Suggested Answers to Problem Set 5

1. An Integrated AR(2) Process

(i) A process \( \{X_t\} \) is integrated of order one if the first difference \( \Delta X_t \) is a stationary series.

(ii) Note that

\[
Y_t + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} = Y_t + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} - \phi_2 Y_{t-1} + \phi_2 Y_{t-1}
\]

\[
= Y_t + (\phi_1 + \phi_2)Y_{t-1} - \phi_2(Y_{t-1} - Y_{t-2})
\]

\[
= Y_t - \rho_1 Y_{t-1} - \rho_2 \Delta Y_{t-1}
\]

where \( \rho_1 = -(\phi_1 + \phi_2) \) and \( \rho_2 = \phi_2 \). Therefore,

\[
Y_t = \rho_1 Y_{t-1} + \rho_2 \Delta Y_{t-1} + \epsilon_t
\] (1)

(iii) Note that

\[
Y_t + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} = Y_t - Y_{t-1} + Y_{t-1} + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} - \phi_2 Y_{t-1} + \phi_2 Y_{t-1}
\]

\[
= \Delta Y_t + (1 + \phi_1 + \phi_2)Y_{t-1} - \phi_2(Y_{t-1} - Y_{t-2})
\]

\[
= \Delta Y_t - \psi_1 Y_{t-1} - \psi_2 \Delta Y_{t-1}
\]

where \( \psi_1 = -(1 + \phi_1 + \phi_2) \) and \( \psi_2 = \phi_2 \). Therefore,

\[
\Delta Y_t = \psi_1 Y_{t-1} + \psi_2 \Delta Y_{t-1} + \epsilon_t
\]

(vi) If \( \rho_1 = 1 \) in (1), then we have

\[
\Delta Y_t = \rho_2 \Delta Y_{t-1} + \epsilon_t
\]

which is AR(1) process in first differences.
2. Forecasting DM/USD Exchange Rates

(iv) The estimation results are given in Table 1.

(v) Under a diffuse prior, the posterior distribution of \( c(2) \) in model \( M_2 \) is

\[
c(2) | Y_1, \ldots, Y_T \sim N(\hat{c}(2), \text{var}(\hat{c}(2))) = N(0.9808, 0.0120^2)
\]

Hence, a 95% posterior confidence set is

\[
[0.9808 - 1.96 \times 0.0120, 0.9808 + 1.96 \times 0.0120] = [0.9573, 1.0043]
\]

in which 1 is included. Since \( M_2 \) is equivalent model with \( M_1 \) when \( c(2) = 1 \), \( M_2 \) has no advantage with the data. Model selection by SIC coincides with this observation, i.e., according to SIC, \( M_1 \) is preferred.

(vi) From Table 2, we can see that the residuals from both model is correlated with their own first lag.

(vii) \( M_3 \) has the smallest SIC, -4.2362, and thus it is preferred.

(viii) For static forecast, \( M_3 \) has the smallest root mean squared error (RMSE) and mean absolute error (MAE). And for dynamic forecast, \( M_1 \) has the smallest RMSE and MAE. But note also that the difference between \( M_1 \) and \( M_3 \) is almost negligible. Hence, the selected model by SIC shows good performance in forecasting.

(ix) Note that “no change” forecast is a random walk forecast, i.e.,

\[
E[Y_{T+h}|Y_1, \ldots, Y_T] = Y_T, \quad h > 0
\]

Hence, dynamic forecasts for DM/USD from 1990:01 to 1995:12 are \@elem(dml, “1989 : 12”), i.e., \textit{dml} at 1989:12, and static forecasts are \textit{dml}(−1). Therefore, RMSE for dynamic and static forecasts are 0.1125 and 0.0276, respectively. In both case, “no change” forecasts are worse than the forecasts based on \( M_3 \).
\[ M_1 : Y_t = c(1) + Y_{t-1} + \epsilon_t \]

Dependent Variable: DML  
Method: Least Squares  
Date: 11/08/02   Time: 16:12  
Sample(adjusted): 1973:02 1989:12  
Included observations: 203 after adjusting endpoints  
DML = C(1) + DML(-1)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>-0.003002</td>
<td>-1.450771</td>
<td>0.1484</td>
</tr>
</tbody>
</table>

R-squared 0.970641  
Adjusted R-squared 0.970641  
S.E. of regression 0.029478  
Sum squared resid 0.175526  
Log likelihood 427.8524  
Durbin-Watson stat 1.363116

\[ M_2 : Y_t = c(1) + c(2) * Y_{t-2} + \epsilon_t \]

Dependent Variable: DML  
Method: Least Squares  
Date: 11/09/02   Time: 01:04  
Sample(adjusted): 1973:02 1989:12  
Included observations: 203 after adjusting endpoints  
DML = C(1) + C(2)*DML(-1)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>0.012575</td>
<td>1.265663</td>
<td>0.2071</td>
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<td>C(2)</td>
<td>0.980843</td>
<td>82.05294</td>
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</table>

R-squared 0.971011  
Adjusted R-squared 0.970867  
S.E. of regression 0.029364  
Sum squared resid 0.173312  
Log likelihood 429.1412  
Durbin-Watson stat 1.354334

Table 1: DM/USD Exchange Rates, Estimation Results
\[ M_3 : Y_t = c(1) + Y_{t-1} + c(2) \cdot \Delta Y_{t-1} + \ldots + c(4) \cdot \Delta Y_{t-3} \epsilon_t \]

Dependent Variable: DML  
Method: Least Squares  
Date: 11/08/02 Time: 16:12  
Sample(adjusted): 1973:05 1989:12  
Included observations: 200 after adjusting endpoints  
DML = C(1) + DML(-1) + C(2) \cdot (DML(-1)-DML(-2)) + C(3) \cdot (DML(-2)-DML(-3)) + C(4) \cdot (DML(-3)-DML(-4))

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>-0.001733</td>
<td>0.001985</td>
<td>-0.873374</td>
</tr>
<tr>
<td>C(2)</td>
<td>0.309019</td>
<td>0.071608</td>
<td>4.315425</td>
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<tr>
<td>C(3)</td>
<td>-0.036228</td>
<td>0.074170</td>
<td>-0.488445</td>
</tr>
<tr>
<td>C(4)</td>
<td>0.056136</td>
<td>0.070742</td>
<td>0.793532</td>
</tr>
</tbody>
</table>

R-squared 0.973678 Mean dependent var 0.806356  
Adjusted R-squared 0.973275 S.D. dependent var 0.170532 
S.E. of regression 0.027878 Akaike info criterion -4.302138  
Sum squared resid 0.152331 Schwarz criterion -4.236171  
Log likelihood 434.2138 F-statistic 2416.705  
Durbin-Watson stat 1.978334 Prob(F-statistic) 0.000000

\[ M_4 : Y_t = c(1) + c(2) \cdot Y_{t-1} + \ldots + c(5) \cdot Y_{t-4} + \epsilon_t \]

Dependent Variable: DML  
Method: Least Squares  
Date: 11/08/02 Time: 16:12  
Sample(adjusted): 1973:05 1989:12  
Included observations: 200 after adjusting endpoints  
DML = C(1) + C(2) \cdot DML(-1) + C(3) \cdot DML(-2) + C(4) \cdot DML(-3) + C(5) \cdot DML(-4)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
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<tr>
<td>C(1)</td>
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<td>0.009674</td>
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<td>C(2)</td>
<td>1.296594</td>
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<td>C(3)</td>
<td>-0.343490</td>
<td>0.116484</td>
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<tr>
<td>C(4)</td>
<td>0.092273</td>
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<td>C(5)</td>
<td>-0.063891</td>
<td>0.070639</td>
<td>-0.904480</td>
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</table>

R-squared 0.974013 Mean dependent var 0.806356  
Adjusted R-squared 0.973275 S.D. dependent var 0.170532 
S.E. of regression 0.027771 Akaike info criterion -4.304967  
Sum squared resid 0.150389 Schwarz criterion -4.222509  
Log likelihood 435.4967 F-statistic 1827.195  
Durbin-Watson stat 1.979339 Prob(F-statistic) 0.000000

Table 1: DM/USD Exchange Rates, Estimation Results (continued)
### (a) Residuals from $\mathcal{M}_1$

Date: 11/09/02   Time: 01:35  
Sample: 1973:01 1989:12  
Included observations: 203

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
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<td>1</td>
<td></td>
<td>0.302</td>
<td>0.302</td>
<td>18.781</td>
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<tr>
<td>2</td>
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<td>0.063</td>
<td>-0.031</td>
<td>19.611</td>
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<tr>
<td>3</td>
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<td>0.060</td>
<td>0.055</td>
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<td>21.087</td>
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<tr>
<td>7</td>
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<td>0.051</td>
<td>0.086</td>
<td>21.631</td>
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<td>0.082</td>
<td>0.045</td>
<td>23.056</td>
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### (b) Residuals from $\mathcal{M}_2$

Date: 11/09/02   Time: 01:35  
Sample: 1973:01 1989:12  
Included observations: 203

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>Q-Stat</th>
<th>Prob</th>
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<tbody>
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<td>0.307</td>
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<td>21.411</td>
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</tr>
<tr>
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<td>0.031</td>
<td>0.015</td>
<td>22.005</td>
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<td>27.355</td>
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</table>

Table 2: Autocorrelation Functions of Residuals
Figure 1: DM/USD Exchange Rates, Dynamic and Static Forecasts