Information Spillovers in Debt Auctions*

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Abstract

We uncover an informational channel of bond yield contagion when governments sell debt in primary markets using discriminatory auctions. If investors can acquire information about a country’s fundamental default risk, two informational regimes may coexist: an informed regime with high yields and high volatility, and a Pareto-dominant uninformed regime with low yields and low volatility. Small increases in one country’s default risk can trigger information acquisition in the other country even if there are no fundamental linkages. The presence of a secondary market strengthens information acquisition incentives, increases yields, and magnifies contagion.

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1 Introduction

This paper studies the determinants of sovereign debt spreads in countries that auction their government bonds in primary markets to a common pool of global investors. We are particularly interested in two empirical regularities. First, sovereign debt crises (sharp increases in sovereign debt yields) are often contagious, in that they spill over to other seemingly unrelated countries. Examples include the Russian crisis of 1998, the Mexican crisis of 1994, the Latin American crises of the 1980s, and the recent Eurozone crisis. Second, crises typically lead to a retrenchment of capital flows and increased market segmentation (see, for example, Milesi-Ferretti et al. (2011) and Lane (2012)). If cross-country diversification improves risk-sharing, then increased market segmentation further impairs sovereign bond markets during crises. We present a theory of government bond pricing that is consistent with the evidence on spillovers across seemingly unrelated countries and increased market segmentation.

Our theory differs from existing work in three ways. First, since primary market prices are the ones that directly determine government revenue, we focus on the contagion of primary rather than secondary market prices. Second, we identify the sovereign price effect of the interaction between primary and secondary markets. Finally, we establish a new informational channel of contagion, which can impact not only on the level of prices but also their volatility.¹

In our model there is a group of wealth-constrained risk-averse investors who buy government bonds in multiple countries and can acquire information about each country’s fundamental default risk at a cost. In equilibrium, shocks to default risk trigger information acquisition by some investors, and the resulting information asymmetry generates a sharp increase in average bond yields and volatility. Since investors prefer to invest in countries in which they are informed and other investors are uninformed, moreover, markets become endogenously segmented. This retrenchment hampers cross-country diversification and further reduces bond prices.

In contrast to much of the existing literature, which focuses on competitive secondary markets, but in line with current practice in many countries, we model pri-

¹For a literature that links a country’s fundamentals to sovereign spreads see Reinhart, Rogoff, and Savastano (2003), Tomz and Wright (2007), Broner, Martin, and Ventura (2010), Tomz and Wright (2013) and Aguiar and Amador (2014)). For a quantitative literature that accounts the effect of default on sovereign spreads see Aguiar and Gopinath (2006), Arellano (2008), Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009). Aguiar et al. (2016) surveys this literature.
mary markets for government debt explicitly as discriminatory price auctions. Following Cole, Neuhann, and Ordonez (2018), who provide a comprehensive analysis of the effects of asymmetric information on a single country’s bond price under different auction protocols, we study the “Walrasian” limit where bidders act as price-takers. This leads to a tractable framework for studying information acquisition incentives without resorting to noise traders or other shocks. In discriminatory auctions, investors submit multiple bids consisting of a price and a commitment to buy a number of bonds at that price. The government executes bids at the bid price in descending order of prices until it raises the required revenue. This leads to a lowest-accepted, or marginal, price, with all bids at prices above the marginal price also accepted. Informed investors know the state (the default probability of the country) and thus the marginal price, while uninformed investors do not know which of the state-contingent marginal prices will obtain. As a result, the uninformed face price risk: bids made at high prices will be accepted even if the bond is risky and the marginal price turns out to be low. This source of risk is specific to the auction protocol, and does not obtain in competitive markets with asymmetric information.

In equilibrium, the severity of the price risk (and thus the value of information) can be measured by the difference in bond prices across states. This price difference is partly exogenous, due to differences in fundamental default risk, and partly endogenous because prices reflect fundamentals more closely when many investors are informed. Hence price risk increases with the number of informed investors. This has several implications. First, uninformed investors do not participate in the auction if price risk is too high. This forces the remaining bidders to buy larger per-capita quantities, which leads to lower prices and a further increase in price risk. Second, there is a strategic complementarity in information acquisition: since price risk is increasing in the fraction of informed investors, it is better to be informed when many other investors are.

This complementarity leads to multiple equilibria when fundamental volatility is moderate: there exists both an uninformed equilibrium without any information acquisition and an informed equilibrium in which information is asymmetric because a fraction of investors acquire information. The informed equilibrium has higher average yields (because fewer investors participate) and higher volatility (because prices more closely reflect fundamentals) than the uninformed equilibrium. Import-

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2Brenner, Galai, and Sade (2009) find that the majority of their sample of 83 countries, including 83% of OECD countries, use discriminatory auctions to sell bonds in primary markets.
tantly, both the government and investors prefer the uninformed equilibrium. For investors, this is because information acquisition amounts to rent-seeking at the expense of other investors, with rents exactly offset by the cost of acquiring information; for the government, it is because the price discount it must offer to risk-averse investors in bad states more than outweighs the premium it can charge in good states. This multiplicity of welfare-ranked equilibria is an attractive feature of a theory of crises because it implies that small shocks to fundamental default risk may trigger a sharp and socially inefficient increase in yields.

The same forces give rise to cross-country spillovers in prices, both directly through portfolio re-allocation and indirectly through information acquisition incentives. To the extent that country fundamentals are not perfectly correlated (indeed, we assume that they are uncorrelated), investors optimally diversify default risk by holding bonds in both countries. If investors are prudent \( u''(c) > 0 \), as is the case for CRRA utility functions), shocks to default risk in one country increase investors’ background risk of holding sovereign debt, reducing risk appetite and increasing required returns in all countries. Information acquisition amplifies this mechanism: since prices are more volatile when more investors are informed, background risk is increasing in the fraction of informed investors, and so required returns increase more sharply in an informed equilibrium than in an uninformed equilibrium. Since the value of information is increasing in price risk, moreover, information in one country increases the incentives to acquire information in the other country as well. Hence there is contagion in information regimes that amplifies direct price contagion.

For an individual investor, the incremental benefit of becoming informed in a first country is higher than the value of becoming informed in a second country. Moreover, uninformed investors tilt their portfolios towards countries with few informed investors. Hence information acquisition leads to specialization and, ultimately, segmentation. This implies inefficiently low cross-country diversification, and further increases yields. Our model thus features two complementary amplification mechanisms for a small initial shock to default risk: (i) an increase in information acquisition that increases spreads and volatility, and (ii) endogenous market segmentation that hampers risk-sharing and further boosts yields. These features allow our

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3This result is driven in part by the fact that information does not affect real production or allocation decisions in our setting. Accordingly, there are no real benefits of information. More generally, of course, information may have benefits in terms of disciplining governments or allocating funds to productive investment opportunities. We assume away those benefits to focus on the forces behind information acquisition, but they would make the informed equilibrium more desirable.
Given that the macroeconomic literature typically focuses on bond prices determined in secondary markets, we extend the model to include a competitive secondary market and study its impact on primary market prices. The dual market structure allows us to separate information acquisition incentives from risk-sharing. To do so, we assume that auction prices are publicly observed, allowing uninformed investors to back out the true default probability from realized marginal prices. Hence secondary market trading takes place under symmetric information, and the only remaining motive for trade is the sharing of default risk. Importantly, investors’ risk exposure prior to the secondary market is determined by their bids at auction. This means that expected secondary market prices affect bids and information acquisition in the primary market.

In the equilibrium of the model with secondary markets, informed investors buy a large number of bonds at auction and then sell a fraction of their portfolio in the secondary market at pure arbitrage profit (that is, prices are strictly higher in the secondary market than at auction). Price risk discourages uninformed investors from taking advantage of this arbitrage: because information is asymmetric only in the auction, uninformed investors wait for the secondary market in order to avoid being adversely selected at auction. Since government revenue is affected only by auction prices, this is costly to the government: primary market prices are lower because fewer investors participate in the auction. Moreover, secondary markets also raise information acquisition incentives. This is because the option to resell allows informed investors to exploit their information advantage in the auction without having to tilt their final portfolios toward the asset about which they are informed. That is, information rents are now risk-free pure arbitrage profits rather than bundled with inefficient cross-country diversification. This effect further lowers primary market prices. Hence we identify an adverse feedback from secondary market trading to primary market prices that should be weighed against other potential benefits of secondary markets that are absent in our model (such as increased liquidity.)

Previous work has also explored contagion in financial and sovereign debt markets, but not from the perspective of endogenous asymmetric information and the interplay between primary and secondary markets. Even though the most common view of contagion relies on real linkages, such as trade in goods or financial assets, that may transmit negative shocks from one country to the next, it is often difficult to em-
pirically identify linkages that are plausibly powerful enough to induce the degree of contagion observed in many debt crisis episodes. This lead to new set of explanations that rely on self-fulfilling debt crises either through feedback effects as in Calvo (1988) and Lorenzoni and Werning (2013) or rollover problems, as in Cole and Kehoe (2000), Aguiar et al. (2015), and Bocola and Dovis (2015). In this paper we explore another novel form of linkages between countries that stem not from country fundamentals (the supply side) but rather from the investment and information acquisition decisions of common investors (the demand side). Closer to our insight, Van Nieuwerburgh and Veldkamp (2009) also use a model of information acquisition to study home bias and segmentation in financial markets, but without focusing on primary markets they find that information acquisition is a strategic substitute. The endogenous price risk generated by the auction protocol leads to a strategic complementarity in our model, and thus equilibrium multiplicity and contagion in information regimes.

Our contagion result relies only on investor prudence and the fact that there is a common pool of investors for all countries. The magnification of contagion through segmentation relies on the possibility of endogenously asymmetric information. Hence it does not rely on changes in investors’ wealth (as in Kyle and Xiong (2001) or Goldstein and Pauzner (2004)), borrowing constraints (as in Yuan (2005)) or short-selling constraints (as in Calvo and Mendoza (1999)).

We describe the model and equilibrium concept in Section 2. Section 3 studies the equilibrium without secondary markets and Section 4 the full model with secondary markets. Section 5 concludes. All proofs are in Appendix A.

2 Model

2.1 Environment

We consider a two-country variant of the Cole, Neuhann, and Ordonez (2018) model of sovereign debt auctions. Specifically, we study a two-period economy featuring a measure one of ex-ante identical risk-averse investors and two governments indexed by subscript $j$. Governments are modeled mechanically: each needs to raise $D_j$ units

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4Broner, Gelos, and Reinhart (2004) provide empirical evidence about the importance of portfolio effects for contagion, while Lizarazo (2013) and Broner, Lorenzoni, and Schmukler (2013) discuss the importance of risk aversion for explaining the behavior of sovereign spreads.

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of the numeraire good in period one by auctioning a bond that promises repayment in period two. The volume of bonds that must be sold to raise the required revenue $D_j$ is therefore pinned down by investor bids. This allows us to characterize the impact of funding needs on the government’s cost of financing.

Bonds are risky because they deliver a unit of the numeraire only if the issuing government does not default. If the government defaults, then investors cannot recover any of their investment. Country $j$’s probability of default, $\kappa_j(\theta_j)$, is a random variable that depends only on the realization of a country-specific fundamental $\theta_j \in \{b, g\}$. Without loss of generality, $\kappa_j(g) < \kappa_j(b)$ for all $j$. The ex-ante probabilities of these states are $f_j(g)$ and $f_j(b)$ respectively, with $f_j(g) + f_j(b) = 1$, and the unconditional default probability of country $j$ is

$$\kappa^u_j = f_j(b)\kappa_j(b) + f_j(g)\kappa_j(g).$$

Since the default probability determines the expected repayment of the bond, we refer to the realization $\theta_j$ as a quality shock. We say that the bond is of good quality if $\theta_j = g$ and of bad quality if $\theta_j = b$.\(^5\) To focus on information-based contagion in the absence of real linkages, we assume that default risk is independent across countries.

Governments sell bonds in an auction in period one. If the amount of money raised at auction falls short of $D_j$ then we assume that the government simply defaults on any bonds that it sold in period one.\(^6\) In addition to the primary market auction, investors trade bonds after the auction in a competitive secondary market. We provide a detailed description of the auction protocol and the secondary market structure below.

The objective of investors is to maximize expected second period utility as measured by a strictly increasing and strictly concave flow utility function $U$ over their second period consumption that satisfies the Inada conditions. Each investor has wealth $W$ in period one and can either invest in a risk-free bond (storage) or buy risky government bonds in the auction.

To transparently characterize how changes in default risk in one country affect bond yields in the other country, we assume that asset markets are partially seg-

\(^5\)It is straightforward to think of the $\kappa_j(\theta_j)$ realizations being themselves governed by an aggregate public shock $\nu_j$ at the beginning of the period. Then information would not be about $\kappa_j$ but about $\nu_j$, but the analysis that follows remains the same.

\(^6\)We can also interpret this to mean that they defaulted on the bonds coming due in period one.
mented. At the beginning of period one, each investor is split into two traders, with each trader tasked with trading in one specific country. Investor $i$ allocates trading capital $\omega_j^i$ to trader $j$, the trader in charge of investing in country $j$, where $\sum_j \omega_j^i = W$. Trader $j$ then either buys country $j$’s risky bond (at auction or in the secondary market) or invests in risk-free storage. When the secondary market closes, traders’ bond holdings are pooled to form the investor’s final portfolio, and preferences are over the final portfolio. When there is no risk of confusion, we refer to traders associated with a particular investor simply as the investor.

In each auction, there are two types of traders: those who are informed about $\theta_j$ and those who are not. Hence there are four types of investors in the economy: those who are uninformed in both countries, those who are informed in the first country but not in the second, those who are informed in the second country but not in the first, and those who are informed in both countries. We denote the type of investor by $i \in \{UU, IU, UI, II\}$, and use $n_i$ to denote the associated share of investors, with $\sum_i n_i = 1$. Because investors are otherwise identical (save for their information), we can refer to a representative investor of type $i$. Consistent with our mechanical modeling of the supply of bonds, we assume that governments do not observe $\theta$. We assume that markets are segmented in that each trader cannot condition his bids on the information of the other trader who is part of the same investor family (neither in the primary nor in the secondary market). Put differently, investors cannot share information obtained in different countries prior to the close of all trading. This assumption reduces the number of equilibrium prices from 16 to 8 without affecting the basic mechanisms.

Information about $\theta_j$ can be acquired at a cost. If $a_j \in \{0, 1\}$ denotes the decision to acquire information in country $j$, then the type of an investor with information acquisition profile $\{a_1, a_2\}$ is

$$\iota(a_1, a_2) = \begin{cases} 
II & \text{if } \{a_1, a_2\} = \{1, 1\} \\
IU & \text{if } \{a_1, a_2\} = \{1, 0\} \\
UI & \text{if } \{a_1, a_2\} = \{0, 1\} \\
UU & \text{if } \{a_1, a_2\} = \{0, 0\}
\end{cases}$$

The cost of information $C(a_1, a_2)$ is the same for all investors, and we restrict attention
to the flexible functional form

\[ C(a_1, a_2) = a_1 K_1 + a_2 K_2 + a_1 a_2 \times K_{12}, \quad K_j \geq 0, K_{12} \geq -\max\{K_1, K_2\}, \]

where \( K_{12} \) captures economies of scale in information acquisition.

### 2.2 Auction

We focus on discriminatory (pay-your-bid) auctions, which are the primary market protocol used by the majority of developed economies. Each trader in country \( j \) can submit multiple bids at auction. A bid is a price and quantity pair \( \{P, B\} \) representing a commitment to purchase \( B \) units of the bond at price \( P \) should the government decide to execute the bid. Bids are executed at the bid price. The government treats each bid independently, sorts all bids from the highest to the lowest bid price, and accepts all bids in descending order and stops at the highest price that generates revenue \( D_j \). We refer to this lowest accepted price as the marginal price \( \bar{P}_j \), and to bids above the marginal price as bids in the money.\(^7\) If the marginal price does not exactly clear the auction (i.e. the auction generates revenue strictly greater than \( D_j \) at the marginal price) then only a fraction of the bids at the marginal price are accepted and bonds are rationed pro-rata among investors. A unit of the bond is a claim to a real unit of the numeraire good in period two. This claim pays either 1 or 0, hence the range of prices is \( P \in [0, 1] \).

Investors lack commitment in two important dimensions. First, they cannot commit to honor any inter-temporal contracts. We will take this to mean that they cannot borrow, and that they cannot submit short sale (negative) bids at the auction. Second, they cannot commit to credibly share their information about \( \theta \), which we take to mean that there is no market for information. Investor \( i \)'s strategy in country \( j \) is a bid function \( B^i_j(P|\theta_j) \) satisfying the short-sale constraint

\[ B^i_j(P|\theta_j) \geq 0. \tag{1} \]

and the measurability constraint

\[ B^i_j(P|g) = B^i_j(P|b) \quad \text{if} \quad \alpha^i_j = 0, \tag{2} \]

\(^7\)Our auction protocol ensures that this is the highest possible marginal price given the bids.
which imposes that bids do not depend on information that has not been acquired.

**Remark 1 (Government Revenue)** If \( \bar{P}_j(\theta_j) \) is the marginal price in state \( \theta_j \), then the amount that government \( j \) raises in state \( \theta_j \) is

\[
\int_{P_j(\theta_j)}^{1} \sum_i n_i B_j^i(P|\theta_j) P dP
\]

This amount is declining in the marginal price \( \bar{P}_j(\theta_j) \) because the prices at which bids are executed are fixed while the number of accepted bids is decreasing in the marginal price. This implies that a government with higher funding needs will face a lower marginal price.

### 2.2.1 Bidding and Inference

Investors have rational expectations regarding the equilibrium set of marginal prices, the states associated with each marginal price, and their probabilities. Specifically, uninformed investors correctly forecast the set of possible prices, but are uninformed about the realized marginal price because they do not know the realized state.

Once the auction has taken place, uninformed investors can draw inferences with respect to the state of the world using the realized marginal price (informed investors already know \( \theta \) and thus do not need to rely on inference from the price). In our setting, this implies that uninformed investors will be able to perfectly infer the state of the world in country \( j \) if \( \bar{P}_j(g) \neq \bar{P}_j(b) \), but remain uninformed if \( \bar{P}_j(g) = \bar{P}_j(b) \). This ex-post information is of limited use in the auction because all investors must commit to their bids prior to observing the marginal price\(^8\). However, investors will be able to use the inferred information in the secondary market (as we describe below).

Since the auction is discriminatory, it is a dominant strategy to bid only at the possible marginal prices \( \bar{P}(\theta) \). Any bid slightly above \( \bar{P}(\theta) \) is accepted in the same states of the world but costs more. We therefore take as given that investors do not bid at non-marginal prices. Given this restriction, we no longer need to differentiate marginal and non-marginal prices, and we drop the notation \( \bar{P} \) and just refer to \( P \), the bid and realized prices. We also use a starker specification for bids.

\(^8\)This differentiates our model from rational expectation models in the tradition of (Grossman and Stiglitz 1980)
**Definition 1** The marginal price in country $j$ and state $(\theta_j) \in \{g, b\}$, is denoted by $P_j(\theta_j)$, and the set of marginal prices by $\mathcal{P}_j$. An action for an investor of type $i$ is a pair of functions \(\{B^i_j(\theta_j|\theta_j)\}_{j=1,2}\), which denote the number of units bid in country $j$ at marginal price $P_j(\theta_j)$ when the realized state in country $j$ is $\hat{\theta}_j$. If the investor is not informed in country $j$, then the bid function must satisfy the measurability constraint $B^i_j(\theta_j|g) = B^i_j(\theta_j|b)$.

By this definition, bids in country $j$ are conditional on the state in country $j$ only. This is due to the assumption that investors cannot share information across countries. Let $B^i_{R,j}(\theta_j)$ and $B^i_{RF,j}(\theta_j)$, respectively, denote the number of risky bonds and risk-free assets acquired by investor $i$ in country $j$ and state $\theta_j$. An informed investor bids only at the true marginal price (the one corresponding to the realized state); an uninformed investor bids only at possible marginal prices, but may bid at prices other than the true marginal price. Since the government accepts all bids above the true marginal price, risky purchases are

\[
B^i_{R,j}(\theta_j) = \begin{cases} 
B^i_j(\theta_j|\theta_j) & \text{if } a^i_j = 1 \\
\sum_{\theta': P_j(\theta') \geq P_j(\theta_j)} B^i_j(\theta'|\theta_j) & \text{if } a^i_j = 0.
\end{cases}
\]

Risk-free purchases are:

\[
B^i_{RF,j}(\theta_j) = \begin{cases} 
\omega^i_j - P_j(\theta)B^i_j(\theta_j|\theta_j) & \text{if } a^i_j = 1 \\
\omega^i_j - \sum_{\theta': P_j(\theta') \geq P_j(\theta_j)} P_j(\theta')B^i_j(\theta'|\theta_j) & \text{if } a^i_j = 0.
\end{cases}
\]

Investor’s total expenditures on risky bonds $X^i_j(\theta_j)$ satisfy the budget constraint

\[
X^i_j(\theta_j) = \omega^i_j - B^i_{RF,j}(\theta_j),
\]

and the auction-clearing condition condition in country $j$ and state $\theta_j$ is

\[
\sum_i X^i_j(\theta_j) = D_j.
\]

**Remark 2 (Price Risk)** Focus on a single country, and suppose that informed investors bid $B^I(g)$ in the good state and $B^I(b) > B^I(g)$ in the bad state. To replicate the same portfolio of risky bonds, an uninformed investor would need to bid $B^I(g)$ at marginal price $P(g)$ and $B^U(b) - B^I(g)$ at marginal price $P(b) < P(g)$. In the bad state, the expenditure of an informed investor is $X^I(b) = P(b)B^I(b)$, while the expenditure of an uninformed investor is $X^U(b) = \ldots$
\[ P(g)B^I(g) + P(b)(B^I(b) - B^I(g)) = X^I(b) + (P(g) - P(b))B^I(g). \]

Uninformed investors thus overpay in the bad state because they do not know whether the true marginal price is \( P(g) \) or \( P(b) \). Hence there is price risk, and it is increasing in the price spread \( P(g) - P(b) \).

### 2.3 Secondary Market

The secondary market opens in each country once the auction is over. It is perfectly competitive with market-clearing price \( q_j(\theta_j) \). Auction prices are publicly observable, hence the information set of a secondary market trader in country \( j \) consists of \( P_j(\theta_j) \) if the investor is uninformed, and \( \{P_j(\theta_j), \theta_j\} \) if the investor is informed. If a strictly positive mass of investors is informed in country \( j \), then \( P_j(g) \neq P_j(b) \) and the observed auction price is fully revealing of the state.\(^9\) Hence investors who do not acquire information in the auction are informed in the secondary market if information is impounded into auction prices. That all investors are symmetrically informed in secondary markets, however, does not imply that investors are symmetric in the secondary market, as informed and uninformed investors have previously bid differently at the auction.

We assume that the two traders in an investor family cannot share information or financial resources between the primary and secondary markets. If they could share information and resources, then the secondary market price in a country would also depend on the state in the other country. This linkage would increase the number of prices to be determined but in a largely uninteresting manner.\(^10\)

Investor \( i \) enters the secondary market in country \( j \) given state \( \theta_j \), with portfolio \( A^i_j(\theta_j) \equiv \{B^i_{R,j}(\theta_j), B^i_{RF,j}(\theta_j)\} \). Since auction prices are a function of \( \theta_j \), we denote the secondary market bids of an investor by \( b^i_j(\theta_j, A^i_j(\theta_j)) \). Noting that \( A^i_j(g) = A^i_j(b) \) if \( P_j(g) = P_j(b) \), we can write the secondary market measurability constraint bids as

\begin{equation}
  b^i_j(g, A^i_j(\theta_j)) = b^i_j(b, A^i_j(\theta_j)) \quad \text{if} \quad \alpha^i_j = 0 \quad \text{and} \quad P_j(g) = P_j(b). \quad (5)
\end{equation}

\(^9\)If no investor is informed at the auction, prices do not convey any information.

\(^10\)To see this, note that if preferences were CRRA, for instance, all investors would hold similar portfolio shares in each asset and the level of prices would be the same as in a representative investor model where investors just own the per capita level of each bond. Thus, the price vector would just be a function of the state in each country and the number of each type of risky bond sold.
In the secondary market, bonds are in zero net supply and market-clearing is
\[
\sum_i b^i_j(\theta_j, A^i_j(\theta_j)) = 0 \quad \text{for all } j \text{ and } \theta_j. \tag{6}
\]

Since investors cannot borrow or short-sell, secondary market bids must satisfy
\[
q_j(\theta_j) b^i_j(\theta_j, A^i_j(\theta_j)) \leq B^i_{RF,j}(\theta_j) \tag{7}
\]
\[
b^i_j(\theta_j, A^i_j(\theta_j)) \geq -B^i_{R,j}(\theta_j) \tag{8}
\]

When there is no risk of confusion, we suppress the dependence of secondary market bids on \(A^i_j(\theta_j)\) and simply write \(b^i_j(\theta_j)\). The final portfolio that determines investor \(i\)'s payoffs in country \(j\) and state \(\theta_j\) is
\[
\bar{B}^i_{R,j}(\theta_j) = B^i_{R,j}(\theta_j) + b^i_j(\theta_j), \quad \text{and} \quad \bar{B}^i_{RF,j}(\theta_j) = B^i_{RF,j}(\theta_j) - q_j(\theta_j)b^i_j(\theta_j).
\]

### 2.4 Equilibrium Concept

The expected payoff to an investor of type \(i\) is
\[
V^i = \sum_{\theta_1 \in \Theta_1} \sum_{\theta_2 \in \Theta_2} f(\theta_1)f(\theta_2) \left\{ \kappa_1(\theta_1)\kappa_2(\theta_2)U \left( \bar{B}^i_{RF,1}(\theta_1) + \bar{B}^i_{RF,2}(\theta_2) \right) \right. \\
+ \kappa_1(\theta_1)(1 - \kappa_2(\theta_2))U \left( \bar{B}^i_{RF,1}(\theta_1) + \bar{B}^i_{RF,2}(\theta_2) + \bar{B}^i_{R,1}(\theta_1) \right) \\
+ (1 - \kappa_1(\theta_1))\kappa_2(\theta_2)U \left( \bar{B}^i_{RF,1}(\theta_1) + \bar{B}^i_{R,1}(\theta_1) + \bar{B}^i_{RF,2}(\theta_2) \right) \\
+ (1 - \kappa_1(\theta_1))(1 - \kappa_2(\theta_2))U \left( \bar{B}^i_{RF,1}(\theta_1) + \bar{B}^i_{R,1}(\theta_1) + \bar{B}^i_{RF,2}(\theta_2) + \bar{B}^i_{R,2}(\theta_2) \right) \}.
\tag{9}
\]

**Definition 2** Investor \(i\)'s bidding problem is to choose primary market and secondary market bid functions to maximize (9) subject to budget constraints (3) and (7), short-sale constraints (1) and (8), and measurability constraints (2) and (5).

The value of the objective function (9) at the optimal portfolio is \(\bar{V}^i\).

**Definition 3** An investor’s information acquisition problem is \(\max_{a_1,a_2} V^i(a_1,a_2) - C(a_1, a_2)\).
Definition 4 (Equilibrium)  An equilibrium is defined as pricing functions $P : \{b, g\}^2 \rightarrow [0, 1]$ and $q : \{b, g\}^2 \rightarrow [0, 1]$, bid functions $B^i : \{b, g\}^2 \rightarrow [0, \infty)$ and $b^i : \{b, g\}^2 \rightarrow [0, \infty)$ for investor type $i$, and information acquisition decisions $\{\alpha_1, \alpha_2\}$ for each investor such that:

1. bid functions solve type $i$’s bidding problem,
2. information acquisition decisions solve each investor’s information acquisition problem,
3. primary markets clear in all states in accordance with (4),
4. secondary markets clear in all states in accordance with (6).

3 Equilibrium without Secondary Markets

We begin by studying equilibrium without secondary markets. We first assume that the share of informed investors is fixed, and show that negative shocks to one country’s fundamentals increase spreads in both countries through portfolio reallocation contagion as long as investors exhibit prudence ($u''(c) > 0$). We then introduce information acquisition and show that there are cross-country complementarities in the incentives to acquire information. This leads to informational contagion: shocks that induce local information acquisition may trigger global information acquisition, generating lower prices on average and a higher sensitivity of prices to fundamentals.

We use the following notation. The total number of risk-free assets held by investor $i$ in state $\{\theta_1, \theta_2\}$ is

$$\tilde{W}^i(\theta_1, \theta_2) \equiv \sum_j B^i_{RF,j}(\theta_j).$$

If country $j$ defaults, investor $i$’s expectation over marginal utility $U^i$ given $\{\theta_1, \theta_2\}$ is

$$\mathbb{E}[U^i|\theta_1, \theta_2, j \text{ default}] = \kappa_{-j}(\theta_{-j}) U'(\tilde{W}^i(\theta_1, \theta_2)) + (1 - \kappa_{-j}(\theta_{-j})) U'\left(\tilde{W}^i(\theta_1, \theta_2) + B^i_{R,-j}(\theta_{-j})\right),$$

(10)

If country $j$ repays, it is

$$\mathbb{E}[U^i|\theta_1, \theta_2, j \text{ repay}] = \kappa_{-j}(\theta_{-j}) U'(\tilde{W}^i(\theta_1, \theta_2) + B^i_{R,j}(\theta_j))$$

$$+ (1 - \kappa_{-j}(\theta_{-j})) U'\left(\tilde{W}^i(\theta_1, \theta_2) + B^i_{R,j}(\theta_j) + B^i_{R,-j}(\theta_{-j})\right).$$

(11)
If investor $i$ is uninformed about country $j$, we write $\theta_j = \tilde{\theta}_j$ and $\kappa_j(\tilde{\theta}_j) = \kappa^u_j$ to indicate that the expected default probability is the unconditional probability.

### 3.1 Contagion through Portfolio Reallocation

To establish that contagion arises just because investors optimally adjust their portfolios after shocks to default risk, assume first that no investor is informed. The marginal price in each country must then be independent of the state. Without loss of generality, restrict attention to a single price in each country, and let investors choose two bid quantities at these prices, $B_1$ and $B_2$. Then $B_{R,j}(\theta_j) = B_j$, $B_{RF,j}(\theta_j) = \omega_j - P_j B_j$ and $\tilde{W} \equiv B_{RF,j}(\theta_1, \theta_2) = W - \sum_j P_j B_j(\theta_j)$ for all $j$ and $\theta_j$. The first-order condition for bids in country $j$ is

$$
(1 - \kappa^u_j) \times \mathbb{E}[U'_j|\tilde{\theta}_1, \tilde{\theta}_2, j \text{ repay}] = \frac{P_j}{1 - P_j}.
$$

The next proposition shows that the model generates contagion (increases in default risk in one country affect prices in the other) for a general and commonly used class of utility functions, even though default risk is independent across countries.

**Proposition 1** If $U(\cdot)$ satisfies decreasing absolute risk aversion, then $\frac{\partial P_j}{\partial \kappa_{-j}} < 0$.

The intuition follows from equations (10) and (11). Prudence ($u'' > 0$) is a necessary condition for decreasing absolute risk aversion, and prudent investors purchase fewer risky assets when they are subject to more background risk. Since increased default risk in one country increases investors’ background risk when trading in the other country, both countries must lower prices to raise the required revenue.

Figure 1 plots equilibrium prices as a function of Country 1’s unconditional default probability $\kappa^u_1$ when no investor is informed. Prices are declining in both countries, and fall more steeply in Country 1, which suffers a direct increase in default risk. Prices are lower and contagion is stronger when Country 1 has more outstanding debt. This is because a country’s impact on investors’ background risk depends on the extent to which portfolios are exposed to that country.
We now characterize contagion stemming from asymmetric information about a country’s default risk. For simplicity, we do so under the presumption that some investors acquire information in one country (the informed country), while no investor acquires information in the other country (the uninformed country). Without loss of generality, let Country 1 be the informed country. Then \( n^{IU} = n > 0, n^{UU} = 1 - n \) and \( n^{UI} = n^{II} = 0 \). For simplicity, we use superscript \( I \) to refer to investors informed in Country 1, and superscript \( U \) to refer to all other investors.\(^{11}\)

Start by considering the optimal portfolio of uninformed investors. Since no investors is informed about Country 2, there is a single marginal price \( P_2 \) in that country. Hence realized asset holdings in Country 2 are independent of \( \theta_2 \), satisfy \( B^{U}_{R2}(\theta_2) = B^{U}_2 \) and \( B^{U}_{RF2}(g) = \omega_2 - P_2 B^{U}_2 \) for all \( \theta_2 \), and can be evaluated using unconditional default probability \( \kappa^U_2 \). In Country 1, instead, there are two state-contingent marginal prices: \( P_1(b) \) and \( P_1(g) > P_1(b) \). Because bids submitted at \( P_1(g) \) will also be

\(^{11}\)As will become clear, the benefits of information are continuous in \( n \). For any \( n \), we can thus find parameters \( K_1, K_2, K_{1,2} \) such that the proposed information allocation is a solution to the underlying information acquisition problem.
accepted if \( \theta_1 = b \), uninformed investors’ realized portfolios depend on \( \theta_1 \),

\[
B^U_{R,1}(g) = B^U_1(g), \quad B^U_{R,1}(b) = B^U_1(b) + B^U_1(g),
\]

\[
B^U_{RF,1}(g) = \omega_1 - P_1(g)B^U_1(g), \quad \text{and} \quad B^U_{RF,1}(b) = \omega_1 - P_1(b)B^U_1(b) - P_1(g)B^U_1(g),
\]

and their expected value must be assessed using the state-contingent default probability \( \kappa_1(\theta_1) \). Specifically, the first-order condition for bids at the high price is evaluated using a weighted average of marginal utilities under default and repayment across both states. Hence the first-order condition for \( B^U_1(g) \) is

\[
\frac{\sum_{\theta_1} f_1(\theta_1) \times (1 - \kappa_1(\theta_1)) \times \mathbb{E}[U^U_1|\theta_1, \tilde{\theta}_2, \text{1 repay}]}{\sum_{\theta_1} f_1(\theta_1) \times \kappa_1(\theta_1) \times \mathbb{E}[U^U_1|\theta_1, \tilde{\theta}_2, \text{1 default}]} \leq \frac{P_1(g)}{1 - P_1(g)}.
\]

(13)

and with equality whenever the short-sale constraint does not bind.

Since low-price bids are accepted only if \( \theta_1 = b \), uninformed investors face no uncertainty about the state in which such bids are accepted. Nevertheless, the fact that their high-price bids are also accepted in the bad state alters marginal incentives to bid at the low price, generating portfolio differences across investors in the bad state even though all investors can bid at the low price as if they are informed.\(^{12}\) The first-order condition for \( B^U_1(b) \) is

\[
\frac{(1 - \kappa_1(b)) \times \mathbb{E}[U^U_1|b, \tilde{\theta}_2, \text{1 repay}]}{\kappa_1(b) \times \mathbb{E}[U^U_1|b, \tilde{\theta}_2, \text{1 default}]} = \frac{P_1(b)}{1 - P_1(b)}.
\]

(14)

While uninformed investors face no uncertainty with respect to final asset holdings in Country 2, the presence of informed investors in Country 1 leads to uncertainty with respect to the overall realized portfolio. The first-order condition for bids in Country 2 thus weighs marginal utilities by the possible states in Country 1, i.e.

\[
\frac{(1 - \kappa^u_2) \sum_{\theta_1} f_1(\theta_1) \mathbb{E}[U^U_1|\theta_1, \tilde{\theta}_2, \text{2 repay}]}{\kappa^u_2 \sum_{\theta_1} f_1(\theta_1) \mathbb{E}[U^U_1|\theta_1, \tilde{\theta}_2, \text{2 default}]} = \frac{P_2}{1 - P_2}.
\]

(15)

Next consider informed investors, who differ from uninformed investors only in that they know \( \theta_1 \) when bidding in Country 1. The associated first-order conditions

\(^{12}\) A caveat applies if the short-sale constraint binds on bids at the high price, and the uninformed do not bid anything at those prices. In this case, there are no overhanging high-price bids to be accepted in the bad state, and marginal bidding incentives conditional on \( B^U_1(b) \) are the same for all investors.
for $B_1^I(g)$ and $B_1^I(b)$ are

$$\frac{(1 - \kappa_1(g)) \times \mathbb{E}[U'_1|g, \hat{\theta}_2, 1 \text{ repay}]}{\kappa_1(g) \times \mathbb{E}[U'_1|g, \hat{\theta}_2, 1 \text{ default}]} = \frac{P_1(g)}{1 - P_1(g)},$$

(16)

$$\frac{(1 - \kappa_1(b)) \times \mathbb{E}[U'_1|b, \hat{\theta}_2, 1 \text{ repay}]}{\kappa_1(b) \times \mathbb{E}[U'_1|b, \hat{\theta}_2, 1 \text{ default}]} = \frac{P_1(b)}{1 - P_1(b)}.$$  

(17)

Since traders cannot share information acquired in one country prior to bidding in the other country, informed investors still face uncertainty about the realized portfolio in Country 1 when bidding in Country 2. Hence the first-order condition for $B_2^I$ is

$$\frac{(1 - \kappa_2^u) \sum_{\theta_1} f_1(\theta_1) \mathbb{E}[U'_1|\theta_1, \hat{\theta}_2, 2 \text{ repay}]}{\kappa_2^u \sum_{\theta_1} f_1(\theta_1) \mathbb{E}[U'_1|\theta_1, \theta_2, 2 \text{ default}]} = \frac{P_2}{1 - P_2}.$$  

(18)

3.2.1 Contagion from Endogenous Information Segmentation

We now show that changing the share of informed investors in one country affects prices in both countries. The combination of discrete payoffs (default or repay), decreasing absolute risk aversion and four assets (three risky bonds and one risk-free asset) makes it difficult to analytically characterize optimal portfolios in full generality. To develop analytical intuition, we assume that the utility function satisfies the constant relative risk aversion (CRRA) property, and adapt a “small risk” approach to optimal portfolios by studying a second-order approximation of investors’ decision problem around the limit $\{ D_1, D_2 \} \to 0$. In this limit, market-clearing bids are small relative to total wealth, and we are able to express optimal portfolios purely in terms of expected rates of return and volatility for the three risky bonds (i.e. their Sharpe ratios). This allows us to compare investors by studying differences in the risk-return trade-off induced by information acquisition. We call the resulting optimal portfolio the optimal approximate portfolio.

Define $\delta_j(\theta_j)$ to be equal to 1 if country $j$ defaults in state $\theta_j$, and 0 otherwise. Then the realized rate of a return on a bond sold by $j$ in state $\theta_j$ is

$$r_j(\delta_j|\theta_j) = \frac{1 - \delta_j(\theta_j) - P_j(\theta_j)}{P_j(\theta_j)}.$$  

Denote investor $i$’s expected default probability for a bond purchased in country $j$ at marginal price $P_j(\theta_j)$ by $\tilde{\kappa}_j^i(\theta_j)$. Since no investor is informed in Country 2, $\tilde{\kappa}_2^i(\theta_j) = \kappa_2^u$.
for all \( i \) and \( j \). There is also no difference across investors if \( \theta_1 = b \); since all bids at \( P_1(b) \) are accepted if and only if \( \theta_1 = b \), \( \tilde{k}_i^1(b) = \kappa_1(b) \) for all \( i \). There are differences if \( \theta_1 = g \), however: since uninformed bids at \( P_1(g) \) are accepted in both states, \( \tilde{k}_i^U(g) = \kappa_1^U(g) \) for informed investors and \( \tilde{k}_i^U(g) = \kappa_1^u \) for uninformed investors. The bond’s expected return from the perspective of investor \( i \) is then

\[
\tilde{r}_j^i(\theta_j) = \frac{1 - \tilde{k}_j^i(\theta_j) - P_j(\theta_j)}{P_j(\theta_j)}.
\]

Hence uninformed investors expect to earn a strictly lower return per unit of investment than informed investors if they buy at \( P_1(g) \) and the same return if they buy at \( P_1(b) \) or in Country 2. Given standard deviation of returns \( \sigma_j^i(\theta_j) \), the Sharpe ratio is

\[
s_j^i(\theta_j) = \frac{\tilde{r}_j^i(\theta_j)}{\sigma_j^i(\theta_j)}.
\]

The next result shows that asymmetric information generates segmentation. When default probabilities are not very high (less than 50%) , informed investors always face a more favorable risk-return profile in Country 1 than uninformed investors, and thus optimally allocate fewer funds to Country 2. Because segmentation hampers risk-sharing, the existence of informed investors in one country lowers prices in both countries. It also directly implies that changing the composition of investors affects prices in both country. The result can be stated purely as a function of Sharpe ratios because there are no differences in marginal utilities at the point of approximation. The cutoff for the default probability is 50% simply because payoffs are binary.

**Proposition 2** \( s_j^I(\theta_j) = s_j^U(\theta_j) \) if \( j = 2 \) or if \( j = 1 \) and \( \theta_1 = b \). If \( k_1^u < \frac{1}{2} \), then \( s_1^I(g) > s_1^U(g) \) and the optimal approximate portfolio displays segmentation, \( B_2^I < B_2^U \).

Figure 2 illustrates equilibrium bond prices in both countries as function of \( n \), the fraction of investors informed in Country 1. We do this exercise far from the approximation that we used above to provide analytical insights, and we obtain the same qualitative relations. As a benchmark, the black horizontal lines in both panels show bond prices when there is no information in either country (this is for \( n = 0 \)).

In Country 1 the price \( P_1(g) \) is increasing in the fraction \( n \) of informed investors bidding. The reason is that informed bidders are willing to pay more when they know the state is good. The price \( P_1(b) \) decreases for analogous reasons. Finally,
prices $P_2$ in Country 2 do not change monotonically with $n$ but are always lower than the benchmark in which $n = 0$ in both countries. Hence, a country’s bonds price is the highest when no investor is informed in another country, which shows that there is information contagion.

Several properties of model are relevant to understanding how information in Country 1 affects prices. First, when there are many informed investors ($n > 0.77$), uninformed investors choose not to participate in the auction at the high price $P_1(g)$. This means that all investors have the same ex-ante exposure to Country 1 when bidding at the low price (i.e. the uninformed have not already had some high-price bids accepted), they choose the same low-price bids, and $P_1(b)$ is independent on $n$ in this range. Second, when there are few informed investors ($n \to 0$), $P_1(g)$ is very close to the price in the uninformed equilibrium because uninformed investors dominate the market and have to purchase almost all bonds in the good state in order for markets to clear. The price $P_1(b)$, however, is discontinuously lower than the price in the uninformed equilibrium because prices have to drop enough to induce those few informed investors to buy in the bad state. Finally, the price $P_2$ is lower than in the uninformed equilibrium because of segmentation: informed investors pull back from Country 2 to spend more in Country 1 (where they are informed) and, even though uninformed investors bid relatively more in Country 2 where they do not face price risk, their absolute demand in Country 2 declines because of the higher background
risk introduced by the price risk in Country 1.

Governments naturally prefer a higher price because it lowers debt burdens. In Country 1 the light-blue dashed line shows the unconditional expected bond price (averaging across good and bad states). For most of the range of \( n \) both countries are worse off because of information in Country 1, except when \( n \) is sufficiently close to one.\(^{13}\) The intuition is that information has two effects, (i) it is costly, and that cost partly passes through to prices and (ii) it induces segmentation, which worsens risk sharing and increases background risk.

### 3.2.2 Contagion of Informational Regimes

We now show that information acquisition in one country may induce information acquisition in the other country. First, we describe the possibility of multiple equilibria in a single country, then we show how information in one country affects the incentives to become informed in another country.

The solid blue line of Figure 3 shows the difference in expected utility between informed and uninformed investors in the corresponding equilibrium, gross of information costs. This difference is non-monotonic due to the interaction of two forces. On the one hand an increase of \( n \) raises the price spread \( P_1(g) - P_1(b) \) and, thus, the price risk faced by uninformed investors (see Country 1 prices in Figure 2). On the other hand, an increase in \( n \) strengthens competition for good bonds among informed investors, dissipating rents on infra-marginal bond purchases. The first force dominates if \( n \) is small, and the second force dominates if \( n \) is large (since uninformed investors do not bid in the good state when price risk is too severe.)

With endogenous information acquisition, the equilibrium share of informed investors is such that the utility difference equals the information cost. The utility difference increases discontinuously at \( n = 0 \) (from the black horizontal line to the blue curve). If no investor is informed \( (n = 0) \) an informed investor faces a single non-contingent price. This limits the investor’s ability to exploit his information advantage vis-a-vis the uninformed. In the limit of the informed equilibrium as \( n \to 0 \), instead, they face two distinct prices. For an appropriate range of costs (between the

\(^{13}\)In this case Country 1 is better off in expectation because \( P_1(g) \) grows while \( P_1(b) \) remains constant, and then average prices reaches a point above the uninformed equilibrium price because of the strong competition of good bonds among informed investors.
black horizontal line and the blue curve at \( n = 0 \) this creates two possible equilibria, one with \( n^* = 0 \) and the other with \( n^* > 0 \).

Figure 3: Value of information in informed and uninformed equilibrium. Preferences: \( U(\cdot) = \log(\cdot) \). Parameters: \( W = 800, \kappa_1^U = 0.1, \kappa_1^b = 0.35, \kappa_2^U = \kappa_2^b = 0.275, D_1=D_2 = 300 \).

Next, we discuss how \( n \) affects the incentives to acquire information in a second country. The dashed green line of Figure 3 shows the incremental expected utility obtained by a single deviating investor who is uninformed in Country 1 (where a share \( n \) is informed) and considers acquiring information in Country 2. The horizontal black line shows the incremental expected utility of acquiring information in Country 2 when no investor is informed in Country 1. The dotted red line shows the incremental utility of acquiring information in Country 2 for an investors who is already informed in Country 1. Hence the incentive to acquire information in Country 2 is always strictly higher when there is some information in Country 1, and the additional incentive to become informed in a second country is smaller than the incentive to become informed in a first country.

Taken together, these results show that the presence of informed investors in Country 1 always increases uninformed investors’ incentives to acquire information in Country 2. The intuition is that a country without informed investors becomes a “safe haven” where uninformed investors can invest without facing price risk. Information acquisition thus leads to a migration of uninformed investors to the other country. Because this country now represents a higher share of uninformed investors’
portfolio, this in turn increases incentives to acquire information in the uninformed country. When there are informed investors in both countries, the remaining uninformed investors cannot avoid paying excessive prices in some states of the world unless they stop bidding. This further increases their incentives to acquire information. The existence of informed investors thus begets further information acquisition, creating a novel channel of contagion through spillovers in the informational regime.

Which shocks can induce information acquisition in one country and trigger spillovers? One trivial possibility is that the cost of information falls. A more interesting possibility is that the value of information increases because default risk rises. As an example, Figure 4 plots equilibrium prices and the value of information in Country 1 when \( n \approx 0 \), as a function of \( \kappa_1^b \). An increase in the bad-state default probability in Country 1 reduce prices in all states and in all countries. \( P_1(b) \) falls because default is more likely in this state. \( P_1(g) \) falls because the decline in \( P_1(b) \) increases the price risk faced by uninformed investors, as they overpay by more when their high-price bids are accepted in the bad state, which deters their bidding at the high price. \( P_2 \) declines because of prudence and direct price contagion. This is shown in the second panel. The third panel shows that an increase in price risk also increases the value of information (both in informed equilibrium, and as a deviation from the uninformed equilibrium). Hence an increase in default risk can attract information, triggering a decrease in bond prices and a switch in information regimes in both countries. This is the case even though countries are not fundamentally linked.

4 Equilibrium with Secondary Markets

We now show that opening a competitive secondary market in each country induces more information acquisition and reduces auction prices even further. We have already argued that information is symmetric in the secondary market because auction prices are observable. So what is the benefit of acquiring information before the auction when it can freely inferred from realized prices ex-post? We show that information still has value because informed investors can earn arbitrage profits by buying...
low in the primary market and selling high in the secondary market. Indeed, the value of information may be higher when there are secondary markets, because informed investors can use the secondary market to partially re-balance their exposure to default risk ex-post. The dual market structure thus offers a tractable way of separating trading for information purposes from trading for risk-sharing purposes. The next proposition compares equilibrium prices and bids in these two markets.

**Proposition 3** Fix any equilibrium with information acquisition in country $j$. Prices satisfy $P_j(b) = q_j(b)$ and $P_j(g) < q_j(g)$. If investor $i$ is informed in $j$, then $B^i_j(g) = \frac{\omega^i_j}{P_j(g)}$ and $b^i_j(g) < 0$. If no information is acquired in country $j$, then $P_j(\theta_j) = q_j(\theta_j) = P^U_j$ for all $\theta_j$.

There can be no cross-market arbitrage opportunities in the low state because even uninformed investors face no price risk when bidding at the low auction price. Hence any investor could trade to eliminate such rents, and primary and secondary market prices are identical given $\theta_1 = b$. There is, however, a novel trade-off when deciding on the number of bids to submit in the high state. Uninformed investors face a **quality-price trade-off**: buying at the auction offers the benefit of paying a low price for good bonds, but investors run the risk that high-price bids are also accepted in the bad state. Buying in the secondary market is expensive, but eliminates this price risk. Informed investors face an **arbitrage-diversification trade-off**: they can earn more arbitrage profits if they allocate more capital to the country with information (say Country 1, as above), but doing so forces them to spend less in Country 2, lowering the degree of cross-country diversification.

![Figure 4: Effects of $\kappa^1_i$ on prices and the value of information. Preferences: $U(\cdot) = \log(\cdot)$. Parameters: $W = 800, \kappa^2_i = 0.1, \kappa^1_i \in [0.4, 0.5], \kappa^{U/2} = 0.275, D_1 = D_2 = 300, n = 10^{-6}$](attachment:image.png)
The next result shows that the limits to arbitrage are endogenous: when there are enough informed investors, there is sufficient informed capital to fully eliminate the cross-market arbitrage and prices and final allocations are equivalent to the full-information equilibrium. In this case, informed investors take the entire primary market in the good state, and then sell these good bonds to uninformed investors in the secondary market at zero markup. Since waiting for the secondary market is now costless, the uninformed can buy bonds as if they are informed. Hence there are no differences in investors’ final portfolios.

**Proposition 4** The equilibrium with secondary markets is such that \( P_1(g) = q_1(g) \) if and only if \( n \geq \bar{n} = \frac{D_1}{W-D_2} < 1 \). In this case, the equilibrium is equivalent to the full information equilibrium \( n = 1 \) without secondary markets.

Figure 5 plots equilibrium prices. The dotted line shows the secondary market price \( q_1(g) \) in the good state (there is no difference between primary and secondary market prices in the bad state). The dashed line shows the auction price in the presence of a secondary market; the solid line shows the auction price in the absence of a secondary market. The black horizontal line plots the uninformed equilibrium price as a reference point. Figures 8 to 10 in the Appendix show the associated total bond holdings of informed and uninformed investors, the allocation of bids across auction and secondary markets, and secondary market volumes.

The first, and perhaps most relevant, observation is that auction prices are strictly lower in all states compared to both the uninformed equilibrium and the informed equilibrium without secondary markets as long as the share of informed investors is sufficiently small \( n < 0.55 \) in this parametric example. The intuition is as follows. When \( n \) is small, there are not enough informed investors to clear the primary market supply of good bonds, and so uninformed investors must also bid at high prices. But uninformed investors now have the option of waiting for the secondary market, and hence auction prices must fall to induce uninformed participation. Even so, uninformed investors allocate at least some of their purchases to secondary market, else the secondary market would not clear. For this reason (and because \( P_1(g) \) falls), there are fewer high-price bids that the government can execute in the bad state, implying that there is more residual supply that needs to be cleared at the low price. Hence \( P_1(b) \) must also fall.

A second observation is that the effect of secondary markets is not isolated to Country 1, but also spills over to lower prices in Country 2. This is because informed
Investors earn arbitrage profits in Country 1, so that it is optimal for them to reallocate more funds from Country 2 to Country 1. This mechanism is reminiscent of Proposition 2 where we showed that the informed spend less in Country 2 in order to take advantage of a more favorable risk-return trade-off in Country 1. With secondary markets, this effect is amplified because arbitrage profits are risk-free. When \( n \) is high, uninformed investors eventually stop bidding at the high auction price, and instead only purchase good bonds in the secondary market (in the parametric example, this occurs at around \( n = 0.55 \)). When only informed investors participate in the auction, arbitrage opportunities are competed away more rapidly, leading to sharp decline in arbitrage profits. Informed investors are thus less willing to forego the benefits of diversification to capture arbitrage rents, and they respond by shifting a share of their portfolio back to Country 2. In contrast to the case without secondary markets, moreover, uninformed investors now benefit from a high \( n \) as cross-market price differences shrink. This allows them to bypass price risk in the auction, and lowers the background risk they face when investing in Country 2, leading to a relative increase in their demand in Country 2 as well. Both effects combine to generate a reversal in the comparative statics of \( P_2 \) and push \( P_2 \) higher than in the
absence of secondary markets for $n$ sufficiently high.\footnote{Notice that prices in the bad state in this range are constant because only informed investors participate in the high-state auction, which implies that uninformed investors bid as if they were informed.}

When the fraction of informed investors is large enough such that it reaches $\bar{n}$ (in this example around $n = 0.6$), there is enough informed capital to fully arbitrage price differences across markets in the good state. As a result, uninformed investors can buy bonds as if they were fully informed in all states: they bid in auctions only in the bad state, and rely on the secondary markets in the high state. This implies that the “full information” equilibrium obtains for all $n \geq 0.6$ even though a fraction of investors is not informed.

Taken together, the impact of asymmetric information on auction prices in the presence of secondary markets thus changes dramatically around intermediate levels of $n$. When there are not many informed investors, secondary markets generate arbitrage opportunities for informed investors that magnify their reallocation of funds towards the informed country and allow the uninformed to partially avoid price risk by moving funds from the auction to the secondary market. Both effects depress prices in both countries. On the other hand, uninformed investors benefit from secondary markets because they can buy bad bonds in auctions and good bonds in secondary markets as if they were informed. This allows the uninformed to take on more risk exposure overall, and leads to a better aggregate allocation of risk. The latter effect dominates when $n$ is high and arbitrage spreads are low.

With endogenous costly information acquisition, we must have the equilibrium share of informed investors satisfies $n^* < 0.6$, as information does not have any value in equilibrium for $n \geq 0.6$. As long as the cost of information is not too low (so that $n^* < 0.55$ and uninformed investors must be induced to participate in the auction if $\theta_1 = g$), the presence of a secondary market thus leads to strictly lower prices in the auction in all states and all countries. Since government revenues are determined by the auction price, our model provides a channel by which liquid aftermarket can depress government revenue. One way to interpret this result is that secondary market trading forces a transfer of resources from the government to informed investors. The next result shows that the presence of secondary markets also makes it more likely that investors do acquire information in equilibrium and induce such an outcome.

**Proposition 5** Let $n \to 0$. The difference in expected utility between informed and uninformed investors is strictly higher when there are secondary markets.
It follows that there is a range of information costs for which there exists an in-
formed equilibrium in the presence of secondary markets, but not in their absence. 
While our model abstracts from liquidity-based benefits of secondary markets that 
may lower yields, we establish a channel through which secondary markets may raise 
government’s financing costs and contribute to spillovers in sovereign bond markets. 
This result is due to the interaction of endogenous adverse selection in the primary 
market and limited arbitrage capital assigned.

5 Conclusion

This paper constructs a simple model of portfolio choice with information acquisi-
tion by a global pool of risk-averse investors who can buy sovereign debt issued by a 
number of different countries in primary markets. There are three novelties in our ap-
proach. First, we allows for endogenous asymmetric information about fundamental 
default risk. Second, we focus on primary markets and the role of commonly-used 
discriminatory price protocols in determining the equilibrium degree of information 
asymmetry and its impact on yields and spillovers. Third, we explore the implica-
tions of secondary markets, and their interaction with primary markets and asym-
metric information.

In this setting we uncover three important sources of contagion in sovereign 
bond spreads: First, contagion does not require fundamental linkages or common 
factors, just a common pool of prudent investors who re-balance portfolios in re-
sponse to country-specific default risk shocks. Second, asymmetric information gener-
ates contagion through endogenous market segmentation: informed investors tend 
to invest more in the country in which they are informed, which generates price risk 
that increases background risk and affects bond prices globally. In this regard, we 
also show that endogenous price risk leads to complementarities in information ac-
quision. Finally, information is contagious: if investors acquire information about 
fundamentals in one country, this increases the likelihood that investors also want to 
become informed about the fundamentals in other countries even when there are no 
economies of scale in information acquisition. As information asymmetries lead to 
lower prices and higher volatility, all these novel sources of spillovers reinforce each 
other.

By introducing secondary markets and analyzing their interaction with primary
markets in the presence of endogenous asymmetric information, we have shown that aftermarkets introduce risk-free arbitrage opportunities for informed investors, thereby encouraging information acquisition and discouraging the participation of uninformed investors in primary markets. Both effects combine to reduce auction prices and government revenues in all states and in all countries.

Our results highlight it is not straightforward to interpret changes in sovereign debt prices as informative about changes in country fundamentals, as they depend not only on publicly observable information but also on privately acquired information. Moreover, they depend not only on the particular country’s informational regime, but also on the information regime in other countries.

We purposefully made several assumptions to isolate the effects of auction protocols and asymmetric information on bond prices and spillovers. Relaxing some of these assumptions would likely magnify the effects we uncover. Examples include allowing default probabilities to respond endogenously to bond prices, introducing fundamental linkages across countries or time-varying prudence, allowing for exogenous market segmentation, or assuming economies of scale in the production of information. Relaxing other assumptions, such allowing information to affect real choices and allocations, would likely introduce benefits of information acquisition that are absent in our setting.
References


Appendix: Proofs

A.1 Proof of Proposition 1:

Assume there are no informed investor in either country. From equation (12), an uninformed investor is indifferent toward bidding an additional unit in country 1 when

$$(1 - \kappa_1^u)(1 - P_1) \times \mathbb{E}[U'_u|\bar{\theta}_1, \bar{\theta}_2, \text{1 repays}] - \kappa_1^u P_1 \times \mathbb{E}[U'_u|\bar{\theta}_1, \bar{\theta}_2, \text{1 defaults}] = 0,$$  \hspace{1cm} (19)

and indifferent toward bidding an additional unit in country 2 when

$$(1 - \kappa_2^u)(1 - P_2) \times \mathbb{E}[U'_u|\bar{\theta}_1, \bar{\theta}_2, \text{2 repays}] - \kappa_2^u P_2 \times \mathbb{E}[U'_u|\bar{\theta}_1, \bar{\theta}_2, \text{2 defaults}] = 0.$$  \hspace{1cm} (20)

In what follows we take Country 1 as the reference country, but the proof is symmetric for Country 2. The uninformed equilibrium market clearing in country 1, from equation (4), is simply

$$B_{R,1}^i(\bar{\theta}_1) = \frac{D_1}{P_1},$$

Given we are focusing in the case in which all investors are uninformed, we can rewrite the expected marginal utilities in a simpler and more convenient form. For the case of repayment, for instance

$$\mathbb{E}[U'_u|\bar{\theta}_1, \bar{\theta}_2, \text{1 repays}] = \mathbb{E}(U'(r_1, \cdot)) = \kappa_2 U'_{(r_1,d_2)} + (1 - \kappa_2) U'_{(r_1,r_2)}$$

$$= \kappa_2 U' \left( \hat{W} + \frac{D_1}{P_1} \right) + (1 - \kappa_2) U' \left( \hat{W} + \frac{D_1 + D_2}{P_1} \right)$$

where $U'_{(r_1,d_2)}$ represents the marginal utility in the case country 1 repays and country 2 defaults (the other three expected marginal utilities are constructed and represented similarly), while

$$\hat{W} = W - D_1 - D_2$$

is a parametric constant that represent the available wealth after investment (this is, all investors are symmetric and the mass one of representative investors has to buy the debt of both countries).

We can write the system of equations (19) and (20) in implicit form as

$$F_1(\kappa_1, \kappa_2, P_1, P_2) = 0$$

$$F_2(\kappa_1, \kappa_2, P_1, P_2) = 0$$

and determine how prices change with $\kappa_1$. Taking derivatives with respect to $\kappa_1$ and
applying the implicit function theorem,

\[
\begin{bmatrix}
\frac{\partial F_1}{\partial \kappa_1} & \frac{\partial F_1}{\partial \kappa_2} \\
\frac{\partial F_2}{\partial \kappa_1} & \frac{\partial F_2}{\partial \kappa_2}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial P_1}{\partial \kappa_1} & 0 \\
0 & \frac{\partial P_2}{\partial \kappa_2}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial F_1}{\partial \kappa_1} & \frac{\partial F_2}{\partial \kappa_2}
\end{bmatrix}
\]

where

\[
\frac{\partial F_1}{\partial \kappa_1} = - [(1 - P_1)\mathbb{E}(U'_{(r_1, \cdot)}) + P_1\mathbb{E}(U'_d)] < 0.
\]

Intuitively the direct effect of the default probability in country \( j \) is negative because, when the probability of default in a country increases it is more likely the investor loses the investment.

The direct effect of the default probability \( \kappa_1 \) in country 2, is

\[
\frac{\partial F_2}{\partial \kappa_1} = (1 - \kappa_2)(1 - P_2) \left[ U'_{(d_1, r_2)} - U'_{(r_1, r_2)} \right] - \kappa_2 P_2 \left[ U'_{(d_1, d_2)} - U'_{(r_1, d_2)} \right].
\]

This is also negative, but only when the utility function displays decreasing absolute risk aversion (DARA). To see this in a transparent way, take the limit \( D_1 \to 0 \). In the limit we can simply write this derivative as

\[
\lim_{D_1 \to 0} \frac{\partial F_2}{\partial \kappa_1} = (1 - \kappa_2)(1 - P_2) \left[ -U''_{(\cdot, r_2)} \right] - \kappa_2 P_2 \left[ -U''_{(\cdot, d_2)} \right]
\]

where \( U'_{(\cdot, d_2)} \) and \( U''_{(\cdot, d_2)} \) represent that, in the limit \( D_1 \to 0 \), it is irrelevant whether the derivatives are evaluated at \( d_1 \), at \( r_1 \) or at the expectation. The first equality comes from the first-order condition from investing in country 2. Then this condition is satisfied as long as

\[
\frac{(1 - \kappa_2)(1 - P_2)}{\kappa_2 P_2} = \frac{U'_{(\cdot, d_2)}}{U'_{(\cdot, r_2)}} < \frac{-U''_{(\cdot, d_2)}}{-U''_{(\cdot, r_2)}},
\]

where the first equality comes from the first-order condition from investing in country 2. Then this condition is satisfied as long as

\[
ARA(r_2) \equiv \frac{-U''_{(\cdot, r_2)}}{U'_{(\cdot, r_2)}} < \frac{-U''_{(\cdot, d_2)}}{U'_{(\cdot, d_2)}} \equiv ARA(d_2),
\]

which is the definition of decreasing absolute risk aversion. Notice that DARA requires that \( U''(\cdot) > 0 \) (prudence). Thus DARA requires the necessary (but not sufficient) condition of prudence (see Kimball (1993)).

Now we can obtain the coefficients of the matrix that multiplies the eigenvector. The diagonal coefficients are given by the derivatives of the first-order condition of investing in a country with respect to the price in that country. The first element,

\[16\] As long as the utility function also respects global regularity conditions (no kinks, for instance) this is also a global property.
corresponding to country 1, is

\[
\frac{\partial F_1}{\partial P_1} = -\mathbb{E}(U'_{1,\cdot}) + (1 - \kappa_1)(1 - P_1) \frac{D_1}{P_1^2} \mathbb{E}(-U''_{(r_1, \cdot)}) < 0 \tag{21}
\]

These derivatives are negative in the equilibrium with the highest equilibrium price in each country. To see this, notice that \(F_1(P_1 = 1) < 0\) and \(F_1(P_1 = 0) > 0\) (from replacing \(P_1 = 1\) and \(P_1 = 0\) in equation 19). This implies that the highest \(P_1^*\) in equilibrium, for which \(F_1(P_1^*) = 0\) should be such that \(\frac{\partial F_1(P_1^*)}{\partial P_1} < 0\). This equilibrium displays the comparative statics that arise when the equilibrium price is unique.

The off-diagonal coefficients are given by the derivative of the first-order condition of investing in a country with respect to the price in the other country. The element corresponding to country 1 responding to the price in country 2 is,

\[
\frac{\partial F_1}{\partial P_2} = \frac{D_2}{P_2^2} (1 - \kappa_2) \left[ (1 - \kappa_1)(1 - P_1)(-U''_{(d_1, r_2)}) - \kappa_1 P_1 (-U''_{(r_1, r_2)}) \right]. \tag{22}
\]

This cross derivative is negative when

\[
\frac{(1 - \kappa_1)(1 - P_1)}{\kappa_1 P_1} = \frac{\mathbb{E}(U'_{(d_1, \cdot)})}{\mathbb{E}(U'_{(r_1, \cdot)})} < \frac{-U''_{(d_1, r_2)}}{-U''_{(r_1, r_2)}},
\]

which is always the case when the utility function displays DARA. To see this take the limit as \(D_2 \to 0\). In this case the expression can be approximated in terms of absolute risk aversion as follows

\[
ARA(r_1) \equiv \frac{-U''_{(r_1, \cdot)}}{-U''_{(r_1, \cdot)}} < \frac{-U''_{(d_1, \cdot)}}{-U''_{(d_1, \cdot)}} \equiv ARA(d_1).
\]

The two derivatives of interest in the eigenvector, \(\frac{\partial F_1}{\partial r_1}\) and \(\frac{\partial F_1}{\partial r_2}\), are negative if and only if both eigenvalues are negative. The eigenvalues come from the solution to

\[
\begin{vmatrix}
\frac{\partial F_1}{\partial P_1} - \lambda & \frac{\partial F_1}{\partial P_2} \\
\frac{\partial F_2}{\partial P_1} & \frac{\partial F_2}{\partial P_2} - \lambda
\end{vmatrix} = 0
\]

and are both negative if and only if,

\[
\frac{\partial F_1 \partial F_2}{\partial P_1 \partial P_2} > \frac{\partial F_2 \partial F_1}{\partial P_1 \partial P_2}
\]

To analyze this expression, notice that we can write the indirect derivative (\(\frac{\partial F_1}{\partial P_2}\) for instance) as a function of the direct derivative (this is, \(\frac{\partial F_2}{\partial P_2}\)). To see this, rewrite
Define $F = \frac{\partial}{\partial P} (1 - \kappa_2) (1 - \kappa_1) (U''_{(r_1, r_2)}) - P_1 \frac{D_2}{P_2^2} (1 - \kappa_2) \left[ (1 - \kappa_1) (U''_{(r_1, r_2)}) + \kappa_1 P_1 (U''_{(d_1, r_2)}) \right]$, 

and rewrite equation (21), but the one corresponding to country 2, as 

$$(1 - P_2) \frac{D_2}{P_2^2} (1 - \kappa_2) E(-U''_{(r_2, t8)}) = \frac{\partial F_2}{\partial P_2} + \mathbb{E}(U'_{(c, \cdot)}).$$

Then, 

$$\frac{\partial F_1}{\partial P_2} = \frac{D_2}{P_2^2} (1 - \kappa_2) (1 - \kappa_1) (U''_{(r_1, r_2)}) - \frac{P_1}{1 - P_2} \left[ \frac{\partial F_2}{\partial P_2} + \mathbb{E}(U'_{(c, \cdot)}) \right]$$

(23)

We are now ready to show that the absolute value of $\frac{\partial F_j}{\partial P_j}$ is always larger than the absolute value of $\frac{\partial F_j}{\partial P_j}$, for both $j = \{1, 2\}$, and then the eigenvalues are negative. Formally, this condition for the effect of price 1 (a symmetric condition holds for the effect of price 2) is, 

$$| \frac{\partial F_1}{\partial P_1} | > | \frac{\partial F_2}{\partial P_2} |.$$ 

Define $Y_1 \equiv \frac{D_2}{P_2} (1 - \kappa_1)(1 - \kappa_2) E(-U''_{(r_1, r_2)})$. This condition on absolute values, using expressions (21) and (23) and their signs, can be written as 

$$\mathbb{E}(U'_{(c, \cdot)}) - \frac{1 - P_1}{1 - \kappa_2} Y_1 > \frac{P_2}{1 - P_1} \left[ \frac{1 - P_1}{1 - \kappa_2} Y_1 \right] - \frac{D_1}{P_1} (1 - \kappa_1)(1 - \kappa_2)(-U''_{(r_1, r_2)}).$$

In the limit $\lim_{D_2 \to 0} \frac{D_2}{P_1} (1 - \kappa_1)(1 - \kappa_2)(-U''_{(r_1, r_2)}) \to Y_1$ and this condition becomes 

$$\mathbb{E}(U'_{(c, \cdot)}) - \frac{1 - P_1}{1 - \kappa_2} Y_1 > \frac{P_2 - (1 - \kappa_2)}{1 - \kappa_2} Y_1,$$

or 

$$\mathbb{E}(U'_{(c, \cdot)}) > \left[ \frac{P_2 + \kappa_2 - P_1}{1 - \kappa_2} \right] Y_1.$$ 

This condition always holds because 

$$\mathbb{E}(U'_{(c, \cdot)}) > \left[ \frac{1 - P_1}{1 - \kappa_2} \right] Y_1 > \left[ \frac{P_2 + \kappa_2 - P_1}{1 - \kappa_2} \right] Y_1,$$

where the first inequality comes from $\frac{\partial F_1}{\partial P_1}$ being negative from equation (21) and the second inequality from the fact that $1 > P_2 + \kappa_2$ (as $P_2 < 1 - \kappa_2$ from risk aversion).

**Q.E.D.**
A.2 Proof of Proposition 2

Let \( n \in (0, 1) \). There are 8 possible states: for each \( \theta_1 \in \{ g, b \} \), each country may default (\( d \)) or repay (\( r \)). The associated consumption levels are \( \{ c^i_{rr}(\theta), c^i_{rd}(\theta), c^i_{dr}(\theta), c^i_{dd}(\theta) \} \), and the objective function is

\[
V = a_1 \left\{ \kappa_1(g) \left[ \kappa_2^{u} U(c^i_{dd}(g)) + (1 - \kappa_2^{u}) U(c^i_{dr}(g)) \right] \right\} + (1 - \kappa_1(g)) \left\{ \kappa_2^{u} U(c^i_{rd}(g)) + (1 - \kappa_2^{u}) U(c^i_{rr}(g)) \right\} 
+ (1 - a_1) \left\{ \kappa_1(b) \left[ \kappa_2^{u} U(c^i_{dd}(b)) + (1 - \kappa_2^{u}) U(c^i_{dr}(b)) \right] \right\} + (1 - \kappa_1(b)) \left\{ \kappa_2^{u} U(c^i_{rd}(b)) + (1 - \kappa_2^{u}) U(c^i_{rr}(b)) \right\}
\]

We compute a second-order Taylor approximation of the objective function \( V \) around the “small risk” limit where \( D_1 \rightarrow 0 \) and \( D_2 \rightarrow 0 \), and so when \( B^i_j(\theta_j) \rightarrow 0 \) for all \( i \), all \( j \), and all \( \theta_j \). Hence consumption is approximately equal to \( W \) in the limit. Then, we solve for the optimal approximate portfolio by taking first-order conditions of the approximated objective function with respect to \( B^i_1(g) \), \( B^i_1(b) \) and \( B^i_2 \). For the informed, this yields

\[
0 = a_1(1 - \kappa_1(g) - P_1(g))U'(W) + a_1 \left[ \kappa_1(g)(-P_1(g))^2 + (1 - \kappa_1(g))(1 - P_1(g))^2 \right] U''(W)B^I_1(g) + a_1(1 - \kappa_1(g) - P_1(g))(1 - \kappa_2^{u} - P_2)U''(W)B^I_2 \tag{24}
\]

\[
0 = (1 - a_1)(1 - \kappa_1(b) - P_1^b)U'(W) + (1 - a_1) \left[ \kappa_1(b)(-P_1^b)^2 + (1 - \kappa_1(b))(1 - P_1^b)^2 \right] U''(W)B^I_1(b) + (1 - a_1)(1 - \kappa_1(b) - P_1^b)(1 - \kappa_2^{u} - P_2)U''(W)B^I_2 \tag{25}
\]

\[
0 = (1 - \kappa_2^{u} - P_2)U'(W) + \left[ \kappa_2^{u}(-P_2)^2 + (1 - \kappa_2^{u})(1 - P_2)^2 \right] U''(W)B^I_2 + a_1(1 - \kappa_2^{u} - P_2)(1 - \kappa_1(g) - P_1(g))U''(W)B^I_1(g) + (1 - a_1)(1 - \kappa_2^{u} - P_2)(1 - \kappa_1(b) - P_1^b)U''(W)B^I_1(b) \tag{26}
\]

Define informed investors’ expected rates of return by \( \bar{\tau}^I_1(g) = \frac{1 - \kappa_1(g) - P_1(g)}{P_1(g)} \), \( \bar{\tau}^I_1(b) = \frac{1 - \kappa_1(b) - P_1(b)}{P_1(b)} \) and \( \bar{\tau}^I_2 = \frac{1 - \kappa_2^{u} - P_2}{P_2} \), and let \( \sigma_1^I(g) \), \( \sigma_1^I(b) \), and \( \sigma_2^I \) denote the associated standard deviations. Consider the first two terms on the right-hand side of (24). Multi-
ploying the first by \( \frac{P_1(g)}{P_{1(g)}} \) implies
\[
a_1(1 - \kappa_1(g) - P_1(g))U'(W) = a_1\tilde{r}_1(g)P_1(g)U'(W)
\]
while multiplying the second by \( \left( \frac{P_1(g)}{P_{1(g)}} \right)^2 \) gives
\[
a_1\left[ \kappa_1(g)(-P_1(g))^2+(1-\kappa_1(g))(1-P_1(g))^2 \right]U''(W)B_1^I(g) = a_1\mathbb{E}\left[ (\tilde{r}_1^I(g))^2 \right] P_1(g)^2U''(W)B_1^I(g)
\]
All other terms in equations (24)-(26) can be analogously rewritten. Let \( U(c) = (1-c)^{\frac{1}{1-\gamma}} \), and define the state-contingent portfolio weights \( \omega_1^I(g) = \frac{P_1(g)B_1^I(g)}{W} \), \( \omega_1^I(b) = \frac{P_1(b)B_1^I(b)}{W} \), and \( \omega_2^I = \frac{P_2B_1^I}{W} \). Since \( \text{Var}(x) = \mathbb{E}[x^2] - (\mathbb{E}[x])^2 \) for any random variable \( x \) the system of equations is
\[
\tilde{r}_1^I(g) = \gamma\omega_1^I(g)\left( (\sigma_1^I(g))^2 + (\tilde{r}_1^I(g))^2 \right) + \gamma\omega_2^I\left( \tilde{r}_1^I(g)\tilde{r}_2^I \right) \tag{27}
\]
\[
\tilde{r}_1^I(b) = \gamma\omega_1^I(b)\left( (\sigma_1^I(b))^2 + (\tilde{r}_1^I(b))^2 \right) + \gamma\omega_2^I\left( \tilde{r}_1^I(b)\tilde{r}_2^I \right) \tag{28}
\]
\[
\tilde{r}_2^I = \gamma\omega_2^I\left( (\sigma_2^I)^2 + (\tilde{r}_2^I)^2 \right) + a_1\gamma\omega_1^I(g)\tilde{r}_1^I(g)\tilde{r}_2^I + (1-a_1)\gamma\omega_1^I(b)\tilde{r}_1^I(b)\tilde{r}_2^I \tag{29}
\]
Using the first two equations in the third gives
\[
\omega_2^I\left( \frac{(\sigma_2^I)^2}{\tilde{r}_2^I} \right) = a_1\omega_1^I(g)\left( \frac{(\sigma_1^I(g))^2}{\tilde{r}_1^I(g)} \right) + (1-a_1)\omega_1^I(b)\left( \frac{(\sigma_1^I(b))^2}{\tilde{r}_1^I(b)} \right).
\]
The second and third condition jointly imply
\[
\frac{\omega_1^I(g)}{\omega_1^I(b)} = \left( \frac{\tilde{r}_1^I(g)}{\tilde{r}_1^I(b)} \right) \left( \frac{(\sigma_1^I(b))^2 + \tilde{r}_1^I(b)^2}{(\sigma_1^I(g))^2 + \tilde{r}_1^I(g)^2} \right).
\]
Define the reduced-form parameter
\[
\phi_1(\theta_1) = \frac{(\sigma_1^I(\theta_1))^2}{(\sigma_1^I(\theta_1))^2 + (\tilde{r}_1^I(\theta_1))^2}
\]
and note that it is a uni-variate function of the Sharpe ratio,
\[
\phi_1^I(\theta_1) = \frac{1}{1 + (s_1^I(\theta_1))^2}
\]

Then we can write the optimal bids in Country 2 as

$$\omega_2^* = \left( \frac{s_2^l}{a_2^l} \right) \left[ a_1 \phi_1^U(g) + (1 - a_1) \phi_1^I(b) \right] \frac{1 + s_2^l}{1 + s_2^l} \left[ a_1 \phi_1^U(g) + (1 - a_1) \phi_1^I(b) \right]$$

(30)

To compute the optimal uninformed bids, note that the only difference between investors types is that uninformed bids at $P_1(g)$ are accepted even if $\theta_1 = b$. Hence

$$\frac{\partial c_{rd}^U(b)}{\partial B_1^U(g)} = \frac{\partial c_{rr}^U(b)}{\partial B_1^U(g)} = (1 - P_1(g)) \quad \text{and} \quad \frac{\partial c_{dd}^U(b)}{\partial B_1^U(g)} = \frac{\partial c_{dr}^U(b)}{\partial B_1^U(g)} = -P_1(g)$$

While

$$\frac{\partial c_{rd}^I(b)}{\partial B_1^U(g)} = \frac{\partial c_{rr}^I(b)}{\partial B_1^U(g)} = \frac{\partial c_{dd}^I(b)}{\partial B_1^U(g)} = \frac{\partial c_{dr}^I(b)}{\partial B_1^U(g)} = 0$$

Since there are no differences in state-contingent marginal utilities at the point of approximation, we can solve for the optimal uninformed portfolio using the same approach as above as long as we appropriately redefine the expected return in Country 1 given $\theta_1 = g$ to

$$\tilde{r}_1^U(g) = \frac{1 - \kappa_1^u - P_1(g)}{P_1(g)} < \tilde{r}_1^I(g)$$

Let $\sigma_1^U(g)$ and $s_1^U(g)$ denote the associated standard deviation and Sharpe ratio, and

$$\phi_1^U(\theta_1) = \frac{1}{1 + (s_1^U(\theta_1))^2}$$

Observe further that $\tilde{r}_1^U(b) = \tilde{r}_1^I(b)$ and $\tilde{r}_2^U = \tilde{r}_2^I$. Then the $B_2^U$ satisfies

$$\omega_2^* = \left( \frac{s_2^l}{a_2^l} \right) \left[ a_1 \phi_1^U(g) + (1 - a_1) \phi_1^I(b) \right] \frac{1 + s_2^l}{1 + s_2^l} \left[ a_1 \phi_1^U(g) + (1 - a_1) \phi_1^I(b) \right]$$

(31)

Since $\phi_1^U(g)$ is strictly decreasing in $s_1^U(g)$, so is $\omega_2^U$. Moreover, bids in Country 2 depend on Country 1 variables only through the relevant Sharpe ratios. Since $\kappa_1^u > \kappa_1(g)$, the uninformed face a strictly higher default probability than the informed but pay the same price. Hence it is sufficient to show that $s_1^I(g)$ is strictly increasing in $\kappa_1(g)$ on the interval $[0, \kappa_1^u]$. For a generic default probability $\kappa$ and price $P$, the return
variance and the Sharpe ratio are, respectively,
\[ \sigma^2 = \frac{\kappa(1 - \kappa)}{P^2} \quad \text{and} \quad s_g = \frac{\bar{r}}{\sigma} = \frac{1 - \kappa - P}{\sqrt{\kappa(1 - \kappa)}}. \]

Hence \( \frac{\partial \sigma^2}{\partial \kappa} \geq 0 \) if and only if \( \kappa \leq \frac{1}{2} \), and the Sharpe ratio is decreasing in \( \kappa \) if
\[ \kappa(1 - \kappa) + \left( \frac{1}{2} - \kappa \right) (1 - \kappa - P) \geq 0 \] (32)

A sufficient condition for the Sharpe ratio to decrease in \( \kappa \), and hence a sufficient condition for uninformed investors to invest more in Country 2 than informed investors (because \( \kappa_i^u > \kappa_1(g) \)) is that \( \kappa_i^u < \frac{1}{2} \). Q.E.D.

A.3 Proof of Proposition 3

If there were price differences across markets conditional on \( \theta_j = b \), both informed and uninformed investors would trade to arbitrage price differences (recall that uninformed bids at the low price are accepted if and only if \( \theta_1 = b \)). But if all investors take the same side of the arbitrage in the primary market, then the secondary market cannot clear. Now turn to the good state. If \( P_j(g) > q_j(g) \), then all investors find it strictly optimal to wait, and the primary market does not clear. If \( P_j(g) = q_j(g) \), uninformed investors can trade under perfect information in the secondary market at the primary market price. Hence the informed cannot do better than the uninformed, and there are no incentives to acquire information. Hence, when there is information in the auction it must be that \( P_j(g) < q_j(g) \).

Now, turning to bids, informed investors fully exploit the arbitrage opportunity of higher prices in secondary markets when the state is good by using all wealth allocated to country \( j \) to buy bonds in the good state and by selling a fraction to uninformed investors in the secondary market. Uninformed investors cannot exploit the arbitrage to the same extent because they run the risk of overpaying in the bad state. To see why arbitrage can persist, note that the supply of assets in the secondary market is bounded above by \( \sum_{i=1}^{n} \frac{W_i}{P_j(g)} \), while the demand for bonds in the primary market is decreasing in the fraction of informed investors. All else equal, reducing the number of informed investors thus widens the gap between primary and secondary market prices in the high state.

Q.E.D.

A.4 Proof of Proposition 4

Prices differ across primary and secondary markets if and only if there is arbitrage capital is limited (not enough informed investors in the auction). We show here that
there is enough informed capital to fully arbitrage prices if and only if \( n \geq \bar{n} \). Let \( \hat{P}_j, \hat{B}_1(g) \), and \( B_2^I \) denote the equilibrium good-state price and informed bids in the equilibrium in which all investors are informed and there are no secondary markets. In this equilibrium, informed investors spend \( \hat{P}_2 \hat{B}_2^I \) in Country 2. By auction-clearing, \( \hat{P}_2 \hat{B}_2^I = D_2 \) and then from the budget constraint, informed investors have \( W - D_2 \) in capital to invest in Country 1. In order for informed buy the entire supply of bonds in Country 1 at price \( \hat{P}_1 \) if \( \theta_1 = g \), we require that \( n(W - D_2) \geq \hat{P}_1 B_1^I(g) = D_1 \), where the last equality follows from auction clearing. This condition is equivalent to \( n \geq \bar{n} \).

If \( n \geq \bar{n} \), we can construct an equilibrium in which informed investors buy the entire supply of bonds in the primary market when \( \theta_1 = g \), and then sell some of these bonds to uninformed investors in the secondary market at the same price. This implies that uninformed investors can buy bonds as if they were informed and choose not to participate in primary markets. Hence the equilibrium must be such that all prices are identical to the fully informed equilibrium.

**Q.E.D.**

### A.5 Proof of Proposition 5

Consider first the informed equilibrium in the presence of secondary markets. In the limit, as \( n \to 0 \), uninformed investors’ bids must clear the auction in all states. Market clearing in the good state implies \( \lim_{n \to 0} P_1(g) B_1^U(g) = D_1 \). Since high-price bids are also accepted in the bad state \( \lim_{n \to 0} P_1(g) = P_1^U \). To ensure market-clearing in the secondary market when \( \theta_1 = g \), we require that \( \lim_{n \to 0} b_1(g) = 0 \). By the first-order condition for \( b_1(g) \), this condition is satisfied if and only if \( \lim_{n \to 0} q_1(g) = \frac{n}{1-\alpha} \), where \( \alpha = \frac{\kappa_1 P_1^U(1-\kappa_1(g))}{(1-\kappa_1(g))(1-P_1^U)\kappa_1(g)} \). It is then easy to verify that \( \lim_{n \to 0} q_1(g) > \lim_{n \to 0} P_1(g) \).

Let’s now \( \{B_1(g)^*, B_1^*(b), B_2^*\} \) denote the optimal portfolio of an informed investor in informed equilibrium in the absence of secondary markets. By the Inada conditions, we must have that informed investors hold at least some risk-free bonds. Hence an informed investor with access to secondary markets could make pure arbitrage profits by investing all the wealth initially allocated to risk-free assets to good-state bonds in the primary market, and then selling them in the secondary market. This would allow the investor to hold the same portfolio of risky assets as in the absence of secondary markets, and increase the number of risk-free assets. Hence the utility is strictly higher. Since the utility of informed investors is unchanged, the result follows. **Q.E.D.**
B Appendix (for online publication): Additional Figures

B.1 Bids behind incentives to acquire information in Figure 3

Figure 6: Informed bids in informed equilibrium and upon a deviation. Preferences: $U(\cdot) = \log(\cdot)$.
Parameters: $W = 800$, $\kappa^g_1 = 0.1$, $\kappa^h_1 = 0.35$, $\kappa^U_1 = \kappa^U_2 = 0.275$, $D_1 = D_2 = 300$. 
Figure 7: Uninformed bids in informed equilibrium and upon a deviation. Preferences: $U(\cdot) = \log(\cdot)$.
Parameters: $W = 800$, $\kappa_1^g = 0.1$, $\kappa_1^h = 0.35$, $\kappa_1^l = \kappa_2^l = 0.275$, $D_1 = D_2 = 300$. 

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B.2 Bids behind prices in Figure 5

Figure 8: Informed portfolio with and without secondary markets. Preferences: $U(\cdot) = \log(\cdot)$. Parameters: $W = 800, \kappa_1^g = 0.1, \kappa_1^b = 0.35, \kappa_2^U = \kappa_2^U = 0.275, D_1 = D_2 = 300$.

Figure 9: Uninformed portfolio with and without secondary markets. Preferences: $U(\cdot) = \log(\cdot)$. Parameters: $W = 800, \kappa_1^g = 0.1, \kappa_1^b = 0.35, \kappa_2^U = \kappa_2^U = 0.275, D_1 = D_2 = 300$. 
Figure 10: Allocation of bids across auction and secondary market given $\theta_1 = g$. Preferences: $U(\cdot) = \log(\cdot)$. Parameters: $W = 800$, $\kappa^b_1 = 0.1$, $\kappa^b_2 = 0.35$, $\kappa^U_1 = \kappa^U_2 = 0.275$, $D_1=D_2 = 300$. 