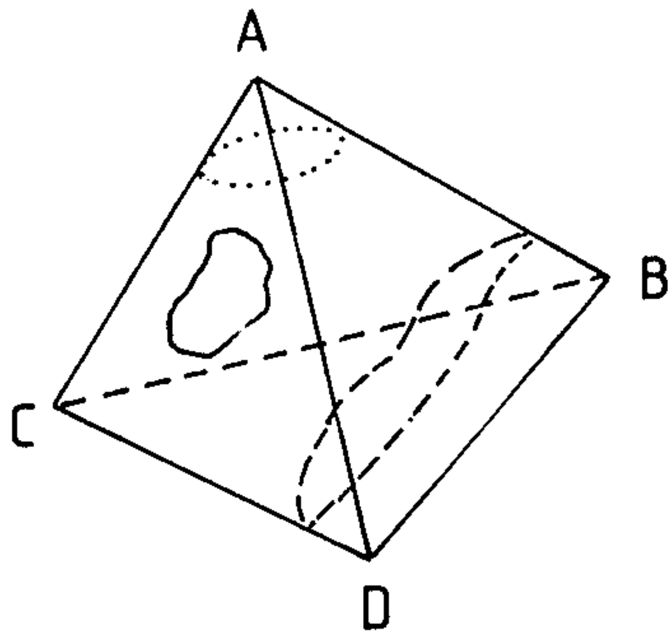
Neutrinos and strings



- **Introduction**
- **Semi-realistic string constructions**
- **The Z_3 heterotic orbifold**
- **Outlook**

(In collaboration with J. Giedt, G. Kane, B. Nelson.)

Neutrino mass

- Nonzero mass may be first break with standard model
- Enormous theoretical effort: GUT, family symmetries, bottom up
  - Majorana masses may be favored because not forbidden by SM gauge symmetries
  - GUT seesaw (heavy Majorana singlet), often combined with family symmetries. Usually ordinary hierarchy.
  - Higgs triplets (“type II seesaw”), often assuming GUT, Left-Right relations

- Very little work from string constructions, even though probably Planck scale
  - E. Witten, Nucl. Phys. B 268, 79 (1986). (E_6 difficulties.)
  - C. Coriano and A. E. Faraggi, Phys. Lett. B 581, 99 (2004); A. E. Faraggi and M. Thormeier, Nucl. Phys. B 624, 163 (2002). (Heterotic inspired. Extended seesaw with extra dynamical assumptions.)
  - J. R. Ellis, G. K. Leontaris, S. Lola and D. V. Nanopoulos, Eur. Phys. J. C 9, 389 (1999). (Flipped $SU(5)$. May be seesaw, but nonstandard and non-GUT-like Majorana, Dirac matrices. Flatness?)
  - L. E. Ibanez, F. Marchesano and R. Rabadan, JHEP 0111, 002 (2001). (Intersecting brane. L conserved.)
  - I. Antoniadis, E. Kiritsis, J. Rizos and T. N. Tomaras, Nucl. Phys. B 660, 81 (2003). (D-brane. L conserved.)
  - J. Giedt, G. Kane, PL, B. Nelson, hep-th/0502032. (Systematic study of heterotic Z_3 orbifolds.)

- **Key ingredients of most bottom up models forbidden in known constructions (heterotic or intersecting brane)**

  (Due to string symmetries or constraints, not simplicity or elegance)

  - "Right-handed" neutrinos may not be gauge singlets
  - Large representations difficult to achieve (bifundamentals, singlets, or adjoints)
  - GUT Yukawa relations broken
  - String symmetries/constraints severely restrict couplings, e.g., Majorana masses, or simultaneous Dirac and Majorana masses
  - L may be conserved
  - Small Dirac masses from HDO, extended (TeV-scale) seesaw, or triplet seesaw (with inverted hierarchy) should be considered very seriously

Semi-realistic string constructions

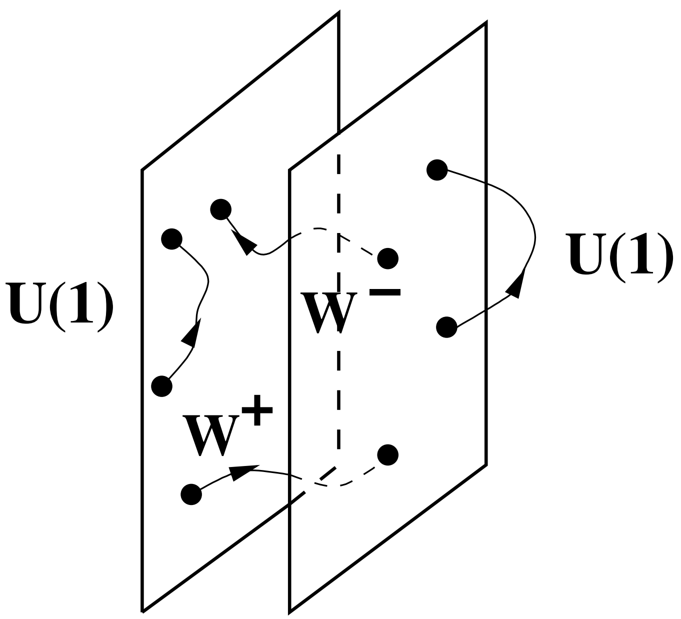
- Quasi-realistic models: contain MSSM gauge group and spectrum and quasi-hidden sector
  - Heterotic $E_8 \times E_8$ (closed strings)
  - Intersecting brane (open strings ending on branes for matter)
- May be additional matter/gauge factors surviving to low energy
- Stringy constraints/selection rules may forbid coupings allowed by $4d$ symmetries
- Will focus on $M_s \sim M_{Pl}$ (gauge couplings on toroidal) with TeV-scale supersymmetry

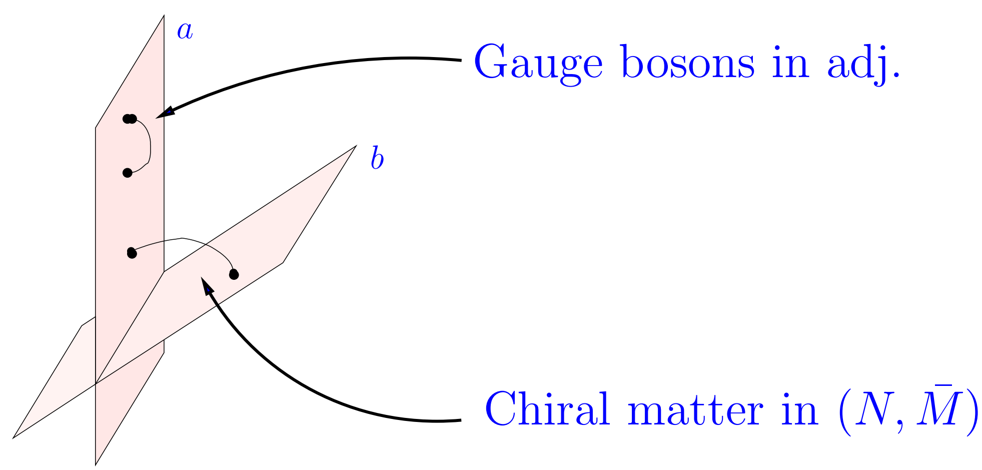
Intersecting Branes

- **Intersecting D-brane**
  - Closed strings (gravitons) and open strings ending on D-branes
  - D6-branes: fill ordinary space and 3 of the 6 extra dimensions
  - Stringy implementation of "brane world" ideas

- **Gauge interactions from strings beginning/ending on stack of parallel branes** (one for each group factor) **→moduli-dependent gauge couplings and no simple unification unless $SU(5)$ stack**

- **Chiral matter: strings at intersection of branes, e.g., $SU(N)\times SU(M)\rightarrow$ bifundamental $(N,\bar{M})$**



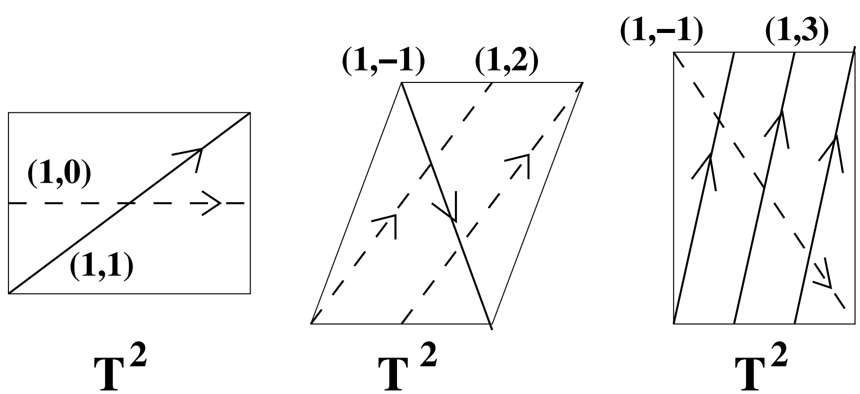


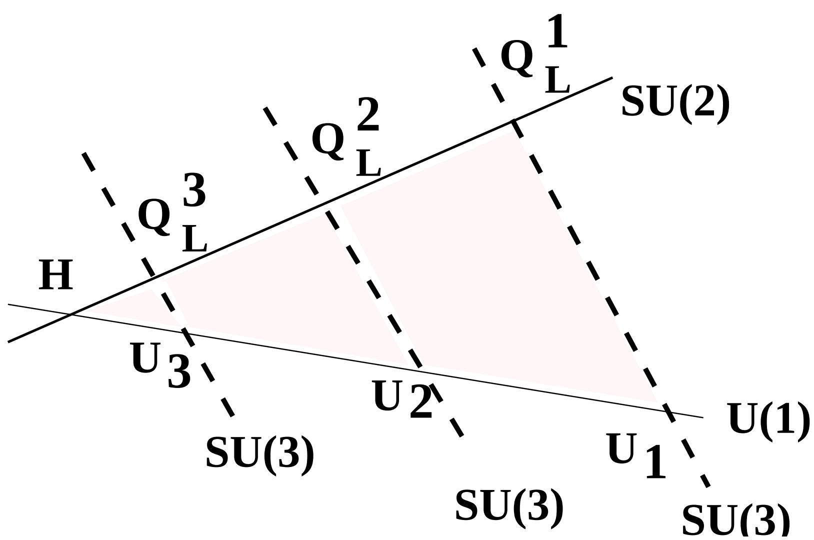
- **Family replication from multiple intersections on compactified geometry**

- **Yukawa interactions** $\sim \exp(-A_{ijk}) \rightarrow$ **hierarchies**

- **Existing models: additional gauge factors, Higgs, chiral matter**

- **Existing models: conserved L; no diagonal (Majorana) triangles**



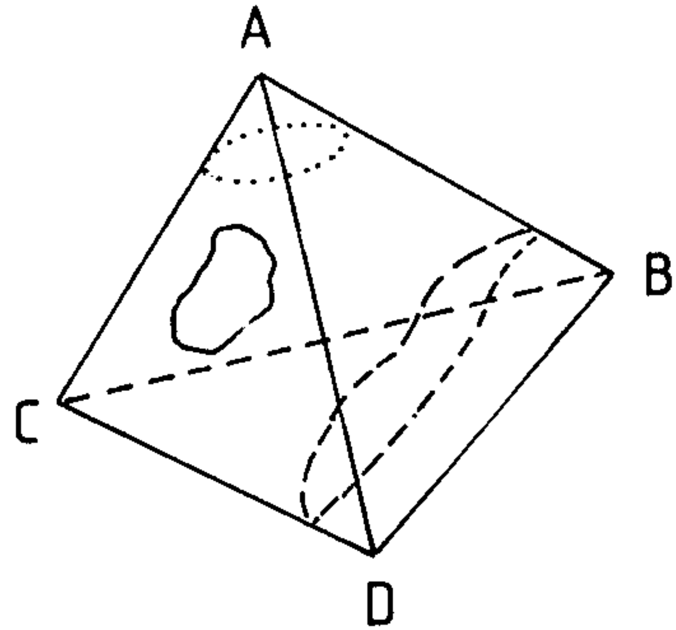


The $E_8 \times E_8$ Heterotic String

- $E_8 \times E_8 \rightarrow G \supset SU(3) \times SU(2) \times U(1)$ by compactification, background gauge fields (Wilson lines)

- Usually $G = SU(3) \times SU(2) \times U(1)^n$ rather than GUT

- For $G =$ GUT, hard to obtain adjoints and high dimensional representations



- Families may have multiplet rearrangement $\rightarrow$ GUT Yukawa relations lost or modified

- Stringy constraints/selection rules may forbid coupings allowed by $4d$ symmetries

- Usually an anomalous $U(1)_A$ compensated by large VEVs (F and D flatness) $\longrightarrow$ vacuum restabilization $\longrightarrow$ symmetries/spectrum further reduced

- Yukawa hierarchies by higher dimensional terms

$$W \sim \left(\frac{S}{M_{Pl}}\right)^p \psi\psi^c H, \quad \Rightarrow h_{eff} \sim \left(\frac{\langle S\rangle}{M_{Pl}}\right)^p$$

  e.g. $\langle S\rangle/M_{Pl} \sim 1/10$ if driven by $U(1)_A$; $\langle S\rangle/M_{Pl} \ll 1$ for intermediate scale (D-flat) $U(1)'$.

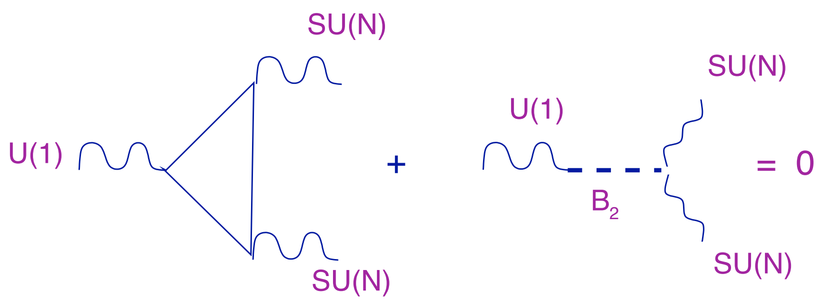
- Existing models: additional gauge factors, Higgs, chiral matter

- Gauge unification, but often non-canonical Kač-Moody level, especially for $U(1)$, and additional Higgs/exotics (Compensation?)

- Families may have different embeddings $\longrightarrow$ FCNC for extra $U(1)'$

Anomalous $U(1)_A$; F and D flatness; vacuum restabilization

- Typically, $U(1)^n$. One linear combination may be anomalous
- Green-Schwarz mechanism cancels anomaly in 4d



- Fayet-Iliopoulous term added to the D- term of $U(1)_A$

$$\xi_{\rm FI} = \frac{g^2_{\rm STR} {\rm Tr}\ Q^A}{192\pi^2} M^2_{\rm PL}$$

- Supersymmetry is restored when certain scalar fields acquire VEV's such that D- and F flatness conditions are satisfied:

$$
\begin{aligned}
D_{\mathrm{A}} \equiv \sum_i Q_i^{(A)} |S_i|^2 + \xi_{\mathrm{FI}} &= 0 \\
D_{\mathrm{a}} \equiv \sum_i Q_i^{(a)} |S_i|^2 &= 0 \\
F_i \equiv \frac{\partial W}{\partial S_i} = 0; \; W &= 0
\end{aligned}
$$

- VEVs $|S_i|$ reduce gauge symmetries, give masses (restabilization)

- Other S_i VEVs can acquire intermediate scale masses by radiative breaking

Dirac masses

- Can achieve small Dirac masses (neutrino or other) by higher dimensional operators

$$L_\nu \sim \left(\frac{S}{M_{Pl}}\right)^p L N_L^c H_2, \quad \langle S \rangle \ll M_{Pl}$$

$$\Rightarrow m_D \sim \left(\frac{\langle S \rangle}{M_{Pl}}\right)^p \langle H_2 \rangle$$

- Large $p \Rightarrow \langle S \rangle$ close to M_{Pl} (e.g., anomalous $U(1)_A$)

- Small $p \Rightarrow$ intermediate scale $\ll M_{Pl}$

- Similar HDO may give light steriles and ordinary/sterile mixing

Majorana masses

- Can one generate large effective m_S from

$$W_\nu \sim c_{ij} \frac{S^{q+1}}{M_{Pl}^q} N_i N_j \quad \Rightarrow (m_S)_{ij} \sim c_{ij} \frac{\langle S \rangle^{q+1}}{M_{Pl}^q},$$

  consistent with D and F flatness?

- Can one have such terms simultaneously with Dirac couplings, consistent with flatness and other constraints?

- Are bottom-up model assumptions for relations to quark, charged lepton masses maintained?

The Z_3 Heterotic Orbifold

- No completely realistic constructions
- Existing constructions usually focus on quark sector
  - Neutrino masses rarely considered, and then as afterthought
  - No construction has yielded GUT-like seesaw
- Study Z_3 heterotic orbifolds (semi-realistic 3- family models), focussing on neutrino sector (Joel Giedt, G. Kane, PL, Brent Nelson)
- $E_8 \times E_8(\text{hidden}) \rightarrow SU(3) \times SU(2) \times U(1)^5 \times E_8(\text{hidden})$
- Large number of possible vacua:
  - Is the minimal seesaw generic?
  - Is some subclass of vacua favored?
  - Any clue about hierarchies, mixings, etc?

Search for Minimal Seesaw

- Look for structure in Z_3 heterotic orbifold:

$$W_{\text{eff}} = (\nu_i \quad N_i) \begin{pmatrix} 0 & (m_D)_{ij} \\ (m_D)_{ji} & (m_M)_{ij} \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j \end{pmatrix}$$

- Require simultaneous Majorana *and* Dirac couplings, and appropriate hypercharge
- *Don't* insist on realistic quark sector
- Majorana mass from $\langle S_1 \cdots S_{n-2} \rangle NN/M_{\text{PL}}^{n-3}$
- Dirac mass from $\langle S_1' \cdots S_{d-3}' \rangle NLH_u/M_{\text{PL}}^{d-3}$
- Only 5 embeddings into $E_8 \times E_8$, 4 realistic hidden sector groups $\longrightarrow$ 175 models in 20 patterns with same ξ_{FI} (Giedt)

Pattern	No.	$G_{\rm hid}$	$r_{\rm FI}$	Species
1.1	7	$SO(10)\times U(1)^3$	No $U(1)_A$	51
1.2	7	$SO(10)\times U(1)^3$	0.15	76
2.1	10	$SU(5)\times SU(2)\times U(1)^3$	0.09	64
2.2	10	$SU(5)\times SU(2)\times U(1)^3$	0.10	66
2.3	7	$SU(5)\times SU(2)\times U(1)^3$	0.10	65
2.4	7	$SU(5)\times SU(2)\times U(1)^3$	0.13	60
2.5	6	$SU(5)\times SU(2)\times U(1)^3$	0.14	61
2.6	6	$SU(5)\times SU(2)\times U(1)^3$	0.12	51
3.1	12	$SU(4)\times SU(2)^2\times U(1)^3$	0.07	58
3.2	5	$SU(4)\times SU(2)^2\times U(1)^3$	0.12	57
3.3	10	$SU(4)\times SU(2)^2\times U(1)^3$	0.12	57
3.4	5	$SU(4)\times SU(2)^2\times U(1)^3$	0.13	53
4.1	7	$SU(3)\times SU(2)^2\times U(1)^4$	0.10	61
4.2	12	$SU(3)\times SU(2)^2\times U(1)^4$	0.09	62
4.3	7	$SU(3)\times SU(2)^2\times U(1)^4$	0.07	63
4.4	15	$SU(3)\times SU(2)^2\times U(1)^4$	0.12	59
4.5	17	$SU(3)\times SU(2)^2\times U(1)^4$	0.11	61
4.6	13	$SU(3)\times SU(2)^2\times U(1)^4$	0.12	60
4.7	6	$SU(3)\times SU(2)^2\times U(1)^4$	0.11	62
4.8	6	$SU(3)\times SU(2)^2\times U(1)^4$	0.12	53

- Classified superpotential terms of degree ≤ 9
- Large number ($O(50)$) fields in each, $\sim$ half are SM singlets
- *None* are singlets under all $U(1)$'s
- Huge number of terms, but small wrt number of fields due to symmetries/selection rules
- $r_{\mathrm{FI}} = \sqrt{|\xi_{\mathrm{FI}}|}/M_{\mathrm{PL}} \sim \langle S_i \rangle / M_{\mathrm{PL}}$

Pattern	3	4	6	7	8	9
1.1	113	24	21329	23768	1697	3380308
1.2	97	12	13968	4418	498	1552812
2.1	67	10	5188	3515	162	342186
2.2	80	11	7573	3066	272	582326
2.3	75	10	6508	2874	250	467020
2.4	53	0	2795	360	0	119454
2.5	58	6	3363	688	26	150838
2.6	31	0	642	0	0	10976
3.1	54	4	2749	768	21	119973
3.2	43	2	1758	291	9	59182
3.3	48	4	2187	393	20	81497
3.4	31	8	750	375	42	15074
4.1	50	3	2090	693	14	81222
4.2	62	6	3206	793	38	143257
4.3	55	5	2516	613	15	100793
4.4	38	2	1137	147	3	28788
4.5	48	0	1872	0	0	62597
4.6	47	0	1738	50	0	51970
4.7	53	0	2219	0	0	76244
4.8	21	0	301	0	0	4120

- Require F and D flatness

- Examined 3 models from each pattern (conjecture: all models in pattern equivalent)

- Studied subset of flat directions with 1d D flatness and minimal F-flatness (more general directions very complicated)

- Huge number of D-flat directions, reduced drastically by F-flatness

Pattern	w/o	w/3	w/3-9
1.1	1486616	16283	489
1.2	11656	188	28
2.1	155555	1239	245
2.2	96932	737	249
2.3	43884	670	115
2.4	5195	114	12
2.5	12	0	0
2.6	825	9	9
3.1	16927	80	27
3.2	2443	18	10
3.3	9871	74	22
3.4	1303	59	41
4.1	17413	106	26
4.2	78819	513	199
4.3	14715	310	163
4.4	26	0	0
4.5	5126	32	25
4.6	128	8	5
4.7	5285	15	15
4.8	49	1	1

- **For each surviving direction, looked for candidate Majorana mass terms $\langle S_1 \cdots S_{n-2} \rangle NN$, where the $\langle S_i \rangle \neq 0$ for that direction**

- **Only two patterns out of 20 (2.6 and 1.1) have candidate Majorana mass terms**

- **Must still check:**
  - **Is there a surviving hypercharge Y with $Y_N = 0$?**
  - **Are there candidate Dirac couplings $\langle S'_1 \cdots S'_{d-3} \rangle NLH_u$ at low enough order?**
  - **Do L, H, and quark candidates have correct Y?**

Pattern 2.6

- Six directions have Majorana mass terms of form

$$\begin{aligned} \mathbf{I-monomial:} &\quad \mathbf{(4,4,6,7,18,35,43,43),} \\ \mathbf{Eff.\ Maj.\ mass:} &\quad \mathbf{(\underline{4},5,5)} \end{aligned}$$

  - Numbers refer to a classification of the chiral matter superfields
  - I-monomial lists S_i fields with VEVs (of order $r_{\rm FI} M_{\rm PL} \sim 0.1 M_{\rm PL}$)
  - Underlined fields are the S_i, others (N_5) are Majorana neutrinos
  - Family indices suppressed

- However, *no* Dirac couplings involving N_5 through degree $d \leq 6$, i.e., none through order $S'^{d-3} N_5 L H_u$

- Light seesaw masses would be of order

$$m_\nu \sim \frac{(r_{\rm FI}^{d-3} v_u)^2}{r_{\rm FI} M_{\rm PL}} \sim r_{\rm FI}^{2d-7} \times 10^{-5}\ {\rm eV} \underbrace{\rightarrow}_{d>6} < 10^{-10}\ {\rm eV}$$

- Also eight directions of form

$$\begin{aligned} \mathbf{I-monomial:} &\qquad (4,4,7,18,19,27,43,43), \\ \mathbf{Eff.\ Maj.\ mass:} &\qquad (\underline{7,7,19,27,43,43,43},34,34) \end{aligned}$$

- However, no Dirac couplings of degree $< 9 \Rightarrow\ m_\nu \leq 10^{-10}$ eV

Pattern 1.1

- No anomalous $U(1)_A$; VEVs may still be determined, e.g., by radiative breaking of non-anomalous, typically at intermediate scale

- Two classes of directions with Majorana masses, but first has no Dirac couplings through (needed) degree 6. Second class promising:

$$\mathbf{I-monomial:} \quad (\mathbf{3,3,8,21,22,29,46,72}),$$
$$\mathbf{Eff.\ Maj.\ mass:} \quad (\underline{\mathbf{8,22,46,72}},\mathbf{9,9})$$

- There is also a candidate Dirac mass: $N_9 L_{36} L_{64}$, where L_{36}, L_{64} are two $SU(2)$ doublets

- Can define appropriate hypercharge for all fields $\rightarrow L_{36} = L$, $L_{64} = H_u$ (family indices suppressed)
  - A second set of Majorana and Dirac couplings of higher degree also present (not shown)
  - No realistic quark Yukawas (and no GUT-type relations)
  - Undesired doubling of leptons and Higgs

- *Apparently*, we have found an example of a seesaw, even if not fully realistic!

- We were about to study family structure (scale, hierarchy, mixings)

The Fatal Flaw

- The same direction has degree 3 mass terms coupling N_9 to other fields $\tilde{N}$:

$$W_{\rm mix} = \lambda \underline{S_8} N_9 \tilde{N}_{14} + \lambda \underline{S_{22}} N_9 \tilde{N}_{27} + \lambda \underline{S_{72}} N_9 \tilde{N}_{50} + \lambda \underline{S_{46}} N_9 \tilde{N}_{81}$$

$$L = (\nu_L \;\; \tilde{N} \;\; N) \begin{pmatrix} 0 & 0 & A \\ 0 & 0 & B \\ A & B & C \end{pmatrix} \begin{pmatrix} \nu_L \\ \tilde{N} \\ N \end{pmatrix},$$

  with $B \gg C \gg A$

- Three massless and six supermassive neutrinos! (no additional terms generated to needed order)

- This could also occur for other apparent seesaws

Outlook

- Neutrino mass likely due to large or Planck scale effects, but little previous work in string context

- *No* viable examples of minimal seesaw in huge class of Z_3 orbifold vacua
  - Could consider more general vacua (two independent VEVs, cancellations of F terms)
  - Other types of orbifolds and heterotic constructions? Will also have strong gauge and stringy constraints. (L conserved in existing intersecting brane)

- Even if a few examples are found, they don't appear generic

- Consider alternatives seriously
  - Small Dirac masses from high degree terms (very common in constructions) (could also give light sterile ν's and mixing)
  - Extended seesaws, $m_\nu \sim m_D^{2+k}/M^{1+k}$, with $k \geq 1$ and low (e.g., TeV) scale M
  - Higgs triplet models: non-trivial to embed in strings (higher level), but very predictive (e.g., inverted hierarchy with nearly bi-maximal mixing) (B. Nelson, PL)

Extended (TeV) Seesaw?

- $m_\nu \sim m^{p+1}/m_S^p, \quad p > 1$ (e.g., $m \sim 100$ MeV, $m_S \sim 1$ TeV for $p = 2$)

- ν_L, N_R, N'_R (3 flavors each)

$$L = \frac{1}{2}\left(\bar{\nu}_L \;\; \bar{N}_L^c \;\; \bar{N'}_L^c\right)\begin{pmatrix} 0 & \mathrm{m}_D & \mathrm{m}_{D'} \\ \mathrm{m}_D^T & 0 & \mathrm{m}_{SS'} \\ \mathrm{m}_{D'}^T & \mathrm{m}_{SS'}^T & 0 \end{pmatrix}\begin{pmatrix} \nu_R^c \\ N_R \\ N'_R \end{pmatrix} + \mathrm{hc}$$

or

$$L = \frac{1}{2}\left(\bar{\nu}_L \;\; \bar{N}_L^c \;\; \bar{N'}_L^c\right)\begin{pmatrix} 0 & \mathrm{m}_D & 0 \\ \mathrm{m}_D^T & 0 & \mathrm{m}_{SS'} \\ 0 & \mathrm{m}_{SS'}^T & m_{S'} \end{pmatrix}\begin{pmatrix} \nu_R^c \\ N_R \\ N'_R \end{pmatrix} + \mathrm{hc}$$

(Faraggi et al.: may occur in specific heterotic model, with dynamical assumptions.)

Triplet models

- Introduce Higgs triplet $T = (T^{++}\ T^{+}\ T^{0})^{T}$ with weak hypercharge $Y = 1$
- Majorana masses m_T generated from $L_\nu = \lambda^T_{ij} L_i T L_j$ if $\langle T^0 \rangle \neq 0$
- Old Gelmini-Roncadelli model: $\langle T^0 \rangle \ll$ EW scale with spontaneous L violation
  - Excluded by $Z \rightarrow$ Majoron $+$ scalar (equivalent to $\Delta N_\nu = 2$)
- Modern triplet models (type II seesaw) break L explicity by THH couplings, giving large Majoron mass (Lazarides, Shafi, Wetterich, Mohapatra, Senjanovic, Schechter, Valle, Ma, Hambye, Sarkar, Rossi, ...)
- Often considered in $SO(10)$ or LR context, with both ordinary and triplet mechanisms competing and with related parameters, but can consider independently.

- **General SUSY case**

$$W_\nu = \lambda^T_{ij} L_i T L_j + \lambda_1 H_1 T H_1 + \lambda_2 H_2 \bar{T} H_2 + M_T T \bar{T} + \mu H_1 H_2$$

T, $\bar{T}$ are triplets with $Y = \pm 1$, $M_T \sim 10^{12} - 10^{14}$ GeV. Typically,

$$\langle T^0 \rangle \sim -\lambda \langle H_2^0 \rangle^2 / m_T \quad \Rightarrow$$

$$\mathrm{m}^\nu_{ij} = -\lambda^T_{ij} \lambda_2 \frac{v_2^2}{M_T}$$

String constructions

- Expect $\lambda^T_{ij} = 0$ for $i = j$ (off-diagonal) $\Rightarrow m^\nu_{ii} = 0$
- Also, need multiple Higgs doublets $H_{1,2}$ with $\lambda_{1,2}$ off diagonal
- Partial explanation: $SU(2)$ triplet with $Y \neq 0$ requires higher level embedding, e.g., of $SU(2) \subset SU(2) \times SU(2)$ (Have Z_3 constructions with some but not all of the features.)

$$W \sim \lambda^T_{1j} L_1(2,1) T(2,2) L_j(1,2), \; j = 2,3$$

yields

$$m^\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

- Typical string case: $|a| = |b|$

- HDO (or $SU(2) \subset SU(2) \times SU(2) \times SU(2)$) can give $m^\nu_{23} \neq 0$

- For

$$m^\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$$

can take a, b, c real w.l.o.g. by redefinition of fields (not true for general m^ν)

- $\mathrm{Tr}\ m^\nu = 0$ and $m^\nu = m^{\nu\dagger} \Rightarrow m_1 + m_2 + m_3 = 0$

- $|\Delta m^2_{\rm Atm}| \sim 2\times10^{-3}$ eV^2, $\Delta m^2_\odot \sim 8\times10^{-5}$ eV$^2 \Rightarrow$ two solutions
  - For $\Delta m^2_\odot$ =0
    - (a) $m_i \propto 1,\ -\frac{1}{2},\ -\frac{1}{2}$ (ordinary, with shifted masses)
    - (b) $m_i \propto 1,\ -1,\ 0$ (inverted)
  - With $\Delta m^2_\odot \neq 0$
    - (a) $m_i = 0.054,\ -0.026,\ -0.026$ eV ($\sum |m_i| = 0.107$ eV (cosmology))
    - (b) $m_i = 0.046,\ -0.045,\ -0.001$ eV ($\sum |m_i| = 0.092$ eV (cosmology))

$$m^\nu_a \sim \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \qquad m^\nu_b \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

  - (a) leads to unrealistic mixing matrix $\Rightarrow$ consider (b)

A special texture

- The $L_e - L_\mu - L_\tau$ conserving texture

$$m^\nu \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

has been considered phenomenologically by *many* authors (Zee; Barbieri, Hall, Smith, Strumia, Weiner; King, Singh; Ohlsson; Barbieri, Hambye, Romanino; Lebed, Martin; Babu, Mohapatra; Lavignac, Masina, Savoy; Feruglio, Strumia, Vissani; Altarelli, Feruglio, Masina)

$$m^{\nu} \sim \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

- New aspects
  - Strong string motivation
  - Motivation for special case $|a| = |b|$
  - Most likely perturbation in 23 element from HOT
- Diagonalization: $\tan\theta_{\rm Atm} = b/a \Rightarrow$ need $|b| = |a|$ for maximal
- $\tan^2\theta_{\odot} = 1$ (maximal) (experiment $\tan^2\theta_{\odot} = 0.40^{+0.09}_{-0.07}$)

- **Majorana mass matrix**

$$m^\nu \sim \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

- **Inverted hierarchy**

- **Bimaximal mixing for $U_e = I$:**

$$U_\nu \sim \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- **Perturbations on m^ν cannot give both $\Delta m^2_\odot$ and $\frac{\pi}{4} - \theta_\odot \sim \theta_C \sim 0.23$ without fine-tuning between terms, e.g.,**

$$\frac{1}{4\sqrt{2}}\frac{\Delta m^2_\odot}{\Delta m^2_{\rm Atm}} = -\frac{\epsilon_{23}}{4} \sim 0.007 \neq \frac{\pi}{4} - \theta_\odot \sim 0.23$$

- However, $U_e \neq I$ with small angles (comparable to CKM) can can give agreement with experiment (Frampton, Petcov, Rodejohann; Romanino; Altarelli, Feruglio, Masina)

$$U_e^\dagger \sim \begin{pmatrix} 1 & -s_{12}^e & 0 \\ s_{12}^e & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

yields

$$\begin{aligned} \theta_\odot &\sim \frac{\pi}{4} - \frac{s_{12}^e}{\sqrt{2}} = 0.56^{+0.05}_{-0.04} \\ |U_{e3}|^2 &\sim \frac{(s_{12}^e)^2}{2} \sim (0.023 - 0.081),\ 90\% \ (\exp : < 0.03) \\ m_{\beta\beta} &\sim m_2(\cos^2\theta_\odot - \sin^2\theta_\odot) \sim 0.020 \text{ eV} \end{aligned}$$

Conclusions

- Neutrino mass likely due to large or Planck scale effects, but little work in string context

- Specific orbifold string constructions (heterotic, intersecting brane) not consistent with common GUT and bottom up assumptions for m_ν

- No examples of minimal seesaw in large class of heterotic Z_3 orbifold vacua

- Small Dirac, extended seesaw, Higgs triplet (inverted hierarchy in string context) may be more likely