On The Cosmological Production of Light Sterile Neutrinos

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ABSTRACT

The cosmological production of light sterile neutrinos via mixing with ordinary doublet neutrinos is discussed, including the role of coherent production, neutrino oscillations, and rescattering. It is shown that if a simple condition on the neutrino masses and mixing angle θ is satisfied the sterile neutrinos would have been in equilibrium in the early Universe, and the decoupling temperature is calculated in terms of θ. The implications for nucleosynthesis, the energy density of the present Universe, the Solar neutrino problem, and laboratory experiments are discussed. It is argued that nucleosynthesis most likely already excludes a large region of interesting parameters.

There are a number of cosmological and astrophysical limits on the number and properties of neutrinos with normal weak interactions [1]. For example, from the abundance of primordial helium and other light elements one has [2]

$$N_\nu \leq 3.8, \tag{1}$$

where N_ν is the number of ordinary (SU_2 - doublet) two component neutrinos (such as ν_e, ν_μ, ν_τ, and possible fourth family neutrinos) with mass $\leq O(1\ MeV)$. Similarly, limits on the cosmological energy density imply [1]

$$\sum_i m_{\nu_i} < 40\ eV, \tag{2}$$

where the sum extends over the light neutrinos which are stable or have lifetimes longer than the age of the Universe. There are also a variety of limits on unstable neutrinos decaying via ordinary weak interactions ($\nu_2 \rightarrow \nu_1 \gamma$, $\nu_1 \, e^+e^-$). Combined with laboratory limits [3, 4], these imply [5] m_{ν_μ}, $m_{\nu_\tau} < 40 \ eV$ unless new physics is invoked to allow fast and largely invisible decays or annihilations [6].

However, many theoretical models predict the existence of additional two-component sterile neutrinos, which do not have full-strength weak interactions (*i.e.,* which are SU_2 singlets). For example, if the ordinary neutrinos are massive Dirac particles they must have sterile right-handed partners. Similarly, many models, such as grand unified theories based on SO_{10} or E_6, involve sterile Majorana neutrinos. These are often assumed to be superheavy, as in seesaw models [7], but they could also be very light. For example, superstring-inspired models usually predict that such neutrinos are light [8]. Mixing between light sterile neutrinos and ordinary doublet neutrinos leads to a number of interesting phenomenological consequences: in some versions there are natural cancellations between the contributions of Majorana mass eigenstates in neutrinoless double beta decay [9, 10]; there can be "second class" vacuum oscillations between ordinary and sterile neutrinos [11]; neutrinos can decay via flavor-changing neutral currents [11]; and there can be matter-enhanced Mikheyev-Smirnov -Wolfenstein (MSW) transitions between ordinary and sterile neutrinos in the Sun or in supernovae [12]. In fact, a Dirac neutrino can be thought of as a pair of degenerate Majorana neutrinos with opposite CP parities and a conserved total lepton number. In conventional $SU_2 \times U_1$ models these mass eigenstates are 45^o mixtures of an SU_2 doublet and singlet [3]. Similarly, a pseudo-Dirac neutrino [13] is a perturbation on a Dirac neutrino in which the two masses are not quite degenerate.

In the near future measurements of the total Z width and the partial width for $Z \rightarrow$ (invisible) will precisely determine the number of doublet neutrinos with mass $< M_Z/2$ (or provide an upper limit if there are other invisible particles). However, these results will not be sensitive to sterile neutrinos [14]. It is therefore important to determine the extent to which the cosmological limits such as (1) and (2) apply to sterile neutrinos. Several authors [15] have discussed the production of sterile ν's via superweak interactions mediated by heavy W' or Z' bosons. Typically, such neutrinos are not produced in cosmologically relevant numbers unless $M_{W',Z'}$ are lighter than $O(1 \ TeV)$. In this letter I will discuss the production by mass and by mixing with ordinary neutrinos. I will concentrate on light ($m < O(1\ MeV)$) sterile neutrinos. In this case the temperature was large compared to the neutrino masses during the relevant cosmological period, so neutrino oscillation and rescattering effects must be considered. Special cases have been discussed before [16, 10], but here I will give the general condition for the sterile neutrinos to have been in equilibrium, the decoupling temperature, and the implications for nucleosynthesis and the present energy density. The implications for the Solar neutrino problem and laboratory oscillation experiments are also discussed. In particular, it will be shown that nucleosynthesis most likely already excludes a region of parameters much larger than will ever be probed in the laboratory, as well as a significant fraction of the parameters for which MSW conversions into sterile neutrinos could solve the Solar neutrino problem.

The decoupling temperature for neutrinos (or other particles) is obtained by comparing the production rate [1]

$$\Gamma = \langle \sigma v \rangle n_t \tag{3}$$

with the expansion rate

$$H = [8\pi G_N \rho/3]^{1/2} \simeq 1.66 g_*^{1/2} T^2/m_p. \tag{4}$$

In (3) and (4) $\langle \sigma v \rangle$ is the thermally averaged production cross section times relative velocity, $n_T \propto T^3$ is the number density of target particles, $m_p = G_N^{-1/2} = 1.22 \times 10^{19}\ GeV$ is the Planck scale, and $g_*(T) = g_B(T) + \frac{7}{8} g_F(T)$, where g_B and g_F are respectively the number of relativistic boson and fermion spin states present at temperature T. For doublet neutrinos, the relevant reactions are

$$f\bar{f} \leftrightarrow \nu\bar{\nu} \qquad l^- f \leftrightarrow \nu f' \qquad \nu f \rightarrow \nu f, \tag{5}$$

where f is a fermion and l^- is a charged lepton. The elastic process $\nu f \rightarrow \nu f$ maintains kinetic but not number equilibrium; however, it is important for the production of sterile neutrinos in the following. These reactions yield $\langle \sigma v \rangle \sim G_F^2 T^2$ for $T < O(M_W)$, so that

$$\Gamma_\nu = C(T) G_F^2 T^5, \tag{6}$$

where the coefficient $C(T)$ includes a weighted average over the various reactions, and particles in (5). The T dependence depends on the number of species present at T, so that $C(T)$ is roughly proportional to $g_*(T)$. Comparing (4) and (5), one has the well known result [1] $\Gamma_\nu/H \sim G_F^2 m_p T^3$, so the neutrinos were in equilibrium for $T > T_\nu$, where $T_\nu \sim (G_F^2 m_p)^{-1/3} \sim 1\ MeV$ is the decoupling temperature.

For Dirac neutrinos the left-handed component ν_L is a doublet, but the right-handed component N_R is sterile. Assuming no new interactions [15] N_R can only be produced by mass effects or Higgs mediated interactions. In particular, N_R can be produced in the reactions (5), but with a rate,

$$\Gamma_N \sim \left(\frac{m_\nu}{T}\right)^2 \Gamma_\nu, \tag{7}$$

where the m_ν/T suppression is a helicity flip factor. Hence, $\Gamma_N/H \sim m_\nu^2 T/T_\nu^3$, which is $\sim 10^{-5}$ for $m_\nu \sim 10\ eV$ and $T \sim M_W$. For larger T, the ratio falls rapidly because (a) G_F in the weak cross section is replaced by g^2/T^2; and (b) m_ν in (7) is really the T dependent effective mass. Assuming m_ν is generated by the ordinary Higgs mechanism, $m_\nu(T) \to 0$ above the electroweak phase transition at $T \sim O(M_W)$ when $SU_2 \times U_1$ symmetry is restored. Hence, helicity flip never leads to an equilibrium for the right-handed components of light ($m_\nu < KeV$) Dirac neutrinos, so they were not produced in cosmologically-significant numbers. Similar remarks apply to N_R production via Higgs exchange – this is suppressed by small Yukawa couplings, so that $\Gamma_N/H \ll 1$ for all T for $m_\nu < O(KeV)$.

However, for pseudo-Dirac neutrinos or for the general case of mixing between doublets and singlets the sterile neutrinos can be produced via mixing effects. Consider a left-handed doublet neutrino ν_L (ν_{eL}, $\nu_{\mu L}$, or $\nu_{\tau L}$) and a right-handed singlet N_R. These are necessarily related by CPT to a right-handed doublet antineutrino $\nu_R^c \equiv C\bar{\nu}_L^T$ and a left-handed sterile antineutrino $N_L^c \equiv C\bar{N}_R^T$, where C is the charge conjugation matrix. In general, the two left-handed states ν_L and N_L^c can mix, so that

$$\begin{aligned} \nu_L &= \nu_{1L}\cos\theta + \nu_{2L}\sin\theta \\ N_L^c &= -\nu_{1L}\sin\theta + \nu_{2L}\cos\theta, \end{aligned} \tag{7}$$

where ν_{1L} and ν_{2L} are the two mass eigenstate neutrinos and θ is the mixing angle. (I consider mixing between only two states for simplicity.) A similar formula describes $\nu_R^c - N_R$

mixing. For a Dirac neutrino, one has $\theta = \pi/4$, $m_1 = m_2$, and the theory can be rewritten in such a way that lepton number conservation is manifest [3]. For the pseudo-Dirac case [13] there is a small splitting between m_1 and m_2 and $\theta \sim \pi/4$. In most other models, θ is small and m_1 and m_2 are usually non-degenerate.

It is well known that if ν_L (or ν_R^c) is produced in a reaction with energy $E \gg m_{1,2}$ at time $t = 0$, then, since the coherent components ν_1 and ν_2 evolve with different phases, the state will evolve into

$$\nu_L(t) = a(t)\nu_L + b(t)N_L^c, \tag{8}$$

at a later time t, where

$$|b(t)| = \sin 2\theta \ \sin \frac{\Delta m^2 t}{4E} \tag{9}$$

and $\Delta m^2 \equiv m_1^2 - m_2^2$. That is, there is a probability $P(t) = |b(t)|^2$ that ν_L will oscillate into N_L^c. Since N_L^c is sterile, such "second class" oscillations [11] are only observable in disappearance experiments. The current laboratory limits are shown in Figure 1.

Now, consider the cosmological production of ν_L or ν_R^c via any of the reactions in (5). For $T \gg m_{1,2}$ the produced state will again evolve [17] according to (8). If there were no subsequent rescattering the $\nu_L - N_L^c$ oscillations would not produce any extra particles. However, prior to ν_L decoupling, the ν_L component of $\nu_L(t)$ will rescatter by one of the reactions in (5) after a time t_c, where the average collision time is

$$\langle t_c \rangle = \Gamma_\nu^{-1} = \frac{1}{C(T) G_F^2 T^5}, \tag{10}$$

and Γ_ν is given by (6). This rescattering destroys the phase relation between the components of $\nu_L(t)$ and thus constitutes a measurement of the state. Hence, the probability that the originally produced ν_L ends up as a N_L^c after the rescattering is $P(t_c) = |b(t_c)|^2$, and the N_L^c production rate is [18, 19, 20]

$$\Gamma_N \simeq \Gamma_\nu \langle |b(t_c)|^2 \rangle, \tag{11}$$

where the average is over both t_c and the neutrino energy E. Thus

$$\frac{\Gamma_N}{H} = \sin^2 2\theta \left\langle \sin^2 \frac{\Delta m^2 t_c}{4E} \right\rangle \frac{\Gamma_\nu}{H}, \tag{12}$$

and the production of N_L^c depends on the three time scales H^{-1}, $\langle t_c \rangle$, and $\langle \tau_{osc} \rangle \equiv \langle 4E/\Delta m^2 \rangle \simeq 12.6T/\Delta m^2$ (this is $8.3 \times 10^{-9} sec$ for $T = 1\ MeV$, $\Delta m^2 = 1\ eV^2$), as well as on θ. It is convenient to write

$$\frac{\Gamma_\nu}{H} = \left(\frac{T}{T_\nu}\right)^3 \frac{C(T)/C(T_\nu)}{g_*^{1/2}(T)/g_*^{1/2}(T_\nu)} \tag{13}$$

using (4) and (6), where T_ν is the ν decoupling temperature for kinetic equilibrium. In principle T_ν differs for ν_e as opposed to ν_μ or ν_τ (due to the charged current $\nu_e e$ interaction), but it is sufficient here to take $T_\nu \sim 1\ MeV$ for all cases.

For high temperatures one has $\langle t_c \rangle \ll \langle \tau_{osc} \rangle$, so that

$$\langle \sin^2 \frac{t_c}{\tau_{osc}} \rangle \sim \frac{\langle t_c \rangle^2}{\langle \tau_{osc} \rangle^2} \sim \left(\frac{\Delta m^2}{G_F^2 T^6}\right)^2 \rightarrow 0. \tag{14}$$

That is, the rescattering occurs too rapidly to allow a significant N_L^c component to develop. On the other hand, for sufficiently low T one has $\langle t_c \rangle \gg \langle \tau_{osc} \rangle$. Thus, many oscillations occur on the average before rescattering. In this limit, $\sin^2(t_c/\tau_{osc})$ averages to $1/2$ and one has

$$\frac{\Gamma_N}{H} \simeq \frac{1}{2} \sin^2 2\theta \quad \frac{\Gamma_\nu}{H}, \tag{15}$$

which for small θ is twice the result one would have obtained for ν_{2L} production if one (incorrectly) interpreted (7) as leading to the incoherent production of ν_1 and ν_2.

From (12) - (15) one has that $\Gamma_N/H \propto T^3$ for small T and T^{-9} for large T. (I am only concerned with the regime $T \ll M_W$.) The maximum of Γ_N/H occurs at the temperature T_M at which $\langle t_c \rangle / \langle \tau_{osc} \rangle = O(\pi/2)$. If $\Gamma_N/H > 1$ at T_M then the sterile neutrinos would have been in equilibrium for $T > T_N$, where the decoupling temperature T_N is defined by $\Gamma_N(T_N)/H(T_N) = 1$. Otherwise, the N were never in equilibrium and are cosmologically unimportant [21]. One obtains

$$\left(\frac{T_M}{T_\nu}\right)^3 \left(\frac{C(T_M)}{C(T_\nu)}\right)^{\frac{1}{2}} \sim 1.1 \times 10^4 \left(\frac{\Delta m^2}{1\ eV^2}\right)^{\frac{1}{2}} \left(\frac{1\ MeV}{T_\nu}\right)^{\frac{3}{2}}, \tag{16}$$

which yields the condition for equilibrium (*i.e.,* $\Gamma_N(T_M)/H(T_M) > 1$)

$$\sin^4 2\theta \left(\frac{\Delta m^2}{1\ eV^2}\right) > \frac{3.5 \times 10^{-8}}{\lambda(T_M)} \left(\frac{T_\nu}{1\ MeV}\right)^3. \tag{17}$$

The quantity

$$\lambda(T) \equiv \frac{C(T)/C(T_\nu)}{g_*(T)/g_*(T_\nu)} \tag{18}$$

is very close to unity. Condition (17) is satisfied for a wide range of interesting parameters, as shown in Figure 1. In particular, the right-handed components of pseudo-Dirac neutrinos would have once been in equilibrium for $\Delta m^2 > 3.5 \times 10^{-8} eV^2$ for $T_\nu = 1\ MeV$.

If (17) is satisfied, then it usually suffices to use the slow rescattering approximation in (15) to calculate the sterile neutrino decoupling temperate T_N. (This is not adequate for Γ_N/H very close to unity at T_M.) From (13) and (15) one obtains

$$\left(\frac{T_N}{T_\nu}\right)^3 \frac{g_*^{1/2}(T_N)\lambda(T_N)}{g_*^{1/2}(T_\nu)} \simeq \frac{2}{\sin^2 2\theta}. \tag{19}$$

That is, for $\Gamma_N(T_M)/H(T_M) \gg 1$ the decoupling temperature depends only on θ.

For $T_N < m_\mu$ one has [15] $g_*(T_N) = g_*(T_\nu) = 43/4$; while for $m_\mu < T_N < m_\pi$, $g_*(T_N) = 57/4$; for $m_\pi < T_N < T_c$, $g_*(T_N) = 69/4$, where $T_c \sim 200 - 400\ MeV$ is the temperature of the quark-gluon phase transition. The values of $g_*(T_N)$ for other T_N are listed in Table 1, along with the corresponding $\sin^2 2\theta$ ranges.

If $T_N > m_\mu$ then subsequent to decoupling the annihilation of $\mu^+\mu^-$ and other species reheated the ν temperature but not that of the sterile N_L^c. Standard arguments based on entropy conservation [15] imply that the ratio of N_L^c temperature to that of ν_L is $[g_*(T_\nu)/g_*(T_N)]^{1/3}$. Hence, at the time of nucleosynthesis or later

$$\frac{n_N}{n_\nu} = \frac{g_*(T_\nu)}{g_*(T_N)} \tag{20}$$

$$\frac{\rho_N}{\rho_\nu} = \left[\frac{g_*(T_\nu)}{g_*(T_N)}\right]^{\frac{4}{3}}, \tag{21}$$

where n_N/n_ν is the ratio of N_L^c to ν_L (or N_R to ν_R^c) number densities, while ρ_N/ρ_ν is the ratio of their energy densities while they are still relativistic ($T >> m_{1,2}$). These ratios, which are also given in Table 1, determine the cosmological effectiveness of sterile neutrinos. For example, (1) is replaced by

$$N_\nu + \sum_i \frac{\rho_{N_i}}{\rho_\nu} \leq 3.8. \tag{22}$$

Similarly, if the mixing angles are small then one can approximately identify the two mass eigenstates with ν_L and N_L^c, respectively. Then, (2) becomes

$$\sum_i \left(m_{\nu_i} + \frac{n_{N_i}}{n_\nu} m_{N_i} \right) < 40\ eV, \tag{23}$$

where m_{ν_i} and m_{N_i} are respectively the masses of the (predominantly) doublet and singlet mass eigenstates of the i^{th} family.

From Table 1 we see that sterile neutrinos are important cosmologically even for very small mixings, provided (17) is satisfied. In fact, $n_N/n_\nu \simeq 1$ for $\sin^2 2\theta > 1.7 \times 10^{-6}$. The contours for various T_N as a function of Δm^2 and $\sin^2 2\theta$ are compared with laboratory oscillation limits in Figure 1. The nucleosynthesis limit in (22) nominally excludes a region much larger than the laboratory limits. However, caution is advised because the exact upper limit depends on observations which are dominated by systematic uncertainties, so the confidence level significance is obscure [2]. Nevertheless, the results strongly suggest that there are no sterile neutrinos in this parameter range.

The Solar neutrino problem could be solved by MSW conversions [12] of ν_e into a sterile neutrino rather than into a ν_μ or ν_τ. This would have implications for SN1987A [12]. A comparison of the parameters for the Solar neutrino problem with the nucleosynthesis limits in Figure 1 suggests that a portion of the parameter range (including the large angle solution and much of the adiabatic solution) is excluded but the nonadiabatic solution is allowed.

These comments were for the production of a single sterile neutrino. If, as seems likely, there are three sterile neutrinos (one per family) with significant mixing, then their cosmological importance is enhanced, *i.e.,* the effects are additive (eqns (22), (23)).

In conclusion, oscillations and rescattering are an efficient way to produce light sterile neutrinos cosmologically, provided the equilibrium condition (17) is satisfied. The number densities and relativistic energy densities relative to ordinary neutrinos are given in Table 1. Constraints from nucleosynthesis suggest that the observation of oscillations into sterile neutrinos is unlikely to be observed in the laboratory, but that MSW conversion into sterile neutrinos remains as a viable solution to the Solar neutrino problem for part of the parameter space.

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[19] For small $P(t_c)$ the subsequent oscillation of N_L^c is unimportant.

[20] The coherence time, for which the ν_1 and ν_2 wave packets overlap, is larger than t_c by $(\Delta v)^{-1} \sim E^2/\Delta m^2 \gg 1$.

[21] There may be small regions of parameters for which neutrinos with mass $> 40\ eV$ may be important for the present energy density even though they were never fully in equilibrium. I am grateful to G. Steigman for this observation.

[22] The MSW curve in Fig. 1 is actually for $\nu_e \rightarrow \nu_\mu$ or ν_τ. For sterile neutrinos, one must replace the electron density n_e in the MSW formalism by $n_e - n_n/2$, where n_n is the neutron density, which is no more than a 6% change for the Sun.

$\sin^2 2\theta$	T_N	$g_*(T_N)$	n_N/n_ν	ρ_N/ρ_ν
$> 1.7 \times 10^{-6}$	$< m_\mu$	43/4	1	1
$(0.71 - 1.7) \times 10^{-6}$	$m_\mu - m_\pi$	57/4	0.75	0.69
$(2.0 - 7.1) \times 10^{-7}$	$m_\pi - T_c$	69/4	0.62	0.53
$(2.1 - 20) \times 10^{-8}$	$T_c - m_s$	205/4	0.21	0.12
$(2.5 - 210) \times 10^{-10}$	$m_s - m_c$	247/4	0.17	0.097

Table 1. Ranges of $\sin^2 2\theta$ and the corresponding decoupling temperatures (assuming the equilibrium condition (17) is satisfied). Also listed are g_* at T_N and the ratios of number densities and relativistic energy densities of N relative to ν. The calculations assume $T_c = 200\ MeV$, $m_s = 350\ MeV$, and $m_c = 1.5\ GeV$.

Figure 1. The equilibrium condition (17) and the contours of fixed T as a function of Δm^2 and $\sin^2 2\theta$. The region above the equilibrium line and to the right of the $T_N = m_\mu$ line is nominally excluded by nucleosynthesis. Also shown are the parameters (shaded region) for which the Solar neutrino problem could be accounted for by MSW conversion into sterile neutrinos [12, 22]. The curves on the upper right are the laboratory disappearance limits (the regions inside the contours are excluded): $\bar{\nu}_e \not\to \bar{\nu}_X$ (Gösgen), dotted line; $\nu_\mu \not\to \nu_X$ (dashed lines), (a) CCFR, (b) CDHS, (c) CHARM [10].