Fighting Crises with Secrecy*

Gary Gorton†  Guillermo Ordoñez‡

October 2018

Abstract

How does central bank lending during a crisis restore confidence? Emergency lending facilities which are opaque (in that names of borrowers are kept secret) raise the perceived average quality of bank assets in the economy, creating an information externality that prevents runs. Stigma (the cost of a banks participation at the lending facility becoming public) is desirable as it keeps banks participating in the lending facilities from revealing their participation. The central bank’s key policy instrument for limiting the amount lent while maintaining secrecy is the haircut on bank assets used as collateral at the emergency lending facility.

*We thank Huberto Ennis, Mark Gertler, Todd Keister, Pamela Labadie, Ricardo Reis, Eric Rosen-gren, Geoffrey Tootell and seminar participants at the Boston Fed, Wharton, the AEA Meetings in Boston, the NAESM in Philadelphia, the SED Meetings in Toulouse and the “Innovations in Central Banking” Conference at the St. Louis Fed for comments and suggestions. This paper previously circu-lated under the title “How Central Banks End Crises.” The usual waiver of liability applies.

†Yale University and NBER (e-mail: gary.gorton@yale.edu)
‡University of Pennsylvania and NBER (e-mail: ordonez@econ.upenn.edu)
1 Introduction

How does central bank emergency lending during a financial crisis restore confidence? During the financial crisis of 2007-2008 the Federal Reserve introduced a number of new emergency lending programs, including the Term Auction Facility, the Term Securities Lending Facility, and the Primary Dealers Credit Facility, which provide public funds to financial intermediaries in exchange for private assets. Also as part of their Emergency Economic Stabilization Act, the U.S. Treasury implemented similar programs, such as the Troubled Asset Relief Program (TARP). These facilities were specifically designed to hide borrowers’ identities.\(^1\) Secrecy was also integral to the special crisis lending programs of the Bank of England and the European Central Bank.\(^2\) Plenderleith (2012), asked by the Bank of England to review their Emergency Lending Facilities (ELA) during the financial crisis, wrote: “Was secrecy appropriate in 2008? In light of the fragility of the markets at the time . . . it was right to endeavor to keep ELA operations covert. . . in conditions of more systemic disturbance, as in 2008, ELA is likely to be more effective if provided covertly” (p. 70). Even before the Federal Reserve came into existence, private bank clearinghouses would also open an emergency lending facility during banking panics keeping the identities of borrowing banks secret. See Gorton and Tallman (2018).

The secrecy of all these lending facilities has been widely criticized for hiding the identities of weak or insolvent banks, resulting in fierce calls for transparency. During the recent crisis, for instance, Bloomberg and Fox News sued the Fed (under the Freedom of Information Act) to obtain the identities of borrowers.\(^3\) In this paper, however, we show that lending facilities that replace “dubious private assets” with “good public bonds” in secret are indeed optimal. Secrecy creates an information externality by raising the average quality of assets in the banking system without replacing all assets under suspicion, mitigating the desire of depositors to examine banks’ assets, thereby avoiding a withdrawal of funds (runs hereafter) from those banks that are

---

\(^1\)Bernanke (2010): “. . . [because of] the competitive format of the auctions, the TAF [Term Auction Facility] has not suffered the stigma of the conventional discount window” (p.2). Also, see Armantier et al. (2015). In the case of TARP there were fierce criticisms after their implementation by the Senate Congressional Oversight Panel about its lack of transparency. See http://www.jec.senate.gov/public/index.cfm/2009/3/03.1.09.


found to have less than average asset quality. A crisis here is an information event, as in Dang, Gorton, and Holmström (2013) and Gorton and Ordonez (2014). Lending facilities recreate confidence and avoid inefficient examination of banks’ portfolios. Secrecy minimizes the social cost of resorting to lending facilities.

Our view of a bank run is in contrast to the more standard view of a run as a coordination failure that occurs even if private assets are solvent and focuses instead on the process of examining the bank’s assets before choosing whether to withdraw funds. Our view captures both depositors seeking to withdraw from banks but also repo lenders wanting to withdraw via higher haircuts on the collateral or not rolling over their loans at all. Evidence that not all banks experience runs comes from the recent financial crisis is provided by Perignon, Thesmar and Vuillemey (2017) who show that in the European market for unsecured wholesale certificates of deposit, some banks experiences funding dry-ups while other banks did not.

As these runs are motivated by suspicions about the bank’s backing collateral, the central bank may lend cash against collateral using an emergency lending facility, or lend Treasury bonds against collateral as in the Term Securities Lending Facility (TSLF) to raise the perceived average quality of the backing collateral. Our arguments below apply in either case, but we will, for consistency, speak throughout of a facility like the TSLF where government bonds are lent against private collateral. Additionally, we will speak of the “central bank”, but we also think of that as including fiscal authorities’ facilities, as with TARP.

In this paper, we claim that restoring confidence during crises means successfully creating this externality of increasing the expected quality of banks’ private portfolio with some government backing. Furthermore, what makes this policy implementable is the existence of “stigma”, which refers to the cost to a bank if its participation in lending facilities becomes known. A bank may want to reveal its participation so that it is known that its liabilities are backed by government debt or cash. Stigma is desirable as it keeps banks participating in the lending facilities from revealing their participation. The central banks key policy instrument for limiting the amount lent is the haircut on bank assets offered as collateral at the emergency lending facility.

In our model households are born with endowments already deposited in a bank. They can withdraw immediately to consume or keep the deposits in the bank. A bank is an institution that also has its own funds and has the proprietary access to two productive investment opportunities. One of those opportunities generates an
asset that is pledgeable and can be used as collateral. The other is a stochastic project that is not pledgeable. The loan has to be collateralized. We denote the first investment collateral and the second investment project. The first will be financed with own resources and the second with deposits as long as households do not withdraw first.

A collateral has both a common component and an idiosyncratic component that determine their probability of generating cash flows at the end of the period (its quality). While the quality of the collateral is privately known by each bank, it is unknown to depositors. We assume, however, that depositors can privately discover whether the collateral is good (generates cash flows) or bad (it does not generate cash flows) by acquiring information at a cost.

The underlying problem in the economy is a scarcity of good collateral (“safe debt” to back repo, for example). The bank’s own collateral will be used to back deposits (e.g., the land on which a building is being built). When good collateral is scarce, an efficient substitute is ignorance about which private assets used as collateral are good and which are bad, as in the absence of information even banks with a bad collateral can avoid early withdrawals and invest in the project. In that case, good and bad collateral are pooled in an informational sense. When this pooling results in a high enough perceived average value of banks’ portfolios, depositors do not examine their own bank’s portfolio (no run), maintain their deposits in the bank and banks efficiently invest in their projects.

A crisis happens when an (exogenous) event occurs (a fall in home prices, for instance) causing depositors to run, examining the bank’s portfolio and withdrawing if the bank has bad collateral. If this happens, banks can react by reducing the investment scale to avoid information acquisition, or give in to the run, be examined by depositors and hope the depositors find out that the collateral is good so it can invest at the optimal scale. In either case, absent government intervention, aggregate consumption falls during a crisis.

The government’s goal is to prevent runs, which means avoiding information acquisition about banks’ portfolios. How does a central bank end a crisis in our model? First, the central bank opens an emergency lending facility (called the discount window throughout the paper). As banks have heterogeneous collateral (because of the idiosyncratic shocks), which banks go to the discount window depends on their private information about their portfolio quality and on the haircut (discount throughout
the paper) on the private assets they deliver as collateral at the discount window in exchange for government bonds.

The choice of the haircut on the value of a bank’s asset controls which banks participate in the discount window and by doing so determines the perceived average quality of private collateral remaining in the economy. But, banks which go to the discount window might be tempted to reveal that they have exchanged dubious private collateral for good government bonds (public collateral), so that they can maintain their deposits. But, this reveals information about their own collateral quality which will affect them next period. If by revealing that they went to the borrowing facility banks are perceived as having low quality collateral (because they exchanged low quality collateral for high quality government bonds), then they may be stigmatized. “Stigma” is the cost, in terms of future fund raising, of revealing that the bank borrowed from the emergency facility. We show how the stigma costs arise endogenously and are necessary for the policy of opacity to work, as banks participating in lending facilities do not have incentives to reveal this information. Stigma aligns the secrecy incentives of the government with those of individual banks.

In contrast to the standard view that opacity prevents stigma, in our paper stigma prevents transparency. Stigma plays an important role in sustaining secrecy, allowing the central bank to generate an information externality. If the central bank is successful, the result is a high enough perceived average value of banks’ assets in the economy such that runs do not occur. The threat of stigma is critical for opacity to be sustainable in equilibrium. The optimal haircut is given by the point at which there are enough banks participating at the lending facility to avoid runs on all banks, and no bank faces stigma in equilibrium.

A prominent branch of the literature has studied the effects of lending facilities in helping the economy during crises. Most of this work is based on the premise that a crisis is given by an exogenous tightening in credit constraints (see Gertler and Kiyotaki (2010) for a discussion) and lending facilities are set up to provide loans directly (increasing the role of governments as credit providers), open a discount window (mostly to improve inter-banking operations) and injecting equity (to increase the net worth of banks). In our model the tightening of credit constraints is endogenous and created by an informational reaction in credit markets to changes in fundamentals, and then we emphasize the role of lending facilities in relaxing the informational tightening of credit conditions.
There is also a large literature on the lender-of-last-resort summarized by Freixas, Giannini, Hoggarth and Soussa (1999 and 2000) and by Bignon, Flandreau, and Ugolini (2009).\textsuperscript{4} Recent historical work also includes Flandreau and Ugolini (2011) and Bignon, Flandreau, and Ugolini (2009) who document the development of the lender-of-last-resort role at the Bank of England and at the Bank of France. Unlike the existing literature, we focus on why secrecy surrounds interventions during crises, the roles of stigma, and a determination of how haircuts are set during a crisis.

Our paper also contributes to the recent discussion about the optimal disclosure of information about financial variables by governments and regulators. A clear example is the literature that studies stress tests and focus on the possible impact of information disclosure about a bank’s portfolio on funding (Alvarez and Barlevy (2015) and Leitner and Williams (2017)), on runs (Bouvard, Chaigneau, and De Motta (2015), Faria-e Castro, Martinez, and Philippon (2017) and Williams (2017)) and on risk-sharing (Goldstein and Leitner (2018)). Our work differs from that literature in that we do not study how regulators disclose information about the realized portfolio and health of financial intermediaries, but instead on how the central bank discloses information about its efforts to modify the portfolio of financial intermediaries in distress. In this sense we consider the effects of government’s disclosure on the effectiveness of government’s interventions and not on the behavior of banks in order to avoid interventions.

The paper proceeds as follows. In Section 2 we specify the model, including the households’ choice to run or not to run and the banks’ choice of whether to invest or not. In Section 3 we describe crises and the structure of interventions that we consider. Section 4 concerns the equilibrium when the economy is in a crisis and the central bank opens a lending facility. First, we determine the equilibrium for a fixed collateral haircut and a disclosure policy, and then we allow the central bank to choose the haircut level and the disclosure policy that maximize welfare. Section 5 concludes. All proofs are in the Appendix.

2 Model

2.1 Environment

We study a discrete-time economy with two periods that is composed of a central bank, a mass 1 of banks, and a mass 1 of households. Households live for a single period, are risk-neutral and are indifferent between consuming at the beginning or at the end of the period in which they live. At the beginning of the second period another mass of households is born. Each household is born with an endowment $D$ of consumption good (numeraire), which is already deposited in a bank at the beginning of the period (accordingly we refer to them as depositors), which is available for withdrawal at no cost at the beginning of the period.

Banks live for two periods and are also risk-neutral. Banks will have two one-period investment opportunities, one of these projects can be self-financed (with bank equity, for example) and used as collateral in the borrowing and lending market (for example because their proceedings are observable and verifiable in court), and hence will be called the collateral. The other will be called the project.

With probability $p_{it}$ the collateral is of good type, in which case it delivers $C$ units of numeraire at the end of period $t$. With the complementary probability, the collateral is of bad type and does not deliver any numeraire at the end of the period. The collateral types are iid across periods and across banks. We refer to the ex-ante probability that bank $i$’s collateral in period $t$ is good, $p_{it}$, as the quality of the collateral held by bank $i$ in period $t$. At the end of a period, this project backing the collateral matures and it is sold to the households. Then at the start of the next period the bank receives a new endowment of own funds and invests in a new project, creating new collateral.\(^5\)

The quality of the collateral depends on an aggregate component and on an idiosyncratic component, $p_{it} = \bar{p}_t + \eta_i$. The aggregate component, $\bar{p}_t$, is time-varying and captures the average quality of collateral in the economy. The idiosyncratic component, $\eta_i$, is persistent and captures heterogeneity in the banks’ ability to generate collateral, where $\eta_i \sim F[-\bar{\eta}, \bar{\eta}]$, with $E(\eta_i) = 0$ and $\bar{\eta}$ such that $p_i \in [0, 1]$.\(^6\)

\(^5\)It is simplest way to think of the collateral project as being financed by the bank’s own endowment and the other project as being financed by deposits, but this is not strictly necessary.

\(^6\)An example of collateral in our setting is the construction of a building, financed by the banker’s own funds and then sold to a property manager in charge of selling the units and dealing with tenants.
to the bank’s collateral, no agent (neither the bank nor the depositor) knows the collateral type at the beginning of the period but there is asymmetric information about its quality: while both banks and depositors know the average quality of collateral in the economy, $p_t$, each bank also privately observes its own idiosyncratic component $\eta_i$, so the bank has a better assessment of its own collateral quality, $p_{it}$.

In addition, each bank $i$ is matched with a single depositor every period. The deposits are used to finance another investment opportunity (in what follows we refer to it simply as the project). The project pays $A \min\{K, K^*\}$ with probability $q$ and 0 otherwise, where $K^*$ is the maximum feasible scale of production. In terms of the information structure, the result of the project (whether it succeeds or fails) is known by the bank but it is neither observable nor verifiable by the depositors.

### 2.2 Information Production

Even though no agent knows the collateral type at the beginning of the period, depositors have a technology to *privately investigate the collateral type at the beginning of the period* at a cost $\gamma$ in terms of numeraire. Using this technology they can determine whether the collateral can deliver $C$ units of numeraire when managed at the end of the period or not.\(^7\) We assume that acquiring information is a private activity, but the depositor can credibly disclose (certify) his private information immediately upon acquisition if it is in his best interest to do so. This information acquisition technology introduces incentives for depositors to learn about the bank’s collateral type before deciding to keep their funds in the bank or not, taking advantage of such private information before putting their deposits at risk.

At the end of the period, *after all contracts are settled*, the collateral type becomes public knowledge. As it will become clear later, this assumption allows us to isolate the market for collateral at the end of the period (in which banks sell the collateral, this is the project that has been completed, for management to households) from the contracting problem at the beginning of the period, allowing us to focus on the activity that has real implications in our setting.

---

\(^7\)Assuming that banks can also learn the bank’s collateral type does not modify the main insights. See Gorton and Ordonez (2018) for double-sided information acquisition in a model without banks.
2.3 Projects

We make the following parametric assumptions about the project’s payoffs:

**Assumption 1  Project payoffs**

- $qA > 1$. *It is ex-ante optimal to finance the project up to a scale $K^*$.*
- $D > K^* + \gamma$. *It is feasible to implement the optimal investment just using deposits.*
- $C > K^*$. *Good collateral is sufficient to cover the optimal loan size.*

Note that it follows immediately that the bank can use the proceeds of the good collateral to finance the project up to $K^*$, but this is not possible because the bank has to finance the collateral at the beginning of the period and these proceeds arise at the end of the period. In other words, the bank needs deposits to finance the project.

2.4 Timing

The timing during a single period is as follows. At the beginning of the period, a bank offers a contract to the households that consists of an investment plan (the commitment to invest a certain amount $K$ in the project), an interest rate for deposits (a promised repayment $R \geq D$ at the end of the period), and the collateral that secures the deposits (a fraction $x$ of the collateral project to deliver to the depositor in case the bank defaults). Conditional on this contract, depositors choose whether or not to examine the bank’s collateral and then decide whether or not to withdraw the deposits at the beginning of the period. At the end of the period, two sequential movements take place. First investments payoffs are determined and loan contracts are fulfilled. Then, collateral types are revealed and banks sell their collateral projects (collateral that they did not hand out to lenders because of default) to households (making a take-it or leave-it offer to a randomly matched household).

---

8This is done for simplicity. The bank will receive a new endowment in the second period, but the new collateral will inherit the same idiosyncratic component that determines its quality.
2.5 Withdrawal Choices

In what follows we say there is a run on a bank when depositors examine the bank’s collateral at the beginning of the period, and withdraw the funds upon finding out that the collateral is bad. In stark contrast with the rest of the literature, in our setting a run is not a coordination failure (as there is a single depositor per bank) but instead an information failure. Our interpretation of a run is not one in which the depositor suddenly says “give me the money!” but rather says “show me the money!” Even though coordination issues are relevant, we abstract from them and focus on a novel dimension of distrust on banks that leads to close examination of their portfolio.

Next, we characterize the contracts that banks choose to maximize their profits conditional on depositors’ participation (that is, conditional on depositors leaving their deposits in the bank for investment purposes). We show that by announcing their investment strategies, banks can knowingly cause a run (examination of the collateral before the withdrawal decision) or not (no examination of the collateral). In each situation the bank will offer a repayment promise and collateral in case of default such that the depositors will be indifferent between withdrawing their deposits at the beginning of the period or not. Depending on which contract delivers higher profits in expectation, the bank will optimally choose whether or not to induce a run and be examined. Notice that this is a sequential game between a bank and depositors in which a bank chooses a contract to maximize profits conditional on the constraints imposed by the participation, information acquisition and withdrawal choices of depositors.

While the bank knows its own collateral quality $p_{it}$, households just have an expectation about the probability that a particular bank’s collateral is good, which we denote as $E(p_i|\mathcal{I})$, where $\mathcal{I}$ is the information set of depositors at the time of deciding whether to withdraw at the beginning of the period. In this section we focus on a single bank and a single period, so we dispense with the subindex $it$.

2.5.1 Run

Assume the depositor examines the bank’s collateral at the beginning of the period, spending $\gamma$ of numeraire. As information acquisition is essentially specified by the contract and the information can be certified, both the depositor and the bank will have access to the truth about the collateral type. If the collateral is good, the bank
can credibly guarantee that the depositor will recover the deposits at the end of the period, even if the bank finances the project at the optimal scale $K^*$ and the project fails. This is because $C + D - K^* > D$ by assumption. If the collateral is bad, the bank will always claim the project fails, paying less than what was deposited. This implies that the depositor would always withdraw the deposits at the beginning of the period if the collateral is bad.

What is the bank contract triplet $(R_r, x_r, K_r)$ (the promised repayment at the end of the period, the fraction of collateral that backs the loan and the investment scale) that induces the depositor to examine the collateral and allows for investment if the collateral is good instead of just withdrawing the deposits at the beginning of the period?

Before acquiring information the depositor knows that, conditional on facing a bank that is offering a contract that induces a run, the ex-ante probability of finding good collateral is $E^r(p) \equiv E(p|\text{run})$. As we will discuss, $E^r(p)$ will be determined in equilibrium according to the banks that optimally choose to offer this contract. Upon finding information that the collateral is good, the depositor’s expected payoffs are $qR_r + (1 - q)x_rC - \gamma$. Before acquiring information the depositor also knows that the ex-ante probability of finding bad collateral is $(1 - E^r(p))$, in which case the depositor will always withdraw, just losing the cost of examination, then obtaining $D - \gamma$.

Before using these elements to write the expression for the expected depositor’s payoffs, there is an additional constraint: a truth-telling condition $R_r = D - K_r + x_rC$. If it were the case that $R_r < D - K_r + x_rC$ the bank would always sell the collateral to another household at $C$ to repay the loan (notice that a buying household can infer the collateral is good by observing that a loan that presumes examination has been issued). In contrast, if it were the case that $R_r > D - K_r + x_rC$ the bank would always default on the depositor.

The expected payoffs for a depositor of participating in this contract are then $E^r(p)(R_r - \gamma) + (1 - E^r(p))(D - \gamma)$, independent of $K_r$. We assume depositors do not have negotiation power, so they break even, being indifferent between participating in the contract or withdrawing without examining the collateral and just consuming at the beginning of the period if:

$$D = (1 - E^r(p))D + E^r(p)R_r - \gamma.$$
This participation constraint pins down the interest rate:

\[ R_r = D + \frac{\gamma}{E^r(p)}, \]  

(1)

which is independent of \( q \) (the probability the project pays off).

We have characterized the contract that maximizes the bank’s payoffs under runs as a triplet, \((R_r, x_r, K_r)\), where \( R_r \) is given by equation (1), \( x_r = \frac{R_r - D + K_r}{C} \) (from the truth telling condition) and \( K_r = K^* \) (from feasibility of optimal investment upon finding the collateral is good). Now we can compute the bank’s ex-ante expected profits given this contract. As the bank knows its collateral is good with probability \( p \), and that it will suffer a withdrawal in case its collateral is bad, its ex-ante expected profits are \( p(D + qAK^* - K^* - R_r) + pC \). Substituting \( R_r \) in equilibrium, the period expected profits (net of the expected price of the collateral at the end of the period, \( pC \)) from inducing a run are:

\[ E^r(\pi|p, E^r(p)) = \max \left\{ \frac{pK^*(qA - 1)}{E^r(p)} - \frac{p}{E^r(p)}\gamma, 0 \right\}. \]  

(2)

These expected profits depend not only on the probability that the bank’s collateral is good, \( p \), but also on the average quality of the collateral of all other banks inducing a run. This implies that there is cross-subsidization among banks that face a run: banks with \( p > E^r(p) \) end up paying more to compensate depositors for the information costs in expectation, as \( \frac{p}{E^r(p)} > 1 \). The opposite happens for banks with \( p < E^r(p) \).

2.5.2 No Run

Another possible contract is one where the depositor does no produce information. In this case, the depositor expects to obtain \( R_{nr} \) in case of repayment. And in the case of default, a fraction \( x_{nr} \) of the collateral of expected value \( E^{nr}(p)C \) (where \( E^{nr}(p) \equiv E(p|\text{no run}) \) is the expected quality of collateral among banks that offer a contract that does not trigger examination); this is \( D - K_{nr} + x_{nr}E^{nr}(p)C \). In this case the truth telling constraint is given by \( R_{nr} = D - K_{nr} + x_{nr}E^{nr}(p) \). If \( R_{nr} > D - K_{nr} + x_{nr}E^{nr}(p)C \), the bank can sell the collateral at a price \( E^{nr}(p)C \) (recall information about the type is revealed after contracts are settled) and repay always. In contrast, if \( R_{nr} < D - K_{nr} + x_{nr}E^{nr}(p)C \), the bank would always default. Then, \( x_{nr} = \frac{R_{nr} - D + K_{nr}}{E^{nr}(p)C} \).
The depositor will be indifferent between participating in the contract or not as long as \( D = R_{nr} \), which implies that no run is a feasible contract if and only if \( x_{nr} = \frac{K_{nr}}{E^{nr}(p)C} \leq 1 \). This gives the first constraint on investment from this contract,

\[ K_{nr} \leq E^{nr}(p)C. \tag{3} \]

The second constraint on investment comes from guaranteeing that the depositor does not have an incentive to deviate and examine the bank’s collateral privately at the beginning of the period. Depositors want to deviate because they can maintain deposits at an expected gain if they know the collateral received is good and withdraw if the collateral is bad. Depositors want to deviate if the expected gains from acquiring information, evaluated at \( R_{nr} \) (and then at \( x_{nr} \)), are greater than the gains from not acquiring information. This is

\[ (1 - E^{nr}(p)) D + E^{nr}(p) \left[ qR_{nr}^R + (1 - q)[D - K_{nr} + x_{nr}C] \right] - \gamma > D. \]

Substituting in the definitions of \( R_{nr} \) (and then \( x_{nr} \)), there is no incentive to privately deviate and acquire information about the bank’s collateral if:

\[ (1 - E^{nr}(p)) (1 - q)K_{nr} \leq \gamma. \]

Intuitively, by acquiring information the depositor only keeps his deposits in the bank if the collateral is good, which happens with probability \( E^{nr}(p) \). If there is default, which occurs with probability \( (1 - q) \), the depositor gets \( x_{nr}C \) for a unit of collateral that was obtained at a price \( E^{nr}(p)x_{nr}C = K_{nr} \), making a net gain of \( (1 - E^{nr}(p)) x_{nr}C = (1 - E^{nr}(p)) \frac{K_{nr}}{E^{nr}(p)} \) with probability \( E^{nr}(p)(1 - q) \). In other words, when deviating and examining the collateral, the depositor withdraws at the beginning of the period if the collateral is bad and keeps the funds at the bank if the collateral is good.

The condition that guarantees that depositors do not want to deviate and produce information about the bank’s collateral can then be expressed in terms of the project size, \( K_{nr} \), such that

\[ K_{nr} < \frac{\gamma}{(1 - q)(1 - E^{nr}(p))}. \tag{4} \]

If the bank scales back the project (hence the loan), the depositor has less of an incentive to acquire information about the bank’s collateral.
Imposing constraints (3) and (4), the project size that is consistent with a no-run equilibrium is

\[ K_{nr}(E_{nr}(p)) = \min \left\{ K^*, \frac{\gamma}{(1-q)(1-E_{nr}(p))}, E_{nr}(p)C \right\}, \tag{5} \]

and the bank’s expected profits (net of the expected price of the collateral at the end of the period, \( pC \)) are

\[ E_{nr}(\pi|E_{nr}(p)) = K_{nr}(E_{nr}(p))(qA - 1). \tag{6} \]

### 2.6 Investment Choices

By choosing the investment level, a bank can choose between facing a run (examination of its collateral) or not. The bank then selects the option (and related contract) that delivers the highest profits in expectation. The optimal decision of how much to invest in the project is then isomorphic to the bank announcing an investment strategy that either finances the project at optimal scale (triggering a run) or at a restricted scale (avoiding a run).

All the banks’ decisions must be consistent in aggregate. Even though banks are not fundamentally linked to each other, the choices of banks who choose to induce a run or not will affect the inference of depositors about the quality of the collateral that a bank holds as a function of the contract it offers. As we have shown, the expected profits of a bank suffering a run (equation 2) depends both on \( p \) and \( E_r(p) \) while the expected profits of a bank without a run (equation 6) only depends on \( E_{nr}(p) \).

Since \( E_r(\pi) \) increases in \( p \) while \( E_{nr}(\pi) \) is independent of \( p \), conditional on the strategies of all other banks, if a bank with collateral of quality \( p \) announces an investment plan that induces a run, then all \( p' > p \) will also. Similarly, if a bank with a collateral of quality \( p \) announces an investment plan that avoids a run, then all \( p' < p \) will also. Intuitively, a bank with a high quality collateral (high \( p \) so the bank believes its collateral is very likely to be good) is more willing to open the collateral to examination and to face the lottery that a run represents. This is why banks with better collateral would be more inclined to announce investment plans that induce examination.

This implies that the optimal investment strategy is given by a cutoff rule under which all banks with \( p < p^* \) restrict their investments to avoid runs and all banks
with \( p > p^* \) invest at the optimal scale and open themselves to examination of the collateral (a run), where \( p^* \) is the collateral quality that makes a bank indifferent between a run or not.

\[
E_r(\pi|p^*, E^r(p)) = E_{nr}(\pi|E^{nr}(p)),
\]

where \( E^r(p) = E(p|p > p^*) \) and \( E^{nr}(p) = E(p|p < p^*) \).

More formally, allowing for corner solutions in which all banks either face a run or not, the equilibrium cutoff is such that

\[
p^* = \begin{cases} 
\bar{p} + \bar{\eta} & \text{if } E_r(\pi|\bar{p} + \bar{\eta}, \bar{p} + \bar{\eta}) < E_{nr}(\pi|\bar{p}) \\
p^* & \text{s.t. } E_r(\pi|p^*, E(p|p > p^*)) = E_{nr}(\pi|E(p|p < p^*)) \\
\bar{p} - \bar{\eta} & \text{if } E_r(\pi|\bar{p} - \bar{\eta}, \bar{p}) > E_{nr}(\pi|\bar{p} - \bar{\eta})
\end{cases}
\]

(7)

As both \( E^r(\pi|p^*, E(p|p > p^*)) \) and \( E^{nr}(\pi|E(p|p < p^*)) \) increase with \( p^* \) there may be multiple \( p^* \) in equilibrium. In what follows we will focus on the largest \( p^* \) as this represents the best equilibrium, the one that guarantees the highest sustainable output. The next Proposition shows that in such equilibrium the threshold \( p^* \) increases with the average quality of collateral in the economy, \( \bar{p} \). This is, the higher the average quality of collateral in the economy the fewer banks face runs and collateral examination.

**Proposition 1** In the best equilibrium, the threshold \( p^* \) is increasing in \( \bar{p} \). There exist beliefs \( p^H > p^L \) such that if \( \bar{p} > p^H \) no bank faces a run and if \( \bar{p} < p^L \) all banks face runs.

**Remark on off-equilibrium beliefs:** When the threshold \( p^* \) is a corner (either \( p^* = \bar{p} - \bar{\eta} \) or \( p^* = \bar{p} + \bar{\eta} \)), the expected quality of the collateral of a bank following a strategy that is off-equilibrium is not well-defined. If \( p^* = \bar{p} + \bar{\eta} \) no bank faces a run and then \( E^r(p) \) is not well-defined as it is an off-equilibrium strategy. The same is the case for \( E^{nr}(p) \); if \( p^* = \bar{p} - \bar{\eta} \), then all banks are expected to face a run.

Following the Cho and Kreps (1987) criterion, we assume that if a bank follows a strategy that is not supposed to be followed in equilibrium, depositors believe the bank holds a collateral that maximizes its incentives to deviate from the expected strategy. This is, if \( p^* = \bar{p} + \bar{\eta} \) and a depositor observes a bank investing in a large project so as to induce a run, then the depositor believes that the bank has the highest...
available quality collateral, \(E^r(p) = \bar{p} + \eta\). Similarly, if \(p^r = \bar{p} - \eta\) and a depositor observes a bank investing in a project that discourages examination of the collateral, then the household believes that the bank has the lowest available quality collateral, \(E^{nr}(p) = \bar{p} - \eta\).

3 Crises and Interventions

In the previous section we have characterized the contracts and investment levels of all banks in the economy as a function of the aggregate \(\bar{p}_t\) in each period \(t\). In this section we restrict attention to a situation in which the economy suffers a crisis in the first period and gets back to normal in the second period. How the economy fares during the crisis in the first period will also determine output in the second period.

The crisis in the first period comes from a shock to the economy as follows. First, to capture an information crisis, we assume that \(p_{t=1} = p_L < p_H\), such that all banks would face examination (runs). Second, to capture the turbulent nature of crises, we assume that absent interventions, there are potential information leakages; with probability \(\varepsilon\) the bank’s collateral type is revealed. In the second period the economy goes back to normal times, which we capture by assuming that \(\bar{p}_{t=2} = p_H > p_H\) such that, absent a previous crisis there would not be a run on any bank and, according to equation (5), \(K_{nr} = K^*\).

When the economy is in normal times, absent a previous crisis, it achieves the maximum potential consumption: households consume \(D\) and all banks invest at the optimal scale, producing an additional amount \(K^*(qA - 1)\) of numeraire to consume. Consumption is then

\[W_N = D + p_HC + K^*(qA - 1).\]

During crises, however, absent government intervention, all banks face runs, depositors examine the banks’ collateral and only a fraction \(p_L\) of banks retain their deposits, at an informational cost \(\gamma\), while the remaining fraction \((1 - p_L)\) of banks face withdrawals at the beginning of the period and are not able to finance their projects. In this case, consumption is

\[W_C = D + p_LC + p_LK^*(qA - 1) - \gamma.\]
In crises, absent government intervention, consumption is clearly lower than consumption in normal times and the assumption of information leakages become irrelevant as there is information acquisition about all banks anyway.

### 3.1 Lending Facilities

We focus next on analyzing a particular set of lending facilities that can be used by a central bank to relax the incentives to acquire information in the economy and boost investment during the crisis in the first period. This is the timing of these facilities.

1. The central bank opens a **discount window** where it will exchange $B$ government bonds (hereafter “bonds” for short) per unit of collateral, to be paid in numeraire at the end of the period. It also announces whether it will reveal the identities of banks participating at the discount window (a policy of transparency) or whether these identities will be secret (a policy of opacity). The central bank chooses both $B$ (the “price” to pay for a unit of collateral) and its disclosure strategy and can commit to its announced policy.\(^9\)

2. Banks choose whether to go to the discount window or not. Even if the central bank announces a policy of opacity, still a bank may choose to reveal its participation to its depositors. In that case, it becomes public knowledge that the bank has bonds in its portfolio, which will result in an endogenously determined stigma cost, denoted $\chi$. Banks that reveal themselves to have borrowed at the discount window will be more likely to face runs and to be forced to reduce their project size in the future. The size of such “stigma” will be determined later in equilibrium.\(^{10}\)

3. At the end of the first period, discount window participants with successful projects repay deposits using the proceeds from production and repurchase their collateral back from the central bank. Failing banks lose their bonds to depositors, who redeem them. Successful banks that did not borrow from the discount window, repay their deposits with the proceeds from production and

\(^9\)Borrowing a Treasury bond corresponds to using the Fed’s Term Securities Lending Facility; see Hrung and Seligman (2011). But, bonds could also be thought of as cash or reserves.

\(^{10}\)See Armantier et al. (2015), Anbil (2017), Ennis and Weinberg (2010) and Furfine (2003) for other ways to model stigma costs.
retain their collateral. Unsuccessful banks default and hand over their collateral to the depositors, who manage them at the end of the period.

4. The central bank can liquidate the collateral left in its possession by defaulting banks but only imperfectly. The central bank can only extract a fraction $\phi$ of the value of the banks’ collateral in its possession at the end of the period. Then the numeraire generated by the private assets in possession of the central bank plus distortionary taxes (transfers at a social cost $\delta$ per unit of numeraire) are used to redeem the bonds.

We have added three additional parameters in this section: the leakage probability, $\varepsilon$, the stigma cost, $\chi$, and the social cost of distortionary taxation, $\delta$. The leakage probability is exogenous and its role is simply to avoid discontinuous changes in banking participation in lending facilities and to be better able to explain the forces behind opacity. Distortionary costs are also exogenous and are simply introduced to avoid the trivial result that the government should always lend at no discount. Finally, the role of stigma is more fundamental. First, it is endogenously determined: banks that participate in lending facilities will be those more afraid of information revelation, those with worse collateral in expectation. Second, as banks operate in subsequent periods, revealing participation (and then revealing having worse collateral) expose those banks to runs, not during the crisis period but afterwards, when the economy returns to normal times.

Our focus will be on whether the optimal policy of the central bank is one of transparency or one of opacity. This decision of the central bank will take into account the strategies of banks and depositors.\footnote{We are interested in the optimal disclosure policy within the realm of a lending facility intervention. The intervention corresponding to the solution of an optimal mechanism is outside the scope of this paper, but certainly an interesting problem for further research.} Importantly, the central bank does not produce information about the collateral it receives through the discount window.

Step 2 is the critical step for banks if the central bank chooses the opacity policy. If neither the central bank nor the bank reveals participation at the window, the deposit is backed by a portfolio with uncertain composition. Depositors only know that a fraction of banks participated at the discount window in equilibrium. But, under opacity, discount window borrowers may still wish to reveal that they borrowed so that they can show that their portfolio consists of bonds guaranteed by the government (public collateral) instead of private collateral of uncertain type. As this revelation would
make banks vulnerable to stigma in subsequent periods, they are discouraged to reveal their participation, allowing for the pooling of collateral that the central bank seeks to accomplish.

Step 4 is also important because it concerns the costs of intervention. As the central bank is less efficient than private agents at extracting value from collateral, the extra resources that are needed to redeem bonds come from distortionary taxation or transfers. With no costs there would be no trade-off faced by the central bank in determining the optimal disclosure policy.

4 The Roles of Opacity and Stigma in Fighting Crises

We solve the central bank’s problem in two steps. First, we compute the equilibrium and welfare in the economy under opacity ($Op$) and then under transparency ($Tr$), as a function of the bonds $B$ that the central bank exchanges per unit of collateral through the discount window. Then we obtain the optimal $B^*$ that maximizes welfare in each case and allow the central bank to choose the best disclosure policy, $\{Op, Tr\}$.

We will discuss policy in terms of the central bank’s discount. Define

$$B = \tilde{p}C,$$

such that the central bank choosing $\tilde{p}$ implicitly chooses how many bonds $B$ to offer per unit of collateral with quality $p$. Then the discount for a bank with collateral of expected quality $E(p)$ is given by $(E(p) - \tilde{p})C$. For convenience, and based on this one-to-one mapping between $\tilde{p}$ and $B$, we will discuss comparative statics in terms of $\tilde{p}$.

In computing welfare the central bank has to take into account two equilibrium objects, the fraction of banks that participate in the discount window, $y(\tilde{p}|\{Op, Tr\})$ and the probability of runs, $\sigma(\tilde{p}|\{Op, Tr\})$.

---

12We can also define the haircut by the ratio $1 - \frac{B}{E(p)C} = 1 - \frac{\tilde{p}}{E(p)}$. When we refer to discount, it can also be interpreted as the haircut of government bonds.
4.1 Ending a Crisis with Opacity

4.1.1 Runs under Opacity

In a crisis, in the absence of interventions, all banks will face a run (their collateral will be examined). Since we are focusing on conditions under which a central bank can steer these decisions, prevent runs and improve investment, we now characterize the no-run contract in the presence of the interventions described above.

In any equilibrium in which the central bank successfully maintains the anonymity of participating banks, banks make the same investment decisions regardless of their participation at the discount window as long as they do not face runs. Still the banks’ expected payoffs differ according to whether they go to the discount window or not. The cost of no participation is given by the probability that information about the bank’s collateral type leaks and that the bank suffers a withdrawal of funds in the case that collateral is revealed to be bad. This cost decreases in expectation with the quality of the bank’s collateral $p$.

As the cost of not participating decreases with $p$, banks tend to participate more when they have lower $p$. In other words, if a bank with collateral quality $p$ borrows from the discount window, then all other banks with $p' < p$ will do the same. In contrast, if a bank with collateral quality $p$ does not borrow from the discount window, then no bank with $p' > p$ will borrow. Hence, there is a threshold $p_w^*$ in equilibrium such that all banks $p < p_w^*$ participate and all banks $p > p_w^*$ do not. We can redefine the fraction of banks participating at the discount window as

$$y(p^*_w) = Pr(p < p^*_w).$$

The expected collateral quality of banks participating at the discount windows is

$$E^w(p) = E(p|p < p^*_w)$$

and the expected collateral quality of banks not participating is

$$E^{nw}(p) = E(p|p > p^*_w).$$

Depositors know that with probability $y(p^*_w)$ a particular bank has borrowed from the
discount window and obtained $B$ bonds. Then its portfolio expected value is

$$y(p_w^*)B + (1 - y(p_w^*))E^{nw}(p)C.$$  

Given this expectation, depositors are indifferent between withdrawing or not withdrawing at the beginning of a period when

$$D = qR^R_{nr} + (1 - q)R^D_{nr},$$

where $R^D_{nr}$ is now given by the expected return in case the bank defaults,

$$R^D_{nr} = D - K_{nr} + x_{nr}[yB + (1 - y)E^{nw}(p)C],$$

where $x_{nr}$ is again the fraction of the portfolio promised to the depositor in case of default conditional on no run.

In equilibrium the bank will promise in expectation the same in case of repayment or default (for the same reasons as in the previous section), so $D = R^R_{nr} = R^D_{nr}$. We can then obtain the fraction of collateral in the portfolio that go to the depositors in case of default,

$$x_{nr} = \min \left\{ \frac{K_{nr}}{yB + (1 - y)E^{nw}(p)C}, 1 \right\}.$$  

Now we can compute the incentive of a depositor to privately acquire information about the portfolio of the bank. At a cost $\gamma$ the depositor can privately learn whether the bank has bonds or private collateral in its portfolio, and in the case where the bank has private collateral, whether such collateral is of good or bad type.

The benefits of acquiring information are as follows: with probability $y(p_w^*)$ the bank has bonds and the depositor that examines the portfolio does not withdraw his deposits because he finds the bank holds valuable collateral, getting a payoff of:

$$qR^R_{nr} + (1 - q)[D - K_{nr} + x_{nr}B] - \gamma.$$  

With probability $(1 - y)(1 - E^{nw}(p))$ the bank has a bad private collateral and the depositor withdraws at the beginning of the period, getting a payoff of $D - \gamma$. Finally, with probability $(1 - y)E^{nw}(p)$ the bank has a good private collateral, and the
depositor keeps his deposits in the bank, getting a payoff of

\[ qR^{R}_{nr} + (1-q)[D - K_{nr} + x_{nr}C] - \gamma. \]

As \( R^{R}_{nr} = D \), and adding the previous payoffs weighted by the respective probabilities, there are no incentives to acquire information as long as:

\[ D + y(1-q)[x_{nr}B - K_{nr}] + (1-y)E^{nw}(p)(1-q)[x_{nr}C - K_{nr}] - \gamma \leq D. \]

Rearranging

\[ (1-q)x_{nr}(yB + (1-y)E^{nw}(p)C) - (1-q)[y + (1-y)E^{nw}(p)]K_{nr} \leq \gamma. \]

Since \( x_{nr}(yB + (1-y)E^{nw}(p)C) = K_{nr} \), there is no information acquisition as long as

\[ K_{nr}(E^{nw}(p)) \leq \frac{\gamma}{(1-q)(1-y)(1-E^{nw}(p))}. \] (8)

Based on this condition we can obtain \( \sigma(\bar{p}) \), the probability of a run in which depositors privately acquire information about a bank’s portfolio before choosing whether to withdraw

\[ \sigma(\bar{p}) = \begin{cases} 0 & \text{if } K < \frac{\gamma}{(1-q)(1-y)(1-E^{nw}(p))} \\ [0, 1] & \text{if } K = \frac{\gamma}{(1-q)(1-y)(1-E^{nw}(p))} \\ 1 & \text{if } K > \frac{\gamma}{(1-q)(1-y)(1-E^{nw}(p))} \end{cases} \] (9)

The next Proposition is trivial. It comes from comparing the condition for no information acquisition in the absence of intervention (equation 4) and in the presence of intervention (equation 8). It is also straightforward to check that condition (8) is more likely to hold when \( y \) is higher.

**Proposition 2** Runs are less likely with intervention when there are many banks participating at the discount window (i.e., high \( p^*_w \) and then high \( y \)).

### 4.1.2 Discount Window Borrowing under Opacity

The previous discussion focused on the depositors’ incentives to examine a bank’s portfolio conditional on the fraction of banks participating at the discount window,
Now we solve for this fraction $y$ in equilibrium as a function of $\tilde{p}$.

First, define
\[
L(p, \tilde{p}) \equiv pK^*(qA - 1) - \frac{p}{E_{nw}(p|\tilde{p})} \gamma + pC
\]
as the “relatively (L)ow” bank’s expected payoffs when the bank faces a run given that it did not participate in the discount window (as in equation 2).

Second, define
\[
H(K(\tilde{p})) \equiv K(\tilde{p})(qA - 1) + p_LC
\]
as the “relatively (H)igh” bank’s expected payoffs when the bank does not face a run and is able to invest $K(\tilde{p})$ (as in equation 6). Notice that here we are focusing on a situation in which no bank would purposefully choose a run, and then $E(p) = p_L$.

Finally, define
\[
d(\tilde{p}) \equiv (p_L - \tilde{p})C
\]
as the bank’s discount when borrowing from the discount window.

In this setting define stigma, $\chi(p, \tilde{p})$, to be the cost in terms of a higher probability of a run in the second period coming from information that the bank participated at the discount window in the first period. Even though we will derive this value endogenously later, intuitively it captures the idea that if a bank’s participation in the discount window is revealed, future depositors with access to this information will know that the quality of the bank’s private collateral is lower than average, as $p < p^*_w$.\(^{13}\)

Having defined all these elements, we can compare the payoffs of a bank $p$ from participating at the discount window or not. If the discount is $\tilde{p}$, the expected payoffs of a bank $p$ that does not borrow from the discount window are
\[
E_{nw}(\pi|p, \tilde{p}) = \sigma(\tilde{p})L(p, \tilde{p}) + (1 - \sigma(\tilde{p}))[1 - \varepsilon]H(K) + \varepsilon L(p, \tilde{p})
\] (10)
while its expected payoffs from borrowing in the discount window are
\[
E_w(\pi|p, \tilde{p}) = \sigma(\tilde{p})[H(K) - d(\tilde{p}) - \chi(p, \tilde{p})] + (1 - \sigma(\tilde{p}))[H(K) - d(\tilde{p})].
\] (11)

\(^{13}\)There is in principle a symmetric positive stigma, a benefit in terms of reducing bank runs from the revelation that a firm has not participated in the discount window, then having private collateral with quality above the average. We do not introduce any notation for this, as later we show it is zero.
As can be seen, these payoffs depend both on the quality of the bank’s collateral and on the discount from participating in the discount window.

The next four lemmas characterize the optimal depositors’ run (examination) strategies and banks’ participation strategies, as a function of $\tilde{p}$, under opacity.

**Lemma 1** Very low discount region.

There exists a cutoff $\tilde{p}_h < p_L + \bar{\eta}$ such that, for all $\tilde{p} \in [\tilde{p}_h, p_L + \bar{\eta})$ ("very low discount region"), no depositor runs (that is, $\sigma(\tilde{p}) = 0$) and all banks borrow from the discount window (that is, $y(\tilde{p}) = 1$).

This region shows that there are always levels of discount low enough ($\tilde{p}$ large enough) to induce all banks to participate at the discount window and, based on this outcome no depositor would examine the bank’s portfolio, keeping their deposits available for investment.

**Lemma 2** Low discount region.

There exists a cutoff $\tilde{p}_m < \tilde{p}_h$ such that, for all $\tilde{p} \in [\tilde{p}_m, \tilde{p}_h)$ ("low discount region"), no depositor runs (that is, $\sigma(\tilde{p}) = 0$), not all banks participate (this is, $y(\tilde{p}) < 1$) and participation declines with the level of discount (that is, $y(\tilde{p})$ increases with $\tilde{p}$).

Intuitively, when the discount is low ($\tilde{p}$ is large), many banks choose to borrow at the discount window because the cost in terms of exchanging private private collateral for bonds at a low discount more than compensates for the risk of a run and information about the private collateral being revealed. Given this, depositors do not have incentives to run and examine the bank’s portfolio.

The next lemma characterizes the equilibrium for an intermediate discount region.

**Lemma 3** Intermediate discount region.

There exists a cutoff $\tilde{p}_l < \tilde{p}_m$ such that, for all $\tilde{p} \in [\tilde{p}_l, \tilde{p}_m)$ ("intermediate discount region"), depositors run with positive probability but not always (that is, $\sigma(\tilde{p}) \in (0, 1)$) and a constant fraction $\gamma$ of banks go to the discount window (that is, $y(\tilde{p}) = y(\tilde{p}_m)$).

23
In the intermediate discount range the equilibrium cannot involve pure strategies by depositors. Since participation at the discount window when depositors do not run is low, depositors have incentives to run. In contrast, if depositors run, banks have more incentives to borrow from the discount window, which discourages runs. Depositors have to be indifferent between running or not. As the discount increases in this range, banks incentives to borrow from the discount window have to be compensated for by an increase in the probability of runs.

Finally, the next lemma characterizes the case with large discount.

**Lemma 4** High discount region.

There exists a cutoff $\tilde{p}_n > 0$ such that, for all $\tilde{p} \in (0, \tilde{p}_n]$ (“high discount region”), banks do not borrow from the discount window (that is $y(\tilde{p}) = 0$) and depositors always run (that is, $\sigma(\tilde{p}) = 1$).

Intuitively, when the discount is high ($\tilde{p}$ is low), no bank chooses to borrow from the discount window, even when depositors are running. Given this reaction, depositors always run. The economy generates the same consumption as in the case of a crisis without intervention.

The equilibrium strategies derived in Lemmas 1-4 are illustrated in Figure 1. On the horizontal axis we show the average discount $d(p_L, \tilde{p})$, the red solid function shows the fraction of depositors who run, $\sigma(\tilde{p})$, and the black dashed function shows the fraction of banks that borrow from the discount window, $y(\tilde{p})$. We use the average discount instead of $\tilde{p}$ as it is more intuitive to think of the discount as the cost of participation. The strategies in the “very low discount region” $[0, d(p_L, \tilde{p}_h)]$ are shown in Lemma 1, in the “low discount region” $[d(p_L, \tilde{p}_h), d(p_L, \tilde{p}_m)]$ are shown in Lemma 2, in the “intermediate discount region” $[d(p_L, \tilde{p}_m), d(p_L, \tilde{p}_l)]$ in Lemma 3 and in the “high discount region” $[d(p_L, \tilde{p}_l), C]$ in Lemma 4.

**Role of Stigma in Sustaining Opacity:** It is straightforward to check that no bank would like to deviate from the opaque policy of the central bank in terms of disclosing its own participation, or lack thereof, at the discount window. If revealing their own participation banks have to pay the stigma cost without getting any benefit (the best the bank can accomplish by disclosing participation in the discount window is to prevent examination and invest $K^*$, which is what happens by not revealing participation). Similarly, banks not borrowing from the discount window do not want
to reveal their lack of participation, otherwise they have a higher chance of suffering a run (depositors will always try to examine the bank’s portfolio once they know for sure that they hold private collateral instead of bonds).

4.2 Ending a Crisis with Transparency

When the central bank discloses information about the identity of banks participating at the discount window, the information acquisition strategy of depositors is conditional on this information. More specifically, when depositors know a bank has borrowed from the discount window, they never run on it, as they know the bank uses government bonds as collateral. Then $\sigma(\tilde{p}) = 0$ for all $\tilde{p}$, conditional on participation at the discount window, and $E^u(\pi) = H(K^*) - d(p, \tilde{p}) - \chi(p, \tilde{p})$. In contrast, when depositors know a bank has not participated at the discount window, they always run.
on it, as they know the bank has a private asset in its portfolio. Then \( \sigma(\tilde{p}) = 1 \) for all \( \tilde{p} \), conditional on no participation in the discount window, and \( E_{nw}(\pi|p, \tilde{p}) = L(p, \tilde{p}) \).

This implies that the borrower \( p^*_w \) that is indifferent about borrowing from the discount window, when the discount is given by \( \tilde{p} \), is determined by

\[
H(K^*) - d(p^*_w, \tilde{p}) - \chi(p, \tilde{p}) = L(p^*_w, \tilde{p}).
\]

Notice that this is the same condition that determines \( p'_w(\bar{p}_l) \) in Lemma 4. Still this does not imply that policies of opacity and transparency coincide at \( \bar{p}_l \). While the equilibrium with opacity is characterized by only some banks participating in the discount window, \( \bar{y} < 1 \), the equilibrium with transparency have all banks participating. The same discount induces more banks participating under transparency than under opacity because transparency eliminates the strategic incentive of banks to “hide”. This is, under opacity, a bank has incentives not to borrow from the lending facility so as not to pay a discount and still invest \( K^* \) because no bank is examined.

**Lemma 5** In the best equilibrium under transparency, all banks borrow from the discount window \( y(\tilde{p}) = 1 \) for \( \tilde{p} \in [\tilde{p}_T, \bar{p} + \bar{\eta}] \) such that \( \tilde{p}_T < \bar{p}_h \), and \( y(\tilde{p}) = 0 \) for \( \tilde{p} < \tilde{p}_T \).

Intuitively, when all banks participate under a policy of transparency, the alternative of not participating results in suffering a run for sure. This is in stark contrast to the opacity policy in which, if all banks participate, the alternative of not participating has only a slight chance \( \varepsilon \) of an information leakage and a run.

There is no parameter restriction that prevents \( \tilde{p}_T \) from being smaller than \( \bar{p}_l \). In such a case, for all discount levels \( \tilde{p}_T < \tilde{p} < \bar{p}_l \), there is full participation under transparency and no participation under opacity. The reason is that at \( \bar{p}_l \) depositors always run conditional on there being \( \bar{y} \) banks participating. If all banks were participating, depositors would not run and then the alternative gains from not participating under opacity are very large, inducing individuals to deviate, making this equilibrium unsustainable. This is not the case under transparency in which the alternative to not participating always leads to runs, turning the deviation unprofitable.
4.3 Opacity or Transparency?

Given the equilibrium strategies for each $\bar{p}$ under both opacity and transparency, we can compute the total production (or welfare in our setting) for each $\bar{p}$ under each disclosure policy. As banks keep the net gains from production, welfare is an aggregation of the payoffs of all banks with different quality $p$, given by

$$W(e_p) = \int_p [\mathbb{I}_w[H + B - pC] + (1 - \mathbb{I}_w)[\sigma L(p) + (1 - \sigma)((1 - \varepsilon)H + \varepsilon L(p))]] dF(p)$$

$$+ \int_p \mathbb{I}_w[q(pC - B) + (1 - q)(\phi pC - B)] dF(p)$$

$$+ \int_p [\mathbb{I}_w[q(H - \sigma \chi(p)) - (1 - q)\delta(1 - \phi)pC] + (1 - \mathbb{I}_w)H] dF(p),$$

where $\mathbb{I}_w$ is an indicator function that takes the value 1 if the bank participates at the discount window and 0 otherwise.

Taking integrals and rewriting the expression,

$$W(\bar{p}) = y(H + B - E^w(p)C) + (1 - y)[\sigma \hat{L} + (1 - \sigma)((1 - \varepsilon)H + \varepsilon \hat{L})]$$

$$+ y[q(E^w(p)C - B) + (1 - q)(\phi E^w(p)C - B)]$$

$$+ y[q(H - \sigma \hat{\chi}) - (1 - q)\delta(1 - \phi)E^w(p)C] + (1 - y)H,$$

where $H \equiv H(K^*)$, $\hat{L} \equiv \int_{p|\mathbb{I}_w} L(p, \bar{p}) dp$ and $\hat{\chi} \equiv \int_{p} \chi(p, \bar{p}) dp$.

The first two terms (the first line) represent the welfare of banks in the crisis period. A fraction $y$ of banks borrow from the discount window leading to a production of $H$ and exchanging private collateral for bonds at an average discount of $B - E^w(p)C$. A fraction $1 - y$ of banks do not participate and their investments lead to a production level that depends on whether they suffered a run or if there was an informational leak. This first line of the welfare function can be rewritten as

$$H - y(E^w(p)C - B) - (1 - y)(\varepsilon + \sigma(1 - \varepsilon))(H - \hat{L}).$$

The third term (the second line) represents the welfare for the government. From the fraction $y$ of banks borrowing from the discount window, a fraction $q$ has their private collateral seized, which delivers $E^w(p)C$ in expectation while a fraction $1 - q$ defaults and the private collateral has to be liquidated, recovering just $\phi E^w(p)C$. Still in both
cases the government has to repay $B$ for the bonds. This second line of the welfare function can be rewritten as

$$y(E^w(p)C - B) - y(1-q)(1-\phi)E^w(p)C.$$  

The last term (the third line) captures investments in the second period (no discount). Those banks that did not borrow from the discount window and those that did without their participation being revealed can borrow without triggering a run in the second period (then leading to production level $H$). Those banks that participated and their participation was revealed (because they were examined) can potentially suffer a run in the second period because of stigma (captured by $L$). Finally, the government has to repay (facing inefficiency costs $1-\phi$ and distortionary taxation costs $\delta$) the bonds that could not be covered by liquidating private assets in the previous period. The third line can then be rewritten as

$$H - y[q\sigma\hat{\chi} + (1-q)\delta(1-\phi)E^w(p)C].$$

Adding (and canceling) terms, total welfare is

$$W(\tilde{p}) = 2H - (1-y)(\varepsilon + \sigma(1-\varepsilon))(H - \hat{L}) - y[(1-q)(1+\delta)(1-\phi)E^w(p)C - q\sigma\hat{\chi}].$$

Since the unconstrained welfare is $2H$ in both periods, we can denote the distortion from the crisis as

$$Dist(\tilde{p}) = (1-y)(\varepsilon + \sigma(1-\varepsilon))(H - \hat{L}) + y[(1-q)(1+\delta)(1-\phi)E^w(p)C + q\sigma\hat{\chi}].$$  \hspace{1cm} (12)

Notice that all components depend on $\tilde{p}$ and the goal is to compare how distortion depends on different levels of $\tilde{p}$. The first component shows the distortion that comes from lower output from banks that did not borrow from the discount window, either due to information leaks or because they suffered a run. The second component shows the costs of the distortionary taxation that is needed to cover deposits from defaulting banks, and that cannot be covered by liquidating the private collateral. Finally, the third component shows the lower production in the second period that arises from stigma – banks who were discovered borrowing from the discount window and then were revealed to have collateral of relatively lower quality – being more likely to suffer a run and produce less in the second period.
Now, we can compare the welfare levels of distortions for each $\tilde{p}$ under opacity and under transparency, and follow the Ramsey approach of choosing the one that maximizes welfare. The next Proposition characterizes the optimal policy.

**Proposition 3** When the distortionary costs of taxation $(1 + \delta)$ and public liquidation $(1 - \phi)$ of private assets are large relative to the leakage probability $(\varepsilon)$,

$$\frac{(1 + \delta)(1 - \phi)}{\varepsilon} \geq \frac{(1 - p_L)K^*(qA - 1) + \gamma}{(1 - q)p_L C},$$

a discount rate of $\tilde{p}_m$ under opacity dominates transparency.

Figure 2 displays this proposition graphically, showing welfare under both disclosure policies (dashed red for transparency and solid black for opacity) for all discount rates, using a set of parameters for the illustration such that $d(p_L, \tilde{p}_T) < 0.4 < d(p_L, \tilde{p}_T)$.

Notice that the welfare implemented with a transparent intervention is the same for all discounts in the range of this illustration, as we have assumed the discount that
induces banks not to participate in a transparent lending facility is larger than 0.4. This level of welfare under transparency is naturally lower than the unconstrained first best because the government has to rely on distortionary taxation to meet the deposits of insolvent banks.

Under opacity, welfare depends on the discount. In the very low discount region all banks participate at the discount window, which implies that welfare is the same as that obtained under transparency. In the low discount region, some banks prefer to take advantage of pooling and not participate. In this region welfare increases with the discount as no depositor runs, and then the government needs to rely less on distortionary taxation. The sufficient condition for welfare to increase in this region (which is the one displayed in Proposition 3 and assumed in the figure) guarantees that the distortionary costs of interventions are larger than the costs of some banks facing a run because there is a leak, with probability $\varepsilon$.

In the intermediate discount region, only a fraction of banks ($\bar{y}$) borrow from the discount window but more and more depositors run as the discount increases. This reduces welfare because the level of distortionary taxation is lower than under transparency but more and more banks face runs and produce relatively less in the crisis period and face stigma that reduces their production in the second period. In this region, once the discount level is so large that the “stigma” effect dominates the “less distortion” effect, then welfare under opacity is lower than welfare under transparency. Finally, in the high discount region, the level of the discount is so high that there is no participation in equilibrium under opacity, with welfare reaching a no intervention level.

As is clear, the policy that maximizes welfare is imposing an average discount of $d(p_m)$ under opacity. At this discount, and without disclosure, the participation of banks in the discount window (which creates distortionary costs) is minimized, without triggering any runs.

Remarks on Securitization and Deposit Insurance In our setting, if banks were able to sign contracts that perfectly eliminate the idiosyncratic risk of individual portfolios then no depositor would have an incentive to acquire information about a bank’s asset. In other words, if idiosyncratic risk were eliminated by pooling all projects in the economy, there are no runs and no role for intervention. Indeed, with diversification not only would there be no crisis but all banks would be able to invest at the optimal scale.
Even though we have assumed that banks cannot diversify their individual portfolio risk, there could be in principle two institutions that allow for such diversification: securitization (sustained by private contracts) and deposit insurance (imposed by public regulation).

In the case of securitization, a bank can sign a contract at the beginning of the period, selling shares of its own asset and buying shares of the assets of other banks, eliminating the idiosyncratic risk as the value of its portfolio would be deterministic and equal to $p_L C$. This contract discourages depositors in the bank from acquiring information about its portfolio, which is now irrelevant for the probabilities of recovering the deposit. There are no runs and no crisis. These private contracts are difficult to sustain, however. Banks with high $\eta$ subsidize banks with low $\eta$ and may not have incentives to enter into these contracts as a way to signal their high $\eta$. Studying the sustainability of these contracts is interesting to understand the effects of securitization as a stabilizing innovation, but it is outside the scope of this paper.

Assuming securitization is not feasible, the government may have incentives to impose diversification in the form of deposit insurance. In the standard view of bank runs, under which they are triggered by a collective action problem, deposit insurance prevents panics and then it is not used in equilibrium. In our setting a run is not driven by lack of coordination among depositors but instead by individual incentives to investigate the bank’s portfolio, withdrawing the funds if it is found that the portfolio is of low value. The government can prevent the examination of a bank’s portfolio by forcing banks to pay a premium, ex-post, in case their assets are good and to receive insurance in case their assets are bad. As this cross-subsidization is self-financed inside the system, no taxation is used in equilibrium.

One interpretation of what happened during the recent financial crisis is that some banks (commercial) were under deposit insurance (and nothing happened to them). Some others (shadow) were using securitization. Securitization may be a fragile contract. In particular if adverse selection concerns are present among banks, these contracts may not be sustainable and the same problem analyzed in the paper develops. Again, this is a subject that requires more research.

**Remarks on Stress Tests** Stress tests are simulations based on a bank’s balance sheet to assess how the bank would fare against systemic distress scenarios. Even though introduced by Basel in 1996, they were mostly performed internally by banks until recent financial crises. In 2009 the US authorities conducted a macro stress test under
the framework of the Supervisory Capital Assessment Program (SCAP) for 19 bank holding companies (66% of total US banking sector). In 2010 a stress-test exercise was coordinated by the Committee of European Banking Supervisors (CEBS), the ECB and the European Commission for 91 EU banks (65% of total US banking sector). The level of disclosure of these tests was higher than standard supervisory tests performed afterwards. See details in ECB (2010).

Should individual stress test results be disclosed? When should stress tests be performed? Before or during crises? Our model sheds light on these issues. If regulators have information about each bank’s collateral quality, $p$, before depositors choose to run in the first period, they could prevent a run on banks with very high $p$ by disclosing each individual $p$. Regulators could implement a better outcome, however, just by disclosing which banks have a $p$ below a threshold that prevents runs for the rest of banks with higher $p$. Our model also suggests that disclosing each bank’s idiosyncratic component $\eta_i$ after lending facilities have been implemented is detrimental for output, as this information eliminates the critical role of stigma in preventing individual disclosure and making opacity feasible.

In short, even though our model highlights the benefits of government’s interventions secrecy, conditional on a crisis happening, it does not imply that stress tests are not optimal from a precautionary perspective. Indeed, performing stress tests and disclosing which banks do not pass a minimum requirement in collateral quality could be useful in preventing a crisis from happening.

**Comparison with the Bagehot’s Rule** The classic rule for a central bank to follow in a crisis is Walter Bagehot’s (1873) rule that the central bank should lend freely, at a high rate, and on good collateral. In the recent financial crisis, Ben Bernanke, Mervyn King and Mario Draghi, the respective heads of the Federal Reserve System, the Bank of England, and the European Central Bank, reported that they followed Bagehot’s advice; see Bernanke (2014a and 2014b), King (2010) and Draghi (2013). But, in fact, there was more to their responses to the crisis. All three central banks also engaged in anonymous or secret lending to banks.

Indeed, it is not obvious why Bagehot’s advice would work to restore confidence, or would be expected to work. It worked because of secrecy. Bagehot did not mention secrecy because “... a key feature of the British [banking] system, its in-built protective device for anonymity was overlooked [by Bagehot]” (Capie (2007), p. 313). Capie explains that in England geographically between the country banks and the Bank of
England was a ring of discount houses. Also, see Capie (2002). If a country bank needed money during a crisis it could borrow from its discount house, which in turn might borrow from the Bank of England. In this way, the identities of the actual end borrowers was not publicly known.\(^{14}\)

While the identities of discount window borrowers were not publicly known, the Bank of England knew those identities and, in particular, the identities of the non-bank borrowers in the crisis of 1866, the Overend-Gurney Panic. See Flandreau and Ugolini (2011) on the importance of non-bank borrowers (from the ”shadow banking” system). They argue that access to the Bank of England’s discount facility meant that these non-bank borrowers faced increased monitoring by the Central Bank.

5 Conclusions

A financial crisis occurs when some public information causes depositors to worry about the collateral backing their deposits such that they want to produce information about the bank’s portfolio. This is a bank run. Secret public lending during a financial crisis is important to avoid runs and portfolios examination. Bernanke (2009): “Releasing the names of [the borrowing] institutions in real-time, in the midst of the financial crisis, would have undermined the effectiveness of the emergency lending and the confidence of investors and borrowers ” (p. 1). Recreating confidence means raising the perceived average value of collateral in the economy so that it is not profitable to produce information about banks. The government can achieve this by exchanging bonds (or cash) for lower quality assets. Interestingly, there is no need to replace all the bad assets in the economy to discourage information acquisition. There is an informational externality in the use of opacity by pooling assets. Opacity was adopted not only by governments to deal with crises but also by private bank clearinghouses in the U.S. prior to the Federal Reserve System. Clearinghouse banks pooled their assets so that deposits were claims on all the assets not just one bank’s assets.

The Central Bank can choose the haircut optimally to determine the optimal amount of bond collateral that is put into the economy. The ability to adopt a policy of opacity

\(^{14}\)King (1936) provides more discussion on the industrial organization of British banking in the 19\(^{th}\) century. Also see Pressnell (1956). The Bank of England did not always get along with the discount houses, and there is a complicated history to their interaction. See, e.g., Flandreau and Ugolini (2011).
depends on the threat of stigma, to realign its opacity incentives with those of the banks. Stigma is costly for borrowers, as their participation reveals their holding of worse assets in expectation. Opacity does not try to avoid stigma but stigma is crucial to avoid transparency. Stigma is not observed in equilibrium, not because opacity but because participation on lending facilities imply paying a discount rate.

References


A Appendix

A.1 Proof Proposition 1

We first characterize $p^H$. The maximum profits that a bank with the highest collateral quality $p + \bar{\eta}$ expects when inducing a run, conditional on no other bank facing a run (that is, when $E^r(p) = p + \bar{\eta}$), are

$$E_r(\pi|p + \bar{\eta}, p + \bar{\eta}) = (p + \bar{\eta})K^*(qA - 1) - \gamma.$$  

while the expected profits when not inducing a run, conditional on no other bank facing a run, are

$$E_{nr}(\pi|p) = K_{nr}(p)(qA - 1)$$

where, from equation (5)

$$K_{nr}(p) = \min \left\{ K^*, \frac{\gamma}{(1 - q)(1 - p)} \right\}.$$  

There is always a $p$ large enough such that $K^* < \frac{\gamma}{(1 - q)(1 - p)}$ and $K^* < \bar{p}C$ such that $K_{nr}(p) = K^*$. Then no bank would rather face a run and have a positive probability of not being able to invest, given that it can invest at optimal scale without examination. For all $p > p^H$, all banks invest without runs, where $p^H$ is defined by $E_r(\pi|p^H + \bar{\eta}, p^H + \bar{\eta}) = E_{nr}(\pi|p^H)$, or

$$(p^H + \bar{\eta})K^*(qA - 1) - \gamma = \frac{\gamma}{(1 - q)(1 - p^H)}(qA - 1).$$

In this region, $p^* = p + \bar{\eta}$, which trivially increases one for one with $p$.

We now characterize $p^L$. The maximum expected profits that a bank with the lowest collateral quality $p - \bar{\eta}$ can obtain when avoiding a run when all other banks face runs (that is, $E_{nr}(p) = p - \bar{\eta}$) are

$$E_{nr}(\pi|p - \bar{\eta}) = K_{nr}(p - \bar{\eta})(qA - 1)$$

where, from equation (5)

$$K_{nr}(p - \bar{\eta}) = \min \left\{ K^*, \frac{\gamma}{(1 - q)(1 - (p - \bar{\eta}))}, (p - \bar{\eta})C \right\}.$$  

The expected profits when the bank induces a run, conditional on all other banks inducing a run, are

$$E_r(\pi|p - \bar{\eta}, p) = (p - \bar{\eta}) \left[ K^*(qA - 1) - \frac{\gamma}{p} \right].$$
Defining \( p^L \) by the point at which \( E_r(p^L, \eta) = E_{nr}(p^L, \eta) \), such that
\[
(p^L - \bar{\eta}) \left[ K^*(qA - 1) - \frac{\gamma}{p^L} \right] > \frac{\gamma}{(1-q)(1-(p^L-\bar{\eta}))}(qA - 1),
\]
then when \( \bar{p} < p_L \) all banks invest such that there is examination of their collaterals. In this region, \( p^* = \bar{p} - \eta \), which also trivially increases one for one with \( \bar{p} \).

In the best equilibrium and by monotonicity, in the intermediate region of \( \bar{p} \) the threshold \( p^* \) also increases with \( \bar{p} \). QED.

### A.2 Proof Proposition 2

Follows from comparing the condition for no information acquisition in the absence of intervention (equation 4) and in the presence of intervention (equation 8), and on the comparative statics in equation (8) with respect to \( y \).

### A.3 Proof Lemma 1

Assume first that the Central Bank chooses a discount given by \( \tilde{p} = p_L + \eta \). In this case there is no discount for the bank with the highest collateral quality, this is \( d(p_L + \eta, \tilde{p}) = 0 \) and there is indeed a subsidy for all other banks (the next analysis trivially applies then for all discounts such that \( \tilde{p} > p_L + \eta \)). Compare equations (10) and (11) for \( p = p_L + \eta \). It is optimal for the bank with the highest quality, \( p = p_L + \eta \), to borrow from the discount window, and then it is also the optimal strategy for all other banks to also borrow. This implies \( y = 1 \) and there is no stigma (i.e., \( H = 0 \)), confirming that this is indeed the best sustainable equilibrium.\(^{15}\) Hence, for \( \tilde{p} \geq p_L + \eta \), a fraction \( y(\tilde{p}) = 1 \) of banks participate, from equation (9) \( \alpha(\tilde{p}) = 0 \) and from equation (8) \( K(\tilde{p}) = K^* \).

For lower levels of \( \tilde{p} \), this is still an equilibrium as long as the bank with the highest collateral quality finds it optimal to participate, and then all other banks do as well. The critical level \( \tilde{p}_h \) is determined by the point at which the bank with the highest quality is indifferent between participating at the discount window or not,
\[
H(K^*) - d(p_L + \eta, \tilde{p}_h) = (1 - \varepsilon)H(K^*) + \varepsilon L(p_L + \eta, \tilde{p}_h),
\]

\(^{15}\)Notice that this is only one possible equilibrium. If everybody believes that some banks did not borrow from the discount window, \( \chi > 0 \), it may be indeed optimal for those banks not to borrow from the window. This shows how endogenous stigma may induce equilibrium multiplicity and may generate self-confirming collapses in the use of discount windows. Here we focus on the best equilibrium based on intervention, and show its limitations.

38
Using the definitions of $L(p, \tilde{p}), H(K(\tilde{p}))$ and $d(p, \tilde{p})$,
\[
\tilde{p}_h = (p_L + \overline{\eta}) - \frac{\varepsilon}{C} [(1 - p_L - \overline{\eta})K^* (qA - 1) + \gamma].
\]

QED.

A.4 Proof Lemma 2

Assume first the extreme case in which $\tilde{p} = \tilde{p}_h$. From the previous proposition, $y(\tilde{p}_h) = 1$ and $\sigma(\tilde{p}_h) = 0$. For $\tilde{p} = \tilde{p}_h - \varepsilon$ (from the definition of $\tilde{p}_h$),
\[
H(K^*) - d(p_L + \overline{\eta}, \tilde{p}_h - \varepsilon) < (1 - \varepsilon)H(K^*) + \varepsilon L(p_L + \overline{\eta}, \tilde{p}_h - \varepsilon),
\]
and then banks with collateral quality $p_L + \overline{\eta}$ strictly prefer to not participate at the discount window. This implies that $y(\tilde{p}_h - \varepsilon) \equiv Pr(p < p_w^*(\tilde{p}_h - \varepsilon)) < 1$, where $p_w^*(\tilde{p}_h - \varepsilon)$ is given by the indifference condition
\[
H(K^*) - d(p_w^*, \tilde{p}_h - \varepsilon) = (1 - \varepsilon)H(K^*) + \varepsilon L(p_w^*, \tilde{p}_h - \varepsilon),
\]
or
\[
d(p_w^*, \tilde{p}_h - \varepsilon) = \varepsilon [H(K^*) - L(p_w^*, \tilde{p}_h - \varepsilon)],
\]
where $p_w^*$ declines monotonically as we reduce $\tilde{p}$. Notice that this construction relies on the conjecture that $\sigma(\tilde{p}_h - \varepsilon) = 0$, but for relatively low $\varepsilon$ this is the case as long as $y(\tilde{p}_h - \varepsilon)$ and $E^{nw}(p|\tilde{p}_h - \varepsilon)$ are such that
\[
K^* < \frac{\gamma}{(1 - q)(1 - y)(1 - E^{nw}(p))}.
\]
Define by $p_w^*$ the threshold such that $\gamma(p_w^*)$ is the fraction of banks borrowing from the discount window and $E^{nw}(p|p_w^*)$ is the expected quality of the non-participating banks’ collateral, such that depositors are indifferent between running or not when the bank invests $K^*$, i.e.,
\[
K^* = \frac{\gamma}{(1 - q)(1 - y)(1 - E^{nw}(p))}.
\]
The bank with the marginal collateral quality $p_w^* (\tilde{p}_m)$ is determined by
\[
H(K^*) - d(p_w^*, \tilde{p}_m) = (1 - \varepsilon)H(K^*) + \varepsilon L(p_w^*, \tilde{p}_m),
\]
such that
\[
\gamma = Pr(p < p_w^* (\tilde{p}_m)) \quad \text{and} \quad E^{nw}(p) = E(p|p > p_w^* (\tilde{p}_m)).
\]
Finally, the threshold $p^*_w$ is well-defined, as both $y$ and $E^{nw}(p)$ monotonically increase in $p^*_w$, which monotonically decreases in $\bar{p}$. QED.

### A.5 Proof Lemma 3

Assume first the extreme case where $\bar{p} = \bar{p}_m$. From the previous lemma, $y(\bar{p}_m) = \bar{y}$ and $\sigma(\bar{p}_m) = 0$. For $\bar{p} = \bar{p}_m - \epsilon$, the bank that is indifferent about borrowing from the discount window has slightly lower quality, $p^*_w(\bar{p}_m - \epsilon) < p^*_w(\bar{p}_m)$. Then $y(\bar{p}_m - \epsilon) < \bar{y}$ and $E^{nw}(p|\bar{p}_m - \epsilon) < E^{nw}(p)$, and there are incentives to run if investment is $K^*$, as there are relatively few participants at the discount window (low $y$) and the collateral of those not participating at the discount windows are worse in expectation (low $E^{nw}(p)$). Formally,

$$K^* > \frac{\gamma}{(1-q)(1-y(\bar{p}_m - \epsilon))(1 - E^{nw}(p|\bar{p}_m - \epsilon))}.$$ 

One possibility for banks to prevent runs, $\sigma(\bar{p}) = 0$, is to scale back the investment in the project to $K(\bar{p}_m) < K^*$, to avoid information acquisition. The size of the investment $K$, however, also determines $y$, as $p^*_w(\bar{p}_m - \epsilon)$ is pinned down by the condition

$$d(p^*_w, \bar{p}_m - \epsilon) = \epsilon [H(K(p^*_w)) - L(p^*_w, \bar{p}_m - \epsilon)].$$ 

A lower $K$ relaxes the constraint and reduces the incentive to run, but at the same time reduces $p^*_w$ for a given $\bar{p}$, increasing the incentive to run. Intuitively, for a given discount, a reduction in the gains of borrowing from the discount window (from lower $H(K)$) reduces the quality of the marginal bank which is indifferent between borrowing or not, i.e., reducing $p^*_w$ further.

If $H(K(p^*_w))$ declines faster than $L(p^*_w)$, then no participant will go to the discount window if, at the lowest possible $p$, which is $p_L - \bar{\eta}$,

$$H(K(p_L - \bar{\eta})) - L(p_L - \bar{\eta}, p_L) < 0$$

which we have assumed in the definition of a crisis ($p_L$ is low enough such that all banks would rather face examination than restricting their investments to avoid runs). In words, it is not in the best interests of banks to discourage runs by reducing the size of their investments in the project, which is in contrast to what happens in the absence of intervention. Our result here comes from the endogenous participation of banks at the discount window. By reducing $K$, the effect of a lower $y$ in inducing information acquisition is stronger than the effect of a lower $K$ in discouraging information acquisition, thus increasing on net the incentives for depositors to examine a bank’s collateral as $K$ declines.

Under these conditions, the equilibrium should involve either the discount window.
sustaining an investment of $K^*$ (when a fraction $\bar{y}$ of banks borrows from the discount window) or no participation in the discount window at all, which replicates the allocation without intervention. To maintain the fraction $\bar{y}$ constant in this region as $\bar{\rho}$ declines, the marginal bank with collateral quality $p^*_w(\bar{\rho}_m)$ should be indifferent between borrowing from the discount window or not. This is achievable only if depositors choose to run with some probability and examine the portfolio of banks as $\bar{\rho}$ declines, as this increases the incentives to have bonds in the portfolio.

As the fraction of banks participating at the discount windows is constant at $\bar{y}$, depositors are indeed indifferent between running or not, and $\sigma(\bar{\rho}) > 0$ is an equilibrium. To determine $\sigma(\bar{\rho})$ in equilibrium we next discuss the determination of endogenous stigma.

With positive information acquisition ($\sigma(\bar{\rho}) > 0$) there is stigma when the depositor discovers a bank’s participation at the discount window. The reason there is stigma is that those banks borrowing from the discount window are the ones with relatively low collateral quality (relatively low $\eta_i$). Once a bank is stigmatized, it may face withdrawals during normal times in the second period. To be more precise about the endogeneity of stigma, once back in normal times, the bank will face a run when investing at the optimal scale of production if

$$K^* > \frac{\gamma}{(1 - q)(1 - E^w(p))},$$

and the bank will not suffer a run in the second period based on an indifference condition that pins down $p^*_w$ in the second period where

$$E_r(\pi|p^*_w, E^r(p|p < p^*_w)) = E_r(\pi|E^{nr}(p|p < p^*_w)).$$

We denote by $K^w(p, \bar{\rho})$ the investment size that a bank with a collateral of quality $p$ can obtain in the second period conditional on it having been revealed that the bank borrowed from the discount window in the first period.

Then, stigma is given by

$$\chi(p, \bar{\rho}) = [K^* - K^w(p, \bar{\rho})](qA - 1),$$

where $\chi$ is an increasing function of the discount (a decreasing function of $\bar{\rho}$). As the discount increases, $p^*_w$ decreases, $y(p^*_w)$ decreases and $E^w(p)$ decreases. This leads to a decline in $K^w$ and then an increase in stigma from going to the discount window.

Given $\bar{y}$, to maintain the investment size $K^*$ without triggering information, the indifference of the marginal bank $p^*_w$ pins down the probability the depositor runs. This is $E^{nrw}(\pi|p^*_w) = E^w(\pi|p^*_w)$, which implies

$$\sigma L(p^*_w, \bar{\rho}) + (1 - \sigma)[(1 - \varepsilon)H(K^*) + \varepsilon L(p^*_w, \bar{\rho})] = [H(K^*) - d(p^*_w, \bar{\rho})] - \sigma \chi(p^*_w, \bar{\rho})$$
and then
\[
\sigma(\tilde{p}) = \frac{d(\tilde{p}^*, \tilde{p}) - \varepsilon \left[H(K^*) - L(\tilde{p}^*, \tilde{p})\right]}{(1 - \varepsilon) \left[H(K^*) - L(\tilde{p}^*, \tilde{p})\right] - \chi(\tilde{p}^*, \tilde{p})}.
\] (13)

Finally, depositors randomize between running and not running given that the bank is investing \(K^*\) in the project, and a bank with collateral quality \(\tilde{p}^*_w\) is indifferent between borrowing from the discount window or not. QED.

### A.6 Proof Lemma 4

From equation (13), \(\sigma(\tilde{p}^*_w) = 1\) for \(d(\tilde{p}^*_w, \tilde{p}) \geq H(K^*) - L(\tilde{p}^*_w, \tilde{p}) - \chi(\tilde{p}^*_w, \tilde{p})\). Define \(\tilde{p}_T\) to be the discount that solves this condition with equality. Evaluating \(E_{nw}(\pi)\) and \(E(\pi)\) at \(\sigma(\tilde{p}) = 1\) given a project of size \(K^*\) and at the threshold \(\tilde{p}^*_w\) a bank with collateral \(\tilde{p}^*_w\) goes to the discount window whenever
\[
H(K^*) - d(\tilde{p}^*_w, \tilde{p}) - \chi(\tilde{p}^*_w, \tilde{p}) > L(\tilde{p}^*_w, \tilde{p})
\]
which is never the case in this region by the previous condition. Then \(y(\tilde{p}) = 0\) and then it is indeed optimal for depositors to run, that is \(\sigma(\tilde{p}) = 1\). QED.

### A.7 Proof Lemma 5

Define \(\tilde{p}_T\) by
\[
H(K^*) - d(p_L + \bar{\eta}, \tilde{p}_T) = L(p_L + \bar{\eta}, \tilde{p}_T)
\]
. For all \(\tilde{p} > \tilde{p}_T\)
\[
H(K^*) - d(p_L + \bar{\eta}, \tilde{p}) > L(p_L + \bar{\eta}, \tilde{p})
\]
and the bank with collateral of quality \(p_L + \bar{\eta}\) strictly prefers to borrow from the discount window. Notice that as all banks participate, \(\chi(p, \tilde{p}_T) = 0\) for all \(p\).

Recall the condition that pins down \(\tilde{p}_h\) in Lemma 1 is
\[
H(K^*) - d(p_L + \bar{\eta}, \tilde{p}_h) = (1 - \varepsilon)H(K^*) + \varepsilon L(p_L + \bar{\eta}, \tilde{p}_h).
\]
Then, as the right-hand side is larger than in the condition above that pins down \(\tilde{p}_T\), the discount has to be smaller and \(\tilde{p}_T < \tilde{p}_h\). QED.

### A.8 Proof Proposition 3

Here we compare the distortions for different levels of discount \(\tilde{p}\) (in the different regions that we characterized in the previous section) for different disclosure policies.
We consider first the parametric case for which $\tilde{p}_T < \tilde{p}_h$.

Under transparency, all banks participate when $\tilde{p} \geq \tilde{p}_T$. This implies that $y = 1$, $\chi = 0$ and $E^w(p) = p_L$. Distortions in this range are then

$$Dist(\tilde{p}|Tr) = (1 + \delta)(1 - q)(1 - \phi)p_L C.$$ 

In contrast, for all $\tilde{p} < \tilde{p}_T$, the discount window is not sustainable and distortions are the same as without intervention, $H - L \equiv (1 - p_L)K^*(qA - 1) + \gamma$.

Under opacity, distortions also depend on the equilibrium regions. In the “very low” discount region all banks participate, then $y = 1$ and $\chi = 0$, and distortions are the same as under transparency (in the region $\tilde{p} \geq \tilde{p}_T$ above).

In the “low” discount region, $y < 1$ but $\sigma = 0$, then from equation (12)

$$Dist(\tilde{p}|Op) = (1 - y)(H - \tilde{L})\varepsilon + y(1 + \delta)(1 - q)(1 - \phi)E^w(p|Op) C.$$ 

Since $yE^w(p|Op) < p_L$ and $H - L > H - \tilde{L}$, the sufficient condition for the distortion under opacity is lower than the distortion under transparency is

$$(1 + \delta)(1 - q)(1 - \phi)p_L C > \varepsilon(H - L)$$

which is the condition in the Proposition. In words, even though a fraction $\varepsilon$ of non-participating banks produce less, there are fewer banks that participate and need to be covered by distortionary taxation in case they default.

In the “intermediate” discount region, $y = \bar{y}$, but $\sigma > 0$ and increases with the discount. From equation (12) it is clear that in this region the distortion increases with $\sigma$ and then with the discount. While the fraction of banks participating is fixed, there are more runs and then more banks not participating end up producing less, both in the first period (less deposits) and the second period (more stigma).

Finally, in the “high” discount region, the discount window is not sustainable under opacity, so distortions are the same as without intervention, $H - L$.

Summarizing, as the discount increases, distortions under opacity are fixed in the very low discount region, decrease in the low discount region, increase in the intermediate discount region and reach the maximum in the high discount region. In contrast, distortions under transparency are fixed whenever the discount window is sustainable, as either all banks participate or neither does. This implies the optimal discount is $\tilde{p}_m$ under opacity.

Consider finally the parametric case for which $\tilde{p}_T \geq \tilde{p}_h$. In this case the discount window under opacity collapses at lower discount levels than under transparency. Still the optimal policy is a discount of $\tilde{p}_m$ under opacity because $\tilde{p}_T < \tilde{p}_h$, as shown in Lemma 5. QED.