Paul Langacker

The Standard Model and Beyond, Second Edition

Answers to Selected Problems

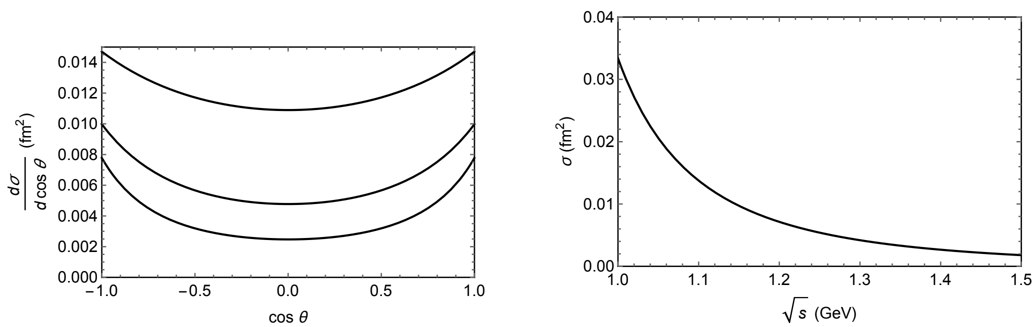
Contents

CHAPTER 2

Review of Perturbative Field Theory

2.1 PROBLEMS

Problem 2.2:



Left: $d\sigma/d\cos\theta$ in units of fm^2 as a function of $\cos\theta$ for $s = 5m^2,\ 6m^2$, and $7m^2$ (from upper to lower). Right: $\sigma(s)$ in units of fm^2 as a function of $\sqrt{s}$

Problem 2.5:

$$M_{fi} = (-i\kappa)^2 \left[\frac{i}{s - m_3^2} + \frac{i}{u - m_3^2} \right]$$

$$\frac{d\sigma}{d\cos\theta} = \frac{|M_{fi}|^2}{32\pi s}, \qquad u = -2p^2(1+\cos\theta), \qquad p = \sqrt{s - 4m_1^2}/2$$

Problem 2.6: $M_{fi} = -24i\sigma_4$

Problem 2.7:

$$\frac{d\Gamma}{d\cos\theta} = \frac{\alpha M_A}{8}\left(1 - \frac{4m^2}{M_A^2}\right)^{3/2}\cos^2\theta, \qquad \bar{\Gamma} = \frac{\alpha M_A}{12}\left(1 - \frac{4m^2}{M_A^2}\right)^{3/2}$$

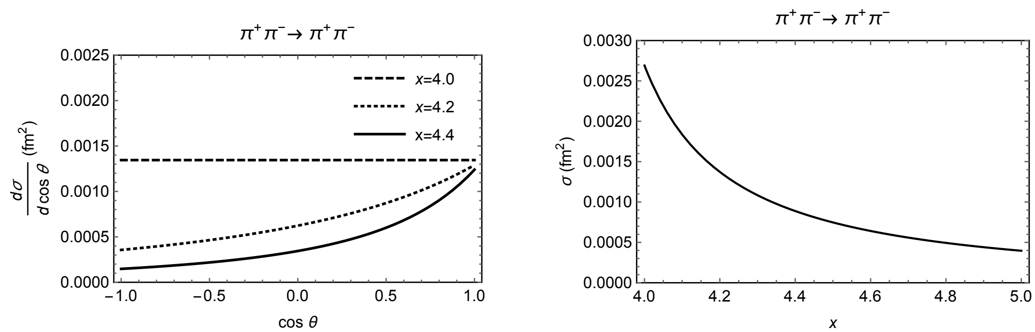
Problem 2.8: (a)

$$M_{fi} = (-ig)^2 \left(\frac{i}{s-\mu^2} + \frac{i}{t-\mu^2} \right),$$

$$t = -2p^2(1-\cos\theta), \qquad p = \frac{\sqrt{s-4m^2}}{2}, \qquad E = \frac{\sqrt{s}}{2},$$

$$\frac{d\sigma}{d\cos\theta} = \frac{|M_{fi}|^2}{32\pi s}, \qquad \sigma = \int_{-1}^{+1} \frac{d\sigma}{d\cos\theta} d\cos\theta.$$

(b,c)



Left: $d\sigma/d\cos\theta$ in units of fm^2 for $x = 4, 4.2$, and 4.4. Right: σ in units of fm^2 vs x.

Problem 2.15: For example, $P_L u(+) \to \frac{m}{2E}\sqrt{2E} \begin{pmatrix} \phi_+ \\ 0 \end{pmatrix}$, so the rates for the wrong helicity are suppressed by $m^2/4E^2$.

Problem 2.18: (a)

$$\frac{d\bar{\sigma}}{d\cos\theta} = \frac{1}{32\pi s}|\bar{M}|^2$$

with

$$|\bar{M}|^2 = h^4 \Bigg[\left(\frac{1}{t-\mu^2}\right)^2 \left(4m^2 - t\right)^2 + \left(\frac{1}{u-\mu^2}\right)^2 \left(4m^2-u\right)^2 - \frac{1}{2}\left(\frac{1}{t-\mu^2}\right)\left(\frac{1}{u-\mu^2}\right)\left[\left(4m^2-u\right)^2 + \left(4m^2-t\right)^2 - \left(s-4m^2\right)^2\right]\Bigg],$$

where

$$E = \frac{\sqrt{s}}{2}, \qquad k = \frac{\sqrt{s-4m^2}}{2}, \qquad t = -2k^2(1-\cos\theta), \qquad u = -2k^2(1+\cos\theta).$$

(b)

$$\frac{d\bar{\sigma}}{d\cos\theta} = \frac{3h^4}{32\pi s}, \qquad \bar{\sigma} = \frac{1}{2}\int_{-1}^{1} d\cos\theta \frac{d\bar{\sigma}}{d\cos\theta} = \frac{3h^4}{32\pi s}.$$

Problem 2.20:

$$\frac{d\bar{\sigma}}{d\cos\theta} = 4\pi\alpha^2 \frac{p^2}{st^2}\left[s + (E + p\cos\theta)^2 - m_\mu^2(1-\cos\theta)\right]$$

Problem 2.21:

$$\frac{d\bar{\sigma}}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2,$$

which displays the singularity from the t-channel pole in the forward direction.

Problem 2.22:

$$\begin{aligned}\frac{d\bar{\sigma}}{d\cos\theta} &= \frac{\pi\alpha^2}{s}\left[\frac{(1+\cos^4\frac{\theta}{2})}{\sin^4\frac{\theta}{2}} + \frac{(1+\sin^4\frac{\theta}{2})}{\cos^4\frac{\theta}{2}} + \frac{2}{\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}}\right] \\ &= \frac{2\pi\alpha^2}{s}\left[\frac{16}{\sin^4\theta} - \frac{8}{\sin^2\theta} + 1\right].\end{aligned}$$

Problem 2.23:

$$\frac{d\bar{\sigma}}{d\cos\theta} = \frac{\pi\alpha^2\beta_f^3}{4s}\sin^2\theta, \qquad \bar{\sigma} = \frac{\pi\alpha^2\beta_f^3}{3s}$$

Problem 2.24:

$$\frac{d\bar{\sigma}}{d\cos\theta} = \frac{(Z\alpha)^2\pi(1-\beta^2\cos^2\frac{\theta}{2})}{2\beta^2p^2\sin^4\frac{\theta}{2}} \xrightarrow[\beta\ll 1]{} \frac{Z^2\alpha^2\pi}{2\beta^2p^2\sin^4\frac{\theta}{2}}$$

Problem 2.25:

$$\Gamma = \frac{p_f}{8\pi M_Z^2}|\bar{M}|^2 \xrightarrow[m\to 0]{} \frac{M_Z}{12\pi}\left(g_V^2 + g_A^2\right).$$

Problem 2.26:

$$\begin{aligned}\frac{d\Gamma}{d\cos\theta} =& \frac{p_f}{8\pi m_\Lambda}\left[\left(|g_S|^2 + |g_P|^2\right)E_p + \left(|g_S|^2 - |g_P|^2\right)m_p \right.\\ &\left. + 2\Re e\left(g_P g_S^*\right)p_f\cos\theta\right].\end{aligned}$$

Problem 2.27:

$$\bar{\sigma}(s) = \frac{12\pi(s/M_V^2)\bar{\Gamma}_{a\bar{a}}\bar{\Gamma}_{b\bar{b}}}{(s-M_V^2)^2 + M_V^2\Gamma_V^2}, \qquad \bar{\Gamma}_{b\bar{b}} = \frac{g_b^2 M_V}{12\pi}.$$

Problem 2.29:

$$M(-,-) = M(+,+) = 8\pi\alpha\, i\left(\frac{1}{\beta_\pi} + 1\right)\frac{\cos\frac{\theta}{2}}{1-\cos\theta}.$$

Problem 2.30: (a)

$$-e\phi_{s_2}^\dagger \left(2m\tilde{A}_0(\vec{q}) - (\vec{p}_1+\vec{p}_2)\cdot\tilde{\vec{A}}(\vec{q}) - \vec{\sigma}\cdot\tilde{\vec{B}}(\vec{q})\,[1+F_2(0)]\right)\phi_{s_1}.$$

(b)

$$\vec{d}_e = \frac{ieG_2(0)}{2m}\vec{\sigma}.$$

Problem 2.31:

$$\tau^{-1} = \frac{1}{4}\delta^2\alpha\, m_e, \qquad \delta < 5.8\times 10^{-28}.$$

Problem 2.32:

$$\begin{aligned}\langle p(2)|H|p(1)\rangle &= \langle p(2)|\int d^3\vec{x}\, eJ_Q^\mu A_\mu |p(1)\rangle \\ &= 2m_p\, e\,\phi_2^\dagger\left[G_E(Q^2)\tilde{A}_0(\vec{q}) - \frac{G_M(Q^2)}{2m_p}\vec{\sigma}\cdot\tilde{\vec{B}}(\vec{q})\right]\phi_1\end{aligned}$$

Problem 2.33: (a)

$$\tilde{V}(\vec{q}) = \lambda, \qquad V(\vec{r}) = \lambda\delta^3(\vec{r}).$$

(b)

$$\tilde{V}(\vec{q}) = -\frac{g_1 g_2}{|\vec{q}|^2 + m_\phi^2}, \qquad V(r) = -\frac{g_1 g_2}{4\pi}\frac{e^{-m_\phi r}}{r}.$$

(c)

$$V(r) = +\frac{g_1 g_2}{4\pi}\frac{e^{-M_V r}}{r}.$$

(d) Sign change for vector, not for scalar.

(e)

$$V(\vec{r}) = \frac{-g_\pi^2 m_\pi^3}{16\pi m_p^2}\left(\vec{\sigma}_p\cdot\vec{\nabla}_x\right)\left(\vec{\sigma}_n\cdot\vec{\nabla}_x\right)\left(\frac{e^{-x}}{x}\right).$$

Problem 2.34:

$$\Gamma = \frac{e^2\, m_\mu^3}{8\pi}\left(|A|^2 + |B|^2\right).$$

CHAPTER 3

Lie Groups, Lie Algebras, and Symmetries

3.1 PROBLEMS

Problem 3.4:

$$\cos\frac{\gamma}{2} = \cos\frac{\alpha}{2}\cos\frac{\beta}{2} - \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\,\hat{\alpha}\cdot\hat{\beta}$$
$$\sin\frac{\gamma}{2}\,\hat{\gamma} = \sin\frac{\alpha}{2}\cos\frac{\beta}{2}\,\hat{\alpha} + \cos\frac{\alpha}{2}\sin\frac{\beta}{2}\,\hat{\beta} + \sin\frac{\alpha}{2}\sin\frac{\beta}{2}\,\hat{\alpha}\times\hat{\beta}.$$

Problem 3.13:

$$\langle\pi^+p|M|\pi^+p\rangle = M_{3/2}, \qquad \langle\pi^-p|M|\pi^-p\rangle = \frac{1}{3}\,M_{3/2} + \frac{2}{3}\,M_{1/2}$$
$$\langle\pi^0n|M|\pi^-p\rangle = \frac{\sqrt{2}}{3}\,M_{3/2} - \frac{\sqrt{2}}{3}\,M_{1/2}, \qquad \langle\pi^0n|M|\pi^0n\rangle = \frac{2}{3}\,M_{3/2} + \frac{1}{3}\,M_{1/2}$$

Problem 3.14:

$$M(\Sigma^+\to\pi^0p) = \frac{\sqrt{2}}{3}\left(M_{3/2} - M_{1/2}\right)$$
$$M(\Sigma^+\to\pi^+n) = \frac{1}{3}M_{3/2} + \frac{2}{3}M_{1/2}$$
$$M(\Sigma^-\to\pi^-n) = M_{3/2}.$$

Problem 3.16:

$$\bar{\sigma}_{\mathcal{A}^+p\to\mathcal{A}^+p} = \bar{\sigma}_{\mathcal{A}^0p\to\mathcal{A}^0p} = \frac{1}{4}\bar{\sigma}_{\mathcal{A}^+n\to\mathcal{A}^0p} = \frac{\lambda^2}{8\pi s}(E_1^2 + m_p^2), \qquad E_1 = \frac{s + m_p^2 - m_{\mathcal{A}}^2}{2\sqrt{s}}$$

Problem 3.19:

$$g_{K^+n\Sigma^-} = 2g_{\pi^+pn} + \sqrt{6}g_{K^+p\Lambda}$$

Problem 3.20:

$$\begin{aligned}
M_p = M_n \equiv M_N &= m_0 - \frac{1}{2\sqrt{3}}m_\beta + \frac{\sqrt{3}}{2}m_\alpha \\
M_{\Xi^0} = M_{\Xi^-} \equiv M_\Xi &= m_0 - \frac{1}{2\sqrt{3}}m_\beta - \frac{\sqrt{3}}{2}m_\alpha \\
M_{\Sigma^\pm} = M_{\Sigma^0} \equiv M_\Sigma &= m_0 + \frac{m_\beta}{\sqrt{3}} \\
M_\Lambda &= m_0 - \frac{m_\beta}{\sqrt{3}}
\end{aligned}$$

Problem 3.22: (b) $g_p/g_n = -3/2$ (exp: -1.46)
(c) $g_p/2 = 3$ (exp: 2.79); $g_n/2 = -2$ (exp: -1.91)

Problem 3.23:

$$\begin{aligned}
M_{\pi^0 p} &= \left(\frac{g_f - g_d}{2}\right)(-ie)\bar{u}_2\left[\gamma^5 \frac{i(\not k_1 + \not p_1 + m)}{(k_1+p_1)^2 - m^2}\not\epsilon + \not\epsilon \frac{i(\not p_1 - \not k_2 + m)}{(p_1 - k_2)^2 - m^2}\gamma^5\right]u_1 \\
M_{K^+\Sigma^0} &= -\left(\frac{g_f + g_d}{2}\right)(-ie)\bar{u}_2\left[\gamma^5 \frac{i(\not k_1 + \not p_1 + m)}{(k_1+p_1)^2 - m^2}\not\epsilon + 2k_2\cdot\epsilon \frac{i}{(k_2 - k_1)^2 - \mu^2}\gamma^5\right]u_1
\end{aligned}$$

Problem 3.25: (a)

$$\left(L^i_A\right)_{ab;cd} = L^i_{ac}\delta_{bd} + L^i_{bd}\delta_{ac} - L^i_{ad}\delta_{bc} - L^i_{bc}\delta_{ad}, \qquad T(L_A) = \frac{m-2}{2}.$$

(b)

$$\left(L^i_S\right)_{ab;cd} = 2c_{ab}\, c_{cb}\left[L^i_{ac}\delta_{bd} + L^i_{bd}\delta_{ac} + L^i_{ad}\delta_{bc} + L^i_{bc}\delta_{ad}\right], \qquad T(L_S) = \frac{m+2}{2}.$$

For $m = 2$, ψ^S_{11}, ψ^S_{12} and ψ^S_{22} correspond to the $J = 1$ angular momentum representation with $J^3 = +1, 0$, and -1, respectively. For example,

$$\left(L^3_S\right)_{11;11} = \frac{1}{4}[4] = 1, \quad \left(L^3_S\right)_{12;12} = \frac{1}{4}[1-1] = 0, \quad \left(L^1_S\right)_{11;12} = \frac{1}{2}\frac{1}{\sqrt{2}}[0+1+1+0] = \frac{1}{\sqrt{2}}.$$

Problem 3.26:

$$\begin{aligned}
J^i_{L\mu} &= \bar{\psi}_L\gamma_\mu L^i_n\psi_L + i\mathrm{Tr}\left(\phi^\dagger L^i_n \overleftrightarrow{\partial}_\mu\, \phi\right) \\
J^i_{R\mu} &= \bar{\psi}_R\gamma_\mu L^i_n\psi_R + i\mathrm{Tr}\left(\phi L^i_n \overleftrightarrow{\partial}_\mu\, \phi^\dagger\right),
\end{aligned}$$

and $L^i_n \to I$ for the $U(1)$ currents.

Problem 3.28:

$$a = \nu = \sqrt{-\mu^2/\lambda}, \qquad b = \sqrt{-\mu^2/2}$$

$$\mathcal{H}(x) = \frac{\mu^4}{2\lambda}\,\mathrm{sech}^4\left(\frac{|\mu|x}{\sqrt{2}}\right), \qquad \int_{-\infty}^{+\infty} \mathcal{H}(x)dx = \frac{2\sqrt{2}}{3}\frac{|\mu|^3}{\lambda}$$

Problem 3.30: (a)

$$M = -2i\lambda\left[1 + \frac{3m_1^2}{t - m_1^2} + m_1^2\left(\frac{1}{s} + \frac{1}{u}\right)\right]$$

(b)

$$M^I = i\left(\frac{m_\psi}{\nu}\right)^2 \bar{u}_4\left[-\frac{m_\eta^2}{m_\psi}\,\frac{1}{t - m_\eta^2} + \gamma^5\frac{1}{\not{p}_1 + \not{p}_2 - m_\psi}\gamma^5 + \gamma^5\frac{1}{\not{p}_2 - \not{p}_3 - m_\psi}\gamma^5\right]u_2$$

Problem 3.32:

$$\psi' = \frac{1}{2}\left\{\left[\sqrt{a-b} + \sqrt{a+b}\,e^{i\phi_m}\right] + \left[-\sqrt{a-b} + \sqrt{a+b}\,e^{i\phi_m}\right]\gamma^5\right\}\psi \to -\gamma^5\psi$$

$$|m| = \sqrt{\frac{c^2+d^2}{a^2-b^2}}, \qquad \tan\phi_m = \frac{d}{c}$$

Problem 3.33:

$$\lambda' > 0 : \phi_1 = \nu_b \equiv \sqrt{-\mu^2/\lambda}, \qquad \phi_2 = 0, \qquad m_1^2 = 2\lambda\nu_b^2, \qquad m_2^2 = \lambda'\nu_b^2.$$

$\phi_2 \to -\phi_2$ survives. $\lambda' = 0$ is similar except there is a circle of degenerate minima. For $\lambda' < 0$, require $2\lambda + \lambda' > 0$.

$$\phi_1 = \phi_2 = \nu_c \equiv \sqrt{-\mu^2/(2\lambda+\lambda')}, \qquad m_1^2 = -2\lambda'\nu_c^2, \qquad m_2^2 = 2(2\lambda+\lambda')\nu_c^2.$$

$\phi_1 \leftrightarrow \phi_2$ survives.

Problem 3.34:

$$\lambda_{12} > 0, \qquad \lambda_3 > 0, \qquad \mu^2 \geq |\mu_{12}^2| \geq 0, \qquad (\text{flat for } \mu^2 = |\mu_{12}^2|).$$

Problem 3.35: (a) $\mu_I^2 + \mu_{II}^2 - 2A > 0$.
(c) Minimum for $\mu_I^2\mu_{II}^2 - A^2 > 0$; saddle point for $\mu_I^2\mu_{II}^2 - A^2 < 0$.
(d,e)

$$\mu_A^2 = \mu_I^2 + \mu_{II}^2 = \frac{2A}{\sin 2\gamma}, \qquad \frac{1}{2}M_Z^2 = \lambda\nu^2 = \frac{\mu_I^2 - \mu_{II}^2\tan^2\gamma}{\tan^2\gamma - 1}.$$

(f)

$$\mu_{Re}^2 = \begin{pmatrix} M_Z^2\cos^2\gamma + \mu_A^2\sin^2\gamma & -(M_Z^2+\mu_A^2)\sin\gamma\cos\gamma \\ -(M_Z^2+\mu_A^2)\sin\gamma\cos\gamma & M_Z^2\sin^2\gamma + \mu_A^2\cos^2\gamma \end{pmatrix}.$$

Problem 3.36: (b)

$$\mu^2 > 0: \qquad m_1 = m_2 = \mu, \qquad m_\psi = 0$$

$$\mu^2 < 0: \qquad m_1^2 = -2\mu^2, \qquad m_2^2 = 0, \qquad m_\psi = h\nu/\sqrt{2}, \text{ with } \nu = \sqrt{-\mu^2/\lambda}$$

(c)

$$\mu^2 > 0: \qquad m_1^2 = \mu^2 + 3\lambda\nu^2, \qquad m_2^2 = \mu^2 + \lambda\nu^2, \qquad m_\psi = \frac{h\nu}{\sqrt{2}}, \text{ with } \nu \sim \sqrt{2}a/\mu^2$$

$$\mu^2 < 0: \qquad m_1^2 = -2\mu^2 + \frac{3\sqrt{2}a}{\nu_0}, \qquad m_2^2 = \frac{\sqrt{2}a}{\nu_0}, \qquad m_\psi = h\frac{\nu_0 + \epsilon}{\sqrt{2}},$$

$$\text{with } \nu = \nu_0 + \epsilon, \qquad \nu_0 = \sqrt{-\mu^2/\lambda}, \qquad \epsilon = -\frac{a}{\sqrt{2}\mu^2}$$

(e)

$$\partial^\mu J_\mu = -i\frac{m_L}{2}(\bar{\psi}_L \mathcal{C} \bar{\psi}_L^T - \psi_L^T \mathcal{C} \psi_L)$$

CHAPTER 4

Gauge Theories

4.1 PROBLEMS

Problem 4.1:

$$A^\mu = \big(0, \frac{n\hat{\theta}}{rg}\big), \qquad \int \left(\vec{\nabla} \times \vec{A}\right) \cdot d\vec{S} = \frac{2\pi n}{g}.$$

Problem 4.11: (a)

$$-g\gamma_\mu/\sqrt{2}, \qquad -g\gamma_\mu/2, \qquad -igC_{\mu\nu\sigma}(-p_3, p_3+p_4, -p_4)$$

(b)

$$M = \frac{-ig^2}{2}\bar{v}_2 \not{\epsilon}_-^* \frac{1}{\not{p}_1 - \not{p}_3 - m_\psi} \not{\epsilon}_+^* u_1 + \frac{ig^2}{2}\frac{1}{s - M_A^2} \bar{v}_2[2 \not{\epsilon}_+^* p_3 \cdot \epsilon_-^* + \epsilon_+^* \cdot \epsilon_-^* (\not{p}_4 - \not{p}_3) - 2 \not{\epsilon}_-^* p_4 \cdot \epsilon_+^*] u_1.$$

(c)

$$M \to \frac{ig^2}{2M_A^2}\bar{v}_2 \not{p}_4 u_1 - \frac{ig^2}{2M_A^2}\bar{v}_2 \not{p}_4 u_1 = 0$$

Problem 4.12: (a) $A_{ijk} = d_{ijk}$.
(b) $A^D_{ijk} = 2md_{ijk}$. One can also show that $A^A_{ijk} = (m-4)d_{ijk}$ and $A^S_{ijk} = (m+4)d_{ijk}$ for the antisymmetric (A) and symmetric (D) products of two fundamentals.

CHAPTER 5

The Strong Interactions and QCD

5.1 PROBLEMS

Problem 5.1: (b) $b = -1$, $c = -1/\sqrt{3}$.
(c) $R = 4(2)$ for HN (QCD); Drell Yan $\propto 5/9$ $(1/3)$; anomaly coefficient $= -4$ for both.

Problem 5.2:

$$|\bar{M}|^2 = \frac{4}{9}\,\frac{g_s^4}{t^2}\,\left[(s-m_c^2)^2 + (u-m_c^2)^2 + 2m_c^2 t\right]$$

$$u - m_c^2 = -2p(E + p\cos\theta), \qquad t = -2p^2(1-\cos\theta), \qquad E = \frac{s+m_c^2}{2\sqrt{s}}, \qquad p = \frac{s-m_c^2}{2\sqrt{s}}$$

Problem 5.3: (a)

$$\begin{aligned}\mathcal{L}_\phi =& \mathrm{Tr}\left[(D_\mu\phi)^\dagger D^\mu\phi\right] - \mu^2 \mathrm{Tr}\,(\phi^\dagger\phi) - \lambda_1\left(\mathrm{Tr}\,(\phi^\dagger\phi)\right)^2 - \lambda_2 \mathrm{Tr}\,(\phi^\dagger\phi\phi^\dagger\phi)\\ &+ \bar{q}_L h\phi q_R + \bar{q}_R h^\dagger\phi^\dagger q_L,\end{aligned}$$

where

$$D^\mu\phi = \partial^\mu\phi + i g_L\,\vec{G}_L^\mu\cdot\vec{L}\,\phi - i g_R\,\phi\,\vec{G}_R^\mu\cdot\vec{L}.$$

(b,c)

$$G^i = \sin\delta\,G_L^i + \cos\delta\,G_R^i, \qquad G_A^i = \cos\delta\,G_L^i - \sin\delta\,G_R^i,$$

where $\tan\delta \equiv g_R/g_L$ and $M_{G_A} = \sqrt{g_L^2+g_R^2}\,v_\phi$.

$$\mathcal{L}_q = -g_s\,\bar{q}\,\vec{\not{G}}\cdot\vec{L}\,q - g_s\cot\delta\,\bar{q}_L\,\vec{\not{G}}_A\cdot\vec{L}\,q_L + g_s\,\tan\delta\,\bar{q}_R\,\vec{\not{G}}_A\cdot\vec{L}\,q_R,$$

where $g_s \equiv g_L\,\sin\delta = g_R\,\cos\delta$; $T^i = T_L^i + T_R^i$.

Problem 5.4:

$$\bar{\sigma} = \frac{8\pi\alpha_s^2}{27s}\,\beta_Q\left(\frac{3-\beta_Q^2}{2}\right) \text{ with } \beta_Q = \sqrt{1-\frac{4m_Q^2}{s}}$$

$$\bar{\sigma} = \frac{2\pi\alpha_s^2\beta_0^3}{27s} \text{ with } \beta_0 = \sqrt{1-\frac{4m_0^2}{s}}.$$

Problem 5.5:

$$\sum_{r=1}^{F} \sigma_{q_0\bar{q}_0\to q_r\bar{q}_r}\ \beta_{rel} = n_f \frac{4\pi\alpha_s^2}{27m_0^2}\ \beta_i^2, \qquad \sigma_{q_0\bar{q}_0\to GG}\ \beta_{rel} = \frac{28}{27}\frac{\pi\alpha_s^2}{m_0^2}.$$

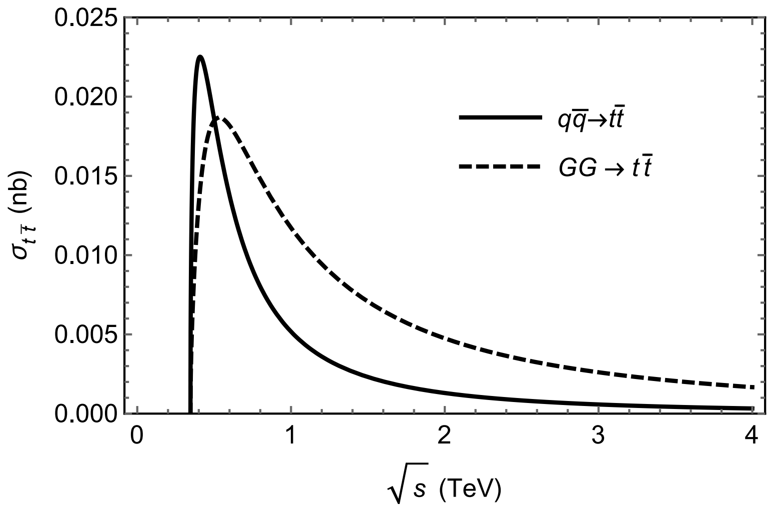
Problem 5.6: (a)

$$|\bar{M}|^2/g_s^4 = \frac{4\left(s^2+u^2-2m_r^2\left(-2m_s^2+s-t+u\right)+2m_r^4-2m_s^2(s-t+u)+2m_s^4\right)}{9t^2}$$

(b)

$$|\bar{M}|^2/g_s^4 = -\frac{\left(7m^4-7m^2t-7m^2u+4t^2-tu+4u^2\right)}{24s^2\left(m^2-t\right)^2\left(m^2-u\right)^2}$$
$$\times\left(6m^8-3m^4t^2-14m^4tu-3m^4u^2+m^2t^3+7m^2t^2u+7m^2tu^2+m^2u^3-t^3u-tu^3\right)$$

(d) Use the expressions for t and u in (2.40) for $m_{1,2}=0, m_{3,4}=m_t$, and the cross section formula in (2.57), integrating $\cos\theta$ from -1 to 1.



Problem 5.8: (a) $B=7$, $\alpha(M_P^2)\sim 0.019$; (b) $B=3$, $\alpha(M_P^2)\sim 0.037$

Problem 5.10:

	charmonium			bottomonium		
	exp	model	difference	exp	model	difference
$1S$	3.097	3.097	–	9.490	9.490	–
$2S$	3.686	3.710	0.64%	10.023	9.927	−0.96%
$3S$				10.355	10.251	−1.01%
$4S$				10.579	10.528	−0.48%

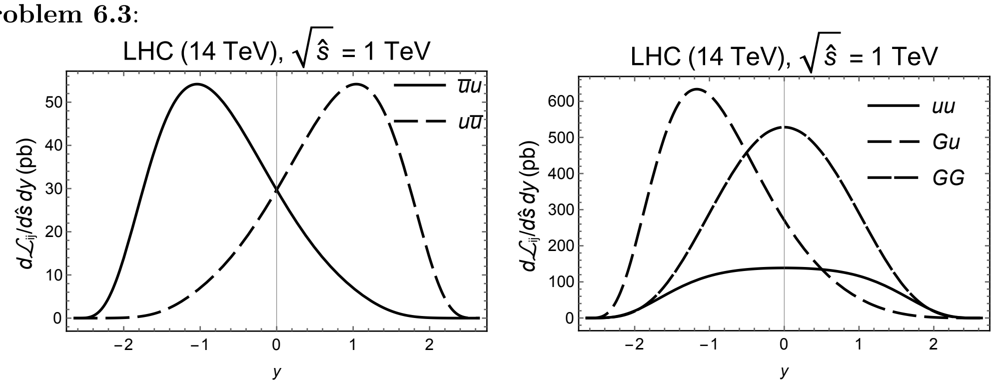
for $m_c \sim 1.27$ GeV and $m_b \sim 4.61$ GeV.

Problem 5.17: (b)Tr $\left(M_1^\dagger M_2\right)$, $\left[\text{Tr}\left(M_1^\dagger M_2\right)\right]\left[\text{Tr}\left(M_a^\dagger M_b\right)\right]$, Tr $\left(M_1^\dagger M_2\ M_1^\dagger M_2\right)$
(c) Tr $(M_a M_b)$, $\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L$
(d) iTr $\left[\hat{n} \cdot \vec{\tau} M_1 M_2^\dagger\right]$.

Collider Physics

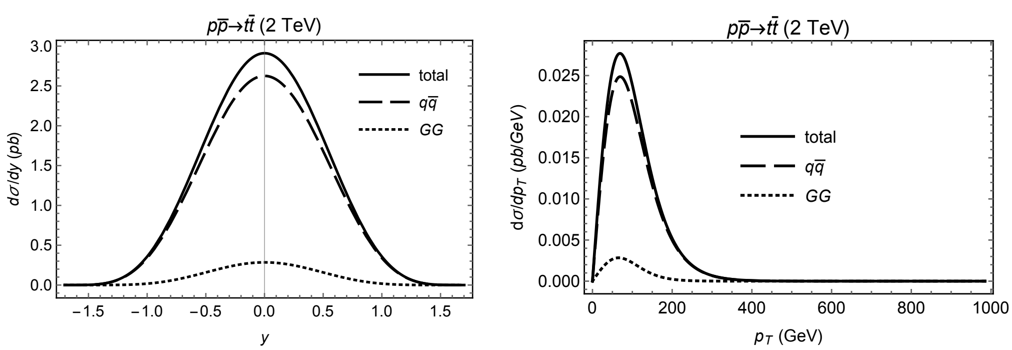
6.1 PROBLEMS

Problem 6.3:



Problem 6.4: Cross sections in pb:

	$p\bar{p}$ (2 TeV)	pp (8 TeV)	pp (14 TeV)
$\sigma_{q\bar{q}}$ (pb)	3.34	22.2	55.6
σ_{GG} (pb)	0.32	78.5	356
σ_{tot} (pb)	3.66	100	412



Problem 6.5:

$$\lambda_{GG} = \frac{\pi^2}{8}\frac{\Gamma_R}{M_R}\frac{d\mathcal{L}_{GG}}{d\hat{s}} \sim 130 \text{ fb}, \qquad \lambda_{u\bar{u}} = \frac{4\pi^2}{9}\frac{\Gamma_R}{M_R}\frac{d\mathcal{L}_{u\bar{u}+\bar{u}u}}{d\hat{s}} \sim 79 \text{ fb}, \qquad \lambda_{d\bar{d}} \sim 51 \text{ fb}.$$

Problem 6.6: $1, \frac{1}{2}, \frac{2}{3}$

The Weak Interactions

7.1 PROBLEMS

Problem 7.2: (a) $12\epsilon^2\,(1-\epsilon)$
(b) $\epsilon^2\left[(3-2\epsilon)+6(1-\epsilon)\right], \qquad \rho=\frac{3}{8}$
(c): $\nu_\mu:\ 2\epsilon^2(3-2\epsilon), \qquad \bar{\nu}_e:\ 12\epsilon^2\,(1-\epsilon)$.

Problem 7.10: (a), (b), (c) in text. (d): $\alpha_{e\nu}=\frac{g_V^2-g_A^2}{g_V^2+3g_A^2}$.

Problem 7.11:

$$\frac{d\Gamma(\pi\nu_\tau)}{d\cos\theta}=\frac{|M|^2}{32\pi m_\tau}=\frac{G_F^2\cos^2\theta_c f_\pi^2 m_\tau^3}{32\pi}(1+\cos\theta)$$
$$\Gamma(\pi\nu_\tau)=\frac{G_F^2\cos^2\theta_c f_\pi^2 m_\tau^3}{16\pi}.$$

The Standard Electroweak Theory

8.1 PROBLEMS

Problem 8.4: (a)

$$V(\phi,\sigma) = \mu_\sigma^2 \sigma^\dagger \sigma + \mu_\phi^2 \phi^\dagger \phi + \lambda_\sigma \left(\sigma^\dagger \sigma\right)^2 + \lambda_\phi \left(\phi^\dagger \phi\right)^2 + \lambda_{\phi\sigma} \sigma^\dagger \sigma \phi^\dagger \phi + \kappa_{\phi\sigma} \sigma \phi^\dagger \tilde{\phi} + \kappa_{\phi\sigma}^* \sigma \tilde{\phi}^\dagger \phi.$$

(c) $M_A \sim \cos\theta_W g' \nu_\sigma$
(d)

$$\bar{\Gamma} = \frac{p_f}{6\pi M_Z^2} g'^2 \sin^2\theta_W M_A^2 \left(2 + \frac{E_A^2}{M_A^2}\right),$$

which survives for $M_A \to 0$ because of the longitudinal mode.

Problem 8.5: (b) $\mathcal{L}_{\ell H} = 0$.

$$\begin{aligned}\mathcal{L}_{\ell(W,A,Z)} = &-\frac{g}{\sqrt{2}} \left(\bar{N}\gamma^\mu E\, W_\mu^+ + \bar{E}\gamma^\mu N\, W_\mu^-\right) + e\bar{E}\gamma^\mu E A_\mu \\ &- \frac{g}{\cos\theta_W} \left[\frac{1}{2}\bar{N}\gamma^\mu N + \bar{E}\gamma^\mu \left(-\frac{1}{2} + \sin^2\theta_W\right) E\right] Z_\mu.\end{aligned}$$

Problem 8.6: $A_L M A_R = \mathrm{diag}(6.46 \quad 3.01 \quad 0.92)$ for

$$A_L = \begin{pmatrix} -0.1 - 0.03i & -0.05 - 0.34i & -0.50 + 0.79i \\ -0.40 + 0.02i & -0.84 + 0.13i & -0.26 - 0.21i \\ -0.9 & 0.4 & 0.1 \end{pmatrix}$$

$$A_R = \begin{pmatrix} 0.05 & -0.12i & -0.93 + 0.35i \\ 0.19 - 0.08i & -0.89 + 0.39i & -0.07 - 0.09i \\ 1. & 0.2 & 0. \times 10^{-1} \end{pmatrix} \times \mathrm{diag}(e^{+3.14i} \quad e^{+0.14i} \quad e^{-0.67i})$$

Problem 8.7:

$$m_1 = m_2 = 3c, \qquad \psi_L = \psi_L^0 = \begin{pmatrix} \psi_{1L}^0 \\ \psi_{2L}^0 \end{pmatrix}, \qquad \psi_R = i \begin{pmatrix} \psi_{2R}^0 \\ \psi_{1R}^0 \end{pmatrix}$$

Problem 8.8: (a)

$$m_1 \sim x, \qquad m_2 \sim B, \qquad A_R \sim I, \qquad A_L^\dagger = \begin{pmatrix} 1 & -\frac{y}{B} \\ \frac{y}{B} & 1 \end{pmatrix}.$$

(b)

$$J_Z^\mu = -\bar{e}_L \gamma^\mu e_L - \frac{y}{B}\left(\bar{e}_L \gamma^\mu E_L + \bar{E}_L \gamma^\mu e_L\right) - \left(\frac{y}{b}\right)^2 \bar{E}_L \gamma^\mu E_L + 2\sin^2\theta_W \left(\bar{e}\gamma^\mu e + \bar{E}\gamma^\mu E\right).$$

Problem 8.9:

$$|\bar{M}|^2 = \frac{g^4 M_Z^2 (g_V^2 + g_A^2)}{2\cos^4\theta_W} \frac{1}{(q^2 - M_Z^2)^2}\left[p_1 \cdot p_3 + \frac{2 p_1 \cdot p_4 \; p_3 \cdot p_4}{M_Z^2}\right]$$

$$\frac{d\bar{\sigma}}{d\cos\theta} = \frac{1}{32\pi s}\,\frac{k_3}{k_1}\,|\bar{M}|^2.$$

$$k_1 = \frac{s - M_H^2}{2\sqrt{s}}, \qquad E_2 = \frac{s + M_H^2}{2\sqrt{s}}, \qquad k_3 = \frac{s - M_Z^2}{2\sqrt{s}}, \qquad E_4 = \frac{s + M_Z^2}{2\sqrt{s}}$$

$$q^2 = (p_3 - p_1)^2 = -2 p_1 \cdot p_3 = -2 k_1 k_3 (1 - \cos\theta)$$

$$p_1 \cdot p_4 = k_1 (E_4 + k_3 \cos\theta), \qquad p_3 \cdot p_4 = \frac{s - M_Z^2}{2}.$$

Problem 8.10: $-\mathcal{L} = -\Gamma^D \kappa \sigma \bar{q}_L \phi d_R / M_D + h.c.$.

Problem 8.13: (a)

$$\frac{d\bar{\sigma}}{dz} = \frac{s}{384\pi}\left[\left(|\epsilon_{LL}|^2 + |\epsilon_{RR}|^2\right)(1+z)^2 + \left(|\epsilon_{LR}|^2 + |\epsilon_{RL}|^2\right)(1-z)^2\right]$$

$$\epsilon_{\mathcal{AB}} = \frac{Q_r e^2}{s} - g_Z^2 D(s) \epsilon_{\mathcal{A}}(\mu) \epsilon_{\mathcal{B}}(r).$$

$z \to -z$ for $\bar{q}_r q_r \to \mu^- \mu^+$.
(b) For $y > 0$

$$\begin{aligned}\frac{d\bar{\sigma}}{dy\,d\hat{z}} =& \lambda_Z \sum_r \Big[\mathfrak{L}_{r\bar{r}}\left(G_N^r (1+\hat{z})^2 + G_F^r (1-\hat{z})^2\right) \\ &+ \mathfrak{L}_{\bar{r}r}\left(G_N^r (1-\hat{z})^2 + G_F^r (1+\hat{z})^2\right)\Big],\end{aligned}$$

($\hat{z} \to -\hat{z}$ for $y < 0$), where

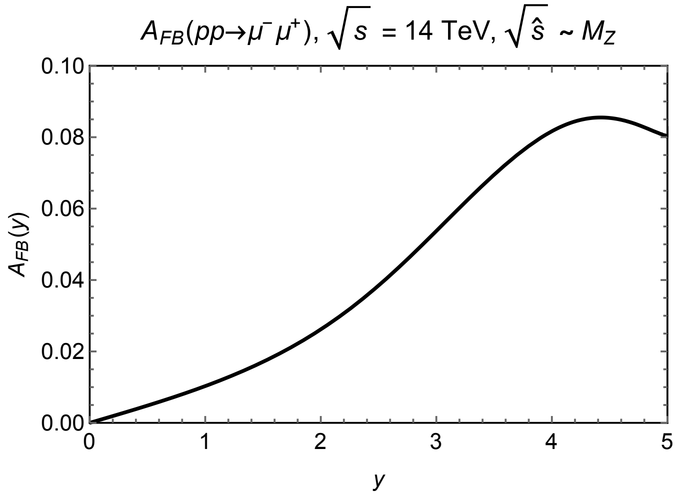
$$\lambda_Z \equiv \frac{g_Z^4}{384 M_Z \Gamma_Z}, \qquad \mathfrak{L}_{r\bar{r}} \equiv \left.\frac{d\mathcal{L}_{q_r \bar{q}_r}}{d\hat{s}\,dy}\right|_{\hat{s}=M_Z^2}, \qquad \mathfrak{L}_{\bar{r}r} \equiv \left.\frac{d\mathcal{L}_{\bar{q}_r q_r}}{d\hat{s}\,dy}\right|_{\hat{s}=M_Z^2}$$

$$G_N^r = \epsilon_L(\mu)^2 \epsilon_L(r)^2 + \epsilon_R(\mu)^2 \epsilon_R(r)^2, \qquad G_F^r = \epsilon_L(\mu)^2 \epsilon_R(r)^2 + \epsilon_R(\mu)^2 \epsilon_L(r)^2.$$

(c)

$$A_{FB}(y) = \frac{3}{4}\frac{\sum_r |\mathfrak{L}_{r\bar{r}} - \mathfrak{L}_{\bar{r}r}|\,(G_N^r - G_F^r)}{\sum_r (\mathfrak{L}_{r\bar{r}} + \mathfrak{L}_{\bar{r}r})\,(G_N^r + G_F^r)}$$

A similar expression holds for $A_{FB}(y_1, y_2)$ except the numerator and denorminator are integrated $\int_{y_1}^{y_2} dy$, or $\left(\int_{y_1}^{y_2} + \int_{-y_2}^{-y_1}\right) dy$. For more detail, see (Han et al., 2013).
(d) $A_{FB}(0, \ln(\sqrt{s}/M_Z)) \sim 0.031$.



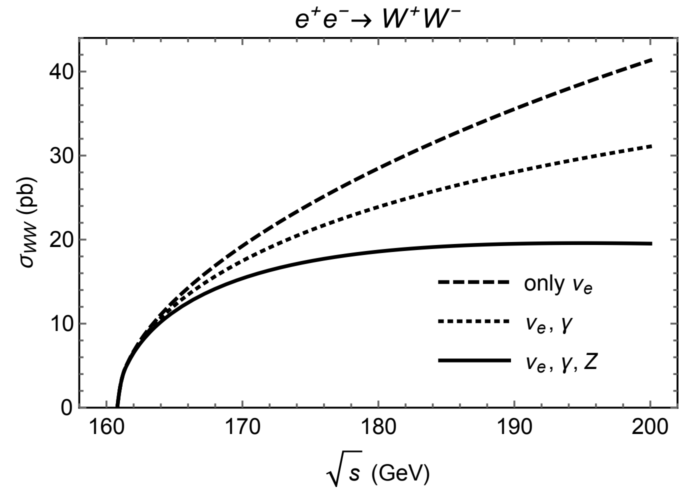
Problem 8.14:

$$M_{\nu_e} = \epsilon^*_{3\mu}\epsilon^*_{4\nu}\;\bar{v}_3\left[\frac{-ig}{\sqrt{2}}\gamma^\nu(1-\gamma^5)\right]\left(\frac{i(\not{p}_1 - \not{p}_3)}{t}\right)\left[\frac{-ig}{\sqrt{2}}\gamma^\mu(1-\gamma^5)\right]u_1$$

$$M_\gamma = \epsilon^*_{3\mu}\epsilon^*_{4\nu}\;\bar{v}_3[ie\gamma^\alpha]u_1\left(\frac{-i}{s}\right) ie\,\mathcal{C}_{\mu\alpha\nu}(-p_3, p_1+p_2, -p_4)$$

$$M_Z = \epsilon^*_{3\mu}\epsilon^*_{4\nu}\;\bar{v}_3\left[\frac{-ig}{2\cos\theta_W}\gamma^\alpha\left(-\frac{1}{2} + 2\sin^2\theta_W - \frac{1}{2}\gamma^5\right)\right]u_1$$

$$\times\left(\frac{-i}{s-M_Z^2}\right) i\frac{e}{\tan\theta_W}\,\mathcal{C}_{\mu\alpha\nu}(-p_3, p_1+p_2, -p_4)$$



Problem 8.15: (a)

$$\frac{d\Gamma}{d\cos\theta} = \frac{\hat{g}^2 E_b^2}{4\pi m_t}\left[2\left(1+\cos\theta\right) + \frac{m_t^2}{M_W^2}\left(1-\cos\theta\right)\right]$$

(c)

$$F_0 = \frac{m_t^2}{2M_W^2 + m_t^2} \simeq 0.70, \qquad F_- = 1 - F_0, \qquad F_+ = 0$$

Problem 8.20: (a)

$$\sigma^{GGF}_{pp\to H} = \frac{\pi^2}{8}\frac{\Gamma_H}{M_H} B(H\to GG)\left.\frac{d\mathcal{L}_{GG}}{d\hat{s}}\right|_{M_H^2} \sim 11 \text{ pb}$$

$$\sigma^{WWF}_{q_r q_s \to q_r q_s H}(\hat{s}) = \frac{\alpha_g^3}{16 M_W^2}\left[\left(1 + \frac{M_H^2}{\hat{s}}\right)\ln\frac{\hat{s}}{M_H^2} - 2 + 2\frac{M_H^2}{\hat{s}}\right]$$

$$\sigma^{WWF}_{pp\to H} = 2\sum_{q_r q_s}\int \frac{d\hat{s}}{\hat{s}}\frac{d\mathcal{L}_{q_r q_s}}{d\hat{s}}\,\hat{s}\,\sigma^{WWF}_{q_r q_s\to q_r q_s H}(\hat{s}) \sim 1.5 \text{ pb}, \qquad \sigma^{ZZF}_{pp\to H} \sim 0.5 \text{ pb}$$

$$\sigma_{q_r\bar{q}_s\to W^\pm H}(\hat{s}) = |V_{rs}|^2\,\frac{\pi\alpha_g^2}{36}\frac{p_f}{p_i}\frac{3M_W^2 + p_f^2}{(\hat{s} - M_W^2)^2}$$

$$\sigma_{pp\to W^\pm H} = 2\sum_{q_r\bar{q}_s}\int\frac{d\hat{s}}{\hat{s}}\frac{d\mathcal{L}_{q_r\bar{q}_s}}{d\hat{s}}\,\hat{s}\,\sigma_{q_r\bar{q}_s\to W^\pm H}(\hat{s}) \sim 0.5 \text{ pb}, \qquad \sigma_{pp\to ZH} \sim 0.3 \text{ pb}.$$

(b)

	σ_{NNLO} (pb)	σ_{LO} (pb)	$N_{\gamma\gamma}$	$N_{4\ell}$
GGF	19	11	258	14
VBF	1.6	2.0	47	2
WH	0.7	0.5	11	0.6
ZH	0.4	0.3	7	0.3
total	22	14	323	17

Problem 8.21: $\Gamma(H \to GG)$ increases by a factor 9, while Γ increases by ~ 1.8.

$$\frac{B(H \to ZZ^*)}{B(H \to ZZ^*)|_{\rm SM}} \sim \frac{1}{1.7} \sim 0.58, \qquad \frac{B(H \to \gamma\gamma)}{B(H \to \gamma\gamma)|_{\rm SM}} \sim \frac{2.8/27.3}{1.7} \sim 0.060,$$

The rates $\sigma_{GG}B$ change by factors of $9 \times 0.58 \sim 5.2$ and $9 \times 0.060 \sim 0.54$ for ZZ^* and $\gamma\gamma$, respectively.

Problem 8.22: (b) Define $V(H+\nu) \equiv c_0 + \sum_{n=2}^{6} c_n H^n$. Then

$$\frac{M_H^2}{2} = c_2 = \lambda\nu^2 + 2\rho\nu^4, \qquad c_3 = \frac{M_H^2}{2\nu}\left(1 + \frac{8\rho\nu^4}{3M_H^2}\right), \qquad c_4 = \frac{M_H^2}{8\nu^2}\left(1 + \frac{16\rho\nu^4}{M_H^2}\right)$$
$$c_5 = \rho\nu, \qquad c_6 = \frac{\rho}{6}.$$

(c) $\kappa^2 \sim 1 - 4\sigma\nu^2$ and $M_H^2/2 \sim \lambda\nu^2(1-4\sigma\nu^2)$. Interactions terms:

$$\sim 2\sigma\left(2\nu H + H^2\right)(\partial_\mu H)^2 - \frac{M_H^2}{2}H^2 - \frac{M_H^2}{2\nu}\left(1 - 2\sigma\nu^2\right)H^3 - \frac{M_H^2}{8\nu^2}\left(1 - 4\sigma\nu^2\right)H^4.$$

Problem 8.23:

$$M = -i\sqrt{2}G_F M_H^2 \frac{s}{s - M_H^2}, \qquad \sigma = \frac{|M|^2}{16\pi s}$$

Problem 8.24: $|\bar{M}|^2 = 4\hat{g}^2\left(\frac{m_t}{M_W}\right)^2 m_t E_b(1-\cos\theta)$

Neutrino Mass and Mixing

9.1 PROBLEMS

Problem 9.2:

$$m_1 = -\epsilon, \qquad \nu_{1L} = \nu_{\mu L} - \nu_{\tau L}$$
$$m_{2,3} = \mp\sqrt{2} + \frac{\epsilon}{2}, \qquad \nu_{2,3L} = \left(\mp\sqrt{2} - \frac{\epsilon}{2}\right)\nu_{eL} + \nu_{\mu L} + \nu_{\tau L},$$

Problem 9.3:

$$2m_T\gamma^\mu(1-\gamma^5)\gamma^\nu \to 2\gamma^\mu\left[(1-\gamma^5)m_T + 2\epsilon\, \not{k}\right]\gamma^\nu + \mathcal{O}(\epsilon^2)$$

Problem 9.5:

$$\text{Dirac: } \bar{\sigma}\beta_{rel} = \frac{G_F^2}{\pi}m_\nu^2\left[\epsilon_L^2 + \epsilon_R^2\right].$$
$$\text{Majorana: } \bar{\sigma}\beta_{rel} = \frac{8\beta^2}{3}\frac{G_F^2}{\pi}m_\nu^2\left[\epsilon_L^2 + \epsilon_R^2\right].$$

CHAPTER 10

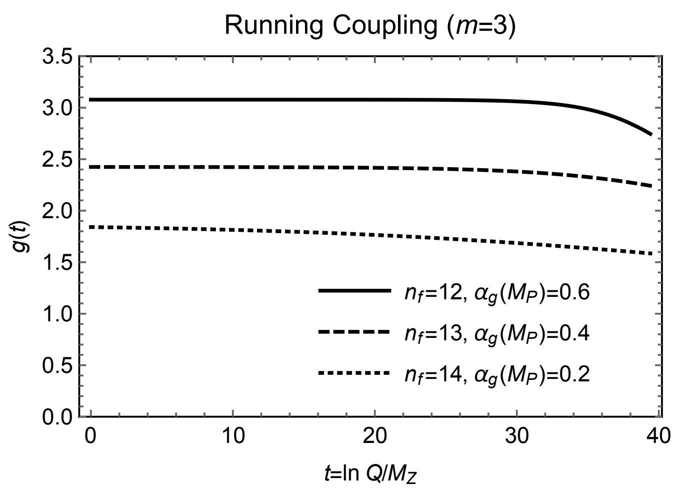
Beyond the Standard Model

10.1 PROBLEMS

Problem 10.1: (a)

$$n_{AF} = \frac{11C_2(G)}{2} = \frac{11m}{2}$$
$$n_{fp} = \frac{17C_2(G)^2}{5C_2(G)+3C_2(L_m)} = \frac{34m^3}{13m^2-3} < n_{AF}$$
$$\alpha_* = -4\pi\frac{\hat{b}_1(m,n_f)}{\hat{b}_2(m,n_f)} = -4\pi\frac{m(11m-2n_f)}{34m^3-13m^2n_f+3n_f}$$

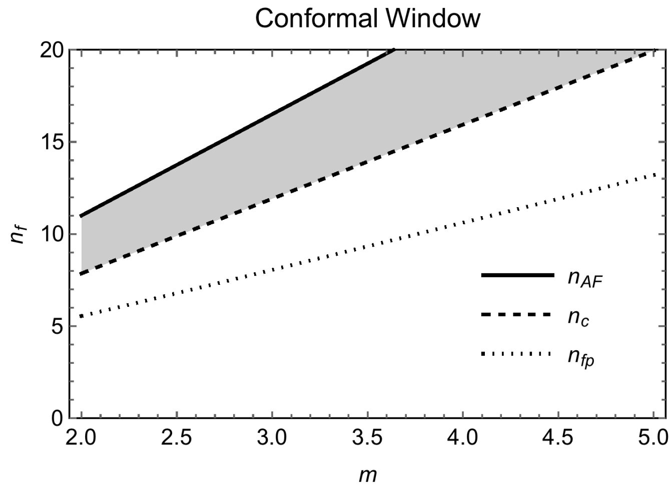
(b)



(c)

$$\alpha_* = \alpha_c \text{ for } n_c = \frac{2}{5}\frac{50m^3-33m}{5m^2-3}.$$

$\alpha_* < \alpha_c$ for $n_f > n_c$.



Problem 10.3: For infinitesimal transformation

$$\mathcal{M}^\dagger \bar{\sigma}^0 \mathcal{M} \sim \bar{\sigma}^0 - \beta^i \bar{\sigma}^i, \qquad \mathcal{M}^\dagger \bar{\sigma}^i \mathcal{M} \sim \bar{\sigma}^i - \epsilon^{ijk}\omega^j \bar{\sigma}^k - \beta^i \bar{\sigma}^0$$

Problem 10.4:

$$\bar{\psi}_{1M}\psi_{2M} = \bar{\xi}_1\bar{\xi}_2 + \xi_1\xi_2, \qquad \bar{\psi}_{1M}\gamma^5\psi_{2M} = \bar{\xi}_1\bar{\xi}_2 - \xi_1\xi_2$$

$$\bar{\psi}_{1M}\gamma^\mu\psi_{2M} = \bar{\xi}_1\bar{\sigma}^\mu\xi_2 + \xi_1\sigma^\mu\bar{\xi}_2, \qquad \bar{\psi}_{1M}\gamma^\mu\gamma^5\psi_{2M} = \bar{\xi}_1\bar{\sigma}^\mu\xi_2 - \xi_1\sigma^\mu\bar{\xi}_2$$

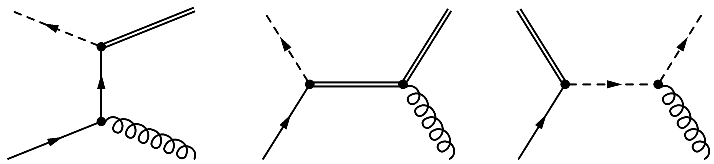
$$\bar{\psi}_{1M}\sigma^{\mu\nu}\psi_{2M} = 2\bar{\xi}_1\,\bar{\mathfrak{s}}^{\mu\nu}\bar{\xi}_2 + 2\xi_1\,\mathfrak{s}^{\mu\nu}\xi_2$$

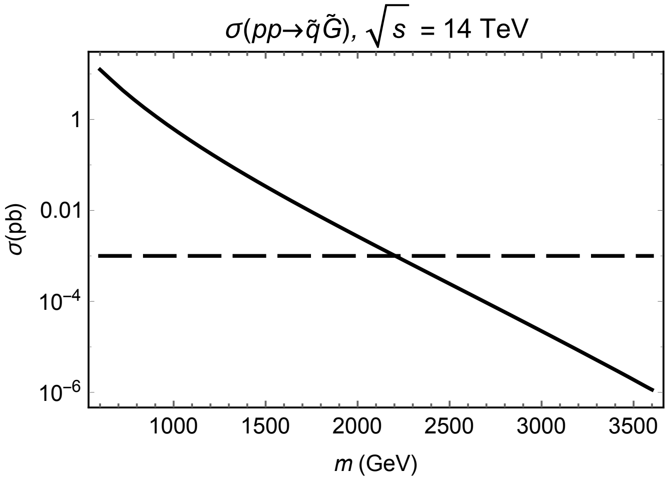
Problem 10.10:

$$\bar{\sigma}(Ah) = \cos^2(\beta-\alpha)\,\frac{g_Z^4}{96\pi}\,\frac{k_{Ah}^3}{k_i}\,\frac{\epsilon_L^2+\epsilon_R^2}{(s-M_Z^2)^2}$$

$$\bar{\sigma}(Zh) = \sin^2(\beta-\alpha)\,\frac{g_Z^4}{96\pi}\,\frac{k_{Zh}^3}{k_i}\,\frac{\epsilon_L^2+\epsilon_R^2}{(s-M_Z^2)^2}\left[1+\frac{3M_Z^2}{k_{Zh}^2}\right]$$

Problem 10.13:





$m \sim 2.2$ TeV

Problem 10.14: (a) $\theta = 2Q_\phi \frac{g_2}{g_1} \frac{M_Z^2}{M_{Z'}^2}$.
(b) $\Gamma_{Z' \to w^+ w^-} = \Gamma_{Z' \to zH} = g_2^2 Q_\phi^2 \frac{M_{Z'}}{48\pi}$.

Problem 10.15: $\left(\tilde{B}\ \tilde{W}^3\ \tilde{h}_d^0\ \tilde{h}_u^0\ \tilde{S}\ \tilde{Z}'\right)$ basis:

$$\begin{pmatrix} m_{\tilde{B}} & 0 & -\frac{g'\nu_d}{2} & \frac{g'\nu_u}{2} & 0 & 0 \\ 0 & m_{\tilde{W}} & \frac{g\nu_d}{2} & -\frac{g\nu_u}{2} & 0 & 0 \\ -\frac{g'\nu_d}{2} & \frac{g\nu_d}{2} & 0 & -\lambda_S s & -\lambda_S \nu_u & g_2 Q_d \nu_d \\ \frac{g'\nu_u}{2} & -\frac{g\nu_u}{2} & -\lambda_S s & 0 & -\lambda_S \nu_d & g_2 Q_u \nu_u \\ 0 & 0 & -\lambda_S \nu_u & -\lambda_S \nu_d & 0 & g_2 Q_S s \\ 0 & 0 & g_2 Q_d \nu_d & g_2 Q_u \nu_u & g_2 Q_S s & m_{\tilde{Z}'} \end{pmatrix}$$

Decoupling limit:

$$\begin{pmatrix} m_{\tilde{B}} & 0 & -\frac{g'\nu_d}{2} & \frac{g'\nu_u}{2} & 0 & 0 \\ 0 & m_{\tilde{W}} & \frac{g\nu_d}{2} & -\frac{g\nu_u}{2} & 0 & 0 \\ -\frac{g'\nu_d}{2} & \frac{g\nu_d}{2} & 0 & -\mu_{eff} & 0 & 0 \\ \frac{g'\nu_u}{2} & -\frac{g\nu_u}{2} & -\mu_{eff} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_2 Q_S s \\ 0 & 0 & 0 & 0 & g_2 Q_S s & 0 \end{pmatrix},$$

where $\mu_{eff} = \lambda_S s$. Vector, Dirac fermion, and scalar masses are all $g_2 Q_S s$

Problem 10.17:

$$(1 + 2\zeta \cos\omega) \sim 0.9999(6)$$

$|\zeta| \lesssim 0.0004$ for $\omega = 0$; $|\zeta| \lesssim 0.02$ for $\omega = \pi/2$

Problem 10.18:

$$V = \frac{15}{4}\mu^2 \nu_\Phi^2 + \frac{15}{16}(15a + 7b)\nu_\Phi^4$$