The Standard Model and Beyond
Answers to Selected Problems

Paul Langacker

September 19, 2016

Contents

Chapter 2

Review of Perturbative Field Theory

2.1 Problems

Problem 2.9: For example, $P_L u(+) \to \frac{m}{2E}\sqrt{2E}\begin{pmatrix} \phi_+ \\ 0 \end{pmatrix}$, so the rates for the wrong helicity are suppressed by $m^2/4E^2$.

Problem 2.12: (a)

$$\frac{d\bar{\sigma}}{d\cos\theta} = \frac{1}{32\pi s}|\bar{M}|^2$$

with

$$|\bar{M}|^2 = h^4\Bigg[\left(\frac{1}{t-\mu^2}\right)^2\left(4m^2-t\right)^2 + \left(\frac{1}{u-\mu^2}\right)^2\left(4m^2-u\right)^2 \\ -\frac{1}{2}\left(\frac{1}{t-\mu^2}\right)\left(\frac{1}{u-\mu^2}\right)\left[\left(4m^2-u\right)^2+\left(4m^2-t\right)^2-\left(s-4m^2\right)^2\right]\Bigg],$$

where

$$E=\frac{\sqrt{s}}{2}, \qquad k=\frac{\sqrt{s-4m^2}}{2}, \qquad t=-2k^2(1-\cos\theta), \qquad u=-2k^2(1+\cos\theta).$$

(b)

$$\frac{d\bar{\sigma}}{d\cos\theta}=\frac{3h^4}{32\pi s}, \qquad \bar{\sigma}=\frac{1}{2}\int_{-1}^{1}d\cos\theta\,\frac{d\bar{\sigma}}{d\cos\theta}=\frac{3h^4}{32\pi s}.$$

Problem 2.14:

$$\frac{d\bar{\sigma}}{d\cos\theta}=\frac{\pi\alpha^2}{2s}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2,$$

which displays the singularity from the t-channel pole in the forward direction.

Problem 2.15:

$$\frac{d\bar{\sigma}}{d\cos\theta} = \frac{\pi\alpha^2}{s}\left[\frac{(1+\cos^4\frac{\theta}{2})}{\sin^4\frac{\theta}{2}} + \frac{(1+\sin^4\frac{\theta}{2})}{\cos^4\frac{\theta}{2}} + \frac{2}{\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}}\right]$$
$$= \frac{2\pi\alpha^2}{s}\left[\frac{16}{\sin^4\theta} - \frac{8}{\sin^2\theta} + 1\right].$$

Problem 2.16:

$$\frac{d\bar{\sigma}}{d\cos\theta} = \frac{\pi\alpha^2\beta_f^3}{4s}\sin^2\theta, \qquad \bar{\sigma} = \frac{\pi\alpha^2\beta_f^3}{3s}$$

Problem 2.17:

$$\frac{d\bar{\sigma}}{d\cos\theta} = \frac{(Z\alpha)^2\pi(1-\beta^2\cos^2\frac{\theta}{2})}{2\beta^2p^2\sin^4\frac{\theta}{2}} \xrightarrow[\beta\ll 1]{} \frac{Z^2\alpha^2\pi}{2\beta^2p^2\sin^4\frac{\theta}{2}}$$

Problem 2.18:

$$\Gamma = \frac{p_f}{8\pi M_Z^2}|\bar{M}|^2 \xrightarrow[m\to 0]{} \frac{M_Z}{12\pi}\left(g_V^2 + g_A^2\right).$$

Problem 2.19:

$$\frac{d\Gamma}{d\cos\theta} = \frac{p_f}{8\pi m_\Lambda}\left[\left(|g_S|^2 + |g_P|^2\right)E_p + \left(|g_S|^2 - |g_P|^2\right)m_p + 2\Re e\left(g_P g_S^*\right)p_f\cos\theta\right].$$

Problem 2.20:

$$\bar{\sigma}(s) = \frac{12\pi(s/M_V^2)\bar{\Gamma}_{a\bar{a}}\bar{\Gamma}_{b\bar{b}}}{(s-M_V^2)^2 + M_V^2\Gamma_V^2}, \qquad \bar{\Gamma}_{b\bar{b}} = \frac{g_b^2 M_V}{12\pi}.$$

Problem 2.22: (a)

$$-e\phi_{s_2}^\dagger\left(2m\tilde{A}_0(\vec{q}) - (\vec{p}_1+\vec{p}_2)\cdot\tilde{\vec{A}}(\vec{q}) - \vec{\sigma}\cdot\tilde{\vec{B}}(\vec{q})\left[1+F_2(0)\right]\right)\phi_{s_1}.$$

(b)

$$\vec{d}_e = \frac{ieG_2(0)}{2m}\vec{\sigma}.$$

Problem 2.23:

$$\begin{aligned}\langle p(2)|H|p(1)\rangle &= \langle p(2)| \int d^3\vec{x}\, eJ^\mu_Q A_\mu |p(1)\rangle \\ &= 2m_p\, e\, \phi_2^\dagger \left[G_E(Q^2)\tilde{A}_0(\vec{q}) - \frac{G_M(Q^2)}{2m_p}\vec{\sigma}\cdot\tilde{\vec{B}}(\vec{q})\right]\phi_1\end{aligned}$$

Problem 2.24: (a)

$$\tilde{V}(\vec{q}) = \lambda, \qquad V(\vec{r}\,) = \lambda\delta^3(\vec{r}\,).$$

(b)

$$\tilde{V}(\vec{q}) = -\frac{g_1 g_2}{|\vec{q}|^2 + m_\phi^2}, \qquad V(r) = -\frac{g_1 g_2}{4\pi}\frac{e^{-m_\phi r}}{r}.$$

(c)

$$V(r) = +\frac{g_1 g_2}{4\pi}\frac{e^{-M_V r}}{r}.$$

(d) Sign change for vector, not for scalar.

(e)

$$V(\vec{r}\,) = \frac{-g_\pi^2 m_\pi^3}{16\pi m_p^2}\left(\vec{\sigma}_p\cdot\vec{\nabla}_x\right)\left(\vec{\sigma}_n\cdot\vec{\nabla}_x\right)\left(\frac{e^{-x}}{x}\right).$$

Problem 2.25:

$$\Gamma = \frac{e^2\, m_\mu^3}{8\pi}\left(|A|^2 + |B|^2\right).$$

Chapter 3

Lie Groups, Lie Algebras, and Symmetries

3.1 Problems

Problem 3.4:

$$\begin{aligned}\cos\frac{\gamma}{2} &= \cos\frac{\alpha}{2}\cos\frac{\beta}{2} - \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\,\hat{\alpha}\cdot\hat{\beta}\\ \sin\frac{\gamma}{2}\,\hat{\gamma} &= \sin\frac{\alpha}{2}\cos\frac{\beta}{2}\,\hat{\alpha} + \cos\frac{\alpha}{2}\sin\frac{\beta}{2}\,\hat{\beta} + \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\,\hat{\alpha}\times\hat{\beta}.\end{aligned}$$

Problem 3.13:

$$\langle\pi^+p|M|\pi^+p\rangle = M_{3/2}, \qquad \langle\pi^-p|M|\pi^-p\rangle = \frac{1}{3}\,M_{3/2} + \frac{2}{3}\,M_{1/2}$$

$$\langle\pi^0n|M|\pi^-p\rangle = \frac{\sqrt{2}}{3}\,M_{3/2} - \frac{\sqrt{2}}{3}\,M_{1/2}, \qquad \langle\pi^0n|M|\pi^0n\rangle = \frac{2}{3}\,M_{3/2} + \frac{1}{3}\,M_{1/2}$$

Problem 3.14:

$$\begin{aligned}M(\Sigma^+\to\pi^0p) &= -\frac{\sqrt{2}}{3}\left(M_{3/2}-M_{1/2}\right)\\ M(\Sigma^+\to\pi^+n) &= \frac{1}{3}M_{3/2} + \frac{2}{3}M_{1/2}\\ M(\Sigma^-\to\pi^-n) &= M_{3/2}.\end{aligned}$$

Problem 3.16:

$$\bar{\sigma}_{\mathcal{A}^+p\to\mathcal{A}^+p} = \bar{\sigma}_{\mathcal{A}^0p\to\mathcal{A}^0p} = \frac{1}{4}\bar{\sigma}_{\mathcal{A}^+n\to\mathcal{A}^0p} = \frac{\lambda^2}{8\pi s}(E_1^2+m_p^2), \qquad E_1 = \frac{s+m_p^2-m_{\mathcal{A}}^2}{2\sqrt{s}}$$

Problem 3.19:

$$
\begin{aligned}
M_p = M_n \equiv M_N &= m_0 - \frac{1}{2\sqrt{3}} m_\beta + \frac{\sqrt{3}}{2} m_\alpha \\
M_{\Xi^0} = M_{\Xi^-} \equiv M_\Xi &= m_0 - \frac{1}{2\sqrt{3}} m_\beta - \frac{\sqrt{3}}{2} m_\alpha \\
M_{\Sigma^\pm} = M_{\Sigma^0} \equiv M_\Sigma &= m_0 + \frac{m_\beta}{\sqrt{3}} \\
M_\Lambda &= m_0 - \frac{m_\beta}{\sqrt{3}}
\end{aligned}
$$

Problem 3.22:

$$
\begin{aligned}
J^i_{L\mu} &= \bar{\psi}_L \gamma_\mu L^i_n \psi_L + i\mathrm{Tr}\left(\phi^\dagger L^i_n \overleftrightarrow{\partial}_\mu \phi\right) \\
J^i_{R\mu} &= \bar{\psi}_R \gamma_\mu L^i_n \psi_R + i\mathrm{Tr}\left(\phi L^i_n \overleftrightarrow{\partial}_\mu \phi^\dagger\right),
\end{aligned}
$$

and $L^i_n \to I$ for the $U(1)$ currents.

Problem 3.24:

$$
a = \nu = \sqrt{-\mu^2/\lambda}, \qquad b = \sqrt{-\mu^2/2}
$$

$$
\mathcal{H}(x) = \frac{\mu^4}{2\lambda} \operatorname{sech}^4\left(\frac{|\mu| x}{\sqrt{2}}\right), \qquad \int_{-\infty}^{+\infty} \mathcal{H}(x) dx = \frac{2\sqrt{2}}{3} \frac{|\mu|^3}{\lambda}
$$

Problem 3.26: (a)

$$
M = -2i\lambda \left[1 + \frac{3m_1^2}{t - m_1^2} + m_1^2 \left(\frac{1}{s} + \frac{1}{u}\right)\right]
$$

(b)

$$
M^I = i\left(\frac{m_\psi}{\nu}\right)^2 \bar{u}_4 \left[-\frac{m_\eta^2}{m_\psi} \frac{1}{t - m_\eta^2} + \gamma^5 \frac{1}{\not{p}_1 + \not{p}_2 - m_\psi} \gamma^5 + \gamma^5 \frac{1}{\not{p}_2 - \not{p}_3 - m_\psi} \gamma^5\right] u_2
$$

Problem 3.28:

$$
\psi' = \frac{1}{2}\left\{\left[\sqrt{a-b} + \sqrt{a+b}\, e^{i\phi_m}\right] + \left[-\sqrt{a-b} + \sqrt{a+b}\, e^{i\phi_m}\right]\gamma^5\right\}\psi \to -\gamma^5 \psi
$$

$$
|m| = \sqrt{\frac{c^2 + d^2}{a^2 - b^2}}, \qquad \tan\phi_m = \frac{d}{c}
$$

Problem 3.29:

$$\lambda' > 0 : \phi_1 = \nu_b \equiv \sqrt{-\mu^2/\lambda}, \qquad \phi_2 = 0, \qquad m_1^2 = 2\lambda\nu_b^2, \qquad m_2^2 = \lambda'\nu_b^2.$$

$\phi_2 \to -\phi_2$ survives. $\lambda' = 0$ is similar except there is a circle of degenerate minima.
For $\lambda' < 0$, require $2\lambda + \lambda' > 0$.

$$\phi_1 = \phi_2 = \nu_c \equiv \sqrt{-\mu^2/(2\lambda + \lambda')}, \qquad m_1^2 = -2\lambda'\nu_c^2, \qquad m_2^2 = 2(2\lambda + \lambda')\nu_c^2.$$

$\phi_1 \leftrightarrow \phi_2$ survives.

Problem 3.30:

$$\lambda_{12} > 0, \qquad \lambda_3 > 0, \qquad \mu^2 \geq |\mu_{12}^2| \geq 0, \qquad (\text{flat for } \mu^2 = |\mu_{12}^2|).$$

Problem 3.31: (a) $\mu_I^2 + \mu_{II}^2 - 2A > 0$.
(c) Minimum for $\mu_I^2\mu_{II}^2 - A^2 > 0$; saddle point for $\mu_I^2\mu_{II}^2 - A^2 < 0$.
(d,e)

$$\mu_A^2 = \mu_I^2 + \mu_{II}^2 = \frac{2A}{\sin 2\gamma}, \qquad \frac{1}{2}M_Z^2 = \lambda\nu^2 = \frac{\mu_I^2 - \mu_{II}^2 \tan^2\gamma}{\tan^2\gamma - 1}.$$

(f)

$$\mu_{Re}^2 = \begin{pmatrix} M_Z^2\cos^2\gamma + \mu_A^2\sin^2\gamma & -(M_Z^2 + \mu_A^2)\sin\gamma\cos\gamma \\ -(M_Z^2 + \mu_A^2)\sin\gamma\cos\gamma & M_Z^2\sin^2\gamma + \mu_A^2\cos^2\gamma \end{pmatrix}.$$

Problem 3.32: (b)

$$\begin{aligned} &\mu^2 > 0: \qquad m_1 = m_2 = \mu, \qquad m_\psi = 0 \\ &\mu^2 < 0: \qquad m_1^2 = -2\mu^2, \qquad m_2^2 = 0, \qquad m_\psi = h\nu/\sqrt{2}, \text{ with } \nu = \sqrt{-\mu^2/\lambda} \end{aligned}$$

(c)

$$\begin{aligned} &\mu^2 > 0: \qquad m_1^2 = \mu^2 + 3\lambda\nu^2, \qquad m_2^2 = \mu^2 + \lambda\nu^2, \qquad m_\psi = \frac{h\nu}{\sqrt{2}}, \text{ with } \nu \sim \sqrt{2}a/\mu^2 \\ &\mu^2 < 0: \qquad m_1^2 = -2\mu^2 + \frac{3\sqrt{2}a}{\nu_0}, \qquad m_2^2 = \frac{\sqrt{2}a}{\nu_0}, \qquad m_\psi = h\frac{\nu_0 + \epsilon}{\sqrt{2}}, \\ &\qquad\qquad \text{with } \nu = \nu_0 + \epsilon, \qquad \nu_0 = \sqrt{-\mu^2/\lambda}, \qquad \epsilon = -\frac{a}{\sqrt{2}\mu^2} \end{aligned}$$

Chapter 4

Gauge Theories

4.1 Problems

Problem 4.1:

$$A^\mu = (0, \frac{n\hat{\theta}}{rg}), \qquad \int \left(\vec{\nabla} \times \vec{A}\right) \cdot d\vec{S} = \frac{2\pi n}{g}.$$

Chapter 5

The Strong Interactions and QCD

5.1 Problems

Problem 5.1:

$$\bar{\sigma} = \frac{8\pi\alpha_s^2}{27s}\ \beta_Q \left(\frac{3-\beta_Q^2}{2}\right) \text{ with } \ \beta_Q = \sqrt{1 - \frac{4m_Q^2}{s}}$$

$$\bar{\sigma} = \frac{2\pi\alpha_s^2\beta_0^3}{27s} \text{ with } \ \beta_0 = \sqrt{1 - \frac{4m_0^2}{s}}.$$

Problem 5.2:

$$\sum_{r=1}^{F} \sigma_{q_0\bar{q}_0 \to q_r\bar{q}_r}\ \beta_{rel} = n_f \frac{4\pi\alpha_s^2}{27m_0^2}\ \beta_i^2, \qquad \sigma_{q_0\bar{q}_0 \to GG}\ \beta_{rel} = \frac{28}{27}\frac{\pi\alpha_s^2}{m_0^2}.$$

Problem 5.7: (b)Tr $\left(M_1^\dagger M_2\right)$, $\left[\text{Tr}\left(M_1^\dagger M_2\right)\right]\left[\text{Tr}\left(M_a^\dagger M_b\right)\right]$, Tr $\left(M_1^\dagger M_2\ M_1^\dagger M_2\right)$
(c) Tr $(M_a M_b)$, $\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$
(d) iTr $\left[\hat{n}\cdot\vec{\tau}M_1M_2^\dagger\right]$.

Chapter 6

The Weak Interactions

6.1 Problems

Problem 6.2: (a) $12\epsilon^2\,(1-\epsilon)$
(b) $\epsilon^2\big[(3-2\epsilon)+6(1-\epsilon)\big]$, $\qquad \rho=\frac{3}{8}$
(c): $\nu_\mu:\ 2\epsilon^2(3-2\epsilon)$, $\qquad \bar{\nu}_e:\ 12\epsilon^2\,(1-\epsilon)$.

Problem 6.10: (a), (b), (c) in text. (d): $\alpha_{e\nu}=\frac{g_V^2-g_A^2}{g_V^2+3g_A^2}$.

Problem 6.11:

$$\frac{d\Gamma(\pi\nu_\tau)}{d\cos\theta}=\frac{|M|^2}{32\pi m_\tau}=\frac{G_F^2\cos^2\theta_c f_\pi^2 m_\tau^3}{32\pi}(1+\cos\theta)$$

$$\Gamma(\pi\nu_\tau)=\frac{G_F^2\cos^2\theta_c f_\pi^2 m_\tau^3}{16\pi}.$$

Chapter 7

The Standard Electroweak Theory

7.1 Problems

Problem 7.4: (a)

$$V(\phi,\sigma) = \mu_\sigma^2 \sigma^\dagger \sigma + \mu_\phi^2 \phi^\dagger \phi + \lambda_\sigma \left(\sigma^\dagger \sigma\right)^2 + \lambda_\phi \left(\phi^\dagger \phi\right)^2 + \lambda_{\phi\sigma} \sigma^\dagger \sigma \phi^\dagger \phi + \kappa_{\phi\sigma} \sigma \phi^\dagger \tilde{\phi} + \kappa_{\phi\sigma}^* \sigma \tilde{\phi}^\dagger \phi.$$

(c) $M_A \sim \cos\theta_W g' \nu_\sigma$
(d)

$$\bar{\Gamma} = \frac{p_f}{6\pi M_Z^2} g'^2 \sin^2\theta_W M_A^2 \left(2 + \frac{E_A^2}{M_A^2}\right),$$

which survives for $M_A \to 0$ because of the longitudinal mode.

Problem 7.5:

$$m_1 = m_2 = 3c, \qquad \psi_L = \psi_L^0 = \begin{pmatrix} \psi_{1L}^0 \\ \psi_{2L}^0 \end{pmatrix}, \qquad \psi_R = i \begin{pmatrix} \psi_{2R}^0 \\ \psi_{1R}^0 \end{pmatrix}$$

Problem 7.6: (a)

$$m_1 \sim x, \qquad m_2 \sim B, \qquad A_R \sim I, \qquad A_L^\dagger = \begin{pmatrix} 1 & -\frac{y}{B} \\ \frac{y}{B} & 1 \end{pmatrix}.$$

(b)

$$J_Z^\mu = -\bar{e}_L \gamma^\mu e_L - \frac{y}{B}\left(\bar{e}_L \gamma^\mu E_L + \bar{E}_L \gamma^\mu e_L\right) - \left(\frac{y}{b}\right)^2 \bar{E}_L \gamma^\mu E_L + 2\sin^2\theta_W \left(\bar{e}\gamma^\mu e + \bar{E}\gamma^\mu E\right).$$

Problem 7.7:

$$|\bar{M}|^2 = \frac{g^4 M_Z^2 (g_V^2 + g_A^2)}{2\cos^4\theta_W} \frac{1}{(q^2 - M_Z^2)^2} \left[p_1 \cdot p_3 + \frac{2p_1 \cdot p_4 \; p_3 \cdot p_4}{M_Z^2} \right]$$

$$\frac{d\bar{\sigma}}{d\cos\theta} = \frac{1}{32\pi s} \; \frac{k_3}{k_1} \; |\bar{M}|^2.$$

$$k_1 = \frac{s - M_H^2}{2\sqrt{s}}, \qquad E_2 = \frac{s + M_H^2}{2\sqrt{s}}, \qquad k_3 = \frac{s - M_Z^2}{2\sqrt{s}}, \qquad E_4 = \frac{s + M_Z^2}{2\sqrt{s}}$$

$$q^2 = (p_3 - p_1)^2 = -2p_1 \cdot p_3 = -2k_1 k_3 \,(1 - \cos\theta)$$

$$p_1 \cdot p_4 = k_1 \,(E_4 + k_3 \cos\theta)\,, \qquad p_3 \cdot p_4 = \frac{s - M_Z^2}{2}.$$

Problem 7.10: (a)

$$\frac{d\Gamma}{d\cos\theta} = \frac{\hat{g}^2 E_b^2}{4\pi m_t} \left[2\,(1 + \cos\theta) + \frac{m_t^2}{M_W^2}\,(1 - \cos\theta) \right]$$

(b)

$$F_0 = \frac{m_t^2}{2M_W^2 + m_t^2} \simeq 0.70, \qquad F_- = 1 - F_0, \qquad F_+ = 0$$

Problem 7.14:

$$M = -i\sqrt{2} G_F M_H^2 \frac{s}{s - M_H^2}, \qquad \sigma = \frac{|M|^2}{16\pi s}$$

Problem 7.15: $|\bar{M}|^2 = 4\hat{g}^2 \left(\frac{m_t}{M_W}\right)^2 m_t E_b (1 - \cos\theta)$

Problem 7.21:

$$m_1 = -\epsilon, \qquad \nu_{1L} = \nu_{\mu L} - \nu_{\tau L}$$

$$m_{2,3} = \mp\sqrt{2} + \frac{\epsilon}{2}, \qquad \nu_{2,3L} = \left(\mp\sqrt{2} - \frac{\epsilon}{2} \right) \nu_{eL} + \nu_{\mu L} + \nu_{\tau L},$$

Problem 7.22:

$$2m_T \gamma^\mu (1 - \gamma^5) \gamma^\nu \to 2\gamma^\mu \left[(1 - \gamma^5) m_T + 2\epsilon \, \not{k} \right] \gamma^\nu + \mathcal{O}(\epsilon^2)$$

Problem 7.24:

$$\text{Dirac: } \bar{\sigma}\beta_{rel} = \frac{G_F^2}{\pi} m_\nu^2 \big[\epsilon_L^2 + \epsilon_R^2\big].$$

$$\text{Majorana: } \bar{\sigma}\beta_{rel} = \frac{8\beta^2}{3}\frac{G_F^2}{\pi} m_\nu^2 \big[\epsilon_L^2 + \epsilon_R^2\big].$$

Chapter 8

Beyond the Standard Model

8.1 Problems

Problem 8.2: For infinitesimal transformation

$$\mathcal{M}^\dagger \bar\sigma^0 \mathcal{M} \sim \bar\sigma^0 - \beta^i \bar\sigma^i, \qquad \mathcal{M}^\dagger \bar\sigma^i \mathcal{M} \sim \bar\sigma^i - \epsilon^{ijk}\omega^j \bar\sigma^k - \beta^i \bar\sigma^0$$

Problem 8.3:

$$\bar\psi_{1M}\psi_{2M} = \bar\xi_1\bar\xi_2 + \xi_1\xi_2, \qquad \bar\psi_{1M}\gamma^5\psi_{2M} = \bar\xi_1\bar\xi_2 - \xi_1\xi_2$$
$$\bar\psi_{1M}\gamma^\mu\psi_{2M} = \bar\xi_1\bar\sigma^\mu\xi_2 + \xi_1\sigma^\mu\bar\xi_2, \qquad \bar\psi_{1M}\gamma^\mu\gamma^5\psi_{2M} = \bar\xi_1\bar\sigma^\mu\xi_2 - \xi_1\sigma^\mu\bar\xi_2$$
$$\bar\psi_{1M}\sigma^{\mu\nu}\psi_{2M} = 2\bar\xi_1\,\bar{\mathfrak{s}}^{\mu\nu}\bar\xi_2 + 2\xi_1\,\mathfrak{s}^{\mu\nu}\xi_2$$

Problem 8.8:

$$\bar\sigma(Ah) = \cos^2(\beta-\alpha)\,\frac{g_Z^4}{96\pi}\,\frac{k_{Ah}^3}{k_i}\,\frac{\epsilon_L^2+\epsilon_R^2}{(s-M_Z^2)^2}$$
$$\bar\sigma(Zh) = \sin^2(\beta-\alpha)\,\frac{g_Z^4}{96\pi}\,\frac{k_{Zh}^3}{k_i}\,\frac{\epsilon_L^2+\epsilon_R^2}{(s-M_Z^2)^2}\left[1+\frac{3M_Z^2}{k_{Zh}^2}\right]$$

Problem 8.11: $\left(\tilde B\ \ \tilde W^3\ \ \tilde h_d^0\ \ \tilde h_u^0\ \ \tilde S\ \ \tilde Z'\right)$ basis:

$$\begin{pmatrix}
m_{\tilde B} & 0 & -\frac{g'\nu_d}{2} & \frac{g'\nu_u}{2} & 0 & 0 \\
0 & m_{\tilde W} & \frac{g\nu_d}{2} & -\frac{g\nu_u}{2} & 0 & 0 \\
-\frac{g'\nu_d}{2} & \frac{g\nu_d}{2} & 0 & -\lambda_S s & -\lambda_S\nu_u & g_2Q_d\nu_d \\
\frac{g'\nu_u}{2} & -\frac{g\nu_u}{2} & -\lambda_S s & 0 & -\lambda_S\nu_d & g_2Q_u\nu_u \\
0 & 0 & -\lambda_S\nu_u & -\lambda_S\nu_d & 0 & g_2Q_S s \\
0 & 0 & g_2Q_d\nu_d & g_2Q_u\nu_u & g_2Q_S s & m_{\tilde Z'}
\end{pmatrix}$$

Decoupling limit:

$$\begin{pmatrix} m_{\tilde{B}} & 0 & -\frac{g'\nu_d}{2} & \frac{g'\nu_u}{2} & 0 & 0 \\ 0 & m_{\tilde{W}} & \frac{g\nu_d}{2} & -\frac{g\nu_u}{2} & 0 & 0 \\ -\frac{g'\nu_d}{2} & \frac{g\nu_d}{2} & 0 & -\mu_{eff} & 0 & 0 \\ \frac{g'\nu_u}{2} & -\frac{g\nu_u}{2} & -\mu_{eff} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_2 Q_S s \\ 0 & 0 & 0 & 0 & g_2 Q_S s & 0 \end{pmatrix},$$

where $\mu_{eff} = \lambda_S s$. Vector, Dirac fermion, and scalar masses are all $g_2 Q_S s$

Problem 8.13:

$$(1 + 2\zeta \cos\omega) \sim 0.9999(10)$$

$|\zeta| \lesssim 0.0005$ for $\omega = 0$; $|\zeta| \lesssim 0.03$ for $\omega = \pi/2$

Problem 8.14:

$$V = \frac{15}{4}\mu^2 \nu_\Phi^2 + \frac{15}{16}(15a + 7b)\nu_\Phi^4$$