a run when it becomes known with certainty that they do not hold bonds in the first, crisis, period.

To show this, consider an opaque intervention with intermediate discount (say $\bar{p} = \bar{p}_m$) such that there is no run in equilibrium and just a fraction of banks participate. Take a participating bank that does not disclose its participation. The first-period profit is
$H(K^*) - d(p, \bar{p})$ and the second period profit is $H(K^*)$ (recall the second period’s aggregate component of collateral is $p_{t=2} = p_H > p^H$, such that the average bank does not suffer a run in the second period). Revealing participation does not increase the first period outcome, as the bank still would not be examined, would still obtain the optimal loan size and would still pay the discount. The second-period profit, however, can be smaller in case the revised belief about the bank’s collateral is such that it triggers a future run, a stigma cost.

Stigma, $\chi(p, \bar{p})$, is the cost in terms of a higher probability of a run in the second period coming from information that the bank participated at the discount window in the first period. Intuitively, since banks participate when their collateral quality is lower than average (specifically $p < p^*_w$), revelation of participation would make those banks more prone to be examined in the second period. Formally, in the second period, the bank suffers a run (obtaining $H(K^*)$ only with probability $p$ and loses the cost of information $\gamma$) if equation (4) is not fulfilled at the optimal scale $K^*$, that is if

$$K^* > \frac{\gamma}{(1 - q)(1 - E^w(p|\bar{p}, p_{t=2}))} \implies p_H + E^w(\eta|\bar{p}) < 1 - \frac{(1 - q)K^*}{\gamma}$$

where $E^w(\eta|\bar{p})$ is the expectation of the idiosyncratic component of collateral, conditional on participation, in which case the bank would lose on net $(1 - p)K^*(qA - 1) + \gamma$. This implies that stigma is given by

$$\chi(p, \bar{p}) = \begin{cases} 
0 & \text{if } E^w(\eta|\bar{p}) \geq 1 - p_H - \frac{(1 - q)K^*}{\gamma} \\
(1 - p)K^*(qA - 1) + \gamma & \text{if } E^w(\eta|\bar{p}) < 1 - p_H - \frac{(1 - q)K^*}{\gamma}.
\end{cases} \quad (12)$$

Notice the stigma cost is more likely to be positive the lower is the aggregate component of collateral (i.e., the weaker is the recovery from the crisis). Intuitively, a weak recovery makes beliefs about the idiosyncratic component more relevant to avoid runs. Similarly, the stigma cost is more likely the higher is the discount $d(\bar{p})$ and the smaller is participation (i.e., the lower is $E^w(\eta|\bar{p})$). Intuitively, when participation is low, participation is a very bad signal about quality. These considerations are summarized in the following proposition, which comes trivially from equation (12).

**Proposition 3** Stigma costs are more likely when the second period’s collateral is weaker (lower $\bar{p}_{t=2}$) and when the discount is larger (smaller $\bar{p}$ such that participation is lower).
That participating banks do not want to disclose their participation is just one side
of the coin. How about non-participating banks? If disclosing participation is costly,
Isn’t it profitable to disclose non-participation? Take a nonparticipating that does not
disclose its lack of participation. The first-period profit is \((1-\varepsilon)H(K^*)+\varepsilon L(p,\bar{p})\) and the
second period profit is \(H(K^*)\). Revealing no participation is costly as it would trigger
a run on the bank. Why? Because if, by disclosing no participation, a bank could
avoid a run every bank would do it instead of paying a discount, and that would
not be an equilibrium. Revealing no participation, however sends a signal that the
bank has better collateral than average. This potential benefit is only useful if in the
second period the average quality of collateral also triggers a crisis, not otherwise. If
the average quality of collateral is such that in the second period there is no run (as we
have assumed), with and without signaling the bank would still obtain \(H(K^*)\).

In short, participating banks do not obtain any additional gain during the crisis but
lose in expectation in future normal periods. In contrast, nonparticipating banks may
lose from disclosure during the crisis, while not gaining anything in future normal
times. The two necessary conditions that generate the implementability of secrecy are:

\section*{ii) there is a single signal (participation or not) that is informative about two variables
(the portfolio composition today and the collateral quality tomorrow) and ii) good and
bad news map asymmetrically into payoffs.}

To be more precise, denote by \(s \in \{s_A, s_B\}\) a binary signal, when \(s_A\) implies good news
\((g)\) about a first information dimension and bad news \((b)\) about a second, while \(s_B\)
implies the opposite. Denote the payoff from each information dimension by \(z \in \{z_1, z_2\}\).
An agent does not want to disclose either signal, neither \(s_A\) nor \(s_B\), then opacity is
sustainable, if and only if

\[ \bar{z}_1 + \bar{z}_2 > z_1(g) + z_2(b) \quad \text{and} \quad \bar{z}_1 + \bar{z}_2 > z_1(b) + z_2(g) \]

where \(\bar{z}\) represents the “status quo”, non-disclosure payoffs.

In our setting, \(s_A\) is participation, \(s_B\) is no participation, the first information dimen-
sion is the portfolio composition after a crisis intervention and the second information
dimension is the idiosyncratic collateral quality. The first condition (no disclosing par-
ticipation) is satisfied because, under a successful intervention, \(z_1(g) = \bar{z}_1\) and because
of stigma \(z_2(b) < \bar{z}_2\). The second conditions (no disclosing non-participation) is satis-
fied because \(z_2(g) = \bar{z}_2\) and because runs to non-participants during crises, \(z_1(b) < \bar{z}_1\).
Notice that there are several applications, beyond banking, which can accommodate these payoff properties and allow for secrecy as an equilibrium outcome. Take, for instance, the case of local and state restaurant inspections. Assume restaurants payoffs increase in today’s average perceived food quality \(z_1\) and decrease in tomorrow’s variance of perceived food quality \((-z_2)\). Assume also that an inspection sends a very bad signal about today’s food quality (if inspection is triggered by regulators concerns about today’s food quality), and non-inspection sends a very bad signal about tomorrow’s expected variance (if inspection today puts restrictions on food quality tomorrow). Under these assumptions, an inspected restaurant would not report inspection, as it may lose many clients who expect low food quality today, while not being compensated with a significant reduction in tomorrow’s variance. A non-inspected restaurant, on the other hand, may not gain much from reporting not being inspected in terms of expecting more clients today, but would potentially generate many doubts among clients about tomorrow’s food quality.\(^\text{16}\)

The next Corollary summarizes this insight in our banking application.

**Corollary 1** Under a successful opaque intervention policy that bypasses a crisis, stigma discourages participating banks from revealing participation and runs discourage nonparticipating banks from revealing non-participation. There are no deviations in equilibrium and the policy of opacity can be successfully implemented.

### 4.3 Ending a Crisis with Transparency

When the central bank discloses the identity of banks participating at the discount window, the information acquisition strategy of depositors is conditional on this information. More specifically, when depositors know a bank has borrowed from the discount window, they never run on it, as they know the bank uses government bonds as collateral, and there is nothing to learn about. Then \(\sigma(\tilde{p}) = 0\) for all \(\tilde{p}\), conditional on participation, and \(E^w(\pi) = H(K^*) - d(p, \tilde{p}) - \chi(p, \tilde{p})\). In contrast, when depositors know a bank has not participated, they always run on it, as they know the bank has a private asset in its portfolio. Then \(\sigma(\tilde{p}) = 1\) for all \(\tilde{p}\), conditional on no participation, and \(E_{nw}(\pi|p, \tilde{p}) = L(p, \tilde{p})\).\(^\text{17}\) The next Lemma characterizes this equilibrium,

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\(^{16}\)We thank a referee for suggesting this example.

\(^{17}\)No participation should always involve a run, otherwise all banks would choose not to participate and depositors would indeed prefer to run, not an equilibrium.
Lemma 5 In the equilibrium under transparency, all banks borrow from the discount window $y(\bar{p}) = 1$ for $\bar{p} \in [\bar{p}_T, \bar{p} + \bar{n}]$ such that $\bar{p}_T < \bar{p}_L$. For $\bar{p} < \bar{p}_T$ is $y(\bar{p}) = Pr(p < p^*_w)$ where $p^*_w$ is given by,

$$L(p^*_w, \bar{p}) + d(p^*_w, \bar{p}) + \chi(p^*_w, \bar{p}) = H(K^*)$$

Intuitively, when banks receive bonds they can invest at optimal scale, at the cost of paying a discount and stigma (only positive in case $y(\bar{p}) < 1$). When discount is not that large all banks choose to participate. In this case, the alternative of not participating is suffering a run for sure. This is in stark contrast to a policy of opacity in which, if all banks participate, a non-participating bank is only examined in case of a leakage. Hence full participation with opacity collapses at lower discount levels.\(^{18}\)

4.4 Opacity or Transparency?

Given the equilibrium strategies for each $\bar{p}$ under both opacity and transparency, we can compute the total production (or welfare in our setting) for each $\bar{p}$ under each disclosure policy.

Since the unconstrained welfare is $2H$ in both periods, a non-treated crisis has a cost

$$Cost\text{ }Crisis = H - L \equiv (1 - p_L)K^*(qA - 1) + \gamma \tag{13}$$

As we show formally in the Online Appendix, and explain intuitively next, the distortion from fighting a crisis is

$$Dist(\bar{p}) = (1 - y)(\varepsilon + \sigma(1 - \varepsilon))(H - \hat{L}) + y[(1 - q)(1 + \delta)(1 - \phi)E^w(p)C] + yQ\sigma\hat{\chi}, \tag{14}$$

where $\hat{L} \equiv \int_{p|nw} L(p, \bar{p})dp$ and $\hat{\chi} \equiv \int_{p|w} \chi(p, \bar{p})dp$ are the loses from runs and stigma among non-participants and participants respectively. The fraction of participants $y$, the fraction of runs $\sigma$ and the level of stigma $\chi$ depend both on the disclosure policy and the lending discount $\bar{p}$. Intuitively, the first component shows the distortion that comes from the lower output of non-participant banks, either due to information leaks

\(^{18}\)If $\bar{p}_T < \bar{p}_L$, for all discount levels $\bar{p}_T < \bar{p} < \bar{p}_L$, there is full participation under transparency and no participation under opacity. With opacity, if all banks were participating, depositors would not run and deviating to no participation is profitable, making this equilibrium unsustainable. In contrast, with transparency the alternative option of not participating always triggers a run, turning the deviation unprofitable.
or because runs. The second component shows the costs of the distortionary taxation that is needed to cover deposits from defaulting participating banks, and that cannot be covered by liquidating their private collateral. Finally, the third component shows the lower production in the second period from the stigma costs of participating banks that are examined. Notice that in the computation of distortions the level of discount is irrelevant, as it just represents a non-distortionary transfer of resources from banks to the central bank (and ultimately households).

The next Proposition characterizes the condition under which intervention under transparency is socially preferred to no intervention,

**Proposition 4** Intervention under transparency is preferred to no intervention only when the distortionary costs of taxation \((1 + \delta)\) and public liquidation \((1 - \phi)\) of private assets are small relative to the gains of intervention, this is,

\[
(1 + \delta)(1 - \phi) < \frac{(1 - p_L)K^*(qA - 1) + \gamma}{(1 - q)p_LC}.
\]

The condition in the Proposition just comes from equation (13) being smaller than equation (14) evaluated at \(y = 1\) and \(\sigma = 0\) (and then \(E^w(p) = p_L\)).

With opacity, welfare depends less trivially on the discount. Figure 2 shows welfare graphically, under both disclosure policies (dashed red for transparency and solid black for opacity) for all discount rates. Notice that the welfare implemented with a transparent intervention is the same for all discounts in the range of this illustration. Under opacity, however, welfare depends on the discount as characterized by Lemmas 1 to 4.

As can be seen, in the very low discount region, all banks participate and there are no runs, so welfare is the same as that obtained under transparency. This implies that there is always a, low enough, discount rate that replicates with opacity the welfare implementable with transparency. This is summarized in the next Corollary.

**Corollary 2** There is always a low enough discount rate that replicates with opacity the welfare obtained with transparency.

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19 Notice that it is enough to use the distortion with full participation for the comparison, as the distortion from partial participation is just a convex combination between these two equations.

20 For illustration purposes we use a set of parameters such that the discount level that prevents full participation in a transparent lending facility is greater than 0.4 (i.e., \(d(p_L, \tilde{p}_t) < 0.4 < d(p_L, \tilde{p}_T)\)).
In the low discount region, some banks prefer to take advantage of pooling and not participate. If the leakage probability is not very large relative to the cost of distortions, welfare increases with the discount. More specifically, as in this region $\sigma = 0$, welfare increases with lower participation (lower $y$) when $\varepsilon(H - \tilde{L}) < (1-q)(1+\delta)(1-\phi)E^w(p)C$. Since $y < 1$, $H - L > H - \tilde{L}$ and $E^w(p) < p_L$, a sufficient condition for distortions to decrease (or welfare to increase) in this region is

$$\frac{(1 + \delta)(1 - \phi)}{\varepsilon} > \frac{H - L}{(1 - q)p_L C}$$  \hspace{1cm} (15)

In the intermediate discount region, only a fraction $\overline{y}$ of banks borrows from the discount window but more and more depositors run as the discount increases, hence more and more banks are forced to produce less in expectation in the first period and suffer from stigma in the second. This reduces welfare. In this region, once the discount level is so large that the “stigma” effect dominates the “less distortion” effect, welfare under opacity is indeed lower than welfare under transparency. Finally, in the high discount region, the level of the discount is so high that there is no participation in equilibrium under opacity, with welfare reaching a no intervention level.
Putting these regions together, it is clear that the policy that maximizes welfare under the sufficient condition (15) is charging a discount \( d(p_m) \) under secrecy about the banks participating in the discount window. At this discount, and without disclosure, the distortionary cost of participation is minimized, still not triggering any runs. The next Proposition 5 characterizes this optimal policy.

**Proposition 5** When the distortionary costs are large relative to leakage probabilities, a discount rate of \( p_m \) under opacity strictly dominates transparency. A sufficient condition for this result is given by equation (15).

When welfare decreases in the low discount region (leakage probabilities are large compared to distortionary costs) it is better to have all banks participating, and then setting the discount window lower than \( d(p_h) \) and operating under opacity generates the same welfare as operating under transparency. Combining Corollary 2 and Proposition 5, we obtain the next Corollary, about the optimality of opacity.

**Corollary 3** If intervention is beneficial, there is always a discount level that turns opaque interventions (at least weakly) preferable to transparent interventions.

### 4.5 Additional Remarks

**Remarks on Implications for Stress Tests** Stress tests are simulations based on a bank’s balance sheet to assess how the bank would fare against different systemic distress scenarios. Even though introduced formally by Basel in 1996, they were mostly performed internally by banks until recent financial crises. Should individual stress test results be disclosed? When should stress tests be performed? Before or during crises? Even though disclosure about stress tests (information about the portfolio quality) and disclosure about participation in lending facilities capture very different forces, our model sheds light on some of these issues.

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21In 2009 the US authorities conducted a macro stress test under the framework of the Supervisory Capital Assessment Program (SCAP) for 19 bank holding companies (66% of total US banking sector). In 2010 a stress-test exercise was coordinated by the Committee of European Banking Supervisors (CEBS), the ECB and the European Commission for 91 EU banks (65% of total US banking sector). The level of disclosure of these tests was higher than standard supervisory tests performed afterwards. See details in ECB (2010).
If by performing stress tests regulators learn each bank’s collateral quality, \( p \), they can strategically choose how and when to disclose it. Regulators could disclose which banks have a \( p \) below a threshold, preventing runs for the rest of banks with higher \( p \). In this sense early disclosure can be relevant from a precautionary perspective, to prevent a crisis from happening. Revealing \( p \) conditional on a crisis happening, however, can be detrimental for the implementation and efficacy of opacity on the use of lending facilities. If banks understand their \( p \) will be disclosed after the crisis, participating banks are not concerned of the stigma from revealing their participation since their low quality collateral will be revealed anyways. In short, when stress tests disclosure eliminates stigma considerations, banks want to individually reveal their participation status, potentially jeopardizing the implementation of opaque lending facilities.

This last point suggests a complementarity between the opacity of stress tests and the opacity of intervention policies. By preventing too much revelation of stress tests it is easier to sustain the role of stigma on sustaining the social advantage of opacity on lending facilities to fight crises. This consideration is absent on the literature about stress tests disclosure, which is silent about the interaction with intervention policies and which is silent about the individual incentives to disclose participation, potentially in opposition to a planner’s optimal disclosure choice.

Remarks on Comparisons with Bagehot’s Rule The classic rule for a central bank to follow in a crisis was provided by Walter Bagehot’s in 1873: central banks should lend freely, at a high rate, and on good collateral. In the recent financial crisis, Ben Bernanke, Mervyn King and Mario Draghi, the respective heads of the Federal Reserve System, the Bank of England, and the European Central Bank, reported that they followed Bagehot’s advice; see Bernanke (2014a and 2014b), King (2010) and Draghi (2013). But, in fact, there was more to their responses to the crisis. All three central banks also engaged in anonymous or secret lending to banks.

Capie (2007) noticed the omission of secrecy by Bagehot: “. . . a key feature of the British [banking] system, its in-built protective device for anonymity was overlooked [by Bagehot]” (p. 313). Capie also proposes a reason of this omission, explaining that if a country bank in England needed money during a crisis it could borrow from its discount house, which in turn might borrow from the Bank of England. In this way, the identities of the actual end borrowers was not publicly known.  

22 See also Capie (2002). King (1936) provides a nice discussion on the industrial organization of British banking in the 19th century, and the complicated interactions between the Bank of England and the
Remarks on Securitization and Deposit Insurance In our setting, if banks were able to sign contracts that eliminate the idiosyncratic risk of individual portfolios then no depositor would have an incentive to acquire information about a bank’s asset. In other words, if idiosyncratic risk were eliminated by pooling all collateral in the economy, there would be no runs, no crisis and no role for intervention. Even though we have assumed that banks cannot diversify their individual portfolio risk, there could be in principle two institutions that allow for such diversification: securitization (sustained by private contracts) and deposit insurance (imposed by public regulation).

In the case of securitization, a bank can sign a contract at the beginning of the period, selling shares of its own asset and buying shares of the assets of other banks, eliminating the idiosyncratic risk as the value of its portfolio would be deterministic and equal to $p_L C$. This contract discourages depositors in the bank from acquiring information about its individual portfolio, which is now irrelevant for the probabilities of recovering the deposit. There are no runs and no crisis. These private contracts are difficult to sustain, however. Banks with high $\eta$ subsidize banks with low $\eta$ and may not have incentives to enter into these contracts, as a way to signal a high $\eta$. Studying the sustainability of these contracts is interesting to understand the effects of securitization as a stabilizing innovation, but it is outside the scope of this paper.

If securitization is not feasible, the government may have incentives to impose diversification in the form of deposit insurance. In the standard view of bank runs, under which they are triggered by a collective action problem, deposit insurance prevents panics on path and then it is not used in equilibrium. In our setting, a run is not driven by lack of coordination among depositors but instead by individual incentives to investigate the bank’s portfolio, withdrawing funds backed by a low value portfolio. The government can prevent the examination of a bank’s portfolio by forcing banks to pay a premium, ex-post, in case their assets are good and to receive insurance in case their assets are bad. As this cross-subsidization can be financed within the system, no taxation is used in equilibrium either.

One interpretation of what happened during the recent financial crisis is that some banks (commercial) were under deposit insurance (and nothing happened to them). Some others (shadow) used securitization, a fragile contract when adverse selection concerns become prevalent among participating banks that may want to cancel the contracts. Again, this is a subject that requires more research.

discount houses. Also see Pressnell (1956) and Flandreau and Ugolini (2011).
5 Conclusions

A financial crisis occurs when some public information causes depositors to worry about the collateral backing their deposits, such that they want to produce information about the bank’s portfolio. This is a bank run, as it triggers withdrawal from banks that are revealed to have bad assets. Recreating confidence means raising the perceived average value of collateral in the economy so that it is not profitable to produce information about any bank. The government can achieve this goal by exchanging bonds (or cash) for lower quality assets, and the cheaper and less distortionary way to achieve this is by replacing only some bad assets in the economy, not all. But for this informational externality to operate, the identity of borrowers should be kept in secret, so also non-borrowers keep operating without runs.

This logic was highlighted by Bernanke (2009): “Releasing the names of [the borrowing] institutions in real-time, in the midst of the financial crisis, would have undermined the effectiveness of the emergency lending and the confidence of investors and borrowers” (p. 1). Opacity was indeed adopted not only by several governments to deal with recent crises but also by private bank clearinghouses in the U.S. prior to the Federal Reserve System. Even though the quick reading of opacity suggests that it is introduced so that participating banks do not suffer from stigma, we show that the causality may reverse: stigma is what allows the implementation of optimal secrecy. It is needed because here banks can potentially disclose their participation (or lack thereof) individually. Opacity does not try to avoid stigma but stigma is crucial to avoid transparency. The tool used by the central bank to fine tune the participation of banks, such that not all participate but enough do to avoid runs on all, is the discount at which lending facilities operate. By restricting participation, the discount also determines the strength of stigma, and then the implementability of opacity.

References


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A  Appendix (for online publication)

A.1  Proof Proposition 1

We first characterize \(p^H\). The maximum profits that a bank with the highest collateral quality \(p + \bar{\eta}\) expects when inducing a run, conditional on no other bank facing a run (that is, when \(E^r(p) = p + \bar{\eta}\)), are

\[
E_r(\pi|p + \bar{\eta}, p + \bar{\eta}) = (p + \bar{\eta})K^*(qA - 1) - \gamma.
\]

while the expected profits when not inducing a run, conditional on no other bank facing a run, are

\[
E_{nr}(\pi|p) = K_{nr}(p)(qA - 1)
\]

where, from equation (5)

\[
K_{nr}(p) = \min \left\{ K^*, \frac{\gamma}{(1 - q)(1 - p)}; pC \right\}.
\]

There is always a \(p\) large enough such that \(K^* < \frac{\gamma}{(1 - q)(1 - p)}\) and \(K^* < pC\) such that \(K_{nr}(p) = K^*\). Then no bank would rather face a run and have a positive probability of not being able to invest, given that it can invest at optimal scale without examination. For all \(p > p^H\), all banks invest without runs, where \(p^H\) is defined by \(E_r(\pi|p^H + \bar{\eta}, p^H + \bar{\eta}) = E_{nr}(\pi|p^H)\), or

\[
(p^H + \bar{\eta})K^*(qA - 1) - \gamma = \frac{\gamma}{(1 - q)(1 - p^H)}(qA - 1).
\]

In this region, \(p^* = p + \bar{\eta}\), which trivially increases one for one with \(p\).

We now characterize \(p^L\). The maximum expected profits that a bank with the lowest collateral quality \(p - \bar{\eta}\) can obtain when avoiding a run when all other banks face runs (that is, \(E_{nr}(p) = p - \bar{\eta}\)) are

\[
E_{nr}(\pi|p - \bar{\eta}) = K_{nr}(p - \bar{\eta})(qA - 1)
\]

where, from equation (5)

\[
K_{nr}(p - \bar{\eta}) = \min \left\{ K^*, \frac{\gamma}{(1 - q)(1 - (p - \bar{\eta}))}; (p - \bar{\eta})C \right\}.
\]

The expected profits when the bank induces a run, conditional on all other banks inducing a run, are

\[
E_r(\pi|p - \bar{\eta}, p) = (p - \bar{\eta}) \left[ K^*(qA - 1) - \frac{\gamma}{p} \right].
\]
Defining $p^L$ by the point at which $E_r(\pi|p^L - \eta, p^L) = E_{nr}(\pi|p^L - \eta)$, such that

$$(p^L - \eta) \left[K^*(qA - 1) - \frac{\gamma}{p^L} \right] > \frac{\gamma}{(1-q)(1-(p^L - \eta))}(qA - 1),$$

then when $p < p^L$ all banks invest such that there is examination of their collaterals. In this region, $p^* = \bar{p} - \eta$, which also trivially increases one for one with $\bar{p}$.

In the best equilibrium and by monotonicity, in the intermediate region of $\bar{p}$ the threshold $p^*$ also increases with $\bar{p}$. QED.

### A.2 Proof Proposition 2

Follows from comparing the condition for no information acquisition in the absence of intervention (equation 4) and in the presence of intervention (equation 8), and on the comparative statics in equation (8) with respect to $y$.

### A.3 Proof Lemma 1

Assume first that the central bank chooses $\bar{p} = p_L + \eta$, and then no discount even for the bank holding the highest collateral quality, this is $d(p_L + \eta, \bar{p}) = 0$. Compare equations (10) and (11) for $p = p_L + \eta$. It is optimal for such bank to participate, but if this is the only bank participating, then stigma is positive in the sense that participation reveals the highest possible idiosyncratic component. Then, participating is also the optimal strategy for all other banks, which implies that there is no learning from participation and no stigma (i.e., $\chi = 0$), confirming that this is indeed the best sustainable equilibrium.\footnote{Notice that this is only one possible equilibrium. If everybody believes that some banks did not borrow from the discount window, $\chi > 0$, and it may be indeed optimal for those banks not to borrow from the window. This shows how endogenous stigma may induce equilibrium multiplicity and may generate self-confirming collapses in the use of discount windows. Here we focus on the best equilibrium based on intervention, and show its limitations.}

Hence, for $\bar{p} \geq p_L + \eta$, a fraction $y(\bar{p}) = 1$ of banks participate, from equation (A.4) $\sigma(\bar{p}) = 0$ and from equation (8) $K(\bar{p}) = K^*$.

For lower levels of $\bar{p}$, this is still an equilibrium as long as the bank with the highest collateral quality finds it optimal to participate, and then all other banks do as well. The critical level $\bar{p}_h$ is determined by the point at which the bank with the highest quality (evaluated at $\chi = 0$) is indifferent between participating or not,

$$H(K^*) - d(p_L + \eta, \bar{p}_h) = (1 - \varepsilon)H(K^*) + \varepsilon L(p_L + \eta, \bar{p}_h).$$

Using the definitions of $L(p, \bar{p})$, $H(K(\bar{p}))$ and $d(p, \bar{p})$,

$$\bar{p}_h = (p_L + \eta) - \frac{\varepsilon}{\zeta} \left[(1 - p_L - \eta)K^*(qA - 1) + \gamma \right].$$

QED
A.4 Proof Lemma 2

Assume first the extreme case in which $\tilde{p} = \tilde{p}_h$. From the previous proposition, $y(\tilde{p}_h) = 1$ and $\sigma(\tilde{p}_h) = 0$. For $\tilde{p} = \tilde{p}_h - \epsilon$ (from the definition of $\tilde{p}_h$),

$$H(K^*) - d(p_L + \bar{\eta}, \tilde{p}_h - \epsilon) < (1 - \epsilon)H(K^*) + \epsilon L(p_L + \bar{\eta}, \tilde{p}_h - \epsilon),$$

and then banks with collateral quality $p_L + \bar{\eta}$ strictly prefer to not participate at the discount window. This implies that $y(\tilde{p}_h - \epsilon) \equiv Pr(p < p^*_w(\tilde{p}_h - \epsilon)) < 1$, where $p_w^*(\tilde{p}_h - \epsilon)$ is given by the indifference condition

$$H(K^*) - d(p_w^*, \tilde{p}_h - \epsilon) = (1 - \epsilon)H(K^*) + \epsilon L(p_w^*, \tilde{p}_h - \epsilon),$$

or

$$d(p_w^*, \tilde{p}_h - \epsilon) = \epsilon [H(K^*) - L(p_w^*, \tilde{p}_h - \epsilon)],$$

where $p_w^*$ declines monotonically as we reduce $\tilde{p}$. Notice that this construction relies on the conjecture that $\sigma(\tilde{p}_h - \epsilon) = 0$, but for relatively low $\epsilon$ this is the case as long as $y(\tilde{p}_h - \epsilon)$ and $E^{nw}(p|\tilde{p}_h - \epsilon)$ are such that

$$K^* < \frac{\gamma}{(1 - q)(1 - y)(1 - E^{nw}(p))}.$$

Define by $p_w^*$ the threshold such that $\tilde{y}(p_w^*)$ is the fraction of banks borrowing from the discount window and $E^w(p|p_w^*)$ is the expected quality of the non-participating banks’ collateral, such that depositors are indifferent between running or not when the bank invests $K^*$, i.e.,

$$K^* = \frac{\gamma}{(1 - q)(1 - \tilde{y})(1 - E^{nw}(p))}.$$

The bank with the marginal collateral quality $p_w^*(\tilde{p}_m)$ is determined by

$$H(K^*) - d(p_w^*, \tilde{p}_m) = (1 - \epsilon)H(K^*) + \epsilon L(p_w^*, \tilde{p}_m),$$

such that

$$\tilde{y} = Pr(p < p_w^*(\tilde{p}_m)) \quad \text{and} \quad \bar{E}^{nw}(p) = E(p|p > p_w^*(\tilde{p}_m)).$$

Finally, the threshold $p_w^*$ is well-defined, as both $y$ and $E^{nw}(p)$ monotonically increase in $p_w^*$, which monotonically decreases in $\tilde{p}$.

Finally, even when there is no examination and then stigma on the equilibrium path, there would be stigma in case participation is leaked or voluntarily disclosed. Stigma, $\chi(p, \tilde{p})$, is the cost in terms of a higher probability of a run in the second period coming from information that the bank participated at the discount window in the first period. Intuitively, since banks participate when their collateral quality is lower than average (specifically $p < p_w^*$), revelation of participation would make those banks more prone
to be examined in the second period. Formally, in the second period, the bank will suffer a run in the second period and obtain $H(K^*)$ only with probability $p$ and lose the cost of information $\gamma$ if equation (4) is not fulfilled at optimal scale $K^*$, this is if

$$K^* > \frac{\gamma}{(1 - q)(1 - E^w(p|\tilde{p}, \tilde{p}_{t-2}))} \quad \text{or} \quad E^w(p|\tilde{p}, \tilde{p}_{t-2}) < 1 - \frac{(1 - q)K^*}{\gamma}$$

where $E^w(p|\tilde{p}, \tilde{p}_{t-2})$ is the expectation of $p$ in the second period, conditional on having participated in the discount window in the first period and conditional on the second period displaying an aggregate component $\tilde{p}_{t-2}$. Since we have assumed that $\tilde{p}_{t-2} = p_H > p_H$, we can rewrite this expectation as

$$E^w(p|\tilde{p}_m, \tilde{p}_{t-2}) = p_H + E^w(\eta|\tilde{p}).$$

in which case the bank would lose in net $(1 - p)K^*(qA - 1) + \gamma$.

This implies that stigma is given by

$$\chi(p, \tilde{p}) = \begin{cases} 
0 & \text{if } E^w(\eta|\tilde{p}) \geq 1 - p_H - \frac{(1 - q)K^*}{\gamma}, \\
(1 - p)K^*(qA - 1) + \gamma & \text{if } E^w(\eta|\tilde{p}) < 1 - p_H - \frac{(1 - q)K^*}{\gamma}.
\end{cases}$$

QED.

### A.5 Proof Lemma 3

Assume first the extreme case where $\tilde{p} = \tilde{p}_m$. From the previous lemma, $y(\tilde{p}_m) = \overline{y}$ and $\sigma(\tilde{p}_m) = \sigma$. For $\tilde{p} = \tilde{p}_m - \epsilon$, the bank that is indifferent about borrowing from the discount window has slightly lower quality, $p^*_w(\tilde{p}_m - \epsilon) < p^*_w(\tilde{p}_m)$. Then $y(\tilde{p}_m - \epsilon) < \overline{y}$ and $E^{nw}(p|\tilde{p}_m - \epsilon) < E^{nw}(p)$, and there are incentives to run if investment is $K^*$, as there are relatively few participants at the discount window (low $y$) and the collateral of those not participating at the discount windows are worse in expectation (low $E^{nw}(p)$).

Formally,

$$K^* > \frac{\gamma}{(1 - q)(1 - y(\tilde{p}_m - \epsilon))(1 - E^{nw}(p|\tilde{p}_m - \epsilon))}.$$  

One possibility for banks to prevent runs, $\sigma(\tilde{p}) = 0$, is to scale back the investment in the project to $K^*(p^*_w) < K^*$, to avoid information acquisition. The size of the investment $K^*$, however, also determines $y$, as $p^*_w(\tilde{p}_m - \epsilon)$ is pinned down by the condition

$$d(p^*_w, \tilde{p}_m - \epsilon) = \epsilon[H(K(p^*_w)) - L(p^*_w, \tilde{p}_m - \epsilon)].$$

A lower $K$ relaxes the constraint and reduces the incentive to run, but at the same time reduces $p^*_w$ for a given $\tilde{p}$, increasing the incentive to run. Intuitively, for a given discount, a reduction in the gains of borrowing from the discount window (from lower
$H(K)$ reduces the quality of the marginal bank which is indifferent between borrowing or not, i.e., reducing $p_w^*$ further.

If $H(K(p_w^*))$ declines faster than $L(p_w^*)$, then no participant will go to the discount window if, at the lowest possible $p$, which is $p_L - \bar{p}$,

$$H(K(p_L - \bar{p})) - L(p_L - \bar{p}, p_L) < 0$$

which we have assumed in the definition of a crisis ($p_L$ is low enough such that all banks would rather face examination than restricting their investments to avoid runs).

In words, it is not in the best interests of banks to discourage runs by reducing the size of their investments in the project, which is in contrast to what happens in the absence of intervention. Our result here comes from the endogenous participation of banks at the discount window. By reducing $K$, the effect of a lower $y$ in inducing information acquisition is stronger than the effect of a lower $K$ in discouraging information acquisition, thus increasing on net the incentives for depositors to examine a bank’s collateral as $K$ declines.

Under these conditions, the equilibrium should involve either the discount window sustaining an investment of $K^*$ (when a fraction $\bar{y}$ of banks borrows from the discount window) or no participation in the discount window at all, which replicates the allocation without intervention. To maintain the fraction $\bar{y}$ constant in this region as $\tilde{p}$ declines, the marginal bank with collateral quality $p_w^*(\tilde{p}_m)$ should be indifferent between borrowing from the discount window or not. This is achievable only if depositors choose to run with some probability and examine the portfolio of banks as $\tilde{p}$ declines, as this increases the incentives to have bonds in the portfolio.

As the fraction of banks participating at the discount windows is constant at $\bar{y}$, depositors are indeed indifferent between running or not, and $\sigma(\tilde{p}) > 0$ is an equilibrium.

To determine $\sigma(\tilde{p})$ in equilibrium we next discuss the determination of endogenous stigma.

With positive information acquisition ($\sigma(\tilde{p}) > 0$) there is stigma when the depositor discovers a bank’s participation at the discount window. The reason there is stigma is that those banks borrowing from the discount window are the ones with relatively low collateral quality (relatively low $\eta_i$). Once a bank is stigmatized, it may face withdrawals during normal times in the second period. To be more precise about the endogeneity of stigma, once back in normal times, the bank will face a run when investing at the optimal scale of production if

$$K^* > \frac{\gamma}{(1 - q)(1 - E^w(p))},$$

and the bank will not suffer a run in the second period based on an indifference condition that pins down $p_2^*$ in the second period where

$$E_r(\pi|p_2^*, E^r(p|p < p_w^*)) = E_r(\pi|E^{nr}(p|p < p_w^*)).$$
We denote by $K^w(p, \bar{\rho})$ the investment size that a bank with a collateral of quality $p$ can obtain in the second period conditional on it having been revealed that the bank borrowed from the discount window in the first period.

Then, stigma is given by

$$\chi(p, \bar{\rho}) = [K^* - K^w(p, \bar{\rho})](qA - 1),$$

where $\chi$ is an increasing function of the discount (a decreasing function of $\bar{\rho}$). As the discount increases, $p^*_w$ decreases, $y(p^*_w)$ decreases and $E^w(p)$ decreases. This leads to a decline in $K^w$ and then an increase in stigma from going to the discount window.

Given $\bar{\rho}$, to maintain the investment size $K^*$ without triggering information, the indifference of the marginal bank $p^*_w$ pins down the probability the depositor runs. This is

$$E_{nw}(\bar{\rho}|p^*_w) = E_w(\bar{\rho}|p^*_w),$$

which implies

$$\sigma L(p^*_w, \bar{\rho}) + (1 - \sigma)[(1 - \varepsilon)H(K^*) + \varepsilon L(p^*_w, \bar{\rho})] = [H(K^*) - d(p^*_w, \bar{\rho})] - \sigma \chi(p^*_w, \bar{\rho})$$

and then

$$\sigma(\bar{\rho}) = \frac{d(p^*_w, \bar{\rho}) - \varepsilon[H(K^*) - L(p^*_w, \bar{\rho})]}{(1 - \varepsilon)[H(K^*) - L(p^*_w, \bar{\rho})] - \chi(p^*_w, \bar{\rho})}. \tag{16}$$

Finally, depositors randomize between running and not running given that the bank is investing $K^*$ in the project, and a bank with collateral quality $p^*_w$ is indifferent between borrowing from the discount window or not. QED.

### A.6 Proof Lemma 4

From equation (16), $\sigma(p^*_w) = 1$ for $d(p^*_w, \bar{\rho}) \geq H(K^*) - L(p^*_w, \bar{\rho}) - \chi(p^*_w, \bar{\rho})$. Define $\bar{\rho}_T$ to be the discount that solves this condition with equality. Evaluating $E^w(\pi)p^*_w$ and $E(\pi)$ at $\sigma(\bar{\rho}) = 1$ given a project of size $K^*$ and at the threshold $p^*_w$, a bank with collateral $p^*_w$ goes to the discount window whenever

$$H(K^*) - d(p^*_w, \bar{\rho}) - \chi(p^*_w, \bar{\rho}) > L(p^*_w, \bar{\rho})$$

which is never the case in this region by the previous condition. Then $y(\bar{\rho}) = 0$ and then it is indeed optimal for depositors to run, that is $\sigma(\bar{\rho}) = 1$. QED.

### A.7 Proof Lemma 5

Define $\tilde{\rho}_T$ by

$$H(K^*) - d(p_L + \eta, \tilde{\rho}_T) = L(p_L + \eta, \tilde{\rho}_T).$$

For all $\tilde{\rho} > \tilde{\rho}_T$

$$H(K^*) - d(p_L + \eta, \tilde{\rho}) > L(p_L + \eta, \tilde{\rho})$$

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and the bank with collateral of quality \( p_L + \eta \) strictly prefers to borrow from the discount window. Notice that as all banks participate, \( \chi(p, \tilde{p}_T) = 0 \) for all \( p \).

For all \( \tilde{p} < \tilde{p}_T \), the fraction of participating banks \( y_T \) in a transparent window is given by the threshold \( p^*_wT \) such that \( H(K^*) - d(p^*_wT, \tilde{p}) - \chi(p^*_wT, \tilde{p}) = L(p^*_wT, \tilde{p}) \) and \( y_T = \Pr(p < p^*_wT) \).

Recall the condition that pins down \( \tilde{p}_h \) in Lemma 1 is

\[
H(K^*) - d(p_L + \eta, \tilde{p}_h) = (1 - \varepsilon)H(K^*) + \varepsilon L(p_L + \eta, \tilde{p}_h).
\]

Then, as the right-hand side is larger than in the condition above that pins down \( \tilde{p}_T \), the discount has to be smaller and \( \tilde{p}_T < \tilde{p}_h \). QED.

### A.8 Derivation of Intervention Distortions

As banks keep the net gains from production, welfare is an aggregation of the payoffs of all banks with different quality \( p \), given by

\[
W(\tilde{p}) = \int_p [\mathbb{I}_w[H + B - pC] + (1 - \mathbb{I}_w)[\sigma L(p) + (1 - \sigma)((1 - \varepsilon)H + \varepsilon L(p))] \] dF(p) \\
+ \int_p \mathbb{I}_w[q(pC - B) + (1 - q)(\phi p C - B)] dF(p) \\
+ \int_p [\mathbb{I}_w[q(H - \sigma \chi(p)) - (1 - q)\delta(1 - \phi)pC] + (1 - \mathbb{I}_w)H] dF(p),
\]

where \( \mathbb{I}_w \) is an indicator function that takes the value 1 if the bank participates at the discount window and 0 otherwise.

Taking integrals and rewriting the expression,

\[
W(\tilde{p}) = y[H + B - E^w(p)C] + (1 - y)[\sigma \hat{L} + (1 - \sigma)((1 - \varepsilon)H + \varepsilon \hat{L})] \\
+ y[q(E^w(p)C - B) + (1 - q)(\phi E^w(p)C - B)] \\
+ y[q(H - \sigma \hat{x}) - (1 - q)\delta(1 - \phi)E^w(p)C] + (1 - y)H,
\]

where \( H \equiv H(K^*), \hat{L} \equiv \int_p \mathbb{I}_w L(p, \tilde{p}) dp \) and \( \hat{x} \equiv \int_p \chi(p, \tilde{p}) dp \).

The first two terms (the first line) represent the welfare of banks in the crisis period. A fraction \( y \) of banks borrow from the discount window leading to a production of \( H \) and exchanging private collateral for bonds at an average discount of \( B - E^w(p)C \). A fraction \( 1 - y \) of banks do not participate and their investments lead to a production level that depends on whether they suffered a run or if there was an informational leak. This first line of the welfare function can be rewritten as

\[
H - y(E^w(p)C - B) - (1 - y)(\varepsilon + \sigma(1 - \varepsilon))(H - \hat{L}).
\]
The third term (the second line) represents the welfare for the government. From the fraction $y$ of banks borrowing from the discount window, a fraction $q$ has their private collateral seized, which delivers $E^w(p)C$ in expectation while a fraction $1-q$ defaults and the private collateral has to be liquidated, recovering just $\phi E^w(p)C$. Still in both cases the government has to repay $B$ for the bonds. This second line of the welfare function can be rewritten as

$$y(E^w(p)C - B) - y(1 - q)(1 - \phi)E^w(p)C.$$

The last term (the third line) captures investments in the second period (no discount). Those banks that did not borrow from the discount window and those that did without their participation being revealed can borrow without triggering a run in the second period (then leading to production level $H$). Those banks that participated and their participation was revealed (because they were examined) can potentially suffer a run in the second period because of stigma (captured by $L$). Finally, the government has to repay (facing inefficiency costs $1-\phi$ and distortionary taxation costs $\delta$) the bonds that could not be covered by liquidating private assets in the previous period. The third line can then be rewritten as

$$H - y[q\sigma\hat{\chi} + (1 - q)\delta(1 - \phi)E^w(p)C].$$

Adding (and canceling) terms, total welfare is

$$W(\tilde{p}) = 2H - (1 - y)(\varepsilon + \sigma(1 - \varepsilon))(H - \tilde{L}) - y[(1 - q)(1 + \delta)(1 - \phi)E^w(p)C - q\sigma\hat{\chi}].$$

Since the unconstrained welfare is $2H$ in both periods, we can denote the distortion from the crisis as equation (14).

The first component shows the distortion that comes from lower output from banks that did not participate, either due to leaks or runs. The second component shows the costs of the distortionary taxation that is needed to cover deposits from defaulting banks, and that cannot be covered by liquidating the private collateral. Finally, the third component shows the lower production in the second period that arises from stigma – banks who were discovered borrowing from the discount window and then were revealed to have collateral of relatively lower quality – being more likely to suffer a run and produce less in the second period.

### A.9 Proof Proposition 5

Here we compare the distortions for different levels of discount $\tilde{p}$ (in the different regions that we characterized in the previous section) for different disclosure policies.

We consider first the parametric case for which $\tilde{p}_T < \tilde{p}_L$. Under transparency, all banks participate when $\tilde{p} \geq \tilde{p}_T$. This implies that $y = 1$, $\chi = 0$ and $E^w(p) = p_L$. Distortions in
this range are then

\[ Dist(\tilde{p}|Tr) = (1 + \delta)(1 - q)(1 - \phi)p_L C. \]

In contrast, for all \( \tilde{p} < \tilde{p}_T \), the discount window is not sustainable and distortions are the same as without intervention, \( H - L \equiv (1 - p_L)K^*(qA - 1) + \gamma. \)

Under opacity, distortions also depend on the equilibrium regions. In the “very low” discount region all banks participate, then \( y = 1 \) and \( \chi = 0 \), and distortions are the same as under transparency (in the region \( \tilde{p} \geq \tilde{p}_T \) above).

In the “low” discount region, \( y < 1 \) but \( \sigma = 0 \), then from equation (14)

\[ Dist(\tilde{p}|Op) = (1 - y)(H - \tilde{L})\varepsilon + y(1 + \delta)(1 - q)(1 - \phi)E^w(p|Op)C. \]

Since \( yE^w(p|Op) < p_L \) and \( H - L > H - \tilde{L} \), the sufficient condition for the distortion under opacity is lower than the distortion under transparency is

\[(1 + \delta)(1 - q)(1 - \phi)p_L C > \varepsilon(H - L).\]

In the “intermediate” discount region, \( y = \overline{y} \), but \( \sigma > 0 \) and increases with the discount. From equation (14) it is clear that in this region the distortion increases with \( \sigma \) and then with the discount. While the fraction of banks participating is fixed, there are more runs and then more banks not participating end up producing less, both in the first period (less deposits) and the second period (more stigma).

Finally, in the “high” discount region, the discount window is not sustainable under opacity, so distortions are the same as without intervention, \( H - L \).

Summarizing, as the discount increases, distortions under opacity are fixed in the very low discount region, decrease in the low discount region, increase in the intermediate discount region and reach the maximum in the high discount region. In contrast, distortions under transparency are fixed whenever the discount window is sustainable, as either all banks participate or neither does. This implies the optimal discount is \( \tilde{p}_m \) under opacity.

Consider finally the parametric case for which \( \tilde{p}_T \geq \tilde{p}_h \). In this case the discount window under opacity collapses at lower discount levels than under transparency. Still the optimal policy is a discount of \( \tilde{p}_m \) under opacity because \( \tilde{p}_T < \tilde{p}_h \), as shown in Lemma 5. QED.