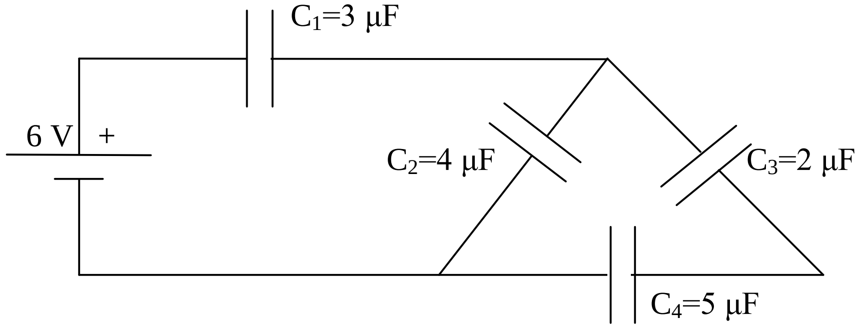
$\mu=10^{-6}$ $\varepsilon_0=8.85\text{x}10^{-12}\ NC^{-2}m^2$

Quiz for February 16th 2005 - Physics 151-001 - Prof. Thomson

1) *Showing your work*, for the network below:



C_{eq}= 1.93 μF for C_1=3 μF
C_{eq}= 1.46 μF for C_1=2 μF
C_{eq}= 0.84 μF for C_1=1 μF

(2 pts)
(1 pt)
(1 pt)
(1 pt)

a. Find the equivalent capacitance.

C_3 and C_4 are in series, so C_{34}=10/7 μF =1.43 μF

$$\frac{1}{C_{34}}=\frac{1}{C_3}+\frac{1}{C_4}$$

$$\frac{1}{C_{34}}=\frac{1}{2\mu F}+\frac{1}{5\mu F}=\frac{7}{10\ \mu F}$$

C_2 and C_{34} are in parallel so $C_{234}=C_2+C_{34}$=38/7 μF =5.43 μF

C_1 and C_{234} are in series so C_{eq} = 114/59 μF = 1.93 μF

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_{234}}$$

$$\frac{1}{C_{eq}}=\frac{1}{3\,\mu F}+\frac{7}{38\,\mu F}=\frac{59}{114\ \mu F}$$

U=34.74 μJ
U=26.28 μJ
U=15.12 μJ

b. How much energy is stored in this system?

Energy stored $U=\frac{1}{2}CV^2$=34.74 μJ

c. How would you rearrange the network to store the maximum amount of energy? Draw your new network and calculate its equivalent capacitance.

C_{max}= 14μF for C_1=3 μF
C_{max}= 13 μF for C_1=2 μF
C_{max}= 12 μF for C_1=1 μF

Place all capacitors in parallel!

d. How much energy is stored in this new network?

Energy stored $U=\frac{1}{2}CV^2$=252μJ

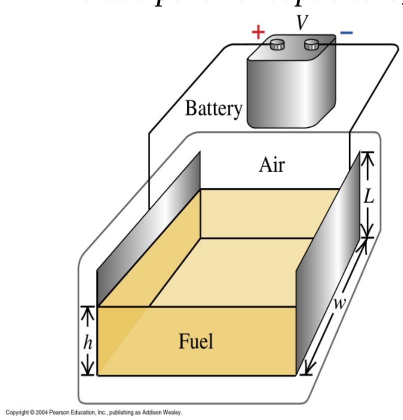
U=252 μJ
U=234 μJ
U=216 μJ

(5 pts)

2) A fuel gauge uses a capacitor to determine the height of the fuel in a tank. The effective dielectric constant K_{eff} changes from a value of 1 when the tank is empty to a value of K, the dielectric constant of the fuel, when the tank is full. The appropriate electronic circuitry can determine the effective dielectric constant of the combined air and fuel between the capacitor plates. Each of the two rectangular plates has a width w and a length L. The height of the fuel between the plates is h. You can ignore any fringing effects.

$$K_{eff} = (1 + \frac{h}{L}(K-1))$$

a. *Showing your work*, derive an expression for K_{eff} as a function of h. *Looks like two parallel capacitors, one filled with air and one filled with fuel.*



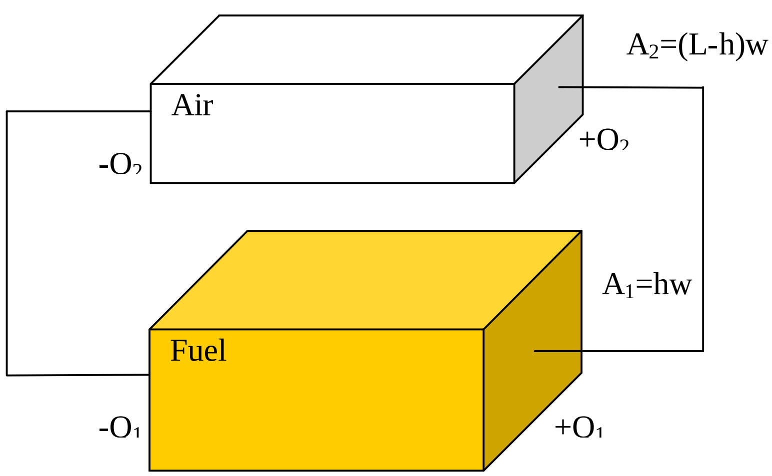
$$C = \frac{A_1\varepsilon_0 K}{d} + \frac{A_2\varepsilon_0}{d}$$

$$C = \frac{hw\varepsilon_0 K}{d} + \frac{(L-h)w\varepsilon_0}{d}$$

$$C = \frac{Lw\varepsilon_0}{d}(\frac{Kh}{L} + 1 - \frac{h}{L})$$

$$C = \frac{Lw\varepsilon_0}{d}(1 + \frac{h}{L}(K-1))$$

$$C = \frac{Lw\varepsilon_0}{d}K_{eff}$$



b. What is the effective dielectric constant for a tank ¼ full, ½ full and ¾ full, if the fuel is gasoline (K=1.95)?

h/L= ¼ full K_{eff}=1.24
h/L=½ full K_{eff}=1.48
h/L=¾ full K_{eff}=1.71