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The Standard Model and Beyond

CRC PRESS
Boca Raton Ann Arbor London Tokyo

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Preface

In the last few decades there has been a tremendous advance in our understanding of the elementary particles and their interactions. We now have a mathematically consistent theory of the strong, electromagnetic, and weak interactions—the standard model—most aspects of which have been successfully tested in detail at colliders, accelerators, and non-accelerator experiments. It also provides a successful framework and has been strongly constrained by many observations in cosmology and astrophysics. The standard model is almost certainly an approximately correct description of Nature down to a distance scale $1/1000^{th}$ the size of the atomic nucleus.

However, nobody believes that the standard model is the ultimate theory: it is too complicated and arbitrary, does not provide an understanding of the patterns of fermion masses and mixings, does not incorporate quantum gravity, and it involves several severe fine-tunings. Furthermore, the origins of electroweak symmetry breaking, whether by the Higgs mechanism or something else, are uncertain. The recent discovery of non-zero neutrino mass can be incorporated, but in more than one way, with different implications for physics at very short distance scales. Finally, the observations of dark matter and energy suggest new particle physics beyond the standard model.

Most current activity is directed towards discovering the new physics which must underlie the standard model. Much of the theoretical effort involves constructing models of possible new physics at the TeV scale, such as supersymmetry or alternative models of spontaneous symmetry breaking. Others are examining the extremely promising ideas of superstring theory, which offer the hope of an ultimate unification of all interactions including gravity. There is a lively debate about the implications of a *landscape* of possible string vacua, and serious efforts are being made to explore the consequences of string theory for the TeV scale. It is likely that a combination of such bottom-up and top-down ideas will be necessary for progress. In any case, new experimental data are urgently needed. At the time of this writing the particle physics community is eagerly awaiting the results of the Large Hadron Collider (LHC) and is optimistic about a possible future International Linear Collider. Future experiments to elucidate the properties of neutrinos and to explore aspects of flavor, and more detailed probes of the dark energy and dark matter, are also anticipated.

The purpose of this volume is to provide an advanced introduction to the physics and formalism of the standard model and other non-abelian gauge theories, and thus to provide a thorough background for topics such as supersymmetry, string theory, extra dimensions, dynamical symmetry breaking, and cosmology. It is intended to provide the tools for a researcher to understand the structure and phenomenological consequences of the standard model, construct extensions, and to carry out calculations at tree level. Some "old-fashioned" topics which may still be useful are included. This is not a text on field theory, and does not substitute for the excellent texts that already exist. Ideally, the reader will have completed a standard field theory course. Nevertheless, Chapter 2 of this book presents a largely self-contained treatment of the complicated technology needed for tree-level calculations involving spin-0, spin-$\frac{1}{2}$, and spin-1 particles, and should be useful for those who have not studied field theory recently, or whose exposure has been more formal than calculational*. It does *not* attempt to deal systematically with the subtleties of renormalization, gauge issues, or higher-order corrections. An introductory-level background in the ideas of particle physics is assumed, with occasional reference to topics such as gluons or supersymmetry before they are formally introduced. Similarly, occasional reference is made to applications to and constraints from astrophysics and cosmology. The necessary background material may be found in the sources listed in the bibliography.

Chapter 1 is a short summary of notations and conventions and of some basic mathematical machinery. Chapter 2 contains a review of calculational techniques in field theory and the status of quantum electrodynamics. Chapters 3 and 4 are concerned with global and local symmetries and the construction of non-abelian gauge theories. Chapter 5 examines the strong interactions and the structure and tests of Quantum Chromodynamics (QCD). Chapters 6 and 7 examine the electroweak interactions and theory, including neutrino masses. Chapter 8 considers the motivations for extending the standard model, and examines supersymmetry, extended gauge groups, and grand unification. There are short appendices on additional topics. The bibliographies list many useful reference books, review articles, research papers, and Web sites. No attempt has been made to list all relevant original articles, with preference given instead to later articles and books that can be used to track down the original ones. Supplementary materials and corrections are available at `http://www.sns.ias.edu/~pgl/SMB/`. Comments, corrections, and typographical errors can also be sent through that site.

I would like to thank Mirjam Cvetič, Jens Erler, Hye-Sung Lee, Gil Paz, Liantao Wang, and Itay Yavin for reading and commenting on parts of the manuscript, Lisa Fleischer for help in the preparation of the manuscript, and my wife Irmgard for her extreme patience during the writing.

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July 4, 2009

*Most calculations, especially at the tree-level, are now carried out by specialized computer programs, many of which are included in the list of Web sites, but it is still important to understand the techniques that go into them.