

Paul Langacker

***The Standard Model and
Beyond, Second Edition***



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Preface

PREFACE TO SECOND EDITION

Much has (or has *not*) happened in the seven years or so since the publication of the first edition of this book. Most notably, a new spin-0 particle with mass ~ 125 GeV was discovered at the LHC by the ATLAS and CMS collaborations, which is either the elementary Higgs boson or something that closely resembles it. The Higgs discovery completes the $\gtrsim 40$ year saga of verifying the standard model (SM). Moreover, its mass is almost maximally interesting; it is near the top of the range predicted by the most popular SM extension, minimal supersymmetry, and near the minimum value consistent with the unextended SM (and then only if the vacuum is metastable)!

However, the notorious problems of the SM are still unresolved. Perhaps the most pressing is the apparently fine-tuned hierarchy between the weak interaction and gravity scales. Extensive searches at the LHC and elsewhere have so far not yielded any compelling evidence for new TeV-scale physics such as supersymmetry, extra space dimensions, or strong coupling that had been proposed to explain or at least stabilize the hierarchy. Equally puzzling is the nature of the dark energy and its incredibly tiny magnitude compared to most theoretical expectations. Similarly, numerous experimental attempts to identify the mysterious dark matter inferred from its gravitational effects have not as yet had any positive results and have excluded much of the parameter space for supersymmetric dark matter. And despite the great experimental success of the SM, it is a very complicated theory, involving several interactions with different properties, and two apparently superfluous heavier copies of the fundamental particles that constitute ordinary matter under ordinary conditions. For these and other reasons, many theorists have started exploring less canonical possibilities, such as a dark matter sector that is at most very weakly coupled to ordinary particles, or, more radically, that the Universe is part of a vast *multiverse* of regions (presumably associated with a superstring landscape of vacua) with different laws of physics.

The existing experimental programs in high energy physics will continue for many years. These include high luminosity running at the LHC; active programs around the world in neutrino, flavor, and dark matter physics; and observational probes of the dark energy. There are also proposed next generation facilities such as new e^+e^- colliders that can serve as Higgs factories, and $\mathcal{O}(100)$ TeV hadron colliders. We will most likely find evidence for any multi-TeV scale physics relevant to the hierarchy problem or that is “just there” as a remnant of a more basic underlying theory; hopefully identify the dark matter and energy and shed light on the origin of the baryon asymmetry; perhaps progress toward a fundamental grand unification, superstring, or other theory that no one has yet imagined; and even reconsider such paradigms as naturalness, uniqueness, and minimality.¹

Like the first edition, this volume is intended to serve as a detailed text and reference on the formalism, technology, phenomenology, and experimental verification of the standard model and its possible extensions. In addition to updating all of the experimental and phenomenological results, it contains expanded discussions of collider, Higgs, neutrino, and dark matter physics, and includes many new problems. The book website at

¹For more extensive speculations along these lines, see, e.g., (Langacker, 2017).

www.sas.upenn.edu/~pg1/SMB2/ includes various supplemental materials, suggestions for use in a one-semester course, and corrections.

I would like to thank Vernon Barger and Jonathan Heckman for critiquing parts of this new edition, all those who have commented on the first one, and Irmgard for her extreme patience during the preparation of this new version.

Paul Langacker
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PREFACE TO FIRST EDITION

In the last few decades there has been a tremendous advance in our understanding of the elementary particles and their interactions. We now have a mathematically consistent theory of the strong, electromagnetic, and weak interactions—the standard model—most aspects of which have been successfully tested in detail at colliders, accelerators, and non-accelerator experiments. It also provides a successful framework and has been strongly constrained by many observations in cosmology and astrophysics. The standard model is almost certainly an approximately correct description of Nature down to a distance scale $1/1000th$ the size of the atomic nucleus.

However, nobody believes that the standard model is the ultimate theory: it is too complicated and arbitrary, does not provide an understanding of the patterns of fermion masses and mixings, does not incorporate quantum gravity, and it involves several severe fine-tunings. Furthermore, the origins of electroweak symmetry breaking, whether by the Higgs mechanism or something else, are uncertain. The recent discovery of non-zero neutrino mass can be incorporated, but in more than one way, with different implications for physics at very short distance scales. Finally, the observations of dark matter and energy suggest new particle physics beyond the standard model.

Most current activity is directed toward discovering the new physics which must underlie the standard model. Much of the theoretical effort involves constructing models of possible new physics at the TeV scale, such as supersymmetry or alternative models of spontaneous symmetry breaking. Others are examining the extremely promising ideas of superstring theory, which offer the hope of an ultimate unification of all interactions including gravity. There is a lively debate about the implications of a *landscape* of possible string vacua, and serious efforts are being made to explore the consequences of string theory for the TeV scale. It is likely that a combination of such bottom-up and top-down ideas will be necessary for progress. In any case, new experimental data are urgently needed. At the time of this writing, the particle physics community is eagerly awaiting the results of the Large Hadron Collider (LHC) and is optimistic about a possible future International Linear Collider. Future experiments to elucidate the properties of neutrinos and to explore aspects of flavor, and more detailed probes of the dark energy and dark matter, are also anticipated.

The purpose of this volume is to provide an advanced introduction to the physics and formalism of the standard model and other non-abelian gauge theories, and thus to provide a thorough background for topics such as supersymmetry, string theory, extra dimensions, dynamical symmetry breaking, and cosmology. It is intended to provide the tools for a researcher to understand the structure and phenomenological consequences of the standard model, construct extensions, and to carry out calculations at tree level. Some “old-fashioned” topics which may still be useful are included. This is not a text on field theory, and does not substitute for the excellent texts that already exist. Ideally, the reader will have completed a standard field theory course. Nevertheless, Chapter 2 of this book presents a largely self-contained treatment of the complicated technology needed for tree-level calculations involving spin-0, spin- $\frac{1}{2}$, and spin-1 particles, and should be useful for those who have not

studied field theory recently, or whose exposure has been more formal than calculational.² It does *not* attempt to deal systematically with the subtleties of renormalization, gauge issues, or higher-order corrections. An introductory-level background in the ideas of particle physics is assumed, with occasional reference to topics such as gluons or supersymmetry before they are formally introduced. Similarly, occasional reference is made to applications to and constraints from astrophysics and cosmology. The necessary background material may be found in the sources listed in the bibliography.

Chapter 1 is a short summary of notations and conventions and of some basic mathematical machinery. Chapter 2 contains a review of calculational techniques in field theory and the status of quantum electrodynamics. Chapters 3 and 4 are concerned with global and local symmetries and the construction of non-abelian gauge theories. Chapter 5 examines the strong interactions and the structure and tests of Quantum Chromodynamics (QCD). Chapters³ 6 and 7 examine the electroweak interactions and theory, including neutrino masses. Chapter 8 considers the motivations for extending the standard model, and examines supersymmetry, extended gauge groups, and grand unification. There are short appendices on additional topics. The bibliographies list many useful reference books, review articles, research papers, and websites. No attempt has been made to list all relevant original articles, with preference given instead to later articles and books that can be used to track down the original ones. Supplementary materials and corrections are available at <http://www.sns.ias.edu/~pjl/SMB/>. Comments, corrections, and typographical errors can also be sent through that site.

I would like to thank Mirjam Cvetič, Jens Erler, Hye-Sung Lee, Gil Paz, Liantao Wang, and Itay Yavin for reading and commenting on parts of the manuscript, Lisa Fleischer for help in the preparation of the manuscript, and my wife Irmgard for her extreme patience during the writing.

Paul Langacker
July 4, 2009

²Most calculations, especially at the tree-level, are now carried out by specialized computer programs, many of which are included in the list of websites, but it is still important to understand the techniques that go into them. Some examples may be found in the notebooks on the book website.

³These chapter numbers refer to the first edition.