# Dynamic Fiscal Policy

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# Preface

In these notes we study fiscal policy in dynamic economic models in which households are rational, forward looking decision units. The government (that is, the federal, state and local governments) affect private decisions of individual households in a number of different ways. Households that work pay income and social security payroll taxes. Income from financial assets is in general subject to taxes as well. Unemployed workers receive temporary transfers from the government in the form of unemployment insurance benefits, and possibly welfare payments thereafter. When retired, most households are entitled to social security benefits and health care assistance in the form of medicare. The presence of all these programs may alter private decisions, thus affect aggregate consumption, saving and thus current and future economic activity. In addition, the government is an important independent player in the macro economy, purchasing a significant fraction of Gross Domestic Product (GDP) on its own, and absorbing a significant fraction of private domestic (and international saving) for the finance of its budget deficit.

We attempt to analyze these issues in a unified theoretical framework, at the base of which lies a simple intertemporal decision problem of private households. We then introduce, step by step, fiscal policies like the ones mentioned above to analytically derive the effects of government activity on the private sector. Consequently these notes are organized in the following way. In the first part we first give an overview over the empirical facts concerning government economic activity and then develop the simple intertemporal consumption choice model. In the second part we then analyze the impact on the economy of given fiscal policies, without asking why those policies would or should be enacted. This positive analysis contains the study of the timing and incidence of consumption, labor and capital income taxes, and the study of social security and unemployment insurance.

In the third part (yet to be written) we then turn to an investigation on how fiscal policy should be carried out if the government is benevolent and wants to maximize the happiness of its citizens. It turns out to be important for this study that the government can commit to future policies (i.e. is not allowed to change its mind later, after, say, a certain tax reform has been enacted). Since this is a rather strong assumption, we then identify what the government can and should do if it knows that, in the future, it has an incentive to change its policy.

Finally, in part 4 (again yet to be written) we will discuss how government policies are formed when, instead of being benevolent, the government decides on policies based on political elections or lobbying by pressure groups. This area of research, called political economy, has recently made important advances in explaining why economic policies, such as the generosity of unemployment benefits, differ so vastly between the US and some continental European countries. We will study some of the successful examples in this new field of research.

# Part I

# Introduction: Facts and the Benchmark Model

In the first part of these notes we want to accomplish two things. First, we want to get a sense on what the government does in modern societies by looking at the data describing government activity in the U.S. We will first display facts that broadly measure the size of the government, relative to overall economic activity. We will then study the structure of the government budget clarify what are the main economic activities the government is engaged in. Then we turn to an investigation of the cyclical properties of government spending and taxation, that is, we analyze how these vary over the business cycle. Finally we take a closer look at government budget deficit and the public debt, both for the U.S. and other economies around the world.

In the second half of the first part we lay the theoretical foundations for our analysis of the role of government fiscal policy in the macro economy. We will construct and analyze the basic intertemporal household consumption-savings problem in the absence of government policy which we will then use extensively, in later parts of these notes, to study the impact of fiscal policy on private decisions of individual households, and thus the entire macro economy. I will first give the simplest version of the model in which households live for only two periods (a model first studied formally by Irving Fisher, to the best of my knowledge), and then extend it to the standard life cycle consumption-savings model pioneered by Franco Modigliani, Albert Ando and Richard Brumberg.[1]

[1] Very related is also the permanent income model of Milton Friedman which stresses the analysis of how households respond with consumption to income shocks, rather than how they allocate consumption over the life cycle.

# Chapter 1

# Empirical Facts of Government Economic Activity

Before proposing theories for the effect and the optimal conduct of fiscal policies it is instructive to study what the government actually does in modern societies. For the most part we will constrain our discussion to the US, but at times we will also explore data from other countries (often Europe and other industrialized economies) to provide a cross-country perspective and comparison.

## 1.1 Data on Government Activity in the U.S.

To organize the data, we start with the familiar decomposition of Gross Domestic Product (GDP) into its components (private consumption, private investment, government spending, net exports), as measured in the National Income and Product Accounts (NIPA). One way to compute nominal GDP is by summing up the total spending on goods and services by the different sectors of the economy.[1]

[1] As you might recall, the other two alternatives for computing GDP are to sum up all sources of income (income side), or to sum up the value added of all sectors of the economy (value added side). After accounting for statistical discrepancies all three methods deliver the same number for nominal GDP.

Formally, let

$$\begin{aligned} C &= \text{Consumption} \\ I &= \text{(Gross) Investment} \\ G &= \text{Government Purchases} \\ X &= \text{Exports} \\ M &= \text{Imports} \\ Y &= \text{Nominal GDP} \end{aligned}$$

Then the well-known spending decomposition of GDP is given by

$$Y = C + I + G + (X - M)$$

Let us turn to a brief description of the components of GDP, acting as a reminder from your intermediate macroeconomics classes.

- Consumption ($C$) is defined as spending of private households on all goods, such as durable goods (cars, TV's, furniture), nondurable goods (food, clothing, gasoline) and services (massages, financial services, education, health care). The only form of household spending that is not included in consumption is spending on new houses.[2] Spending on new houses is included in fixed investment, to which we turn next.

- Gross Investment ($I$) is defined as the sum of all spending of firms on plant, equipment and inventories, and the spending of households on new houses. It is broken down into three categories: *residential fixed investment* (the spending of households on the construction of new houses), *nonresidential fixed investment* (the spending of firms on buildings and equipment for business use) and *inventory investment* (the change in inventories of firms).

- Government spending ($G$) is the sum of federal, state and local government *purchases of goods and services*. Note that government spending does not equal total government outlays: transfer payments to households (such as welfare payments, social security or unemployment benefit payments) or interest payments on public debt are part of government outlays, but *not* included in government spending $G$.

[2] What about purchases of old houses? Note that no production has occured (since the house was already built before). Hence this transaction does not enter this years' GDP. Of course, when the then new house was first built it entered GDP in the particular year.

- As an open economy, the US trades goods and services with the rest of the world. Exports ($X$) are deliveries of US goods and services to the rest of the world, imports ($M$) are deliveries of goods and services from other countries of the world to the US. The quantity $(X - M)$ is also referred to as net exports or the trade balance. We say that a country (such as the US) has a trade surplus if exports exceed imports, i.e. if $X - M > 0$. A country has a trade deficit if $X - M < 0$, which was the case for the US in recent years.

In Table 1 we show the composition of nominal GDP for 2011, broken down to the different spending categories discussed above.[3] The numbers are in billion US dollars. We see that government spending amounts to 20.3 percent of total GDP, with roughly 60% of this coming from purchases of US states and roughly 40% stemming from purchases of the federal government. Thus an important point to notice about US government activity is that, due to its federal structure, in this country a large share of government spending is done at the state and local level, rather than the federal level. However, due to recent increases in expenditures for defence, homeland security and the recent stimulus package during the great recession the share of GDP that accrues to federal government spending has increased. Also, it is important to remember that government spending $G$ only includes the purchase of goods and services by the government (for national defense or the construction of new roads), but not transfer payments such as unemployment insurance, welfare payments and social security benefits. As such, the fraction of $G/Y$ is a first, but fairly incomplete measure of the "size of government".

Table 1.1 also shows other important facts for the US economy which are not directly related to fiscal policy, but will be of some interest in this course. First, more than 70% of GDP goes to private consumption expenditures; this share of GDP has been rising substantially in the 1990's and continues to do so. Within consumption we see that the US economy is now to a large extent a service economy, with almost 66% of overall private consumption expenditures (and thus 47% of overall GDP) going to such services as haircuts, entertainment services, financial services (banking, tax advise etc.) and

[3] As with most of the data in this class, the ones underlying the table come from the Economic Report of the President, which is available online at http://www.whitehouse.gov/administration/eop/cea/economic-report-of-the-President/2013

| | in billion $ | in % of Tot. Nom. GDP |
|---|---|---|
| Total Nom. GDP | 15,075.7 | 100.0% |
| Consumption | 10,729.0 | 71.2% |
| Goods | 3,624.8 | 24.0% |
| Services | 7,104.2 | 47.1% |
| Gross Investment | 1,854.9 | 12.3% |
| Nonresidential | 1,479.6 | 9.8% |
| Residential | 338.7 | 2.3% |
| Changes in Inventory | 36.6 | 0.2% |
| Government Purchases | 3,059.8 | 20.3% |
| Federal Government | 1,222.1 | 8.1% |
| State and Local Government | 1,837.7 | 12.2% |
| Net Exports | -568.1 | -3.8% |
| Exports | 2,094.2 | 13.9% |
| Imports | 2,662.3 | 17.7% |

Table 1.1: Components of GDP, 2011

so forth. The "traditional" manufacturing sector supplying consumer durable goods such as cars and furniture (not shown in detail in the table), now only accounts for about 11% of total consumption expenditures and less than 8% of total GDP.

With respect to investment we note that the bulk of it is investment of firms into machines and factory structures (called nonresidential fixed investment), whereas the construction and purchases of new family homes, called residential fixed investment (for some historical reason this item is not counted in consumer durables consumption), amounts to about 19% of total investment and 2.3% of overall GDP. Note, however, that in 2011 the U.S. economy is still in the largest slump of the housing market in post WWII history, and thus residential fixed investment was abnormally low in that year (it still is, as the housing market is only slowly recovering in 2013 and 2014). A more typically number for residential fixed investment is about 5% of GDP. Finally, changes in inventory, have been slightly positive in 2004, but quantitatively small (as is usually the case).

Finally, the table shows one of the two important deficits the popular economic discussion centers around in recent years. We will talk about the US federal government budget deficit in detail below. The other deficit, the trade deficit (also called net exports or the trade balance), the difference between US exports of goods and services and the value of goods and services the US imports, amounted to about 3.8% of GDP. This means that in 2011 the US population bought \$568 billion worth of goods more from abroad than US firms sold to other countries. As a consequence in 2011 on net foreigners acquired (roughly) \$568 billion in net assets in the US (buying shares of US firms, government debt, taking over US firms etc.). The appendix to this chapter provides further details on the link between the trade deficit and the change in the net foreign asset position of the U.S.

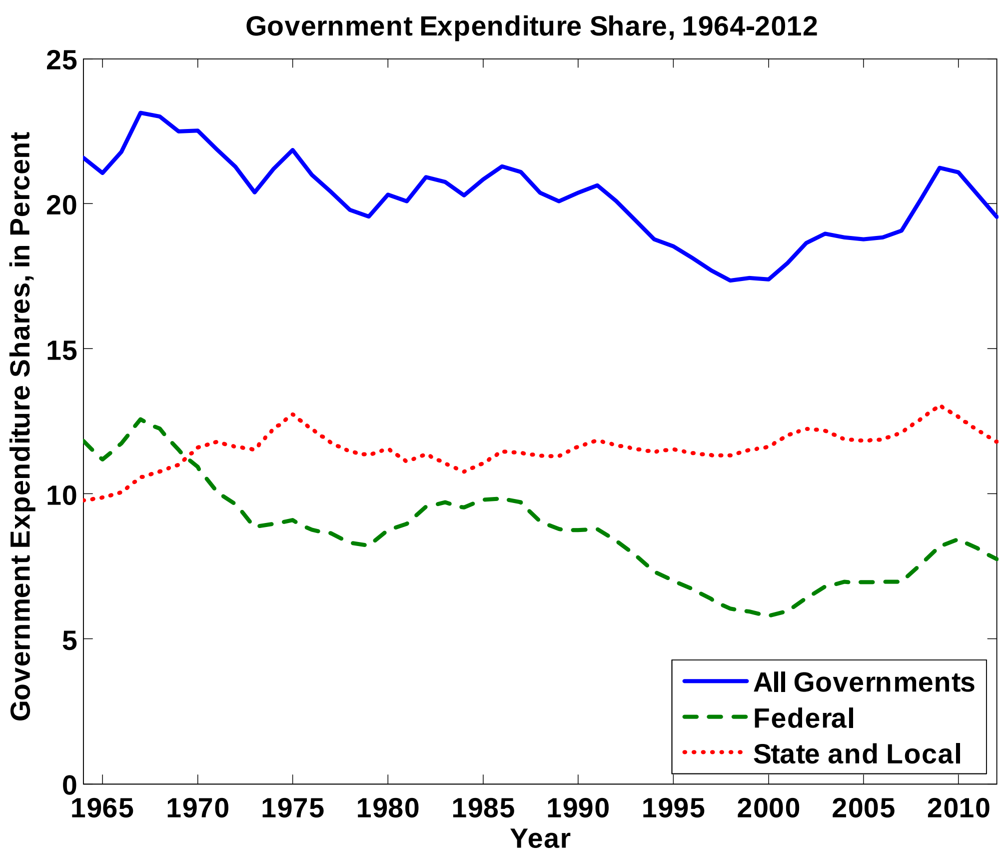


Figure 1.1: Government Spending as Fraction of GDP, 1964-2012

After this little digression we turn back to the size of government spending activity, as a share of GDP. In figure 1.1 we show how this share has developed over time. We observe a substantial decline in the share of GDP devoted to government spending between the 1960's and the year 2001, from about 22% of GDP to about 17% of GDP. This decline is primarily driven by the reduction in federal government spending (as a fraction of GDP), whereas public spending at the state and local level has remained relatively stable over the same time period. From 2001 onwards this trend has been reversed as government expenditures on wars, homeland security and, from 2009 onwards, on the economic stimulus package geared towards softening the great recession. Again, it is federal government spending that is mainly responsible for the reversal of this trend, since homeland security and defense spending is incurred primarily by the federal government which also initiated the stimulus package.

## 1.2 The Structure of Government Budgets

We start our discussion with the federal budget. The federal budget surplus is defined as

$$\begin{aligned}\text{Budget Surplus} &= \text{Total Federal Tax Receipts} \\ &\quad -\text{Total Federal Outlays}\end{aligned}$$

Federal outlays, in turn consist of

$$\begin{aligned}\text{Total Federal Outlays} &= \text{Federal Purchases of Goods and Services} \\ &\quad +\text{Transfers} \\ &\quad +\text{Interest Payments on Fed. Debt} \\ &\quad +\text{Other (small) Items}\end{aligned}$$

The entity "government spending" $G$ that we considered so far equals federal, state and local purchases of goods and services, but does *not include* transfers, such as social security benefits, unemployment insurance and welfare payments, and also does not include interest payments on the outstanding debt. The US federal budget had a deficit every year since 1969 since 1997, then small surpluses between 1998 and 2001, before the increased expenditures for homeland security, the recession of 2001 and the large Bush tax cuts sent the federal budget into deficit again since 2002. The great recession starting in December of 2007[4] and the economic stimulus package

[4] The business cycle dating committee of the National Bureau of Economic Research (NBER) dates the peak of the previous business cycle to December 2007 and the trough

| 2011 Federal Budget (in billion $) | |
|---|---|
| Receipts | 2,303.5 |
| Individual Income Taxes | 1,091.5 |
| Corporate Income Taxes | 181.1 |
| Social Insurance Receipts | 818.8 |
| Other | 212.1 |
| Outlays | 3,603.1 |
| National Defense | 705.6 |
| International Affairs | 45.7 |
| Health | 372.5 |
| Medicare | 485.7 |
| Income Security | 597.4 |
| Social Security | 730.8 |
| Net Interest | 230.0 |
| Other | 435.5 |
| Surplus | –1,299.6 |

Table 1.2: Federal Government Budget, 2011

implied further deep deficits from 2008 onwards. In section 1.4 we will look at data on government deficits and debt in much greater detail.

How can the federal government spend more than it takes in? As with private households, the government can do so by borrowing, i.e. by issuing government bonds that are bought by private banks and households, both in the US and abroad, as well as by foreign central banks. The total federal government debt that is outstanding is the accumulation of past budget deficits. The federal debt and the deficit are related by

$$\begin{aligned}\text{Fed. debt at end of this year} &= \text{Fed. debt at end of last year} \\ &\quad +\text{Fed. budget deficit this year} \qquad (1.1)\end{aligned}$$

Hence when the budget is in deficit, the outstanding federal debt increases, when it is in surplus (as in 1998-2001), the government pays back part of

of the current cycle to June 2009, and thus according to this dating the great recession lasted from December 2007 to June 2009. However, since the recovery since 2009 was very slow and economic activity is still significantly below its long run trend as I write today, I will include the years 2010 through 2013 in the definition of the Great Recession in this class.

its outstanding debt. Now let us look at the federal government budget for the latest year we have final data for, 2011. Table 1.2 contains the exact numbers.

We see that the bulk of the federal government's receipts comes from individual income taxes and social security and unemployment contributions paid by private households (called social insurance receipts in the table), and, to a lesser extent from corporate income taxes (taxes on profits of private companies). The role of indirect business taxes (i.e. sales taxes) which are included in the "Other" category is relatively minor for the *federal* budget as most of sales taxes accrue to the states and cities in which they are levied.

On the outlay side the two biggest posts are national defense, which constitutes about 60% of all federal government purchases ($G$) and transfer payments, mainly social security, medicare and unemployment benefits (which is included in "income security". About 16% of federal outlays go as transfers to states and cities to help finance projects like highways, bridges and other infrastructure projects, as well as to help with social insurance payments of the states. These items are included in the Health, Income Security and Other categories.

A sizeable fraction (6.4%) of the federal budget is devoted to interest payments on the outstanding federal government debt. The outstanding government debt at the end of 2011 was about 100% of GDP. In other words, if the federal government could expropriate all production in the US (or equivalently all income of all households) for the whole year of 2011, it would need all of it to repay the outstanding federal debt at once. The ratio between total government debt (which, roughly, equals federal government debt) and GDP is called the (government) debt-GDP ratio, and is the most commonly reported statistics (apart from the budget deficit as a fraction of GDP) measuring the indebtedness of the federal government. It makes sense to report the debt-GDP ratio instead of the absolute level of the debt because the ratio relates the amount of outstanding debt to the governments' tax base and thus its ability to generate revenue. The broadest measure of the tax base of the government is the GDP of the country.

Let's have a brief look at the budget on the state and local level. The latest official final numbers again stem from the fiscal year 2011. Table 1.3 summarizes the main facts. The premier difference between the federal and state and local governments is the type of revenues and outlays that the different levels of government have, and the fact that states usually have a balanced budget amendment: they are by law prohibited from running

| 2011 State and Local Budgets (in billion $) | |
|---|---|
| Total Revenue | 2,064.7 |
| Personal Taxes | 322.8 |
| Taxes on Production and Sales | 990.4 |
| Corporate Income Taxes | 47.6 |
| Contributions for Soc. Ins. | 18.3 |
| Asset Income | 86.4 |
| Transfers from Federal Gov. | 612.7 |
| Surplus of Gov. Enterprises | -13.8 |
| Total Expenditures | 2,166.3 |
| Govt Spending | 1518.0 |
| Social Insurance Benefits | 538.5 |
| Interest Payments | 109.2 |
| Subsidies | 0.5 |
| Surplus | -102.0 |

Table 1.3: State and Local Budgets, 2011

a deficit, and immediate action is required should a deficit arise.[5] 2011 was one of the fairly rare occasions where the aggregated state and local budgets indeed showed a substantial deficit, driven by revenue shortfalls due to a subdued economic activity in most states in the aftermath of the great recession.

The main observations from the receipts side are that the main source of state and local government revenues stems from indirect sales taxes. A substantial further part of revenues on the state and local level comes about from transfers from the federal government; these transfers are intended to help finance large infrastructure projects and expenditures for homeland security on the state level. Income taxes, although not unimportant for state and local governments, do not nearly comprise as an important share of total revenue as it does for the federal government.

On the outlay side the single most important category is expenditures for government consumption. On the state and local level a large share of this goes to expenditures for public education, in the form of direct purchases of education material and, more importantly, the pay of public school teachers.

[5] The only state in the US that currently does not have a some form of a balanced budget amendment is Vermont.

All payments to state universities and public subsidies to private schools or universities are also part of these outlays. Also part of this category are expenditures for public infrastructure programs such as roads. An important share of expenditures is also used for social insurance, which is comprised mainly of retirement benefits for state employees as well as financial transfers to poor families in the form of welfare and other assistance payments such as state unemployment benefit payments. Finally the state and local governments have to service interest payments on bonds issued to finance certain large infrastructure projects and they give (small) subsidies to attract businesses to their states or cities.

## 1.3 Fiscal Variables and the Business Cycle

In this section we briefly document to what extent actual fiscal policy is correlated with the business cycle. Since we only look at data, all the statements we can make are about correlations[6], not about causality. That is, we certainly at this point of the course do not assert that government economic activity creates or mitigates fluctuations in economic activity.

In Figure 1.2 we plot the unemployment rate as prime indicator of business cycle and purchases of the (federal, state and local) government as a fraction of GDP over time. As already discussed above, one feature that

[6] Remember from basic statistics that the correlation coefficient between two time series $\{x_t, y_t\}_{t=1}^T$ is given by

$$corr(x,y) = \frac{Cov(x,y)}{Std(x) \cdot Std(y)}$$

where

$$\begin{aligned} Cov(x,y) &= \frac{1}{T}\sum_{t=1}^T (x_t - \bar{x})(y_t - \bar{y}) \\ Std(x) &= \sqrt{\sum_{t=1}^T (x_t - \bar{x})^2} \\ Std(y) &= \sqrt{\sum_{t=1}^T (y_t - \bar{y})^2} \end{aligned}$$

are the covariances between the two variables and the standard deviations of the two variables, respectively. A positive correlation coefficient indicates that, on average, the variable $x$ is high at the same time the variable $y$ is high.

appears in the data is that government spending, as a fraction of GDP, has declined over time (see the right scale) from the mid 1960's to 2001 and substantially risen since then. One also can detect that in recessions (in times where the unemployment rises, see the left scale) government spending as a fraction of GDP increases. This is consistent with the view that government spending is being used to a certain degree -successfully or not- to smooth out business cycles.[7]

A similar, even more accentuated picture appears if one plots government transfers (such as unemployment compensation and welfare) against the unemployment rate. The fact that government transfers are countercyclical follows almost by construction: in recessions by definition a lot of people are unemployed and hence more unemployment compensation (and once this runs out, welfare) is paid out. These welfare programs are sometimes called automatic stabilizers, as these programs provide more transfers exactly in times where incomes of households tend to be low on average, hence softening the decline in household disposable incomes and thus possibly consumption expenditures, therefore perhaps mitigating the recessions.

In Figure 1.3 we plot the unemployment rate and government tax receipts as a fraction of GDP against time. We see that tax receipts are strongly procyclical, they increase in booms (low unemployment) and decline during recessions (high unemployment). In this sense taxes act as automatic stabilizers, too, since, due to the progressivity of the tax code, in good times households on average are taxed at a higher rate than in bad times. In this sense the tax system stabilizes after-tax incomes and hence spending over the business cycle. A second reason for declines of taxes in recessions is discretionary tax policy: cutting taxes may provide a stimulus for private consumption and hence may help to lead the economy out of a recession (we will later study a theorem that argues, however, that the timing of taxes is irrelevant for the real economy). For example, the tax cuts in the early 60's under President Kennedy were designed for this purpose; the Bush tax cuts

[7] When one computes the coefficient of correlation between the unemployment rate and the share of government spending in GDP one obtains a positive number, 0.133, suggesting that that government spending is indeed high (relative to GDP) when the unemployment rate is high. This correlation becomes much higher when one ignores the six years from 1966 to 1972, which featured both a strong increase in the unemployment rate and a strong decline in the government expenditure share. Excluding this period one obtains a strongly positive correlation of 0.55, suggesting that government spending was in fact highly countercyclical from the early 70's onward.

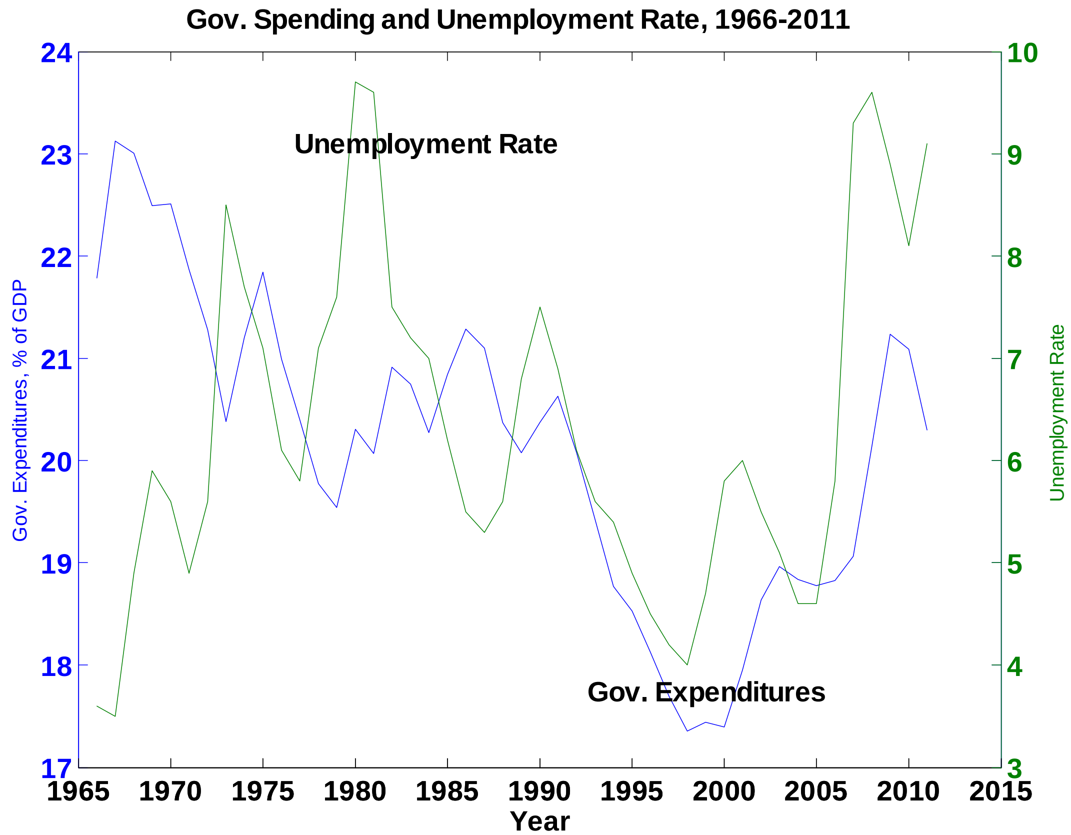


Figure 1.2: Unemployment Rate and Government Spending

were in part motivated by the same reason, and the temporary tax cuts associated with the recent economic stimulus package served a similar purpose. Therefore, in addition to being automatic stabilizers, taxes might be used deliberately in an attempt to fine-tune the business cycle.[8]

Now let us look at the government deficit over the business cycle. Figure 1.4 plots the federal budget deficit as a fraction of GDP and the unemployment rate over time. The first observation is (see the right scale) that the federal budget had small surpluses in the late 60's, then went into (heavy) deficit for the next 35 years or so and only in the late 90's showed surpluses

[8]The correlation between taxes and the unemployment rate is $-0.22$, significantly negative. Remember that a high unemployment rate means bad economic times, with low GDP.

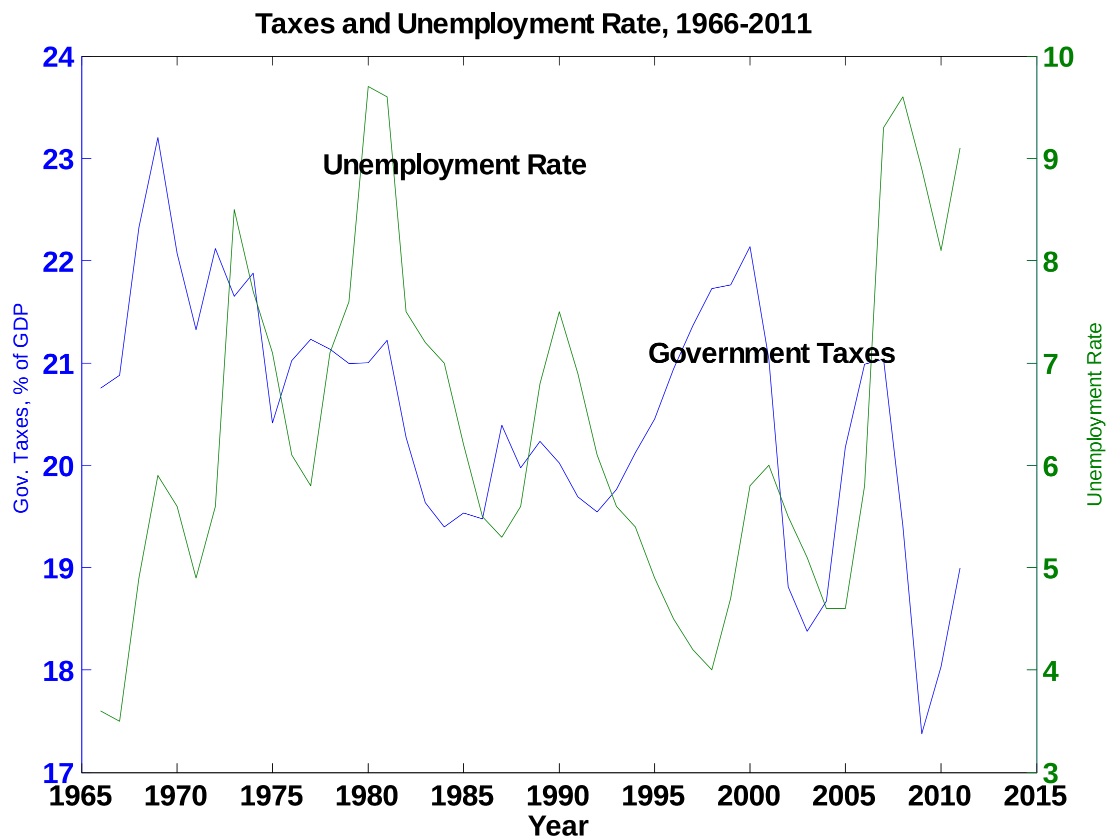


Figure 1.3: Unemployment Rate and Tax Receipts

again, which disappeared in the year 2002 and turned in to large deficits in recent years. One clearly sees the large deficits during the oil price shock recession in 1974-75 and the large deficit during the early Reagan years, due to large increases of defense spending. During the great recession the federal government ran massive budget deficits, in part due to the collapse in tax revenue, but in part also due to the increased expenditures due to the stimulus package.

Overall one observes that the budget deficit is clearly countercyclical: the deficit is large in recessions (as tax revenues decline and government outlays tend to increase) and is small in booms. In fact the extremely long and powerful expansion during the 90's resulted, in combination with federal government spending cuts, in the budget surpluses of the late 1990's.[9]

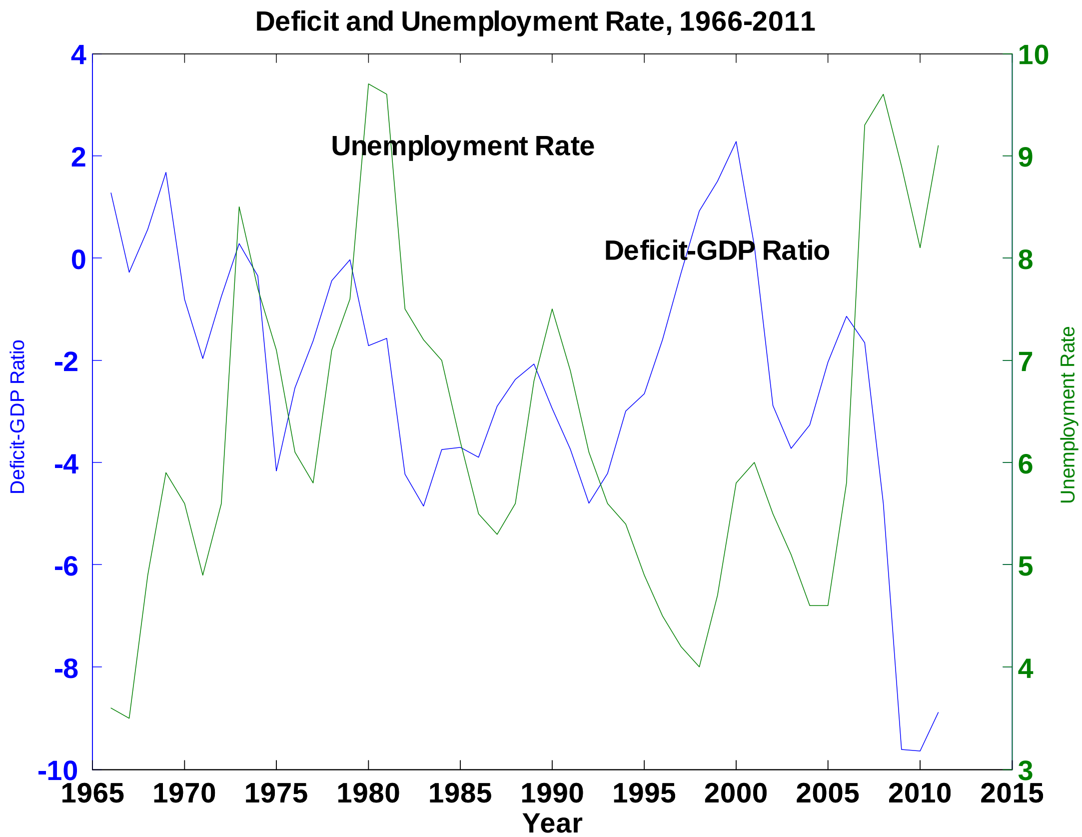


Figure 1.4: Unemployment Rate and Government Deficit

How does one determine whether the federal government is loose or tight on fiscal policy. Just looking at the budget deficit may obscure matters, since the current government may either have generated a large deficit because of loose fiscal policy or because the economy is in a recession where taxes are typically low and transfer payments high, so that the large deficit was beyond the control of the government. Hence economists have developed the notion of the *structural government deficit*: it is the government deficit that would arise if the economy's current GDP equals its potential (or long run trend)

[9] Again, the correlation of unemployment rate and the federal government deficit is strongly negative at $-0.43$: high unemployment rates go hand in hand with large deficits (remember that a deficit is a negative number).

GDP. The structural part of the deficit is *not* due to the business cycle, it is the deficit that on average arises given the current structure of taxes and expenditures. The *cyclical government deficit* is the difference between the actual and the structural deficit: it is that part of the deficit that is due to the business cycle. How loose or restrictive fiscal policy is can then be determined by looking at the structural (rather than the actual) deficit. Currently both the cyclical and the structural component of the U.S. budget show a large deficit, suggesting that simply waiting for a sustained economic recovery and hoping for the deficit problem to disappear is likely not a solution of the problem.

## 1.4 Government Deficits and Government Debt

We previous defined the government budget deficit and related it to the change in the outstanding government debt. In table 1.4 we provide government deficit numbers for a cross-section of industrialized countries, to put the U.S. numbers into an international context.

We observe that, within the Euro area, there is substantial variation in the deficit-GDP ratio in 2010, ranging from a small surplus in Estonia to a massive deficit in Ireland (which bailed out its banking sector using public funds). Comparing the Euro numbers to the US or Japan (or some countries in Europe not (yet) in the Euro area) we observe that deficit figures in the U.S. are high, but not outrageous by international standards. In the midst (or aftermath, depending on the timing one wishes to adopt) of the great recession, the largest macroeconomic downturn since the great recession, very substantial public budget deficits are the norm world-wide. However, note that the budget deficits of the US and Japan are the source of significant concern by policy makers and economists in the respective countries, so the fact the some European counties' substantial deficits are passed by other countries still should not be a sign of comfort.

We now want to take a quick look at the stock of outstanding government debt, both in international comparison as well as over time for the US. For the US the outstanding government debt at the end of 2012 was about \$16.2 trillion, or about 103% of GDP (see above). The ratio between total government debt (which, roughly, equals federal government debt) and GDP is called the (government) debt-GDP ratio, and is the most commonly reported statistics (apart from the budget deficit as a fraction of GDP) measuring the

| International Deficit to GDP Ratios | |
|---|---|
| Country | Deficit/GDP in 2010 |
| Austria | 4.5% |
| Belgium | 3.9% |
| France | 7.1% |
| Germany | 4.2% |
| Luxembourg | 0.9% |
| Netherlands | 5.0% |
| Greece | 10.5% |
| Ireland | 31.2% |
| Italy | 4.5% |
| Portugal | 9.8% |
| Spain | 9.4% |
| Denmark | 2.7% |
| Finland | 2.8% |
| Sweden | 0.1% |
| Czech Republic | 4.8% |
| Estonia | -0.3% |
| Hungary | 4.3% |
| Poland | 7.9% |
| Slovenia | 6.0% |
| Slovakia | 7.7% |
| UK | 10.1% |
| US | 10.6% |
| Japan | 8.4% |

Table 1.4: Federal Government Deficits as fraction of GDP, 2010

indebtedness of the federal government. It makes sense to report the debt-GDP ratio instead of the absolute level of the debt because the ratio relates the amount of outstanding debt to the governments' tax base and thus ability to generate revenue, namely GDP.

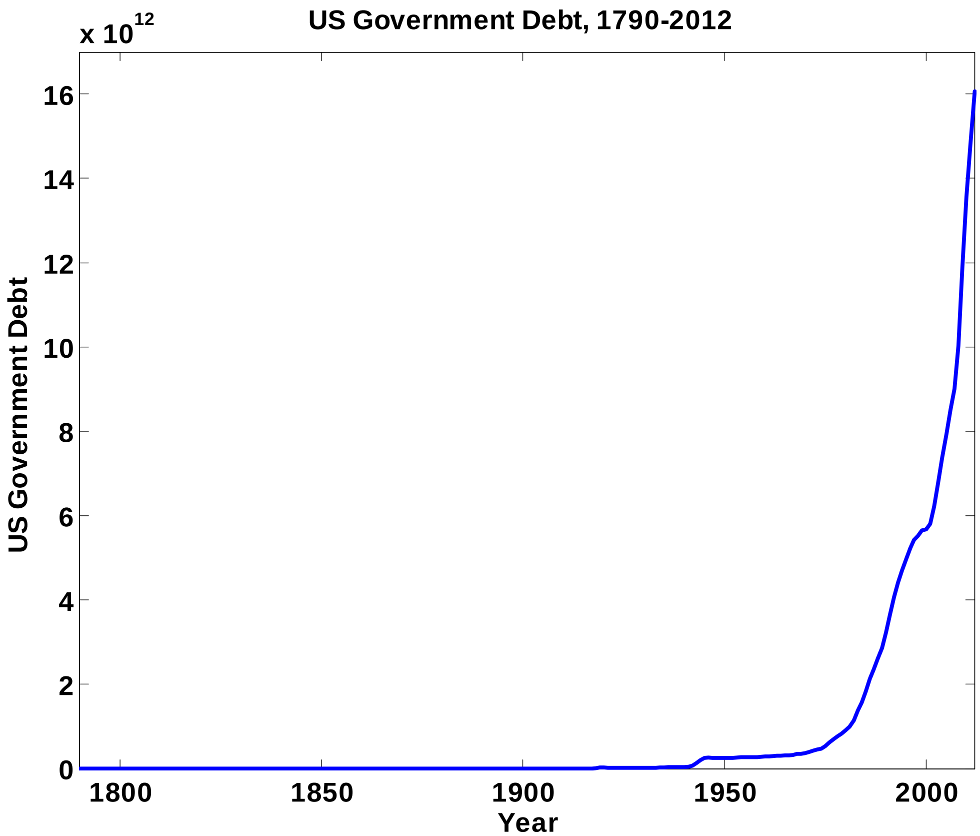


Figure 1.5: US Government Debt

Figure 1.5 plots the outstanding U.S. nominal government debt. It shows the explosion of the debt in the last 70 years. Clearly visible is the sharp increase during World War II and the steep increases in the 1970 and after 2001. .Arguably this picture is somewhat misleading (I selected it nevertheless to emphasize the magnitudes of the numbers). The picture is misleading for two reasons. First, it plots nominal debt and thus does not control for inflation. Second, it does not relate the level of debt to any variable measuring the government's ability to pay for it.

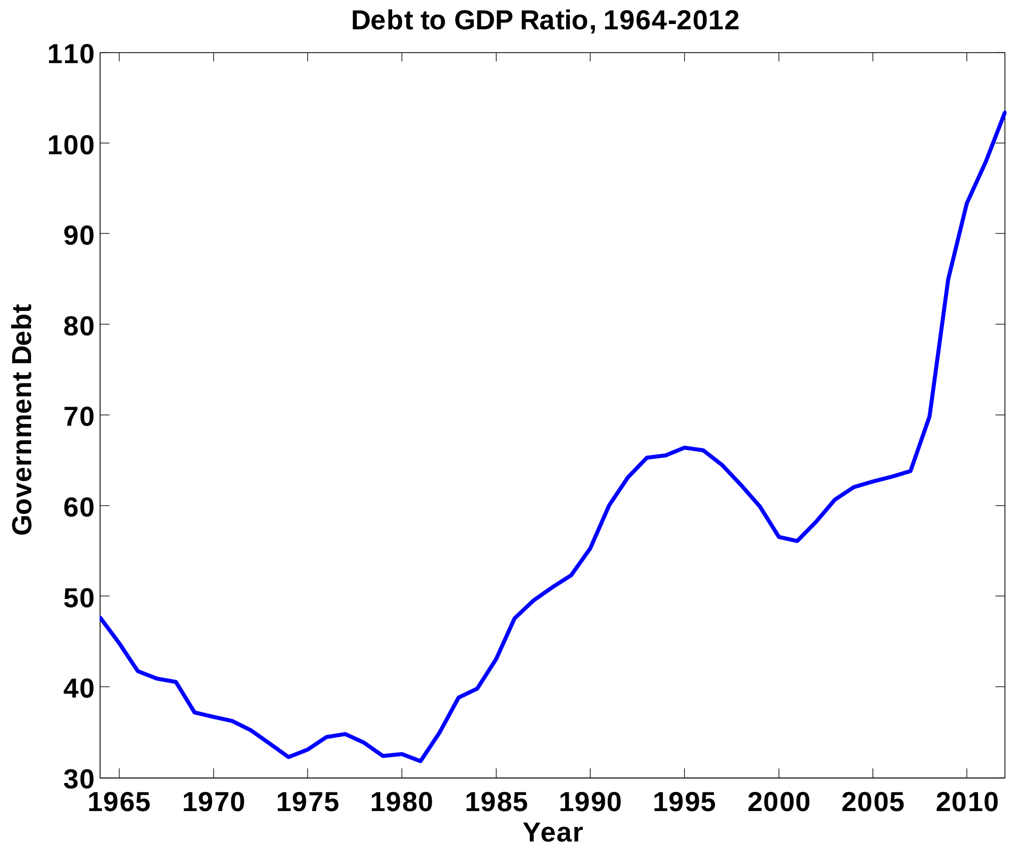


Figure 1.6: Government Debt as a Fraction of GDP, 1964-2012

Therefore it is more informative to plot the debt-GDP ratio in figure 1.6 since it relates the level of public debt to the level of economic activity in a country.[10] The main facts are that during the 60's the US continued to repay part of its WWII debt as the debt grew slower than GDP. Then, starting in the 70's and more pronounced in the 80's, large budget deficits led to a rapid increase in the debt-GDP ratio, a trend that stopped and reversed in the late 1990's (recall that the late 1990's featured budget surpluses, so not only did debt grow slower than GDP, it actually fell in absolute terms). Starting from 2001, and accelerating during the great recession, there has been a massive increase in the debt-to-GDP ratio, which, as indicated above now stands at

[10] For calculating the debt-to-GDP ratio it does not matter whether we divide numinal debt by nominal GDP or real debt by real GDP.

more than 100% of the annual aggregate output and income (that is, GDP) in the U.S.

In order to again place the U.S. numbers into the international context, in table 1.5 we display debt-GDP ratios for various industrialized countries for 2010, the last year for which the data is available for a larger set of countries. Again, the variance of debt-GDP ratios across countries is remarkable, with Japan displaying a ratio of debt to GDP in excess of 200%, many European countries featuring a ratio above 100% (notably Greece, Italy and Belgium). In contrast, Luxembourg and Estonia hardly have any government debt. Finally, Japan's large debt to GDP ratio may help explain the high private sector savings rate in Japan (somebody has to pay that debt, or at least the interest on that debt, with higher taxes sometime in the future). Note that a substantial fraction of this debt was accumulated during the 1990's, when various government spending and tax cut programs where enacted to try to bring Japan out of its decade-long recession.

Furthermore, observe that the former Communist East European countries (such as Estonia, Slovenia, Slovakia and also the Czech Republic) tend to have lower debt-GDP ratios, basically because they started with a blank slate at the collapse of the old regime at the end of the 1980's.

Overall, as with the deficit, the U.S. debt to GDP ratio is not an outlier by international standard, as just about all countries now have a very sizeable debt problem, a large part of which emerged in the aftermath of the great recession, which was a world-wide phenomenon and had similar adverse consequences for tax revenues across the globe. The response of government outlays was somewhat more heterogeneous across countries, which partially explains the cross-country differences in public indebtedness transparent in figure 1.5.

This concludes our brief overview over government spending, taxes, deficits and debt in industrialized countries. Once we have constructed, in the next chapters, our theoretical model that we will use to analyze the effects of fiscal policy, we will combine theoretical analysis with further empirical observations to arrive at a (hopefully) somewhat coherent and complete view of what a modern government does and should do in the economy.

| International Debt to GDP Ratios | |
|---|---|
| Country | Debt/GDP in 2010 |
| Austria | 77.5 |
| Belgium | 100.0 |
| France | 94.9 |
| Germany | 86.9 |
| Luxembourg | 24.5 |
| Netherlands | 71.7 |
| Greece | 123.0 |
| Ireland | 91.7 |
| Italy | 106.2 |
| Portugal | 97.5 |
| Spain | 66.8 |
| Denmark | 54.8 |
| Finland | 56.9 |
| Sweden | 48.0 |
| Czech Republic | 44.5 |
| Estonia | 12.8 |
| Hungary | 86.3 |
| Poland | 62.7 |
| Slovenia | 48.4 |
| Slovakia | 47.1 |
| UK | 88.8 |
| US | 99.1 |
| Japan | 210.2 |

Table 1.5: Government Debt as Fraction of GDP, 2010

## 1.5 Appendix: The Trade Balance

In order to understand the connection between the trade balance and the net foreign asset position of a country we need some more definitions. We already defined what the **trade balance** is: it is the total value of exports minus the total value of imports of the US with all its trading partners. A closely related concept is the **current account balance**. The current account balance equals the trade balance plus **net unilateral transfers**

$$\text{Current Account Balance} = \text{Trade Balance} + \text{Net Unilateral Transfers}$$

Unilateral transfers that the US pays to countries abroad include aid to poor countries, interest payments to foreigners for US government debt, and grants to foreign researchers or institutions. Net unilateral transfers equal transfers of the sort just described received by the US, minus transfers paid out by the US. Usually net unilateral transfers are negative for the US, but small in size (less than 1% of GDP). So for the purpose of this class we can use the trade balance and the current account balance interchangeably. We say that the US has a current account deficit if the current account balance is negative and a current account surplus if the current account balance is positive.

The current account balance thus (roughly) keeps track of import and export flows between countries. The **capital account balance** keeps track of borrowing and lending of the US with abroad. It equals to the **change of the net wealth position** of the US. The US owes money to foreign countries, in the form of government debt held by foreigners, loans that foreign banks made to US companies and in the form of shares that foreigners hold in US companies. Foreign countries owe money to the US for exactly the same reason The net wealth position of the US is the difference between what the US is owed and what it owes to foreign countries. Thus

$$\begin{aligned}\text{Capital Account Balance this year} &= \text{Net wealth position at end of this year} \\ &\quad -\text{Net wealth position at end of last year}\end{aligned}$$

Note that a negative capital account balance means that the net wealth position of the US has decreased: in net terms, wealth has flown out of the US. The reverse is true if the capital account balance is positive: wealth flew into the US.

The current account and the capital account balance are intimately related: they are *always* equal to each other. This is an example of an account-

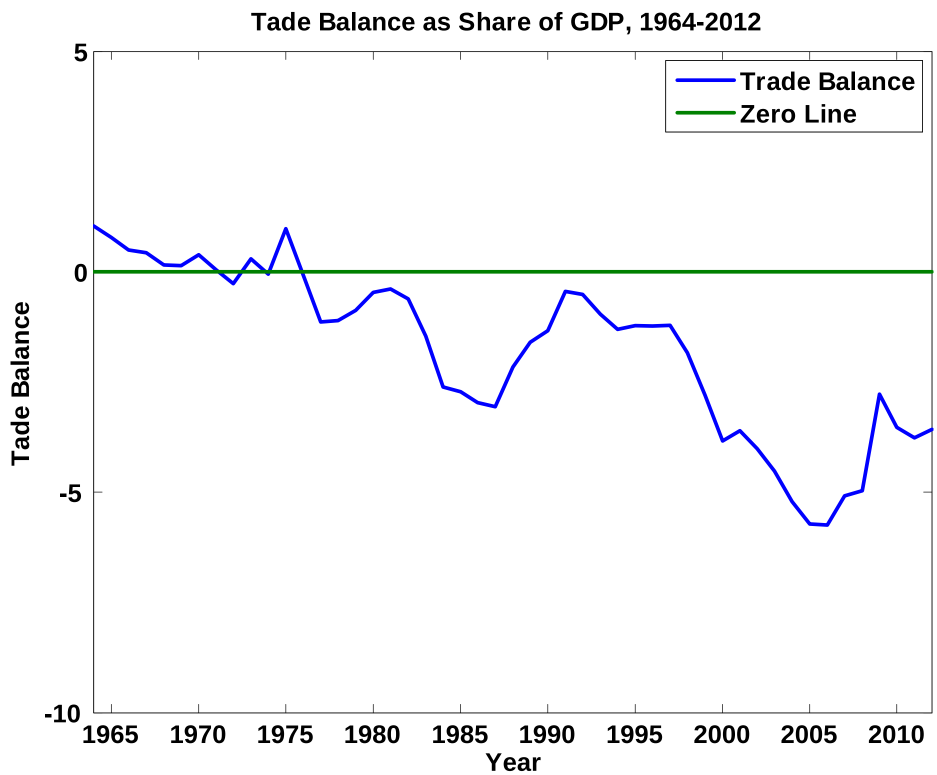


Figure 1.7: Trade Balance as Fraction of GDP, 1963-2011

ing identity.

$$\text{Current Account Balance this year} = \text{Capital Account Balance this year}$$

The reason for this is simple: if the US imports more than it exports, it has to borrow from the rest of the world to pay for the imports. But this change in the net asset position is exactly what the capital account balance captures.

Figure 1.7, which plots the trade balance as a fraction of GDP, shows that the US trade balance was not always negative. In fact, it was mostly positive in the period before the 1980's, before turning sharply negative in the 1990's. Since 1989 the US, traditionally a net lender to the world, has become a net borrower: the net wealth position of the US has become negative in 1989. The US appetite for foreign goods and services also means that, in order to pay for these goods, US consumers have to (directly or indirectly through the companies that import the goods) acquire foreign currency for dollars, which puts pressure on the exchange rate between the dollar and foreign currencies.

# Chapter 2

# A Two Period Benchmark Model

In this section we will develop a simple two-period model of consumption and saving that we will then use to study the impact of government policies on an individual households' consumption and saving decisions (in particular social security, income taxation and government debt). We will then generalize this model to more than two periods and study the empirical predictions of the model with respect to consumption and saving over the life cycle of a typical household. The simple model we present is due to Irving Fisher (1867-1947), and the extension to many periods is due to Albert Ando (1929-2003) and Franco Modigliani (1919-2003) (and, in a slightly different form, to Milton Friedman (1912-present)).

## 2.1 The Model

Consider a single individual, for concreteness call this guy Hardy Krueger. Hardy lives for two periods (you may think of the length of one period as 30 years, so the model is not all that unrealistic). He cares about consumption in the first period of his life, $c_1$ and consumption in the second period of his life, $c_2$. His utility function takes the simple form

$$U(c_1, c_2) = u(c_1) + \beta u(c_2) \tag{2.1}$$

where the parameter $\beta$ is between zero and one and measures Hardy's degree of impatience. A high $\beta$ indicates that consumption in the second period of his life is really important to Hardy, so he is patient. On the other hand, a low $\beta$ makes Hardy really impatient. In the extreme case of $\beta = 0$ Hardy only cares about his consumption in the current period, but not at all about consumption when he is old. The period utility function $u$ is assumed to be at least twice differentiable, strictly increasing and strictly concave. This means that we can take at least two derivatives of $u$, that $u'(c) > 0$ (more consumption increases utility) and $u''(c) < 0$ (an additional unit of consumption increases utility at a decreasing rate).

Hardy has income $y_1 > 0$ in the first period of his life and $y_2 \geq 0$ in the second period of his life (we want to allow $y_2 = 0$ in order to model that Hardy is retired in the second period of his life and therefore, absent any social security system or private saving, has no income in the second period). Income is measured in real terms, that is, in units of the consumption good, not in terms of money. Hardy starts his life with some initial wealth $A \geq 0$, due to bequests that he received from his parents. Again $A$ is measured in terms of the consumption good. Hardy can save some of his income in the first period or some of his initial wealth, or he can borrow against his future income $y_2$. We assume that the interest rate on both savings and on loans is equal to $r$, and we denote by $s$ the saving (borrowing if $s < 0$) that Hardy does. Hence his budget constraint in the first period of his life is

$$c_1 + s = y_1 + A \tag{2.2}$$

Hardy can use his total income in period 1, $y_1 + A$ either for eating today $c_1$ or for saving for tomorrow, $s$. In the second period of his life he faces the budget constraint

$$c_2 = y_2 + (1 + r)s \tag{2.3}$$

i.e. he can eat whatever his income is and whatever he saved from the first period. The problem that Hardy faces is quite simple: given his income and wealth he has to decide how much to eat, $c_1$, in period 1 and how much to save, $s$, for the second period of his life. Consumption $c_2$ in the second period of life is completely determined by his savings choice $s$.

The is a very standard decision problem that you have studied extensively in intermediate microeconomics, with the only difference that the goods that Hardy chooses are not apples and bananas, but consumption today and consumption tomorrow.

For the purpose of analyzing the model and interpreting its outcome we now consolidate (2.2) and (2.3) into one budget constraint, the so-called intertemporal budget constraint (*intertemporal* because it combines income and consumption in both periods). Solving (2.3) for $s$ yields

$$s = \frac{c_2 - y_2}{1+r} \tag{2.4}$$

and substituting this into (2.2) yields

$$c_1 + \frac{c_2 - y_2}{1+r} = y_1 + A$$

or

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} + A \tag{2.5}$$

Let us interpret this budget constraint. We have normalized the price of the consumption good in the first period to 1 (remember from microeconomic theory that we could multiply all prices by a constant and the decision problem of Hardy would not change). The price of the consumption good in period 2 is $\frac{1}{1+r}$, which is also the relative price of consumption in period 2, relative to consumption in period 1. Hence the gross real interest rate $1+r$ is really a price: it is the relative price of consumption goods today to consumption goods tomorrow.[1] So the intertemporal budget constraint says that total expenditures on consumption goods $c_1 + \frac{c_2}{1+r}$, measured in prices of the period 1 consumption good, have to equal total income $y_1 + \frac{y_2}{1+r}$, measured in units of the period 1 consumption good, plus the initial wealth of Hardy. The sum of all labor income $y_1 + \frac{y_2}{1+r}$ is sometimes referred to as human capital or human wealth. Let us denote Hardy's total income, consisting of human capital and initial wealth, by $I$, that is

$$I = y_1 + \frac{y_2}{1+r} + A. \tag{2.6}$$

---

[1] The real interest rate $r$, the nominal interest rate $i$ and the inflation rate are related by the equation

$$1 + r = \frac{1+i}{1+\pi}.$$

Thus

$$i \simeq r + \pi,$$

which is a good approximation as long as $r\pi$ is small relative to $i, r$ and $\pi$. This will be the case if both $r$ and $\pi$ are small. For example, if $r = 0.01 = 1\%$ and $\pi = 0.02 = 2\%$ then $r\pi = 0.0002 = 0.02\%$ which is small relative to $r = 1\%$ and $\pi = 2\%$.

## 2.2 Solution of the Model

Now we can analyze Hardy's consumption decision. He wants to maximize his utility (2.1), but is constrained by the intertemporal budget constraint (2.5). Thus he solves:

$$\begin{aligned} & \max_{c_1,c_2} \{u(c_1) + \beta u(c_2)\} \\ s.t. \qquad c_1 + \frac{c_2}{1+r} &= I \end{aligned} \tag{2.7}$$

In the main text we solve this problem using the Lagrangian approach. To do so we attach a Lagrange multiplier $\lambda$ to the intertemporal budget constraint (2.7) and write the Lagrangian as

$$\mathcal{L} = u(c_1) + \beta u(c_2) + \lambda \left[ I - c_1 + \frac{c_2}{1+r} \right].$$

Taking first order conditions with respect to $c_1$ and $c_2$ yields

$$\begin{aligned} u'(c_1) - \lambda &= 0 \\ \beta u'(c_2) - \frac{\lambda}{1+r} &= 0 \end{aligned}$$

We can rewrite both equations as

$$\begin{aligned} u'(c_1) &= \lambda \\ \beta(1+r)u'(c_2) &= \lambda \end{aligned}$$

Thus combining both equations by eliminating the Lagrange multiplier $\lambda$ we arrive at the perhaps most important equation in all of macroeconomics, the so-called Euler equation, that relates consumption of the typical household in both periods (in our case Hardy Krueger):

$$u'(c_1) = \beta(1+r)u'(c_2) \tag{2.8}$$

or

$$\frac{\beta u'(c_2)}{u'(c_1)} = \frac{1}{1+r}. \tag{2.9}$$

This condition simply states that the consumer maximizes her utility by equalizing the marginal rate of substitution between consumption tomorrow and consumption today, $\frac{\beta(u'(c_2)}{u'(c_1)}$, with relative price of consumption tomorrow to consumption today, $\frac{\frac{1}{1+r}}{1} = \frac{1}{1+r}$. Condition (2.9), together with the budget constraint (2.5), uniquely determines the optimal consumption choices $(c_1, c_2)$, as a function of incomes $(y_1, y_2)$, initial wealth $A$ and the interest rate $r$.[2]

One can solve explicitly for $(c_1, c_2)$ in a number of ways, either algebraically or diagrammatically. We will do both below. We will then doc-

[2] Solving the intertemporal budget constraint for $c_1$ yields

$$c_1 = I - \frac{c_2}{1+r}$$

and plugging into the Euler equation (2.9) yields

$$u'\left(I - \frac{c_2}{1+r}\right) = (1+r)\beta u'(c_2). \tag{2.10}$$

We want to argue that a unique solution to this equaition exists. Strictly speaking, for a unique solution we require another assumption on the utility function, the so-called Inada condition

$$\lim_{c \to 0} u'(c) = \infty$$

that states that marginal utility becomes really large as consumption gets closer and closer to 0. There is another Inada condition that is sometimes useful:

$$\lim_{c \to \infty} u'(c) = 0,$$

but this condition is not needed to prove existence and uniqueness of an optimal solution.

With the first Inada condition it is straightforward to show the existence of a unique solution to (2.10). Either we plot both sides of (2.10) and argue graphically that there exists a unique intersection, or we use some math. The function

$$f(c_2) = u'\left(I - \frac{c_2}{1+r}\right) - (1+r)\beta u'(c_2)$$

is continuous on $c_2 \in (0, (1+r)I)$, strictly increasing (since $u$ is concave) and satisfies (due to the Inada conditions)

$$\begin{aligned} \lim_{c_2 \to 0} f(c_2) &< 0 \\ \lim_{c_2 \to (1+r)I} f(c_2) &> 0. \end{aligned}$$

Thus by the Intermediate Value Theorem from mathematics there exists a (unique, since $f$ is stictly increasing) $c_2^*$ such that $f(c_2) = 0$, and thus a unique solution $c_2^*$ to (2.10).

ument how the optimal solution $(c_1, c_2)$ changes as one changes incomes $(y_1, y_2)$, bequests $A$ or the interest rate $r$. Such analysis is called *comparative statics*. But first we work through an example with a special utility function that leads to a very simple, closed-form solution:

**Example 1** *Suppose that the period utility function is logarithmic, that is* $u(c) = \log(c)$, *and thus the lifetime utility function is given by*

$$\log(c_1) + \beta \log(c_2).$$

*The equation* (2.9) *becomes*

$$\begin{aligned} \frac{\beta * \frac{1}{c_2}}{\frac{1}{c_1}} &= \frac{1}{1+r} \\ \frac{\beta c_1}{c_2} &= \frac{1}{1+r} \\ c_2 &= \beta(1+r)c_1 \end{aligned} \tag{2.11}$$

*Inserting equation* (2.11) *into equation* (2.5) *yields*

$$\begin{aligned} c_1 + \frac{\beta(1+r)c_1}{1+r} &= I \\ c_1(1+\beta) &= I \\ c_1 &= \frac{I}{1+\beta} \\ c_1(y_1, y_2, A, r) &= \frac{1}{1+\beta}\left(y_1 + \frac{y_2}{1+r} + A\right) \end{aligned} \tag{2.12}$$

*Since* $c_2 = \beta(1+r)c_1$ *we find*

$$\begin{aligned} c_2 &= \frac{\beta(1+r)}{1+\beta} I \\ &= \frac{\beta(1+r)}{1+\beta}\left(y_1 + \frac{y_2}{1+r} + A\right) \end{aligned} \tag{2.13}$$

*Finally, since savings* $s = y_1 + A - c_1$

$$\begin{aligned} s &= y_1 + A - \frac{1}{1+\beta}\left(y_1 + \frac{y_2}{1+r} + A\right) \\ &= \frac{\beta}{1+\beta}(y_1 + A) - \frac{y_2}{(1+r)(1+\beta)} \end{aligned}$$

*which may be positive or negative, depending on how high first period income and initial wealth is compared to second period income. So Hardy's optimal consumption choice today is quite simple: eat a fraction* $\frac{1}{1+\beta}$ *of total lifetime income* $I$ *today and save the rest for the second period of your life. Note that the higher is income* $y_1$ *in the first period of Hardy's life, relative to his second period income,* $y_2$*, the higher is saving* $s$*.*

For general utility functions $u(.)$ we can in general not solve for the optimal consumption and savings choices analytically. But even for the general case we can represent the optimal consumption choice graphically, using the standard microeconomic tools of budget lines and indifference curves.

First we plot the budget line (2.5). This is the combination of all $(c_1, c_2)$ Hardy can afford. We draw $c_1$ on the x-axis and $c_2$ on the y-axis. Looking at the left hand side of (2.5) we realize that the budget line is in fact a straight line. Now let us find two points on the line. Suppose $c_2 = 0$, i.e. Hardy does not eat in the second period. Then he can afford $c_1 = y_1 + A + \frac{y_2}{1+r}$ is the first period, so one point on the budget line is $(c_1^a, c_2^a) = (y_1 + A + \frac{y_2}{1+r}, 0)$. Now suppose $c_1 = 0$. Then Hardy can afford to eat $c_2 = (1+r)(y_1 + A) + y_2$ in the second period, so a second point on the budget line is $(c_1^b, c_2^b) = (0, (1+r)(y_1 + A) + y_2)$. Connecting these two points with a straight line yields the entire budget line. We can also compute the slope of the budget line as

$$\begin{aligned} \text{slope} &= \frac{c_2^b - c_2^a}{c_1^b - c_1^a} \\ &= \frac{(1+r)(y_1 + A) + y_2}{-\left(y_1 + A + \frac{y_2}{1+r}\right)} \\ &= -(1+r) \end{aligned}$$

Hence the budget line is downward sloping with slope $(1+r)$. Now let's try to remember some microeconomics. The budget line just tells us what Hardy can afford. The utility function (2.1) tells us how Hardy values consumption today and consumption tomorrow. Remember that an indifference curve is a collection of bundles $(c_1, c_2)$ that yield the same utility, i.e. between which Hardy is indifferent. Let us fix a particular level of utility, say $v$ (which is just a number). Then an indifference curve consists of all consumption bundles $(c_1, c_2)$ such that

$$v = u(c_1) + \beta u(c_2) \tag{2.14}$$

In order to determine the slope of this indifference curve we either find a micro book and look it up, or alternatively totally differentiate (2.14) with respect to $(c_1, c_2)$. To totally differentiate an equation with respect to all its variables (in this case $(c_1, c_2)$) amounts to the following. Suppose we change $c_1$ by a small (infinitesimal) amount $dc_1$. Then the right hand side of (2.14) changes by $dc_1 * u'(c_1)$. Similarly, changing $c_2$ marginally changes (2.14) by $dc_2 * \beta u'(c_2)$. If these changes leave us at the same indifference curve (i.e. no change in overall utility), then it must be the case that

$$dc_1 * u'(c_1) + dc_2 * \beta u'(c_2) = 0$$

or

$$\frac{dc_2}{dc_1} = -\frac{u'(c_1)}{\beta u'(c_2)}$$

which is nothing else than the slope of the indifference curve, or, in technical terms, the (negative of the) marginal rate of substitution between consumption in the second and the first period of Hardy's life.[3] For the example above with $u(c) = \log(c)$, this becomes

$$\frac{dc_2}{dc_1} = -\frac{c_2}{\beta c_1}$$

From equation (2.9) we see that at the optimal consumption choice the slope of the indifference curve and the budget line are equal or

$$-\frac{u'(c_1)}{\beta u'(c_2)} = -(1+r) = \text{slope}$$

or

$$\begin{aligned} \text{MRS} &= \frac{\beta u'(c_2)}{u'(c_1)} = \frac{1}{1+r} & (2.15) \\ u'(c_1) &= (1+r)\beta u'(c_2) & (2.16) \end{aligned}$$

This equation has a nice interpretation. At the optimal consumption choice the cost, in terms of utility, of saving one more unit should be equal

[3] The marginal rate of substitution between consumption in the first and second period is

$$\text{MRS} = \frac{\beta u'(c_2)}{u'(c_1)}$$

and thus the inverse of the MRS between consumption in the second and first period.

to the benefit of saving one more unit (if not, Hardy should either save more or less). But the cost of saving one more unit, and hence one unit lower consumption in the first period, in terms of utility equals $u'(c_1)$. Saving one more unit yields $(1 + r)$ more units of consumption tomorrow. In terms of utility, this is worth $(1 + r)\beta u'(c_2)$. Equality of cost and benefit implies (2.15), which together with the intertemporal budget constraint (2.5) can be solved for the optimal consumption choices. Figure 2.1 shows the optimal consumption (and thus saving choices) diagrammatically

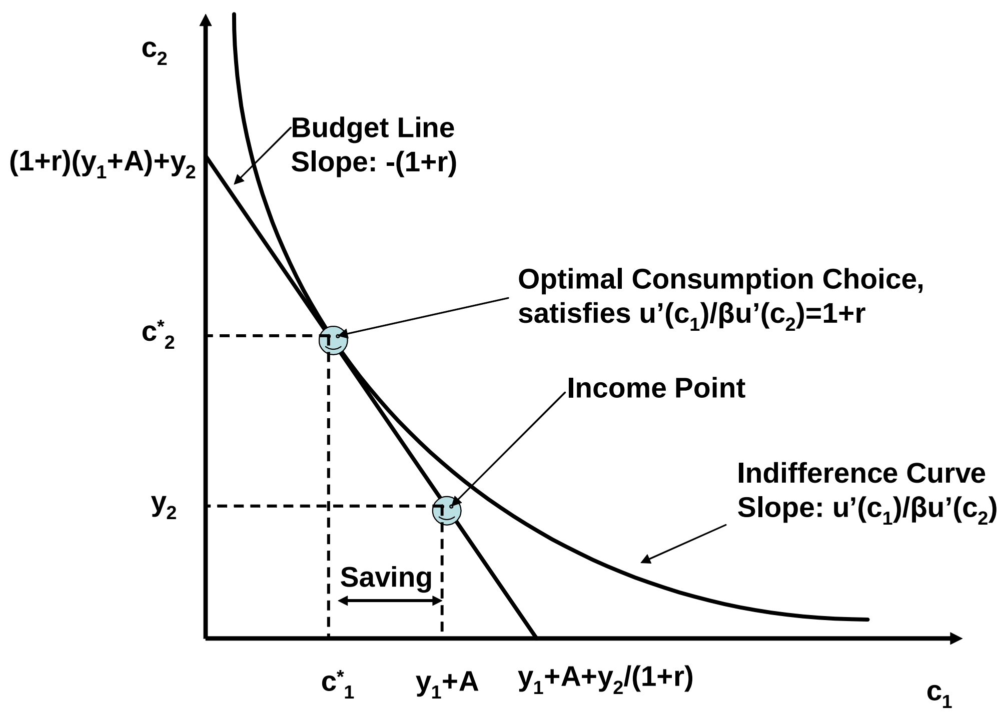


Figure 2.1: Optimal Consumption Choice

## 2.3 Comparative Statics

Government policies, in particular fiscal policy (such as social security and income taxation) affects individual households by changing the level and timing of after-tax income $(y_1, y_2)$. We will argue below that an expansion of the government deficit (and hence its outstanding debt) may also change the real interest rate $r$. In order to study the effect of these policies on the economy it is therefore important to analyze the changes in household behavior induced by changes in after-tax incomes and real interest rates.

### 2.3.1 Income Changes

First we investigate how changes in today's income $y_1$, next period's income $y_2$ and initial wealth $A$ change the optimal consumption choice. First we do the analysis for our particular example 1, then for an arbitrary utility function $u(c)$, using our diagram developed above.

For the example, from (2.12) and (2.13) we see that both $c_1$ and $c_2$ increase with increases in either $y_1, y_2$ or $A$. In particular, remembering that

$$I = y_1 + \frac{y_2}{1+r} + A$$

we have that

$$\begin{aligned} \frac{dc_1}{dI} &= \frac{1}{1+\beta} > 0 \\ \frac{dc_1}{dI} &= \frac{\beta(1+r)}{1+\beta} > 0 \end{aligned}$$

and thus

$$\begin{aligned} \frac{dc_1}{dA} &= \frac{dc_1}{dy_1} = \frac{1}{1+\beta} > 0 \text{ and } \frac{dc_1}{dy_2} = \frac{1}{(1+\beta)(1+r)} > 0 \\ \frac{dc_2}{dA} &= \frac{dc_2}{dy_1} = \frac{\beta(1+r)}{1+\beta} > 0 \text{ and } \frac{dc_2}{dy_2} = \frac{\beta}{1+\beta} > 0 \\ \frac{ds}{dA} &= \frac{ds}{dy_1} = \frac{\beta}{1+\beta} > 0 \text{ and } \frac{ds}{dy_2} = -\frac{1}{(1+\beta)(1+r)} < 0 \end{aligned}$$

The change in consumption in response to a (small) change in income is often referred to as marginal propensity to consume. From the formulas above we see that current consumption $c_1$ increases not only when current income and inherited wealth goes up, but also with an increase in (expected) income tomorrow.[4] Similarly consumption in the second period of Hardy's life increases not only with second period income, but also with income today. Finally, an increase in current income increases savings, whereas an increase in expected income tomorrow decreases saving, since Hardy finds it optimal to consume part of the higher lifetime income already today, and bringing some of the higher income tomorrow into today requires a decline in saving.

For our example we could solve for the changes in consumption behavior induced by income changes directly. In general this is impossible, but we still can carry out a graphical analysis for the general case, in order to trace out the qualitative changes on consumption and saving. In figure 2.2 we show what happens when income in the first period $y_1$ increases to $y_1' > y_1$. As a consequence the budget line shifts out in a parallel fashion (since the interest rate, which dictates the slope of this line does not change). At the new optimum both $c_1$ and $c_2$ are higher than before, just as in the example. The increase in consumption due to an income increase (in either period) is referred to as an income effect. If $A$ increases (which works just as an increase in $y_1$) it sometimes is also called a wealth effect. The income and wealth effects are positive for consumption in both periods for the (separable) utility functions that we will consider in this class, but you should remember from standard micro books that this need not always be the case (remember the infamous inferior goods).

### 2.3.2 Interest Rate Changes

It is more complicated to analyze changes in the interest rate rather than changes in income, since a change in the interest rate will entail three effects. Looking back to the maximization problem of the consumer, the interest rate enters at two separate places. First, on the left hand side of the budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} + A \equiv I(r)$$

[4] Standard static Keynesian consumption functions of the form

$$c_t = \alpha + \gamma(y_t + A_t)$$

typically ignore this later impact of future expected income on current consumption.

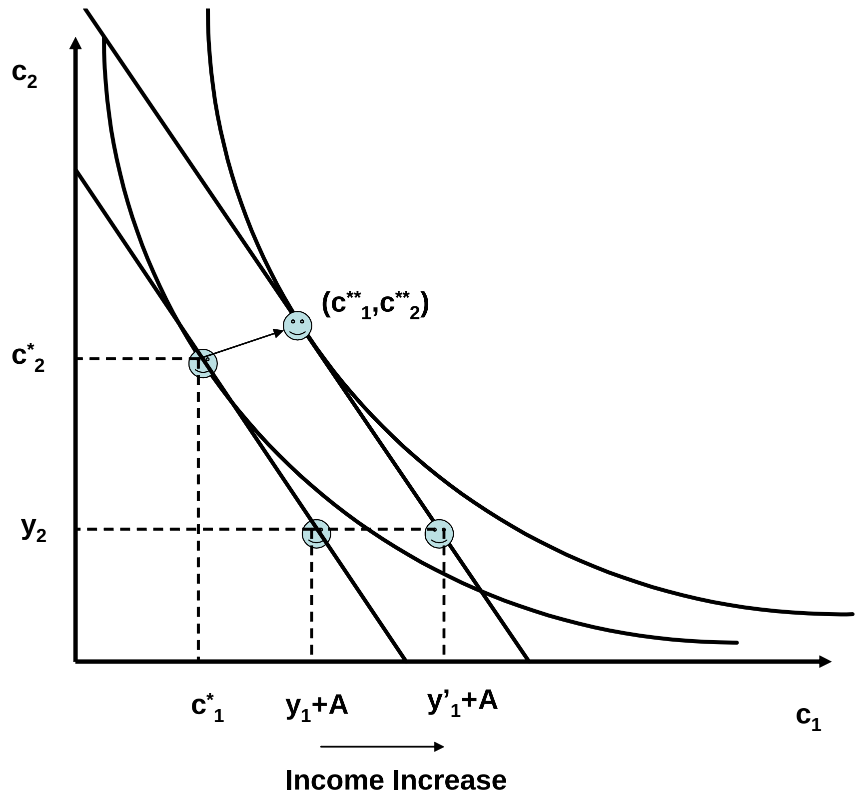


Figure 2.2: A Change in Income

as relative price of the second period consumption, $\frac{1}{1+r}$ and second as discount factor $\frac{1}{1+r}$ for second period income $y_2$. Now for concreteness, suppose the real interest rate $r$ goes up, say to $r' > r$. The first effect comes from the fact that a higher interest rate reduces the present discounted value of second period income, $\frac{y_2}{1+r}$. This is often called a (human capital) wealth effect, as it reduces total resources available for consumption, since $I(r') < I(r)$. The name human capital wealth effect comes from the fact that income $y_2$ is usually derived from working, that is, from applying Hardy's "human capital". Note that this effect is absent if Hardy does not earn income in the second period of his life, that is, if $y_2 = 0$.

The remaining two effects stem from the term $\frac{c_2}{1+r}$. An increase in $r$ reduces the price of second period consumption, $\frac{1}{1+r}$, which has two effects.

| Effect on | Incr. in $r$ $c_1$ | $c_2$ | Decr. in $r$ $c_1$ | $c_2$ |
|---|---|---|---|---|
| Wealth Effect | $-$ | $-$ | $+$ | $+$ |
| Income Effect | $+$ | $+$ | $-$ | $-$ |
| Substitution Effect | $-$ | $+$ | $+$ | $-$ |

Table 2.1: Effects of Interest Rate Changes on Consumption

First, since the price of one of the two goods has declined, households can now afford more; a price decline is like an increase in real income, and thus the change in the optimal consumption choices as result of this price decline is called an income effect. Finally, a decline in $\frac{1}{1+r}$ not only reduces the *absolute* price of second period consumption, it also makes second period consumption cheaper, *relative* to first period consumption (whose price has remained the same). Since second period consumption has become relatively cheaper and first period consumption relatively more expensive, one would expect that Hardy substitutes second period consumption for first period consumption. This effect from a change in the relative price of the two goods is called a substitution effect. Table 2.1 summarizes these three effects on consumption in both periods.

As before, let us first analyze the simple example 1 by repeating the optimal choices from (2.12) and (2.13):

$$\begin{aligned} c_1 &= \frac{1}{1+\beta} * I(r) \\ c_2 &= \frac{\beta(1+r)}{1+\beta} * I(r) \end{aligned}$$

First, an increase in $r$ reduces lifetime income $I(r)$, unless $y_2 = 0$. This is the negative wealth effect, reducing consumption in both periods, ceteris paribus. Second, we observe that for consumption $c_1$ in the first period this is the *only* effect: absent a change in $I(r)$, consumption $c_1$ today does not change. For this special example in which the utility function is $u(c) = \log(c)$, the income and substitution effect (both of which are present) *exactly cancel out*, leaving only the negative wealth effect. In general, as indicated in Table 2.1, the two effects go in opposite direction, but that they exactly cancel out is indeed very special to log-utility. Finally, for $c_2$ we know from the above discussion and Table 2.1 that both income and substitution effect are positive. The term $\frac{\beta(1+r)}{1+\beta}$, which depends positively on the interest rate $r$ reflects this. However, as discussed before the wealth effect is negative, leaving the overall response of consumption $c_2$ in the second period to an interest rate increase ambiguous. However, remembering that $I(r) = A + y_1 + \frac{y_2}{1+r}$, we see that

$$c_2 = \frac{\beta(1+r)}{1+\beta}(A + y_1) + \frac{\beta}{1+\beta}y_2$$

which is increasing in $r$. Thus for our example the wealth effect is dominated by the income and substitution effect and second period consumption increases with the interest rate. However, for general utility functions is need not be true.

Let us now analyze the general case graphically. Again we consider an increase in the interest rate from $r$ to $r' > r$; evidently a decline in the interest rate can be studied in exactly the same form. What happens to the curves in Figure 2.3 as the interest rate increases? The indifference curves do not change, as they do not involve the interest rate. But the budget line changes. Since we assume that the interest rate increases, the budget line gets steeper. And it is straightforward to find a point on the budget line that is affordable with old and new interest rate. Suppose Hardy eats all his first period income and wealth in the first period, $c_1 = y_1 + A$ and all his income in the second period $c_2 = y_2$, in other words, he doesn't save or borrow. This consumption profile is affordable no matter what the interest rate (as the interest rate does not affect Hardy as he neither borrows nor saves). This consumption profile is sometimes called the autarkic consumption profile, as Hardy needs no markets to implement it: he just eats whatever he has in each period. Hence the budget line tilts around the autarky point and gets steeper, as shown in Figure 2.3.

In the figure consumption in period 2 increases, consumption in period 1 decreases and saving increases, just as for the simple example. Note, however, that we could have drawn this picture in such a way that both $(c_1, c_2)$ decline or that $c_1$ increases and $c_2$ decreases (see again Table 2.1). So for general utility functions it is hard to make firm predictions about the consequences of an interest change.

If we know, however, that Hardy is either a borrower or a saver before the interest rate change, then we can establish fairly strong results, both with respect to consumption as well as with respect to welfare, of a change in the interest rate $r$..

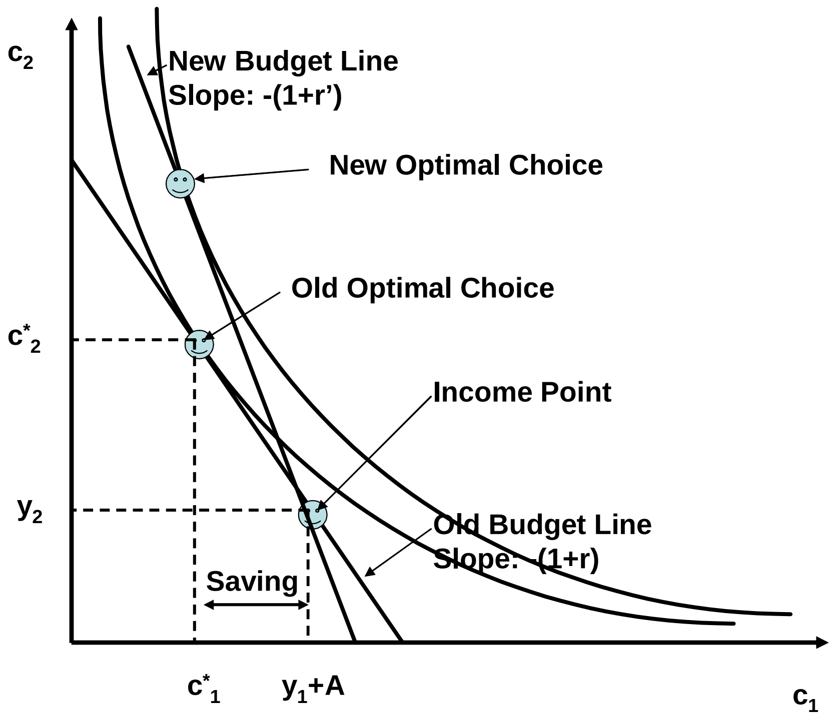


Figure 2.3: An Increase in the Interest Rate

**Proposition 2** *Let* $(c_1^*, c_2^*, s^*)$ *denote the optimal consumption and saving choices associated with interest rate* $r$*. Furthermore denote by* $(\hat{c}_1^*, \hat{c}_2^*, \hat{s}^*)$ *the optimal consumption-savings choice associated with interest* $r' > r$

1. *If* $s^* > 0$ *(that is* $c_1^* < A + y_1$ *and Hardy is a saver at interest rate* $r$*), then* $U(c_1^*, c_2^*) < U(\hat{c}_1^*, \hat{c}_2^*)$ *and either* $c_1^* < \hat{c}_1^*$ *or* $c_2^* < \hat{c}_2^*$ *(or both).*

2. *Conversely, if* $\hat{s}^* < 0$ *(that is* $\hat{c}_1^* > A + y_1$ *and Hardy is a borrower at interest rate* $r'$*), then* $U(c_1^*, c_2^*) > U(\hat{c}_1^*, \hat{c}_2^*)$ *and either* $c_1^* > \hat{c}_1^*$ *or* $c_2^* > \hat{c}_2^*$ *(or both).*

**Proof.** We only prove the first part of the proposition; the proof of the second part is identical. Remember that, before combining the two budget constraints (2.2) and (2.3) into one intertemporal budget constraint they read as

$$\begin{aligned} c_1 + s &= y_1 + A \\ c_2 &= y_2 + (1+r)s \end{aligned}$$

Now consider Hardy's optimal choice $(c_1^*, c_2^*, s^*)$ for an interest rate $r$. Now the interest rate increases to $r' > r$. What Hardy can do (of course it may not be optimal) at this new interest rate is to choose the allocation $(\tilde{c}_1, \tilde{c}_2, \tilde{s})$ given by

$$\begin{aligned} \tilde{c}_1 &= c_1^* > 0 \\ \tilde{s} &= s^* > 0 \end{aligned}$$

and

$$\begin{aligned} \tilde{c}_2 &= y_2 + (1+r')\tilde{s} \\ &= y_2 + (1+r')s^* \\ &> y_2 + (1+r)s^* = c_2^* \end{aligned}$$

This choice $(\tilde{c}_1, \tilde{c}_2, \tilde{s})$ is definitely feasible for Hardy at the interest rate $r'$ and satisfies $\tilde{c}_1 \geq c_1^*$ and $\tilde{c}_2 > c_2^*$ and thus

$$U(c_1^*, c_2^*) < U(\tilde{c}_1, \tilde{c}_2)$$

But the optimal choice at $r'$ is obviously no worse, and thus

$$U(c_1^*, c_2^*) < U(\tilde{c}_1, \tilde{c}_2) \leq U(\hat{c}_1^*, \hat{c}_2^*)$$

and Hardy's welfare increases as result of the increase in the interest rate, if he is a saver. But

$$U(c_1^*, c_2^*) < U(\hat{c}_1^*, \hat{c}_2^*)$$

requires either $c_1^* < \hat{c}_1^*$ or $c_2^* < \hat{c}_2^*$ (or both). ■

## 2.4 Borrowing Constraints

So far we assumed that Hardy could borrow freely at interest rate $r$. But we all (at least some of us) know that sometimes we would like to take out a loan from a bank but are denied from doing so. We now want to analyze how the optimal consumption-savings choice is affected by the presence of borrowing constraints. We will see later that the presence of borrowing constraints may alter the effects that temporary tax cuts have on the economy in crucial way.

As the most extreme scenario, suppose that Hardy cannot borrow at all, that is, let us impose the additional constraint on the consumer maximization problem that

$$s \geq 0. \tag{2.17}$$

Let by $(c_1^*, c_2^*, s^*)$ denote the optimal consumption choice that Hardy would choose *in the absence* of the constraint (2.17). There are two possibilities.

1. If Hardy's optimal unconstrained choice satisfies $s^* \geq 0$, then it remains the optimal choice even after the constraint has been added.[5] In other words, households that want to save are not hurt by their inability to borrow.

2. If Hardy's optimal unconstrained choice satisfies $s^* < 0$ (he would like to borrow), then it violates (2.17) and thus is not admissible. Now with the borrowing constraint, the best he can do is set

$$\begin{aligned} c_1 &= y_1 + A \\ c_2 &= y_2 \\ s &= 0 \end{aligned}$$

   He would like to have even bigger $c_1$, but since he is borrowing constrained he can't bring any of his second period income forward by taking out a loan. Also note that in this case the inability of Hardy to borrow leads to a loss in welfare, compared to the situation in which he has access to loans. This is shown in Figure 2.4 which shows the unconstrained optimum $(c_1^*, c_2^*)$ and the constrained optimum $(c_1 = y_1 + A, c_2 = y_2)$. Since the indifference curve through the

[5] Note that this is a very general property of maximization problems: adding constraints to a maximization problem weakly decreases the maximized value of the objective function and if a maximizer of the unconstrained problem satisfies the additional constraints, it is necessarily a maximizer of the constrained problem. The reverse is evidently not true: an optimal choice of a constrained maximization problem may, but need not remain optimal once the constraints have been lifted.

latter point lies to the left of the indifference curve through the former point, the presence of borrowing constraints leads to a loss in lifetime utility.

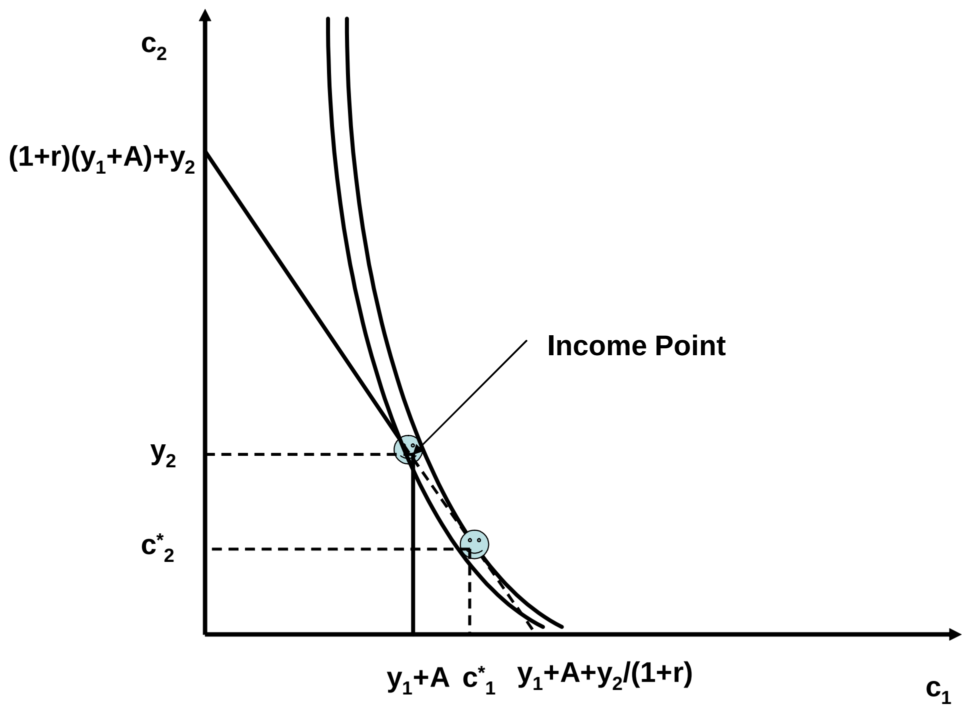


Figure 2.4: Borrowing Constraints

Note that the budget line, in the presence of borrowing constraints has a kink at $(y_1 + A, y_2)$. For $c_1 < y_1 + A$ we have the usual budget constraint, as here $s > 0$ and the borrowing constraint is not binding. But with the borrowing constraint Hardy cannot afford any consumption $c_1 > y_1 + A$, so the budget constraint has a vertical segment at $y_1 + A$, because regardless of what $c_2$, the most Hardy can afford in period 1 is $y_1 + A$. What the figure shows is that, if Hardy was a borrower without the borrowing constraint, then his optimal consumption is at the kink.

Finally, the effects of income changes on optimal consumption choices are potentially more extreme in the presence of borrowing constraints, which may give the government's fiscal policy extra power. First consider a change in second period income $y_2$. In the absence of borrowing constraints we have already analyzed this above. Now suppose Hardy is borrowing-constrained in that his optimal choice satisfies

$$\begin{aligned} c_1 &= y_1 + A \\ c_2 &= y_2 \\ s &= 0 \end{aligned}$$

We see that an increase in $y_2$ does not affect consumption in the first period of his life and increases consumption in the second period of his life one-for-one with income. Why is this? Hardy is borrowing constrained, that is, he would like to take out loans against his second period income even before the increase in $y_2$. Now, with the increase in $y_2$ he would like to borrow even more, but still can't. Thus $c_1 = y_1 + A$ and $s = 0$ remains optimal.

An increase in $y_1$ on the other hand, has strong effects on $c_1$. If, after the increase Hardy still finds it optimal to set $s = 0$ (which will be the case if the increase in $y_1$ is sufficiently small, abstracting from some pathological cases), then consumption in period 1 increases one-for-one with the increase in current income and consumption $c_2$ remains unchanged. Thus, if a government cuts taxes temporarily in period 1, this may have the strongest effects on those individual households that are borrowing-constrained.

## 2.5 A General Equilibrium Version of the Model

So far we took income and initial wealth $(y_1, y_2, A)$ as well as the real interest rate $r$ as exogenously given, that is, determined outside the model. In this section we will briefly demonstrate how to embed it into a model with firms and production in which incomes as interest rates will be determined as part of the analysis. The model is a simplified version of Peter Diamond's (1965) famous overlapping generations model with production.

Households in this model are exactly as described as the Hardy Krueger's in example 1, that is, they have the utility function

$$u(c_1, c_2) = \log(c_1) + \log(c_2)$$

where for simplicity I have set the time discount factor to $\beta = 1$. The general case is analyzed in Appendix B at the end of this chapter.

Hardy maximizes this utility function subject to the budget constraint

$$c_1 + \frac{c_2}{1+r} = I.$$

Now we assume that Hardy starts with no initial wealth, $A = 0$, works for a wage $w$ in the first period of his life, so that his first period income equals $y_1 = w$ and retires in the second period, so that $y_2 = 0$. Then total lifetime income $I = w$ and solving the maximization problem delivers, as shown in example 1 (with $\beta = 1$ and $I = w$), optimal consumption and savings decisions

$$c_1 = \frac{1}{2}w \tag{2.18}$$

$$c_2 = \frac{1}{2}w(1+r) \tag{2.19}$$

$$s = \frac{1}{2}w. \tag{2.20}$$

Now we bring in firms and production in order to determine wages and interest rates $(w, r)$ in equilibrium. A typical firm hires $l$ workers, pays each a wage $w$, leases capital $k$ at a leasing rate $\rho$ and produces consumption goods according to the neoclassical production function[6]

$$y = k^\alpha l^{1-\alpha}. \tag{2.21}$$

The firm takes wages and rental rates of capital $(w, \rho)$ as given and chooses how much labor and capital $(l, k)$ to hire to maximize profits

$$\max_{(k,l)} k^\alpha l^{1-\alpha} - wl - \rho k$$

This maximization problem has the first order conditions

$$(1-\alpha)k^\alpha l^{-\alpha} = w \tag{2.22}$$

$$\alpha k^{\alpha-1} l^{1-\alpha} = \rho. \tag{2.23}$$

Note that there is no dynamic aspect of the firm's problem, it solves exactly the same problem in both periods in which Hardy is alive.

[6] This is exactly the same production function you have seen in your intermediate macroeconomics classes when studying economic growth.

Finally we have to let households such as Hardy and firms interact with each other in an equilibrium, which requires a few more assumptions. First, we assume that when Hardy is young, the total number of *young* households (all of which are identical to Hardy) in the economy is equal to one. Since only young households work, the labor market clearing condition reads as

$$l_1 = 1.$$

and thus Hardy's wages are given from (2.22) by

$$w = (1-\alpha)k_1^\alpha \tag{2.24}$$

where $k_1$ is the capital stock in the economy at the beginning of Hardy's life in period 1, taken as exogenously given in the current analysis.

Next we consider the market clearing condition in the asset market. Households such as Hardy save by purchasing assets. I assume that the only asset available for investment is the physical capital stock in the economy. Thus the savings that all young Hardy's together do have to equal the capital stock $k_2$ that is used in production next period (when Hardy is old).

Thus the asset market clearing condition reads as

$$s = k_2$$

Plugging in for $s$ from (2.20) and exploiting equation (2.24) yields

$$\frac{1}{2}(1-\alpha)k_1^\alpha = k_2. \tag{2.25}$$

This equation determines how the capital stock changes over time. If we are interested in the level of capital that remains constant over time, in economics often referred to as a steady state, then this capital level satisfies $k_1 = k_2 = k$ and thus from (2.25)

$$\begin{aligned} \frac{1}{2}(1-\alpha)k^\alpha &= k \\ k^* &= \left[\frac{1}{2}(1-\alpha)\right]^{\frac{1}{1-\alpha}} \end{aligned}$$

Wages are then given by

$$\begin{aligned} w &= (1-\alpha)\left(k^*\right)^\alpha \\ &= (1-\alpha)\left[\frac{1}{2}(1-\alpha)\right]^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

Finally, the determination of interest rates $r$ requires a bit more thought (and making an thus far implicit assumption explicit). When households save in period 1, they purchase physical capital $k_2$ which is used in production and earns a rental rate $\rho$. The rental rate in turn is given by equation (2.23)

$$\rho = \alpha k^{\alpha-1} l^{1-\alpha} = \alpha \left( \left[ \frac{1}{2}(1-\alpha) \right]^{\frac{1}{1-\alpha}} \right)^{\alpha-1} = \frac{2\alpha}{1-\alpha}$$

If we assume that capital completely depreciates after production, then the only thing households get back from the firm per unit of capital is the rental rate, and thus[7]

$$1 + r = \rho = \frac{2\alpha}{1-\alpha}$$

In appendix B at the end of this chapter we show that exactly the same analysis applies in a more general version of the model in which the economy lasts forever, and in each period a new cohort of households is born that lives for two periods, so that in each time period two generations overlap, one generation being currently young and one generation being currently old. The analysis there also shows how to extend the model to include positive population growth.

[7] Full depreciation is not a bad approximation in a model where one period lasts 30 years. Standard estimates for the annual depreciation rate are about 8%, so after 30 years a fraction

$$(1 - 0.08)^{30} = 0.082$$

of each unit of capital is left undepreciated, implying a depreciation rate of 91.2% over 30 years. Thus assuming 100% depreciation is a good approximation.

With imperfect depreciation the rental rate $\rho$ and the interest rate $r$ are related by

$$1 + r = \rho + (1 - \delta)$$

as households also get returned the undepreciated part $1 - \delta$ of the capital stock they have leased to firms. Thus

$$r = \rho - \delta$$

which, with $\delta = 1$, becomes the formula stated in the text.

## 2.6 Appendix A: Details on the Household Maximization Problem

Here we provide two other, alternative approaches to solve the household maximization problem in section 2.2 of the main text. An alternative to the Lagrangian is to substitute (from the intertemporal budget constraint) $c_1 = I - \frac{c_2}{1+r}$ to get

$$\max_{c_2} \left\{ u\left(I - \frac{c_2}{1+r}\right) + \beta u(c_2) \right\}$$

This is now an unconstrained maximization problem. Let us take first order conditions with respect to $c_2$

$$-\frac{1}{1+r} u'\left(I - \frac{c_2}{1+r}\right) + \beta u'(c_2) = 0$$

or

$$u'\left(I - \frac{c_2}{1+r}\right) = (1+r)\beta u'(c_2)$$

Again using the fact that $c_1 = I - \frac{c_2}{1+r}$ we again obtain equation (2.8) from the main text.

Finally (and this might be the least algebra-intensive approach) we can use a Lagrangian approach, but keep both period budget constraints separate. Attaching Lagrange multipliers $(\lambda_1, \lambda_2)$ to the constraints $(2.2), (2.3)$ the Lagrangian becomes

$$\mathcal{L}' = u(c_1) + \beta u(c_2) + \lambda_1 \left[y_1 + A - c_1 - s\right] + \lambda_2 \left[y_2 + (1+r)s - c_2\right]$$

and is now maximized by choosing $(c_1, c_2, s)$.

The first order conditions for these three choice variables read as

$$\begin{aligned} u'(c_1) - \lambda_1 &= 0 \\ \beta u'(c_2) - \lambda_2 &= 0 \\ -\lambda_1 + \lambda_2(1+r) &= 0. \end{aligned}$$

Rewriting yields

$$\begin{aligned} u'(c_1) &= \lambda_1 \\ \beta u'(c_2) &= \lambda_2 \\ \lambda_2(1+r) &= \lambda_1. \end{aligned}$$

Eliminating the Lagrange multipliers from these equations by combining them again delivers equation (2.8) from the main text.

## 2.7 Appendix B: The Dynamic General Equilibrium Model

Time extends from $t = 0$ forever. In each period $t$ a total number $N_t$ of new young households are born that live for two periods. We assume that the population grows at a constant rate $n$ and thus

$$N_t = (1+n)^t N_0 = (1+n)^t$$

where we normalized the size of the initial young generation to 1.

Young households work for a wage $w_t$ when young, consume $c_{1t}$, save $s_t$ and in the second period of their lives consume the proceeds from their savings, including interest. The interest rate between period $t$ and $t+1$ is denoted by $r_{t+1}$. Thus the household problem reads as

$$\begin{aligned} & \max_{c_{1t},c_{2t+1},s_t} \{\log(c_{1t}) + \beta \log(c_{2t+1})\} \\ c_{1t} + s_t &= w_t \\ c_{2t+1} &= (1 + r_{t+1})s_t. \end{aligned}$$

As we saw in example 1 the solution to this problem is given by

$$\begin{aligned} c_{1t} &= \frac{1}{1+\beta} w_t \\ c_{2t+1} &= \frac{\beta(1+r_{t+1})}{1+\beta} w_t \\ s_t &= \frac{\beta}{1+\beta} w_t \qquad (2.26) \end{aligned}$$

As in the main text, the production function is assumed to be a Cobb-Douglas function. Now, however, we have to distinguish aggregate, economy-wide variables (which we will denote by upper case letters) and per-capita variables (for which we will use lower-case letters). Aggregate output $Y_t$ is given by

$$Y_t = K_t^\alpha L_t^{1-\alpha}$$

and thus

$$w_t = (1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha}$$

The labor market clearing condition now reads as

$$L_t = N_t$$

and thus

$$w_t = (1-\alpha)\left(\frac{K_t}{N_t}\right)^{\alpha} = (1-\alpha)k_t^{\alpha}$$

where now $k_t = \frac{K_t}{N_t}$ is capital per young household.

The capital market clearing condition now equates the saving done by all young households, $s_t N_t$, to the aggregate capital stock in the economy next period, that is:

$$s_t N_t = K_{t+1}$$

or

$$s_t = \frac{K_{t+1}}{N_t} = \frac{K_{t+1}}{N_{t+1}} * \frac{N_{t+1}}{N_t} = k_{t+1}(1+n)$$

Plugging in from the saving function (2.26) yields

$$s_t = \frac{\beta}{1+\beta}w_t = \frac{\beta}{1+\beta}(1-\alpha)k_t^{\alpha} = k_{t+1}(1+n)$$

or

$$k_{t+1} = \frac{\beta(1-\alpha)}{(1+\beta)(1+n)}k_t^{\alpha} \tag{2.27}$$

This equation fully determines the capital stock per young worker, starting from the initial capital stock $k_0 = K_0$ the economy is endowed with at the beginning of time (which is exogenously given, and thus an "initial condition"). Since the per capita capital stock $k_t$ completely determines wages, consumption and saving, equation (2.27) completely characterizes the evolution of the general equilibrium in this economy.

Defining a steady state as a situation in which the per capita capital stock $k_t$ is constant over time and $k_{t+1} = k_t$, from equation (2.27) we see that there are two steady states, first the trivial steady state $k = 0$: if the economy starts with no capital, $k_0 = 0$ it never gets off the ground and remains at

$k = 0$. We will focus instead on the economically meaningful positive steady state capital stock that satisfies

$$k = \frac{\beta(1-\alpha)}{(1+\beta)(1+n)} k^{\alpha}$$

or

$$k = \left[\frac{\beta(1-\alpha)}{(1+\beta)(1+n)}\right]^{\frac{1}{1-\alpha}} \tag{2.28}$$

It is easy to verify that the steady state characterized in section 2.5 of the main text is just a special case of (2.28), with $\beta = 1$ and $n = 0$, that is, no impatience and no population growth.

By plotting $k_{t+1}$ against $k_t$ we can also determine the steady states of the model, and in fact its entire dynamics, graphically. From equation 2.27 we see that, as a function of $k_t$, the curve

$$\frac{\beta(1-\alpha)}{(1+\beta)(1+n)} k_t^{\alpha}$$

starts at the point $(0, 0)$. Since $\alpha$ is between 0 and 1, the curve is strictly concave and thus intersects the 45-degree line once more.[8] At that intersection we have $k_{t+1} = k_t$; but this is exactly the condition for a steady state. Thus this intersection gives us the unique positive per capita capital stock $k^*$. We can also read off the entire dynamics of the model from the figure. Start from an arbitrary initial capital stock $k_0$. The new capital stock $k_1$ can be read off directly from the graph. Going back from $k_1$ on the $y$-axis to the $45^0$ line we find $k_1$ on the $x$-axis. Now we can repeat the procedure to determine $k_2$ and so forth. We also note that if we repeat this procedure we eventually end up at $k^*$, the steady state capital stock. In fact, we end up there independent of the initial capital stock (unless $k_0 = 0$). Thus not only is there a unique positive steady state capital stock $k^*$, but it is approached from every initial condition $k_0$. Thus, to put it in mathematics terms, $k^*$ is globally asymptotically stable.

[8] Mathematically speaking, for this we also need that at $k_t = 0$ the curve is steeper than the $45^0$ line. But the $45^0$ line has a slope of 1, whereas the curve $\frac{\beta(1-\alpha)}{(1+\beta)(1+n)} k_t^{\alpha}$ has a slope that goes to infinity as $k_t$ goes to zero, and thus the curve come out of the origin more steeply than the $45^0$ line.

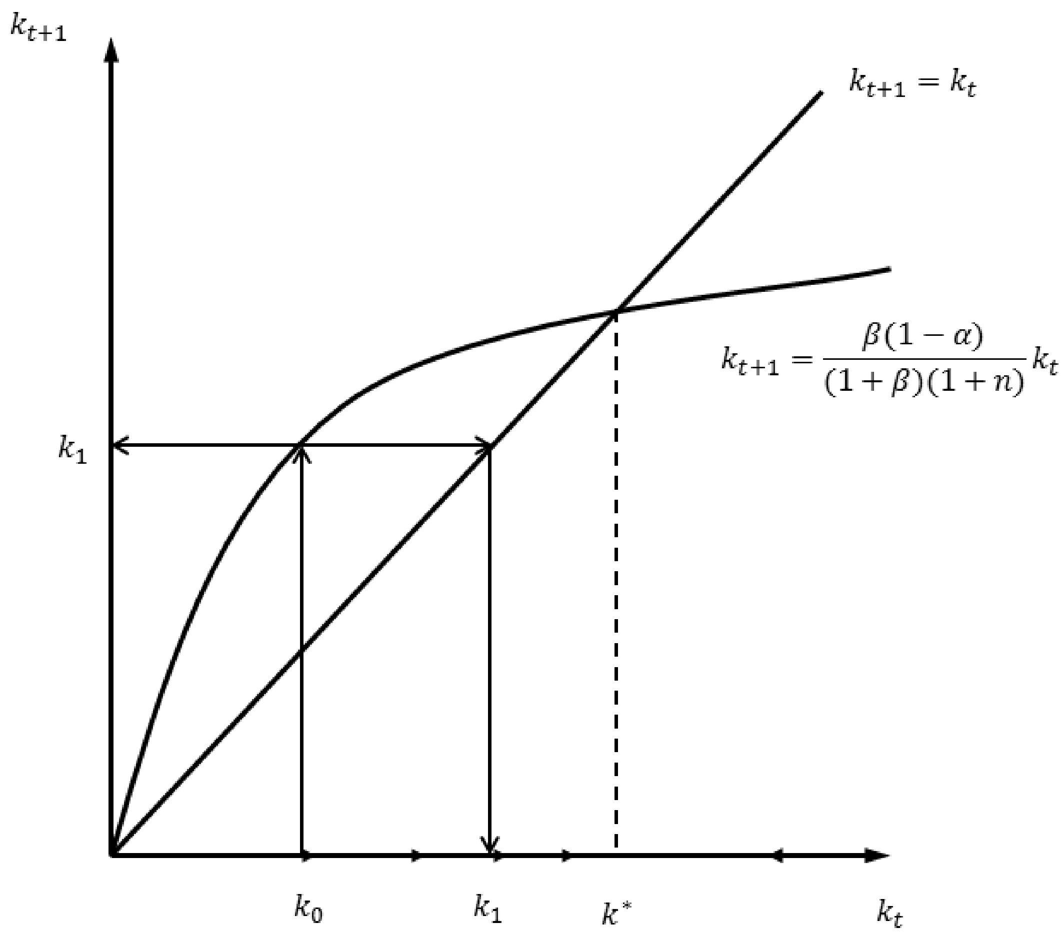


Figure 2.5: Dynamics of the Capital Stock

# Chapter 3

# The Life Cycle Model

The assumption that households live only for two periods is of course a strong one. The generalization of the two period model analyzed in the previous section was pioneered in the 1950's independently by Franco Modigliani and Albert Ando, and by Milton Friedman, with slightly different focus. Whereas Modigliani and Ando used a model with optimal consumption and savings choices over the life cycle to derive implications for consumption and wealth profiles over the life cycle (and thus the results of their analysis were called the life cycle hypothesis), Friedman's focus was on the consumption and savings response of households to changes in income, especially unexpected changes in income (that is, income shocks). His analysis (or better, the results of that analysis) of the impact of the timing and the characteristics of income on individual consumption and savings choices is called the permanent income hypothesis. It is important to note that both sets of results come out of essentially the same model, so we have no need in this section to present two different versions of the model to derive both the Ando-Modigliani life cycle results and the Friedman permanent income results.

We envision a household that lives for $T$ periods. We allow that $T = \infty$, in which case the household lives forever. In each period $t$ of its life the household earns after-tax income $y_t$ and consumes $c_t$. In addition the household may have initial wealth $A \geq 0$ from bequests. In each period the household faces the budget constraint

$$c_t + s_t = y_t + (1 + r)s_{t-1} \tag{3.1}$$

Here $r$ denotes the constant exogenously given interest rate, $s_t$ denotes financial assets carried over from period $t$ to period $t + 1$ and $s_{t-1}$ denotes assets from period $t-1$ carried to period $t$. In the simple model savings and assets were the same thing, now we have to distinguish between them. Savings in period $t$ are defined as the difference between total income $y_t + rs_{t-1}$ (labor income and interest earned) and consumption $c_t$. Thus savings are defined as

$$\begin{aligned} sav_t &= y_t + rs_{t-1} - c_t \\ &= s_t - s_{t-1} \end{aligned} \tag{3.2}$$

where the last inequality comes from (3.1). Thus savings today are nothing else but the change in the asset position of a household between the beginning of the current period and the end of the current period.

In period 1 the budget constraint reads as

$$c_1 + s_1 = A + y_1.$$

It is the goal of the household to maximize its lifetime utility

$$U(c_1, c_2, \ldots, c_T) = u(c_1) + \beta u(c_2) + \beta^2 u(c_3) + \ldots + \beta^{T-1} u(c_T) \tag{3.3}$$

We will often write this more compactly as

$$U(c) = \sum_{t=1}^{T} \beta^{t-1} u(c_t) \tag{3.4}$$

where $c = (c_1, c_2, \ldots, c_T)$ denotes the lifetime consumption profile and the symbol $\sum_{t=1}^{T}$ stands for the sum, from $t = 1$ to $t = T$. If expression (3.4) looks intimidating, you should always remember that it is just another way of writing (3.3).

As above with the simple, two period model we can rewrite the period-by-period budget constraints as a single intertemporal budget constraint. To see this, take the first- and second period budget constraint

$$\begin{aligned} c_1 + s_1 &= A + y_1 \\ c_2 + s_2 &= y_2 + (1+r)s_1 \end{aligned}$$

Now solve the second equation for $s_1$

$$s_1 = \frac{c_2 + s_2 - y_2}{1+r}$$

and plug into the first equation, to obtain

$$c_1 + \frac{c_2 + s_2 - y_2}{1+r} = A + y_1$$

which can be rewritten as

$$c_1 + \frac{c_2}{1+r} + \frac{s_2}{1+r} = A + y_1 + \frac{y_2}{1+r} \tag{3.5}$$

We now can repeat this procedure: from the third period budget constraint

$$c_3 + s_3 = y_3 + (1+r)s_2$$

we can solve for

$$s_2 = \frac{c_3 + s_3 - y_3}{1+r}$$

and plug this into (3.5) to obtain (after some rearrangements)

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} + \frac{s_3}{(1+r)^2} = A + y_1 + \frac{y_2}{1+r} + \frac{y_3}{(1+r)^2}$$

We can continue this process $T$ times, to finally arrive at a single intertemporal budget constraint of the form

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} + \ldots + \frac{c_T}{(1+r)^{T-1}} + \frac{s_T}{(1+r)^{T-1}} = A + y_1 + \frac{y_2}{1+r} + \frac{y_3}{(1+r)^2} \ldots + \frac{y_T}{(1+r)^{T-1}} \tag{3.6}$$

Finally we observe the following. Since $s_T$ denotes the saving from period $T$ to $T+1$, but the household lives only for $T$ periods, she has no use for saving in period $T+1$ (unless she values her children and wants to leave bequests, a possibility that is ruled out for now by specifying a utility function that only depends on one's own consumption, as in (3.3)). On the other hand, we do not allow the household to die in debt (what would happen if we did?) Thus it is always optimal to set $s_T = 0$ and we will do so until further notice. Then (3.6) reads as

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} + \ldots + \frac{c_T}{(1+r)^{T-1}} = A + y_1 + \frac{y_2}{1+r} + \frac{y_3}{(1+r)^2} \ldots + \frac{y_T}{(1+r)^{T-1}} \tag{3.7}$$

or more compactly, as

$$\sum_{t=1}^{T} \frac{c_t}{(1+r)^{t-1}} = A + \sum_{t=1}^{T} \frac{y_t}{(1+r)^{t-1}} = I \tag{3.8}$$

which simply states that the present discounted value of lifetime consumption $(c_1, \ldots, c_T)$ equals the present discounted value of lifetime income $(y_1, \ldots, y_T)$ plus initial bequests which we again denote by $I$.[1]

As in the simple two period model, it is the goal of the household to maximize its lifetime utility (3.3), subject to the lifetime budget constraint (3.7). The choice variables are all consumption levels $(c_1, \ldots, c_T)$. Now the use of graphical analysis is not helpful anymore, since one would have to draw a picture in as many dimensions as there are time periods $T$ (you may want to try for $T = 3$). Thus the only thing we can do is to solve this constrained maximization problem mathematically. We will first do so for the general case, and then consider several important examples.

## 3.1 Solution of the General Problem

In order to maximize the lifetime utility (3.3), subject to the lifetime budget constraint (3.7) we again need to make use of the theory of constrained optimization. That is, we set up the Lagrangian[2], by multiplying the budget constraint by a Lagrange multiplier $\lambda$ and appending it to the objective function (3.3) of the household:

$$\begin{aligned}
\mathcal{L}(c_1, \ldots, c_T) &= u(c_1) + \beta u(c_2) + \beta^2 u(c_3) + \ldots + \beta^{T-1} u(c_T) + \\
&\quad + \lambda \left( A + y_1 + \frac{y_2}{1+r} + \frac{y_3}{(1+r)^2} \ldots + \frac{y_T}{(1+r)^{T-1}} - c_1 - \frac{c_2}{1+r} - \frac{c_3}{(1+r)^2} \ldots \right. \\
&= \sum_{t=1}^{T} \beta^{t-1} u(c_t) + \lambda \left( A + \sum_{t=1}^{T} \frac{y_t}{(1+r)^{t-1}} - \sum_{t=1}^{T} \frac{c_t}{(1+r)^{t-1}} \right)
\end{aligned}$$

Now we take first order conditions with respect to all choice variables $(c_1, \ldots, c_T)$ as well as with respect to the Lagrange multiplier $\lambda$ and set them

[1] If the time horizon $T$ is infinite, this intertemporal budget constraint is still valid as long as the term

$$\frac{s_T}{(1+r)^{T-1}}$$

goes to zero when $T$ goes to infinity. Without proof we assert that any optimal solution to the household problem must have this property.

[2] Named after the French mathematician Joseph Lagrange (1736-1813) who pioneered the mathematics of constrained optimization

to zero. These conditions then determine the optimal solution to the constrained maximization problem.[3] For our example the first order conditions with respect to $c_1$, set to zero, yields

$$u'(c_1) - \lambda = 0$$

or

$$u'(c_1) = \lambda. \tag{3.9}$$

Doing the same for $c_2$ yields

$$\beta u'(c_2) - \lambda \frac{1}{1+r} = 0$$

or

$$(1+r)\beta u'(c_2) = \lambda \tag{3.10}$$

and for an arbitrary $c_t$ we find

$$(1+r)^{t-1}\beta^{t-1}u'(c_t) = \lambda \tag{3.11}$$

The first order condition with respect to the Lagrange multiplier returns back the budget constraint

$$A + \sum_{t=1}^{T} \frac{y_t}{(1+r)^{t-1}} - \sum_{t=1}^{T} \frac{c_t}{(1+r)^{t-1}} = 0 \tag{3.12a}$$

Now we simplify and consolidate the first order conditions. Using (3.9) to (3.11) we have

$$u'(c_1) = (1+r)\beta u'(c_2) = \ldots = [(1+r)\beta]^{t-1} u'(c_t) = [(1+r)\beta]^{t} u'(c_{t+1}) = \ldots = [(1+r)\beta]^{T-1} u'(c_T) \tag{3.13}$$

These equations determine the relative consumption levels across periods, that is, the ratios $\frac{c_2}{c_1}, \frac{c_3}{c_2}$ and so forth.[4] In order to determine the absolute

[3] Those of you with advanced knowledge in mathematics may ask whether we need to check second order conditions. We will only work on problems in this course in which the first order conditions are necessary *and* sufficient (that is, finite dimensional convex maximization problems).

[4] Also note from equation (3.9) that the Lagrange multiplier can be interpreted as the "shadow cost" of the resource constraint: if we had one more unit of income (in period 1), we could buy one more unit of consumption in period 1, with associated utility consequences $u'(c_1) = \lambda$. Thus $\lambda$ measures the marginal benefit from relaxing the intertemporal budget constraint by one unit.

consumption levels we have to use the budget constraint (3.12$a$). Without further assumptions on the interest rate $r$, the time discount factor $\beta$ and income $(y_1, \ldots, y_T)$ no further analytical progress can be made, and thus we will soon make exactly these more specific assumptions.

Before jumping into specific examples let us carefully interpret conditions 3.13. These conditions that determine optimal consumption choices are often called Euler equations, after Swiss mathematician Leonard Euler (1707-1783) who first derived them. Let us pick a particular time period, say $t = 1$. Then the equation reads as

$$u'(c_1) = (1 + r)\beta u'(c_2) \tag{3.14}$$

Remember that this is a condition the optimal consumption choices $(c_1, c_2)$ have to satisfy. Thus the household should not be able to improve his utility by consuming a little less in period 1, save the amount and consume a bit extra in the second period. The cost, in terms of utility, of consuming a small unit less in period 1 is $-u'(c_1)$ and the benefit is computed as follows. Saving an extra unit to period 2 yields $1 + r$ extra units of consumption tomorrow. The extra utility from another consumption unit tomorrow is $\beta u'(c_2)$, so the total utility consequences tomorrow are $(1 + r)\beta u'(c_2)$. Thus the entire consequences from saving a little more today and eating it tomorrow are

$$-u'(c_1) + (1 + r)\beta u'(c_2) \leq 0 \tag{3.15}$$

because the household should not be able to improve his lifetime utility from doing so. Similarly, consuming one unit more today and saving one unit less for tomorrow should also not make the household better off, which leads to

$$-u'(c_1) + (1 + r)\beta u'(c_2) \geq 0. \tag{3.16}$$

Combining the two equations (3.15) and (3.16) yields back (3.14), which simply states that at the optimal consumption choice $(c_1, c_2)$ it cannot improve utility to save either more or less between period 1 and 2.

## 3.2 Important Special Cases

### 3.2.1 Equality of $\beta = \frac{1}{1+r}$

In this case the market discounts income tomorrow, versus income today, at the same rate $\frac{1}{1+r}$ as the household discounts utility today versus tomorrow, $\beta$. In this case, since $\beta(1 + r) = 1$, from (3.13) we find

$$u'(c_1) = u'(c_2) = \ldots = u'(c_t) = \ldots = u'(c_T)$$

But now we remember that we assumed that the utility function is strictly concave (i.e. $u''(c) < 0$), which means that the function $u'(c)$ is strictly decreasing in $c$. We therefore immediately[5] have that

$$c_1 = c_2 = \ldots = c_t = \ldots = c_T$$

and consumption is constant over a households' lifetime. Call that constant consumption level $\bar{c}$. Households find it optimal to choose a perfectly smooth consumption profile, independent of the timing of income. The level of consumption depends solely on the present discounted value of income, plus initial bequests, but the timing of income and consumption is completely de-coupled. The smoothness of consumption over the life cycle and the fact that the timing of consumption and income are completely unrelated are the main predictions of this model and the main implications of what is commonly dubbed the life cycle hypothesis. We can also calculate the level of consumption $\bar{c}$ from the intertemporal budget constraint as

$$\sum_{t=1}^{T} \frac{c_t}{(1+r)^{t-1}} = I$$

and thus

$$\begin{aligned} \bar{c} \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} &= I \\ \bar{c} &= \frac{1}{\sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}}} * I \end{aligned} \quad (3.17)$$

The term $\sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}}$ annuitizes a constant stream of consumption, and thus $\frac{1}{\sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}}} * I$ measures the constant stream of income whose present discounted value is equal to $I$. This is what Friedman called permanent income, and the permanent income hypothesis simply states that optimal consumption in each period equals permanent income. Note that the annuity

[5] If $c_1 > c_2$ we have that $u'(c_1) < u'(c_2)$, since $u'(c)$ is by assumption strictly decreasing. Reversely, if $c_1 < c_2$ then $u'(c_1) > u'(c_2)$. Thus the only possible way to get $u'(c_1) = u'(c_2)$ is to have $c_1 = c_2$.

factor $\sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}}$ is a so-called geometric sum and can be written (provided that $r > 0$) as

$$\sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} = \begin{cases} \frac{1+r-\frac{1}{(1+r)^{T-1}}}{r} & \text{if } T < \infty \\ \frac{1+r}{r} & \text{if } T = \infty \end{cases}.$$

Thus, if households are infinitely lived, the consumption rule takes an especially simple form: consume

$$\bar{c} = \frac{r}{1+r} I$$

which is sometimes the way the literature states the permanent income hypothesis.

Finally, we will derive the implication for life cycle savings and asset accumulation for a version of the model in which households are finitely lived and their income path is composed of a working life phase and a retirement phase. It is most transparent to do so for a simple example.

**Example 3** *Suppose a household lives 60 years, from age 1 to age 60 (in real life this corresponds to age 21 to age 80; before the age of 21 the household is not economically active in that his consumption is dictated by her parents). Also suppose the household inherits nothing, i.e.* $A = 0$*. Finally assume that in the first 45 years of her life, the household works and makes a constant annual income of* \$40,000 *per year. For the last* 15 *years of her life the household is retired and earns nothing; for the time being we ignore social security. Finally we make the simplifying assumption that the interest rate is* $r = 0$*; since in this subsection we assume* $\beta = \frac{1}{1+r}$*, this implies* $\beta = 1$*. We want to figure out the life cycle profile of consumption, saving and asset accumulation. From the previous discussion we already know that consumption over the households' lifetime is constant, that is* $c_1 = c_2 = \ldots = c_{60} = c$*. What we don't know is the level of consumption. But we know that the discounted value of lifetime consumption equals the discounted value of lifetime income. So let us first compute the lifetime value of lifetime income. Here the assumption* $r = 0$ *simplifies matters, because*

$$\begin{aligned} & y_1 + \frac{y_2}{1+r} + \frac{y_3}{(1+r)^2} \cdots + \frac{y_{60}}{(1+r)^{T-1}} \\ = & \ y_1 + y_2 + y_3 \ldots + y_{60} \\ = & \ y_1 + y_2 + y_3 \ldots + y_{45} \\ = & \ 45 * \$40,000 \end{aligned} \tag{3.18}$$

*where we used the fact that for the last* 15 *years the household does not earn anything. The total discounted lifetime cost of consumption, using the fact that consumption is constant at* $c$ *and that the interest rate is* $r = 0$ *is*

$$\begin{aligned} & c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} + \ldots + \frac{c_{60}}{(1+r)^{59}} \\ = & \; c_1 + c_2 + \ldots + c_{60} \\ = & \; 60 * c \end{aligned} \tag{3.19}$$

*Equating* (3.18) *and* (3.19) *yields*

$$\begin{aligned} c &= \frac{45}{60} * \$40,000 \\ &= \$30,000 \end{aligned}$$

*That is, in all his working years the household consumes* \$10,000 *less than her income and puts the money aside for consumption in retirement. With a zero interest rate,* $r = 0$, *it is also easy to compute savings in each period. For all working periods, by definition*

$$\begin{aligned} sav_t &= y_t + rs_{t-1} - c_t \\ &= y_t - c_t \\ &= \$40,000 - \$30,000 \\ &= \$10,000 \end{aligned}$$

*whereas for all retirement periods*

$$\begin{aligned} sav_t &= y_t + rs_{t-1} - c_t \\ &= -c_t \\ &= -\$30,000 \end{aligned}$$

*Finally we can compute the asset position of the household. Remember from* (3.2) *that*

$$sav_t = s_t - s_{t-1}$$

*or*

$$s_t = s_{t-1} + sav_t$$

*That is, assets at the end of period* $t$ *equal assets at the beginning of period* $t$ *(that is, the end of period* $t-1$*) plus the saving in period* $t$. *Since the* *household starts with* 0 *bequests,* $s_0 = 0$. *Thus*

$$\begin{aligned}
s_1 &= s_0 + sav_1 \\
&= \$0 + \$10,000 = \$10,000 \\
s_2 &= s_1 + sav_2 = \$10,000 + \$10,000 = \$20,000 \\
s_3 &= s_2 + sav_3 = \$20,000 + \$10,000 = \$30,000 \\
&\vdots \\
s_{45} &= s_{44} + sav_{45} = \$440,000 + \$10,000 = \$450,000 \\
s_{46} &= s_{45} + sav_{46} = \$450,000 - \$30,000 = \$420,000 \\
s_{47} &= s_{46} + sav_{47} = \$420,000 - \$30,000 = \$390,000 \\
&\vdots \\
s_{60} &= s_{59} + sav_{60} = \$30,000 - \$30,000 = \$0
\end{aligned}$$

*The household accumulates substantial assets for retirement and then runs them down completely in order to finance consumption in old age until death. Note that this household knows exactly when she is going to die and does not value the utility of her children (or has none), so there is no point for her saving beyond her age of sure death. The life cycle profiles of income, consumption, savings and assets are depicted in Figure* 3.1. *Note that the y-axis is not drawn to scale, in order to be able to draw all four variables on the same graph. Also remember that age 1 in our model corresponds to age 21 in the real world, age 45 to age 66 and age 60 to age 80. Again, the crucial features of the model, and thus the diagram, are the facts that consumption is constant over the life time, de-coupled from the timing of income and that the household accumulates assets until retirement and then de-saves until her death.*

The previous example was based on several simplifying assumptions. In exercises you will see that the assumption $r = 0$, while making our life easier, is not essential for the main results. The assumption that $\beta(1 + r)$ (that is, equality of subjective discount factor and market discount factor) however, is crucial, because otherwise consumption is *not* constant over the households' life time.

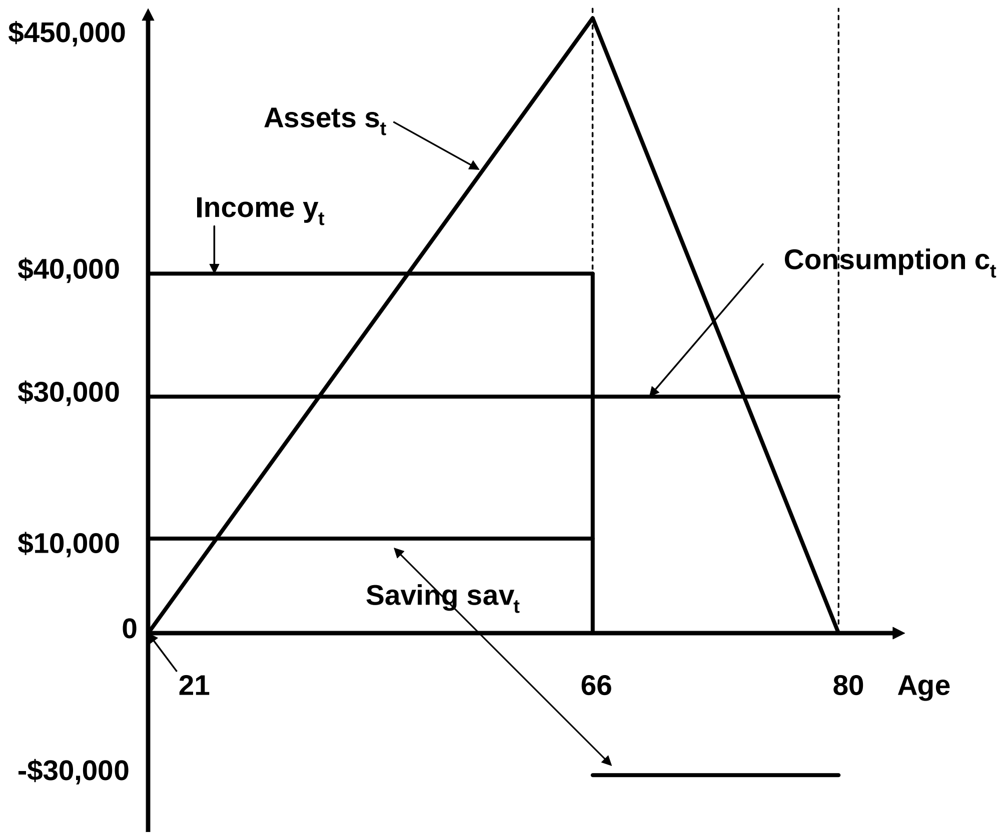


Figure 3.1: Life Cycle Profiles, Model

### 3.2.2 Two Periods and log-Utility

In the case that $\beta \neq \frac{1}{1+r}$, without making stronger assumptions on the utility function we usually cannot make much progress. So now suppose that the household only lives for two periods (that is, $T = 2$) and has period utility $u(c) = \log(c)$. Note that we have solved this problem already; here we merely want to check that our general model boils down to the simple model and yields the same results if we choose $T = 2$. Remembering that for log-utility $u'(c) = \frac{1}{c}$, equation (3.14) yields

$$\frac{1}{c_1} = \frac{(1+r)\beta}{c_2}$$

or

$$c_2 = (1+r)\beta c_1$$

Combining this with the intertemporal budget constraint

$$c_1 + \frac{c_2}{1+r} = A + y_1 + \frac{y_2}{1+r}$$

yields back the optimal solution (2.12) and (2.13).

### 3.2.3 The Relation between $\beta$ and $\frac{1}{1+r}$ and Consumption Growth

We saw in subsection 3.2.1 that if $\beta = \frac{1}{1+r}$, consumption over the life cycle is constant. In this section we will show that if interest rates are high and households are patient (i.e. have a high $\beta$) then they will choose consumption to grow over the life cycle, whereas if interest rates are low and households are impatient, then they will opt for consumption to decline over the life cycle.

**The Case $\beta > \frac{1}{1+r}$**

Now households are patient and the interest rate is high, so that $\beta > \frac{1}{1+r}$ or $\beta(1+r) > 1$. Intuitively, in this case we would expect that households find it optimal to have consumption grow over time. Since they are patient, they don't mind that much postponing consumption to tomorrow, and since the interest rate is high, saving an extra dollar looks really attractive. So one would expect

$$c_1 < c_2 < \ldots < c_t < c_{t+1} < \ldots < c_T. \tag{3.20}$$

Let's see whether this comes out of the math. From (3.13) we have

$$\begin{aligned} u'(c_1) &= (1+r)\beta u'(c_2) = \ldots = [(1+r)\beta]^{t-1} u'(c_t) = [(1+r)\beta]^t u'(c_{t+1}) \\ &= \ldots = [(1+r)\beta]^{T-1} u'(c_T) \end{aligned}$$

Take the first equality, which implies

$$\frac{u'(c_1)}{u'(c_2)} = (1+r)\beta$$

But now we assume $(1+r)\beta > 1$, and therefore

$$\begin{aligned} \frac{u'(c_1)}{u'(c_2)} &> 1 \\ u'(c_1) &> u'(c_2) \end{aligned} \tag{3.21}$$

But again remember that $u'(c)$ is a strictly decreasing function, so the only way that (3.21) can be true is to have $c_1 < c_2$. Thus, consumption is higher in the second than in the first period of a households' life.

For an arbitrary age $t$ equation (3.13) implies

$$\begin{aligned} [(1+r)\beta]^{t-1} u'(c_t) &= [(1+r)\beta]^t u'(c_{t+1}) \\ \frac{u'(c_t)}{u'(c_{t+1})} &= \frac{[(1+r)\beta]^t}{[(1+r)\beta]^{t-1}} = (1+r)\beta > 1 \end{aligned}$$

so that the same argument as for age 1 implies $c_{t+1} > c_t$ for an arbitrary age $t$. Thus consumption continues to rise throughout a households' life time, as proposed in (3.20). The exact growth rate and level of consumption, of course, can only be determined with knowledge of the form of the utility function $u$ and the concrete values for income. The bottom line from this subsection: high interest rates and patience of households makes for little consumption expenditures today, relative to tomorrow.

### The Case $\beta < \frac{1}{1+r}$

Now households are impatient and the interest rate is low, so that $\beta < \frac{1}{1+r}$ or $\beta(1+r) < 1$. Intuitively, we should obtain exactly the reverse result from the last subsection: we now would expect that households find it optimal to have consumption decline over time. Since they are impatient, they don't want to eat now rather than tomorrow, and since the interest rate is low, saving an extra dollar for tomorrow only brings a low return. An identical argument to the above easily shows that now

$$c_1 > c_2 > \ldots > c_t > \ldots > c_T. \tag{3.22}$$

Therefore low interest rates are conducive to high consumption today, relative to tomorrow, even more so if households are very impatient. This discussion concludes our treatment of the basic model which we will use in order to study the effects of fiscal policies. So far our households lived in isolation, unaffected by any government policy. The only interaction with the rest of the economy came through financial markets, on which the household was assumed to be able to borrow and lend at the market interest rate $r$. We will now introduce a government into our simple model and study how simple tax and transfer policies affect the private decisions of households. Before that we have a quick look at consumption over the life cycle from the data.

### Explicit Solution for Constant Relative Risk Aversion Utility

Consider the specific period utility function

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

where $\sigma \geq 0$ is a parameter that measures the curvature of the utility function. This utility function is called the constant relative risk aversion (CRRA) utility function. Notably, for $\sigma = 1$ this utility function becomes[6]

$$u(c) = \log(c).$$

In this case $u'(c) = c^{-\sigma}$ and equation (3.13) implies

$$(c_1)^{-\sigma} = (1+r)\beta(c_2)^{-\sigma} = \ldots = [(1+r)\beta]^{t-1} (c_t)^{-\sigma} = [(1+r)\beta]^{t} (c_{t+1})^{-\sigma} = \ldots = [(1+r)\beta]^{T-1}$$

and thus for any period $t$

$$\begin{aligned} [(1+r)\beta]^{t-1} (c_t)^{-\sigma} &= [(1+r)\beta]^{t} (c_{t+1})^{-\sigma} \\ (c_t)^{-\sigma} &= [(1+r)\beta] (c_{t+1})^{-\sigma} \\ \left(\frac{c_{t+1}}{c_t}\right)^{\sigma} &= (1+r)\beta \end{aligned}$$

[6] This appears surprising but is the consequence of applying L'Hopital's rule to

$$\lim_{\sigma \to 1} \frac{c^{1-\sigma} - 1}{1 - \sigma}.$$

Both numerator and denominator are 0 for $\sigma = 1$. Thus, according to L'Hopital's rule the limit of the ratio is the limit of the ratio of the derivatives (with respect to $\sigma$):

$$\lim_{\sigma \to 1} \frac{c^{1-\sigma} - 1}{1 - \sigma} = \lim_{\sigma \to 1} \frac{-c^{1-\sigma} \log(c)}{-1} = \log(c).$$

or

$$\frac{c_{t+1}}{c_t} = [(1+r)\beta]^{\frac{1}{\sigma}}$$

and thus gross consumption growth $\frac{c_{t+1}}{c_t}$ is constant over the life cycle and equal to $[(1+r)\beta]^{\frac{1}{\sigma}}$, which, depending on $\beta$ and $(1+r)$ is bigger, smaller or equal to 1. As argued before, consumption is falling over the life cycle if $(1+r)\beta < 1$ and rising if $(1+r)\beta > 1$. What the CRRA utility function gives us is the result that consumption rises or falls *at a constant rate* over time.

For this utility function we can in fact explicitly calculate the level of consumption (even for the case $(1+r)\beta \neq 1$). Since

$$\begin{aligned} c_{t+1} &= [(1+r)\beta]^{\frac{1}{\sigma}} c_t = [(1+r)\beta]^{\frac{2}{\sigma}} c_{t-1} = \ldots [(1+r)\beta]^{\frac{t}{\sigma}} c_1 \text{ or} \\ c_t &= [(1+r)\beta]^{\frac{t-1}{\sigma}} c_1 \end{aligned}$$

we can use the intertemporal budget constraint

$$\sum_{t=1}^{T} \frac{c_t}{(1+r)^{t-1}} = I$$

to solve explicitly for consumption in each period. At the risk of algebraic overkill, lugging in for $c_t$ yields

$$\sum_{t=1}^{T} \frac{[(1+r)\beta]^{\frac{t-1}{\sigma}} c_1}{(1+r)^{t-1}} = I$$

But

$$c_1 \sum_{t=1}^{T} \left[\beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}-1}\right]^{t-1} = c_1 \frac{1 - \left[\beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}-1}\right]^T}{1 - \beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}-1}}$$

and thus

$$\begin{aligned} c_1 &= \frac{1 - \beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}-1}}{1 - \left[\beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}-1}\right]^T} * I \\ c_t &= [(1+r)\beta]^{\frac{t-1}{\sigma}} * \frac{1 - \beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}-1}}{1 - \left[\beta^{\frac{1}{\sigma}} (1+r)^{\frac{1}{\sigma}-1}\right]^T} * I \end{aligned}$$

which looks like a pain, but is straightforward to calculate if one assumes specific values for $\beta, r$ and $\sigma$. Furthermore the expression simplifies to (3.17) above if $\beta(1+r) = 1$.

## 3.3 Income Risk

A full treatment of the consumption and savings response to income shocks is beyond the scope of this course.[7] Instead, here I will simply summarize the main results and provide intuition for why they are true. Note, however, that these results require proofs that I am omitting here. The only extra concept we need now is the conditional expectation $E_t$ of an economic variable that is uncertain. Thus, $E_t y_{t+1}$ is the expectation in period $t$ of income in period $t+1$, $E_t y_{t+2}$ is the period $t$ expectation of income in period $t+2$ and so forth. We adopt the timing convention that when expectations are taken in period $t$, income in period $t$ is known (that is, expectations are taken after the random income in period $t$ has been realized). The generalization of the Euler equation to the case with income risk states that for any time $t$

$$u'(c_t) = \beta(1+r)E_t u'(c_{t+1}). \tag{3.23}$$

Since income in period $t+1$ is risky from the perspective of period $t$, so is consumption. Also, in the previous equation we have implicitly assumed that the interest rate is not random (and in fact constant over time). From now on, for the rest of this subsection we will assume $\beta(1+r) = 1$. This assumption is not necessary for most of what follows, but it will make the algebra much less painful. Furthermore, and for the same reason we assume that the lifetime horizon of the household is infinite, $T = \infty$.

Note that in general we cannot pull the expectation into the marginal utility function, since in general

$$E_t u'(c_{t+1}) \neq u'(E_t c_{t+1}).$$

We can pull the expectation into marginal utility if and only if $u'(c_{t+1})$ is linear in $c_{t+1}$, but not for utility functions usually used in macroeconomics (for example the CRRA utility function above does not satisfy this requirement).

But now assume that the utility function is quadratic, that is, let

$$u(c_t) = -\frac{1}{2}(c_t - \bar{c})^2 \tag{3.24}$$

where $\bar{c}$ is the bliss level of consumption. We assume that $\bar{c}$ is so large that given the household's lifetime income this consumption level cannot be

[7] You are welcomend to attend my $2^{nd}$ year PhD course on the topic, though.

attained.[8] We note that for all consumption levels $c_t < \bar{c}$ to the left of $\bar{c}$ we have

$$\begin{aligned} u'(c_t) &= -(c_t - \bar{c}) = \bar{c} - c_t > 0 \\ u''(c_t) &= -1 < 0 \end{aligned}$$

that is, this utility function is strictly increasing and strictly concave for all $c_t < \bar{c}$. Now recall from basic microeconomics that a household with strictly concave utility function is risk averse: the household would be willing to pay a positive insurance premium to get rid of the income risk. We state the key assumptions made thus far explicitly

**Assumption 1:** *The utility function is of quadratic form* (3.24)*, and* $\beta(1+r)$. Furthermore $T = \infty$.

Under this assumption the Euler equation (3.23) becomes

$$-(c_t - \bar{c}) = -E_t(c_{t+1} - \bar{c})$$

or

$$E_t c_{t+1} = c_t \tag{3.25}$$

That is, households arrange consumption in such a way such that, in expectation, it stays constant between today and tomorrow. Of course, in the presence of income risk realized consumption $c_{t+1}$ in period $t+1$ might deviate from this plan: negative income shocks (e.g. becoming unemployed unexpected) will revise consumption downwards from the plan characterized by (3.25).

As in the model without income risk, in order to determine the level of consumption we need the intertemporal budget constraint. Without proof I state that even in the presence of income risk, the budget constraint from any period $t$ onwards is

$$E_t \sum_{s=0}^{\infty} \frac{c_{t+s}}{(1+r)^s} = (1+r)s_{t-1} + E_t \sum_{s=0}^{\infty} \frac{y_{t+s}}{(1+r)^s} \tag{3.26}$$

[8] For example, we can take

$$\bar{c} = 2\frac{1+r}{r} y_{\max}$$

where $y_{\max}$ is the maximum possible income realization and thus $\bar{c}$ is twice the present discounted value of receiving the maximal income forever.

Now (3.25) implies that

$$\begin{aligned} E_t c_{t+1} &= c_t \\ E_t c_{t+2} &= E_t E_{t+1} c_{t+2} = E_t c_{t+1} = c_t \end{aligned}$$

where the second equality is due to the law of iterated expectations, and in general, for all $s$,

$$E_t c_{t+s} = c_t.$$

Thus the left hand side of equation (3.26) becomes

$$E_t \sum_{s=0}^{\infty} \frac{c_{t+s}}{(1+r)^s} = \sum_{s=0}^{\infty} \frac{E_t c_{t+s}}{(1+r)^s} = c_t \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} = \frac{1}{1-\frac{1}{1+r}} c_t = \frac{1+r}{r} c_t.$$

Thus, from (3.26) we obtain as optimal consumption rule

$$c_t = \frac{r}{1+r} \left( (1+r)s_{t-1} + E_t \sum_{s=0}^{\infty} \frac{y_{t+s}}{(1+r)^s} \right) \tag{3.27}$$

that is, consumption is equal to the annuity factor $\frac{r}{1+r}$ times the expected present discounted value of future income plus current savings, or is equal to permanent income. For period 1, thus consumption becomes

$$c_1 = \frac{r}{1+r} \left( A + E_1 \sum_{s=0}^{\infty} \frac{y_{1+s}}{(1+r)^s} \right) = \frac{r}{1+r} E_1 I \tag{3.28}$$

and comparing (3.17) above with (3.28) we observe that the optimal consumption rules are exactly alike: in both cases the household consumes permanent income, that is, the annuity factor times the (expected) present discounted value of future labor income. The only difference is that with income risk we have to take expectations of future labor income.

This result should be quite surprising: despite the presence of income risk the household makes the same planned consumption choices as in the absence of risk. Such behavior is called certainty equivalence behavior: households, in the presence of income risk, make choices that are equivalent to the scenario under no risk. Especially, the household does *not* engage in *precautionary savings behavior* by saving more in the presence than in the absence of future income risk. Another way of saying this is to realize that the only thing that matters for consumption is expected future income; income risk (as measured by the variance of future income) is irrelevant for current consumption choices. This is true despite the fact that the household is risk averse and thus does not like risk; she nevertheless finds it not optimal to adjust her savings behavior to the presence of risk.

Finally, it is important to note that equation (3.25) only says that consumption remains constant between two periods in expectation. *Realized* consumption in period $t+1$ will in general deviate from this plan. In fact, one can show (using tedious algebra) that the realized change in consumption between period $t$ and $t+1$ is given by

$$c_{t+1} - c_t = \frac{r}{1+r} \sum_{s=0}^{\infty} \frac{E_{t+1} y_{t+1+s} - E_t y_{t+1+s}}{(1+r)^s}$$

that is, the realized change in consumption is given by the annuity value $\frac{r}{1+r}$ of the sum of discounted *revisions in expectations* about future income in periods $t+1+s$, that is, $E_{t+1} y_{t+1+s} - E_t y_{t+1+s}$.

How large the realized change in consumption is depends on the type of income shock the household experiences between the two periods. Here we only consider the two extremes, a perfectly permanent shock (such as an unexpected permanent promotion) and a fully transitory shock (such as a one time unexpected bonus). Consider first the permanent promotion, unexpected at time $t$ and materialized at time $t+1$. Suppose the extra income due to the promotion is $p$. The promotion increases income from period $t+1$ on for every period of the households' life, and since it is not expected in period $t$, we have that for all future periods

$$E_{t+1} y_{t+1+s} - E_t y_{t+1+s} = p.$$

Thus

$$c_{t+1} - c_t = \frac{r}{1+r} \sum_{s=0}^{\infty} \frac{p}{(1+r)^s} = p \frac{r}{1+r} \frac{1}{1 - \frac{1}{1+r}} = p$$

that is, consumption goes up by the full amount of the unexpected but permanent income increase between period $t$ and $t+1$.

In contrast now consider a transitory shock, such as a one time unexpected bonus $b$ in period $t+1$. Then

$$E_{t+1} y_{t+1} - E_t y_{t+1} = b$$

and for all future periods beyond $t+1$

$$E_{t+1}y_{t+1+s} - E_t y_{t+1+s} = 0.$$

Then

$$c_{t+1} - c_t = \frac{r}{1+r}\sum_{s=0}^{\infty}\frac{b}{(1+r)^0} = \frac{r}{1+r}b$$

and thus the increase in consumption is only $\frac{r}{1+r}$ of the bonus (of about 2% if the real interest rate is $r = 2\%$).. Instead, most of the bonus is saved and used to increase consumption in *all* future periods by a small bit.

Having characterized the predictions of the life cycle model quite sharply we now confront these predictions with the data.

## 3.4 Empirical Evidence: Life Cycle Consumption Profiles

If one follows an average household over its life cycle, two main stylized facts emerge. First, disposable income follows a hump over the life cycle, with a peak around the age of 45 (the age of the household is defined by the age of the household head). This finding is hardly surprising, given that at young ages households tend to obtain formal education or training on the job and labor force participation of women is low because of child bearing and rearing. As more and more agents finish their education and learn on the job as well as promotions occur, average wages within the cohort increase. Average disposable income at age 45 is almost 2.5 times as high as average personal income at age 25. After the age of 45 disposable income first slowly, then more rapidly declines as more and more people retire and labor productivity (and thus often wages) fall. The average household at age 65 has only 60% of the personal income that the average household at age 45 obtains.

The second main finding is the surprising finding. Not only personal income, but also consumption follows a hump over the life cycle. In other words, consumption seems to track income over the life cycle fairly closely, rather than be completely decoupled from it, as our model predicts. Figure 3.2 (taken from Krueger and Fernandez-Villaverde, 2003) documents the life cycle profile of consumption, with and without adjustment for family size. The key observation from this figure is that consumption displays a hump over the life cycle, and that this hump persists, even after controlling for family size. The figure is constructed using semi-parametric econometric techniques, but the same picture emerges if one uses more standard techniques that control for household age with age dummies.

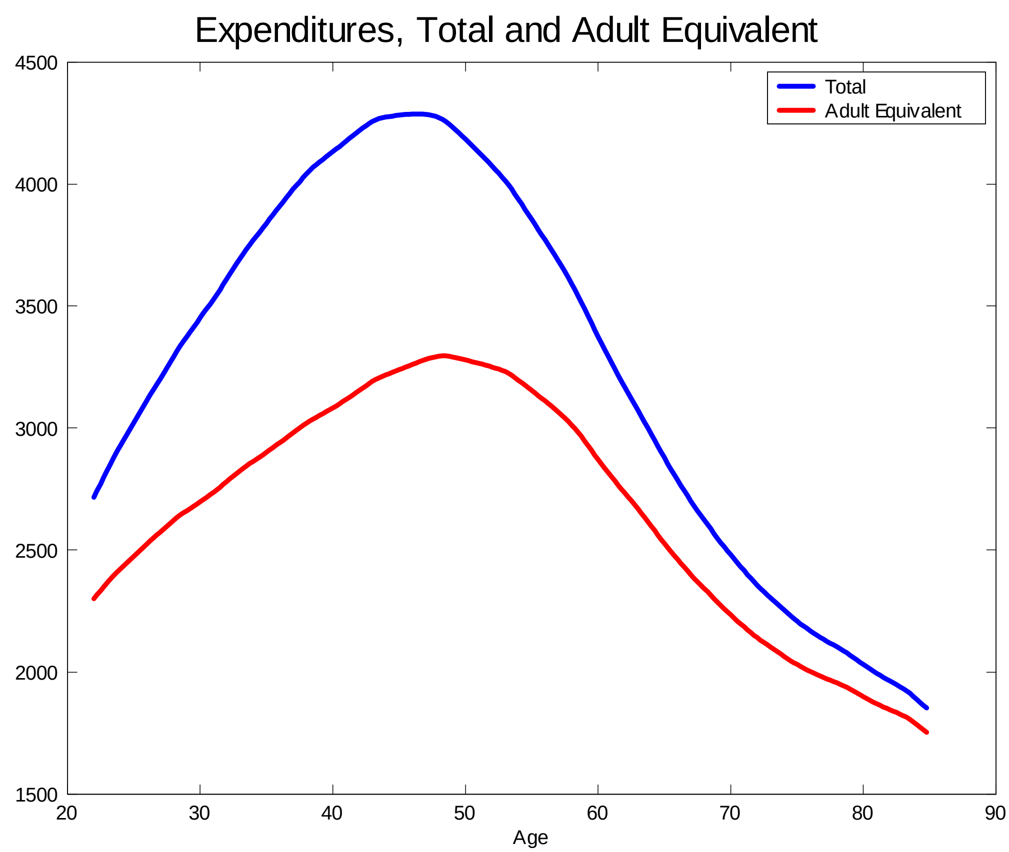


Figure 3.2: Consumption over the Life Cycle

## 3.5 Potential Explanations

There are a number of potential extensions of the basic life cycle model that can rationalize a hump-shaped consumption. So far, the prediction of the model is that consumption is either monotonically upward trending, monotonically downward trending or perfectly flat over the life cycle. So the basic theory can account for at most one side of the empirical hump in life cycle consumption. Here are several other factors that, once appropriately added to the basic model, may account for (part of) the data:

- Changes in household size and household composition: Not only income and consumption follow a hump over the life cycle in the data, but also family size. Our simple model envisioned a single individual composing a household. But if household size changes over the life cycle (people move in together, get married, have children which grow and finally leave the household, then one of the spouses dies), it may be optimal to have consumption follow household size. The life cycle model only asserts that marginal utility of consumption should be smooth over the life cycle, not necessarily consumption expenditures themselves. However, in the previous figure we presented one line that adjusts the consumption data for household size, using so-called household equivalence scales. These scales try to answer the simple question as to how much more consumption expenditures as household have to have in order to obtain the same level of per capita utility, as the size of the household changes. Concretely, suppose that you move in with your boyfriend or girlfriend, the equivalence scale asks: how much more do you have to spend for consumption to be as happy off materially (that is, not counting the joy of living together) as before when you were living by yourself. The number researchers come up with usually is somewhere between 1 and 2, because it requires some additional spending to make you as happy as before (two people eat more than one), but it may not require double the amount (it takes about as much electricity to cook for two people than for one). Technically, this last consideration is called economies of scale in household production. So if one applies household equivalence scales to the data, the size of the hump in lifetime consumption is reduced by about 50%. That is, changes in household size and composition can account for half of the hump, with the remaining part being left unexplained by the life cycle model augmented by changes in family size.[9]

- The life cycle model was presented with exogenous income falling from

[9] The exact fraction demographics can account for is still debated. See Fernandez-Villaverde and Krueger (2004) for a discussion. On a technical note, since there is no data set that follows individuals over their entire life time and collects consumption data, one has to construct these profiles using the synthetic cohort technique, pioneered by Deaton (1985). Again see Fernandez-Villaverde and Krueger (2004) for the details.

the sky. If households have to work to earn their income and dislike work, that is, have the amount of leisure in the utility function, then things get more complicated. Suppose that consumption and leisure are separable in the utility function, that is, suppose that the utility function takes the form

$$\begin{aligned} U(c,l) &= \sum_{t=1}^{T} \beta^{t-1} u(c_t, l_t) \\ &= \sum_{t=1}^{T} \beta^{t-1} \left[ u(c_t) + v(l_t) \right] \end{aligned}$$

where $l_t$ is leisure at age $t$ and $v$ is an increasing and strictly concave function. Then our theory above goes through unchanged and the predictions remain the same. But if consumption and leisure are substitutes (if you work a lot, the marginal utility from your consumption is high), then if labor supply is hump-shaped over the live cycle (because labor productivity is), then households may find it optimal to have a hump-shaped labor supply and consumption profile over the life cycle. This important point was made by Nobel laureate James Heckmen in his dissertation (1974). But Fernandez-Villaverde and Krueger (2004) provide some suggestive evidence that this channel is likely to explain only a small fraction of the consumption hump.

- We saw that the model can predict a declining consumption profile over the life cycle if $\beta(1+r) < 1$. Now suppose that young households can't borrow against their future labor income. Thus the best thing they can do is to consume whatever income whey have when young. Since income is increasing in young ages, so is consumption. As households age, at some point they want to start saving (rather than borrowing), and no constraint prevents them from doing so. But now the fact that $\beta(1+r) < 1$ kicks in and induces consumption to fall. Thus the combination of high impatience and borrowing constraints induces a hump-shaped consumption profile. Empirically, one problem of this explanation is that the peak of the hump in consumption does not occur until about age 45, a point in life where the median household already has accumulated sizeable financial assets, rather than still being borrowing-constrained.

- Finally, we may want to relax the assumption about certain incomes and certain lifetime. If an individual thinks that he will only survive until 100 with certain probability less than one, at age 20 he will plan to save less for age 100 than if she knows for sure she'll get that old. Thus realized consumption at age 100 will be smaller with lifetime uncertainty as without. Since death probabilities increase with age, this induces a decline in optimal consumption as the household ages. The death probabilities act like an additional discount factor in the household's maximization problem. On the other hand, suppose you are 25, with decent income, and you expect your income to increase, but be quite risky. Under the assumption that people have a precautionary savings motive (we will see below that this requires the assumption $u'''(c) > 0$), households will save for precautionary reasons and consume less when young than under certainty, even if income is expected to rise over their lifetime. Then, as the household ages and more and more uncertainty is resolved, the precautionary savings motive loses in importance, households start to consume more, and thus consumption rises over the life cycle, until death probabilities start to become important and consumption starts to fall again, rationalizing the hump in life cycle consumption in the data. Attanasio et al. (1999) show that a standard life cycle model, enriched by changes in household size and uncertainty about income and lifetime is capable of generating a hump in consumption over the life cycle of similar magnitude and timing as in the data.

Rather than discussing these extensions of the model in detail we will now turn to the use of the life cycle model for the analysis of fiscal policy. At the appropriate points we will discuss how the conclusions derived with the simple model change once the model is enriched by some of the elements discussed above.

# Part II

# Positive Theory of Government Activity

# Chapter 4

# Dynamic Theory of Taxation

In this chapter we want to study how government tax and transfer programs that change the size timing of after-tax income streams affect individual consumption and savings choices. We first discuss the government budget constraint, and then establish an important benchmark result that suggests that, under certain conditions, the timing of government taxes, does not affect the consumption choices of individual households. This result, first put forward by David Ricardo (1772-1823), is therefore often called Ricardian Equivalence. After analyzing the most important assumptions for the Ricardian Equivalence theorem to hold, we finally study the impact of consumption taxes, labor income taxes and capital income taxes on individual household decisions, provided that these taxes are not of lump-sum nature.

In chapter 1 we presented data for the government budget. For completeness, we here repeat the federal government budget for the U.S. for the year 2011.

We now want to group the receipts and outlays of the government into three broad categories, in order to map our data into the theoretical analysis to follow. Let government expenditures $G_t$ be comprised of[1]

$$G_t = \text{Defense} + \text{International Affairs} + \text{Health} + \text{Other Outlays}$$

[1] There are small differences between government expenditures $G_t$ as defined in this section and government consumption as measured in NIPA, but this fine distinction is inconsequential for our purposes.

| 2011 Federal Budget (in billion $) | |
|---|---|
| Receipts | 2,303.5 |
| Individual Income Taxes | 1,091.5 |
| Corporate Income Taxes | 181.1 |
| Social Insurance Receipts | 818.8 |
| Other | 212.1 |
| Outlays | 3,603.1 |
| National Defense | 705.6 |
| International Affairs | 45.7 |
| Health | 372.5 |
| Medicare | 485.7 |
| Income Security | 597.4 |
| Social Security | 730.8 |
| Net Interest | 230.0 |
| Other | 435.5 |
| Surplus | –1,299.6 |

Table 4.1: Federal Government Budget, 2011

and net taxes $T_t$ be comprised of

$$\begin{aligned} T_t &= \text{Taxes} + \text{Social Insurance Receipts} + \text{Other Receipts} \\ &\quad \text{- Medicare - Social Security - Income Security} \end{aligned}$$

that is, $T_t$ is all tax receipts from the private sector minus all transfers given back to the private sector. Finally let $r$ denote the interest rate and $B_{t-1}$ (for bonds) denote the outstanding government debt. Then

$$rB_{t-1} = \text{Net Interest}$$

We now will discuss the government budget constraint, using only these symbols $(G_t, T_t, B_{t-1}, r)$. The previous discussion should allow you to always go back from our theory to entities that you see in the data.

## 4.1 The Government Budget Constraint

Like private households the government cannot simply spend money without having revenues. In developed countries the two main sources through which the government can generate revenues is to levy taxes on private households (e.g. via income taxes) and to issue government bonds (i.e. government debt).[2] The main uses of funds are to finance government consumption (e.g. buying tanks), government transfers to private households (e.g. unemployment benefits) and the repayment of outstanding government debt.

Let us formalize the government budget constraint. First assume that when the country was formed, the first government does not inherit any debt from the past. Denote by $t = 1$ the first period a country exists with its own government budget (for the purposes of the US, period 1 corresponds to the year 1776). At time 1 the budget constraint of the government reads as

$$G_1 = T_1 + B_1 \tag{4.1}$$

where $G_1$ is government expenditures in period 1, $T_1$ are total taxes taken in by the government (including payroll taxes for social security) minus transfers to households (e.g. social security payments, unemployment compensation etc.), and $B_1$ are government bonds issued in period 1, corresponding to the outstanding government debt. For an arbitrary period $t$, the government budget constraint reads as

$$G_t + (1 + r)B_{t-1} = T_t + B_t \tag{4.2}$$

where $B_{t-1}$ are the government bonds issued yesterday that come due and need to be repaid, including interest, today. For simplicity we assume that all government bonds have a maturity of one period.

First, we can rewrite (4.2) as

$$G_t - T_t + rB_{t-1} = B_t - B_{t-1}. \tag{4.3}$$

The quantity $G_t - T_t$, the difference between current government spending and tax receipts (net of transfers) is often referred to as the *primary government deficit;* it is the government deficit that ignores interest payments on past debt. This number is often used as a measure of current fiscal responsibility, since interest payments for past debt are inherited from past years (and thus past governments). The current total government deficit is given by the sum of the primary deficit and interest payments on past debt, or

$$def_t = G_t - T_t + rB_{t-1}. \tag{4.4}$$

[2] In addition, the government usually can print fiat currency; the revenue from doing do, called "seigneurage" it a small fraction of total government revenues. It will be ignored from now on.

Equation (4.3) simply states that a government deficit (i.e. $def_t > 0$) results in an increase of the government debt, since $B_t - B_{t-1} > 0$ and thus $B_t > B_{t-1}$. That is, the number of outstanding bonds at the end of period $t$ is bigger than at the end of the previous period, and government debt grows. Obviously, if the government manages to run a surplus (i.e. $def_t < 0$), then it can repay part of its debt.

We now can do with the government budget constraint exactly what we did before for the budget constraint of private households. Equation (4.2), for $t = 2$, reads as

$$G_2 + (1+r)B_1 = T_2 + B_2$$

or

$$B_1 = \frac{T_2 + B_2 - G_2}{1+r}$$

Plug this into equation (4.1) to obtain

$$\begin{aligned} G_1 &= T_1 + \frac{T_2 + B_2 - G_2}{1+r} \\ G_1 + \frac{G_2}{1+r} &= T_1 + \frac{T_2}{1+r} + \frac{B_2}{1+r} \end{aligned}$$

We can continue this process further by substituting out for $B_2$, again using (4.2), for $t = 3$ and so forth. At the end of this we obtain the intertemporal government budget constraint

$$G_1 + \frac{G_2}{1+r} + \frac{G_3}{(1+r)^2} + \ldots + \frac{G_T}{(1+r)^{T-1}} = T_1 + \frac{T_2}{1+r} + \frac{T_3}{(1+r)^2} + \ldots + \frac{T_T}{(1+r)^{T-1}} + \frac{B_T}{(1+r)^{T-1}}$$

We will assume that even the government cannot die in debt and will not find it optimal to leave positive assets, so that $B_T = 0$.[3] Thus the intertemporal government budget constraint reads as

$$G_1 + \frac{G_2}{1+r} + \frac{G_3}{(1+r)^2} + \ldots + \frac{G_T}{(1+r)^{T-1}} = T_1 + \frac{T_2}{1+r} + \frac{T_3}{(1+r)^2} + \ldots + \frac{T_T}{(1+r)^{T-1}}$$

or more compactly, as

$$\sum_{t=1}^{T} \frac{G_t}{(1+r)^{t-1}} = \sum_{t=1}^{T} \frac{T_t}{(1+r)^{t-1}}$$

[3] We could do better than simply assuming this, but this would lead us too far astray.

If the country is assumed to live forever, we write the government constraint as

$$\sum_{t=1}^{\infty} \frac{G_t}{(1+r)^{t-1}} = \sum_{t=1}^{\infty} \frac{T_t}{(1+r)^{t-1}}$$

In short, the government is constrained in its tax and spending policy by a condition that states that the present discounted value of total government expenditures ought to equal the present discounted value of total taxes, just as for private households. The only real difference is that the government may live much longer than private households, but other than that the principle is the same.

## 4.2 The Timing of Taxes: Ricardian Equivalence

### 4.2.1 Historical Origin

How should the government finance a given stream of government expenditures, say, for a war? There are two principal ways to levy revenues for a government, namely to tax in the current period or to issue government debt in the form of government bonds the interest and principal of which has to be paid via taxes in the future. The question then arise what the macroeconomic consequences of using these different instruments are, and which instrument is to be preferred from a normative point of view. The Ricardian Equivalence Hypothesis claims that it makes no difference, that a switch from taxing today to issuing debt and taxing tomorrow does not change real allocations and prices in the economy. It's origin dates back to the classical economist David Ricardo (1772-1823). He wrote about how to finance a war with annual expenditures of £20 millions and asked whether it makes a difference to finance the £20 millions via current taxes or to issue government bonds with infinite maturity (so-called consols) and finance the annual interest payments of £1 million in all future years by future taxes (at an assumed interest rate of 5%). His conclusion was (in "Funding System") that

> in the point of the economy, there is no real difference in either of the modes; for twenty millions in one payment [or] one million per annum for ever ... are precisely of the same value

Here Ricardo formulates and explains the equivalence hypothesis, but immediately makes clear that he is sceptical about its empirical validity

> ...but the people who pay the taxes never so estimate them, and therefore do not manage their affairs accordingly. We are too apt to think, that the war is burdensome only in proportion to what we are at the moment called to pay for it in taxes, without reflecting on the probable duration of such taxes. It would be difficult to convince a man possessed of $\pounds 20,000$, or any other sum, that a perpetual payment of $\pounds 50$ per annum was equally burdensome with a single tax of $\pounds 1,000$.

Ricardo doubts that agents are as rational as they should, according to "in the point of the economy", or that they rationally believe not to live forever and hence do not have to bear part of the burden of the debt. Since Ricardo didn't believe in the empirical validity of the theorem, he has a strong opinion about which financing instrument ought to be used to finance the war

> war-taxes, then, are more economical; for when they are paid, an effort is made to save to the amount of the whole expenditure of the war; in the other case, an effort is only made to save to the amount of the interest of such expenditure.

Ricardo thought of government debt as one of the prime tortures of mankind. Not surprisingly he strongly advocates the use of current taxes. Now we want to use our simple two-period model to demonstrate the Ricardian Equivalence result and then investigate the assumptions on which it relies.

### 4.2.2 Derivation of Ricardian Equivalence

Suppose the world only lasts for two periods, and the government has to finance a war in the first period. The war costs $G_1$ dollars. For simplicity assume that the government does not do any spending in the second period, so that $G_2 = 0$. We want to ask whether it makes a difference whether the government collects taxes for the war in period 1 or issues debt and repays the debt in period 2.

The budget constraints for the government read as

$$\begin{aligned} G_1 &= T_1 + B_1 \\ (1+r)B_1 &= T_2 \end{aligned}$$

where we used the fact that $G_2 = 0$ and $B_2 = 0$ (since the economy only lasts for 2 periods). The two policies are

- Immediate taxation: $T_1 = G_1$ and $B_1 = T_2 = 0$
- Debt issue, to be repaid tomorrow: $T_1 = 0$ and $B_1 = G_1, T_2 = (1 + r)B_1 = (1 + r)G_1$.

Note that both policies satisfy the intertemporal government budget constraint

$$G_1 = T_1 + \frac{T_2}{1+r}$$

Now consider how individual private behavior changes between the two policies. Remember that the typical household maximizes utility

$$u(c_1) + \beta u(c_2)$$

subject to the lifetime budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} + A \tag{4.5}$$

where $y_1$ and $y_2$ are the after-tax incomes in the first and second period of the households' life. Write

$$y_1 = e_1 - T_1 \tag{4.6}$$
$$y_2 = e_2 - T_2 \tag{4.7}$$

where $e_1, e_2$ are the pre-tax earnings of the household and $T_1, T_2$ are taxes paid by the household.

The only thing that the government policies affect are the after tax incomes of the household. Substitute (4.6) and (4.6) into (4.5) to obtain

$$c_1 + \frac{c_2}{1+r} = e_1 - T_1 + \frac{e_2 - T_2}{1+r} + A$$

or

$$c_1 + \frac{c_2}{1+r} + T_1 + \frac{T_2}{1+r} = e_1 + \frac{e_2}{1+r} + A$$

In other words, the household spends the present discounted value of pre-tax income, including initial wealth, $e_1 + \frac{e_2}{1+r} + A$ on the present discounted value of consumption expenses $c_1 + \frac{c_2}{1+r}$ and the present discounted value of income taxes. Two tax-debt policies that imply exactly the same present discounted value of lifetime taxes therefore lead to exactly the same lifetime budget constraint and thus exactly the same individual consumption choices. This is the essence of the Ricardian Equivalence theorem, which we shall state in its general form below.

Before that let us check the present discounted value of taxes under the two policy options discussed above

- For immediate taxation we have $T_1 = G_1$ and $T_2 = 0$, and thus $T_1 + \frac{T_2}{1+r} = G_1$
- For debt issue we have $T_1 = 0$ and $T_2 = (1+r)G_1$, and thus $T_1 + \frac{T_2}{1+r} = G_1$

Therefore both policies imply the same present discounted value of lifetime taxes for the household; that is, the household perfectly rationally sees that, for the second policy, she will be taxed tomorrow because the government debt has to be repaid, and therefore prepares herself correspondingly. The timing of taxes does not matter, as long its lifetime present discounted value is not changed. Consumption choices of the household do not change, but savings choices do. This cannot be seen from the intertemporal household budget constraint (because this constraint was obtained substituting out savings), so let us go back to the period by period budget constraints

$$\begin{aligned} c_1 + s &= e_1 - T_1 \\ c_2 &= e_2 - T_2 + (1+r)s \end{aligned}$$

Let denote $(c_1^*, c_2^*)$ the optimal consumption choices in the two periods; we have already argued that these optimal choices are the same under both policies. Also let $s^*$ denote the optimal saving (or borrowing, if negative) choice under the first policy of immediate taxation. How does the household change its saving choice if we switch to the second policy, debt issue and taxation tomorrow. Let $\tilde{s}$ denote the new saving policy. Again since the optimal consumption choice is the same between the two policies we have (remember $T_1 = 0$ under the second policy)

$$\begin{aligned} c_1^* &= e_1 - T_1 - s^* \\ &= e_1 - \tilde{s} \end{aligned}$$

so that

$$\begin{aligned} e_1 - T_1 - s^* &= e_1 - \tilde{s} \\ \tilde{s} &= s^* + T_1. \end{aligned}$$

That is, under the second policy the household saves exactly $T_1$ more than under the first policy, the full extent of the tax reduction from the second policy. This extra saving $T_1$ yields $(1+r)T_1$ extra income in the second period, exactly enough to pay the taxes levied in the second period by the government to repay its debt. To put it another way, private households under policy 2 know that there will be higher taxes in the future and they adjust their private savings so to exactly be able to offset them with higher saving. Obviously the same argument can be done in a model where households and the government live for more than two periods, and for all kinds of changes in the timing of taxes.

Let us now state Ricardian Equivalence in its general form.

**Theorem 4** *(Ricardian Equivalence) A policy reform that does not change government spending* $(G_1, \ldots, G_T)$*, and only changes the timing of taxes, but leaves the present discounted value of taxes paid by each household in the economy has no effect on aggregate consumption in any time period.*

We could in fact have stated a much more general theorem, asserting that interest rates, GDP, investment and national saving (the sum of private and public saving) are unaffected by a change in the timing of taxes, but for this to be meaningful we would need a model in which interest rates, investment and GDP are determined endogenously within the model, which we have not yet constructed. Also, this theorem relies on several assumptions, which we have not made very explicit so far, but will do so in the next section.

What does this discussion imply for the current government deficit? The theorem says that the timing of taxes (i.e. running a deficit today and repaying it with higher taxes tomorrow) should not matter for individual decisions and the macro economy, so long as government spending is *left unchanged.*

This sounds good news, but one should not forget why the theorem is true: households foresee that taxes will increase in the future and adjust their savings correspondingly; after all, there is a government budget constraint that needs to be obeyed. In addition, the theorem requires a series of important assumptions, as we will now demonstrate.

### 4.2.3 Discussion of the Crucial Assumptions

#### Absence of Binding Borrowing Constraint

You already saw in chapter 1 and homework 1 that binding borrowing constraints can lead a household to change her consumption choices, even if a change in the timing of taxes does not change her discounted lifetime income. In the thought experiment above, if households are borrowing constrained then the first policy (taxation in period 1) leads to a decline in first period consumption by the full amount of the tax. Second period consumption, on the other hand, remains completely unchanged. With government debt finance of the reform, consumption in both periods may go down, since households rationally forecast the tax increase in the second period to pay off the government debt.

**Example 5** *Suppose the French-British war in the U.S. costs* £100 *per person. Households live for two periods, have utility function*

$$\log(c_1) + \log(c_2)$$

*and pre-tax income of* £$1,000$ *in both periods of their life. The war occurs in the first period of these households' lives. For simplicity assume that the interest rate is* $r = 0$. *As before, the two policy options are to tax* £100 *in the first period or to incur* £100 *in government debt, to be repaid in the second period. Since the interest rate is* 0, *the government has to repay* £100 *in the second period (when the war is over). Without borrowing constraints we know from the general theorem above that the two policies have identical consequences. In particular, under both policies discounted lifetime income is* £$1,900$ *and*

$$c_1 = c_2 = \frac{1,900}{2} = 950$$

*Now suppose there are borrowing constraints. The optimal decision with borrowing constraint, under the first policy is* $c_1 = y_1 = 900$ *and* $c_2 = y_2 = 1000$, *whereas under the second policy we have, under borrowing constraints, that* $c_1 = c_2 = 950$ *(since the optimal choice is to consume* 950 *in each period, and first period income is* 1000, *the borrowing constraint is not binding and the unconstrained optimal choice is still feasible, and hence optimal).*

This counter example shows that, if households are borrowing constrained, the timing of taxes may affect private consumption of households and the Ricardian equivalence theorem fails to apply. Current taxes have stronger effects on current consumption than the issuing of debt and implied future taxation, since postponing taxes to the future relaxes borrowing constraints and my increase current consumption.

### No Redistribution of the Tax Burden Across Generations

If the change in the timing of taxes involves redistribution of the tax burden across generations, then, unless these generations are linked together by operative, altruistically motivated bequest motives (we will explain below what exactly we mean by that) Ricardian equivalence fails. This is very easy to see in another simple example.

**Example 6** *Return to the French British war in the previous example, but now consider the two policies originally envisioned by David Ricardo. Policy* 1 *is to levy the* £100 *cost per person by taxing everybody* £100 *at the time of the war. Policy* 2 *is to issue government debt of* £100 *and to repay simply the interest on that debt (without ever retiring the debt itself). Let us assume an interest rate of* 5%. *Thus under policy* 2 *households face taxes of* $T_2 = £5, T_3 = £5$ *and so forth. Now consider a household born at the time of the French British war. Pre-tax income and utility function are identical to that of the previous example. Thus, under policy* 1, *his present discounted value of lifetime income is*

$$I = £1000 - £100 + \frac{£1000}{1.05} = 1852.38$$

*and under policy* 2 *it is*

$$I = £1000 + \frac{£995}{1.05} = 1947.6$$

*Since with the utility function given above we easily see that under policy* 1 *consumption equals*

$$\begin{aligned} c_1 &= 926.19 \\ c_2 &= 972.50 \end{aligned}$$

*and under policy* 2 *it equals*

$$\begin{aligned} c_1 &= 973.8 \\ c_2 &= 1022.5 \end{aligned}$$

*Evidently, because lifetime income is higher under policy 2, the household consumes more in both periods (without borrowing constraints) and strictly prefers policy 2. What happens is that under policy 2, part of the cost of the war is borne by future generations that inherit the debt from the war, at least the interest on which has to be financed via taxation.*[4]

The point that changes in the timing of taxes may, and in most instances will, shift the burden of taxes across generations, was so obvious that for the longest time Ricardian equivalence was thought to be an empirically irrelevant theorem (as a mathematical result it is obviously true, but it was thought to be irrelevant for the real world). Then, in 1974 Robert Barro (then at the University of Chicago, now a professor at Harvard University) wrote a celebrated article arguing that Ricardian equivalence may not be that irrelevant after all. While the technical details are somewhat involved, the basic idea is simple.

First, let us suppose that households live forever (or at least as long as the government). Consider two arbitrary government tax policies. Since we keep the amount of government spending $G_t$ fixed in every period, the intertemporal budget constraint

$$\sum_{t=1}^{\infty} \frac{G_t}{(1+r)^{t-1}} = \sum_{t=1}^{\infty} \frac{T_t}{(1+r)^{t-1}}$$

requires that the two tax policies have the same present discounted value. But without borrowing constraints only the present discounted value of lifetime

[4] Note that even a positive probability of dying before the entire debt from the war is repaid is sufficient to invalidate Ricardian equivalence.

after-tax income matters for a household's consumption choice. But since the present discounted value of taxes is the same under the two policies it follows that (of course keeping pre-tax income the same) the present discounted value of after-tax income is unaffected by the switch from one tax policy to the other. Private decisions thus remain unaffected, therefore all other economic variables in the economy remain unchanged by the tax change. Ricardian equivalence holds.

But how was Barro able to argue that households live forever, when in the real world they clearly do not. The key to his arguments are bequests. Suppose that people live for one period and have utility functions of the form

$$U(c_1) + \beta V(b_1)$$

where $V$ is the maximal lifetime utility your children can achieve in their life if you give them bequests $b$. As before, $c_1$ is consumption of the person currently alive. Now the parameter $\beta$ measures intergenerational altruism (how much you love your children). A value of $\beta > 0$ indicates that you are altruistic, a value of $\beta < 1$ indicates that you love your children, but not quite as much as you love yourself.

The budget constraint is

$$c_1 + b_1 = y_1$$

where $y_1$ is income after taxes of the person currently alive. Bequests are constrained to be non-negative, that is $b_1 \geq 0$. The utility function of the child is given by

$$U(c_2) + \beta V(b_2)$$

and the budget constraint is

$$c_2 + b_2 = y_2 + (1 + r)b_1$$

By noting that $V(b_1)$ is nothing else but the maximized value of $U(c_2) + \beta V(b_2)$ one can now easily show that this economy with one-period lived people that are linked by altruism and bequests (so-called dynasties) is exactly identical to an economy with people that live forever and face borrowing constraints (since we have the restriction that bequests $b_1 \geq 0, b_2 \geq 0$ and so forth). Now from our previous discussion of borrowing constraints we know that binding borrowing constraints invalidate Ricardian equivalence, which leads us to the following

**Conclusion 7** *In the Barro model with one-period lived individuals Ricardian equivalence holds if (and only if) a) individuals are altruistic ($\beta > 0$) and bequest motives are operative (that is, the constraint on bequests $b_t \geq 0$ is never binding in that people find it optimal to always leave positive bequests).*

The key question for the validity of the Barro model (and thus Ricardian equivalence) is then whether the real world is well-approximated with all people leaving positive bequests for altruistic reasons.[5] Thus a big body of empirical literature investigated whether most people, or at least those people that pay the majority of taxes, leave positive bequests. In class I will discuss some of the findings briefly, but the evidence is mixed, with slight favor towards the hypothesis that not enough households leave significant bequests for the infinitely lived household assumption to be justified on empirical grounds.

### Lump-Sum Taxation

A lump-sum tax is a tax that does not change the relative price between two goods that are chosen by private households. These two goods could be consumption at two different periods, consumption and leisure in a given period, or leisure in two different periods. In section 4.4 we will discuss in detail how non-lump sum taxes (often call distortionary taxes, because they distort private decisions) impact optimal consumption, savings and labor supply decisions. Here we simply demonstrate that the timing of taxes is not irrelevant if the government does not have access to lump-sum taxes.

**Example 8** *This example is similar in spirit to the last question of your first homework, but attempts to make the source of failure of Ricardian equivalence even clearer. Back again to our simple war finance example. Households have utility of*

$$\log(c_1) + \log(c_2)$$

*income before taxes of £1000 in each period and the interest rate is equal to 0. The war costs £100. The first policy is to levy a £100 tax on first period*

[5] One can show that if parents leave bequests to children for strategic reasons (i.e. threaten not to leave bequests if the children do not care for them when they are old), then again Ricardian equivalence breaks down, because a change in the timing of taxes changes the severity of the threat of parents (it's worse to be left without bequests if, in addition, the government levies a heavy tax bill on you).

*labor income. The second policy is to issue £100 in debt, repaid in the second period with proportional consumption taxes at rate $\tau$. As before, under the first policy the optimal consumption choice is*

$$\begin{aligned} c_1 &= c_2 = £950 \\ s &= £900 - £950 = -£50 \end{aligned}$$

*The second policy is more tricky, because we don't know how high the tax rate has to be to finance the repayment of the £100 in debt in the second period. The two budget constraints under policy 2 read as*

$$\begin{aligned} c_1 + s &= £1000 \\ c_2(1+\tau) &= £1000 + s \end{aligned}$$

*which can be consolidated to*

$$c_1 + (1+\tau)c_2 = £2000$$

*Maximizing utility subject to the lifetime budget constraint yields*

$$\begin{aligned} c_1 &= £1000 \\ c_2 &= \frac{£1000}{1+\tau} \end{aligned}$$

*We could stop here already, since we see that under the second policy the households consumes strictly more than under the first policy. The reason behind this is that a tax on second period consumption only makes consumption in the second period more expensive, relative to consumption in the first period, and thus households substitute away from the now more expensive to the now cheaper good. The fact that the tax changes the effective relative price between the two goods qualifies this tax as a non-lump-sum tax. For completeness we solve for second period consumption and saving. The government must levy £100 in taxes. But tax revenues are given by*

$$\tau c_2 = \frac{\tau £1000}{1+\tau}$$

*Setting this equal to 100 yields*

$$\begin{aligned} 100 &= 1000 * \frac{\tau}{1+\tau} \\ 0.1 &= \frac{\tau}{1+\tau} \\ \tau &= \frac{0.1}{0.9} = 0.1111 \end{aligned}$$

*Thus*

$$\begin{aligned} c_2 &= 900 \\ s &= 0 \end{aligned}$$

*Finally we can easily show that households prefer the lump-sum way of financing the war (policy 1) than the distortionary way (policy 2), since*

$$\log(950) + \log(950) > \log(1000) + \log(900).$$

*Even though this is just a simple example, it tells a general lesson: with distortionary taxes Ricardian equivalence does not hold and households prefer lump sum taxation for a given amount of expenditures to distortionary taxation.*

## 4.3 An Excursion into the Fiscal Situation of the US

In principal, the projection of the long-run fiscal situation of the government is straightforward. Start with the total debt the U.S. government owes at the beginning of a given year, say 2014, project both outlays and receipts into the future and thus arrive at the level of government debt at any time in the future. Obviously, the forecast of future outlays and revenues is far from a trivial task. It is as hard, and most likely quite harder, to forecast the level of government debt in 2050 than to forecast the weather in the same year.

Fortunately, two prominent economists, Jagadeesh Gokhale and Kent Smetters, have done the work for us and found a concise way to summarize the current long-run fiscal situation of the U.S. government. Their original joint monograph *Fiscal and Generational Imbalances* was published in 2003. In these notes, I draw on the update of the numbers provided in the paper by Gokhale from 2013, *Spending Beyond our Means: How We are Bankrupting Future Generations* to provide an up to date account of the situation (which, as you might imagine, changed quite dramatically between 2003 and 2012, the most recent year for which Gokhale has data).

Before going into the details, a word about the authors. Gokhale was a consultant to the Department of the Treasury from July to December 2002 and is now a senior fellow at the Cato Institute. Smetters was assistant secretary at the U.S. Treasury from 2001 to 2002 and a consultant from August

2002 to February 2003, before returning to his regular job as a Professor of risk and insurance at the Wharton School of Finance.

### 4.3.1 Two Measures of the Fiscal Situation

Gokhale and Smetters define two crucial measures of the fiscal situation

$$FI_t = PVE_t + B_t - PVR_t$$

where $PVE_t$ is the present discounted value of *projected* expenditures under current fiscal policy (that is, current tax policy, current social security policy, current Medicare policy etc.) from period $t+1$ onward, $PVR_t$ stands for the present discounted value of all future projected receipts from period $t+1$ onward and $B_t$ stands for the outstanding government debt at the end of period $t$. Thus the measure $FI_t$, which the authors call fiscal imbalance, measures the aggregate shortfall in the governments' finances, due to past behavior as captured in $B_t$, and future behavior, as captured in $PVE_t - PVR_t$.

In terms of our previous notation

$$PVE_t = \sum_{\tau=t+1}^{\infty} \frac{G_\tau}{(1+r)^{\tau-t}}$$

and

$$PVR_t = \sum_{\tau=t+1}^{\infty} \frac{T_\tau}{(1+r)^{\tau-t}}$$

as well as

$$B_t = \sum_{\tau=1}^{t} \frac{G_\tau}{(1+r)^{\tau-t}} - \sum_{\tau=1}^{t} \frac{T_\tau}{(1+r)^{\tau-t}}$$

Thus our intertemporal budget constraint suggests that a fiscal policy that is feasible must necessarily have $FI_t = 0$; in other words, if one computes a $FI_t > 0$ under current and projected future policy, then fiscal policy *has to* change, absent a default on government debt sometime in the future. Also note that one can compute the measure $FI_t$ for the entire federal government, or for selected programs (such as social security) separately.

In order to assess which generations bear what burden of the total fiscal imbalance, an additional concept is needed. For example, $FI_t$ would not show a change if social security benefits tomorrow would be increased, to be financed with future increases in payroll taxes of the same present discounted value. In order to capture the effects of such policy changes Gokhale and Smetters define as

$$GI_t = PVE_t^L + B_t - PVR_t^L$$

where $GI_t$ is the generational imbalance at the end of period $t$, $PVE_t^L$ is the present discounted value of outlays paid to generations currently alive (thus denoted by $L$) in period $t$, with $PVR_t^L$ defined correspondingly. Thus $GI_t$ is that part of the fiscal imbalance $FI_t$ that results from interactions of the government with past (through $B_t$) and currently living generations; the difference

$$FI_t - GI_t$$

then denotes the projected part of fiscal imbalance due to the difference between spending on, and taxes paid by future generations. Decomposing the fiscal imbalance $FI_t$ into $GI_t$ and $FI_t - GI_t$ thus gives a sense of the degree of intergenerational redistribution between past and current generations on one hand and future generations on the other hand. Note that one could split $FI_t$ much more finely among different generations rather than using the coarse distinction between past, currently living and future generations, that is, once could (and Gokhale presents such calculations) calculate the part of $FI_t$ due to current 43 year old males, 20 year old females etc. In these notes I will focus on the broad summary measures state most clearly Gokhale's main arguments.

### 4.3.2 Main Assumptions

In this subsection we collect the main assumptions underlying Gokhale's results (which are in turn mildly updated assumptions presented by Gokhale and Smetters (2003), and much more clearly discussed there).

- Real interest rate (discount rate for the present value calculations) of 3.68% per annum (average yield on a 30 year Treasury bond in recent years). Note: using a larger discount factor would, other things equal, reduce the importance of future polices on the $FI_t$ measure.

- An annual growth rate of real wages of between 1% and 2%, based on future projections of the Congressional Budget Office (see his endnote 20 for the link to the actual CBO projections).

- As we will see, the crucial assumptions for the numbers are related to health care costs. Here the author uses the assumptions of the Medicare Board of Trustees report to account for the fact that the expenditure (in per capita terms) growth rate in Medicare is projected to be significantly above the projections from growth rates of wages for the immediate future. Part of this gap is accounted for the fact that the prices of medical goods grow faster than the prices of the general Consumer Price Index (CPI).[6] Beyond 2035 the assumption is that the gap in the growth rate between Medicare expenditure growth and wage growth gradually shrinks to zero, which, as one might argue, is an optimistic assumption that favors finding a smaller fiscal imbalance.

- Numbers for two scenarios are presented. The Baseline policy scenario corresponds to current fiscal policy, the Alternative policy scenario factors in likely policy changes. Both the Baseline and the Alternative policy scenario are provided by the CBO. The Alternative scenario recognizes that some spending cuts and tax increases schedule for the next decade (and part of current law) will likely not be implemented. As a consequence $FI_t$ will be larger under the Alternative (more realistic, at least as perceived by Gokhale) relative to the Baseline scenario. Although the exact magnitudes below depend on which scenario is being used, the overall message does not.

- In order to compute $GI$, one needs to break down taxes paid and outlays received by generations, which requires a host of ancillary assumptions, and in particular on generation specific mortality rates, which in turn determine the relative size of current and future generations.

### 4.3.3 Main Results

Table 4.2 presents measures of the fiscal imbalance for the U.S. federal government under the Baseline policy scenario. This table is an abridged version of table 4 in Gokhale (2012).

Table 4.3 repeats the same accounting exercise, but under the Alternative policy scenario. For this scenario Gokhale presents a decomposition of

[6] For the purpose of the current discussion it is irrelevant whether this price increase is due to improvements in the quality of medical goods and services or reflect differential technological progress or increasing market power in the health sector.

**Fiscal Imbalance, Baseline (Billion of 2012 Dollars)**

| Part of the Budget | 2012 | 2017 | 2022 |
|---|---|---|---|
| $FI$ in Social Insurance | $64,853$ | $70,961$ | $82,564$ |
| $FI$ in Social Security | $N/A$ | $N/A$ | $N/A$ |
| $FI$ in Medicare | $N/A$ | $N/A$ | $N/A$ |
| $FI$ in Rest of Federal Government | $-10,502$ | $-10,687$ | $-11,742$ |
| **Total** $FI$ | $54,675$ | $60,274$ | $70,822$ |

Table 4.2: Fiscal and Generational Imbalance

**Fiscal Imbalance, Alternative (Billion of 2012 Dollars)**

| Part of the Budget | 2012 | 2017 | 2022 |
|---|---|---|---|
| $FI$ in Social Insurance (SS+Med.) | $65,934$ | $72,036$ | $83,606$ |
| $FI$ in Social Security | $20,077$ | $22,272$ | $26,660$ |
| $FI$ in Medicare | $45,857$ | $49,764$ | $56,946$ |
| $FI$ in Rest of Federal Government | $25,457$ | $29,826$ | $36,660$ |
| **Total** $FI$ | $91,391$ | $101,862$ | $120,266$ |

Table 4.3: Fiscal and Generational Imbalance

the part of the fiscal imbalance due to Social Security and due to Medicare separately. It is drawn from table 5 and 11 of Gokhale (2012).

Tables 4.2 and 4.3 summarize the fiscal imbalance in absolute dollar terms (in constant 2012 prices). Magnitudes of 100 trillion dollars might be hard to comprehend unless you relate it to total economic activity, as measured by real GDP, for example. In table 4.4 we relate the fiscal imbalance to the present value of GDP. The numbers measure what additional fraction of GDP the government would have to confiscate, starting today and into the indefinite future, to pay for the entire fiscal imbalance (under the CBO projections about future GDP and under the maintained assumption that such confiscation would leave the path of future GDP unchanged). We only report the values for the Alternative policy scenarios, the numbers for the Baseline scenario are qualitatively similar, but lower (since the $FI$ is lower -in fact, negative) for the rest of the government in that scenario.

**Fiscal Imbal., Alt. Scen. (% of Pres. Val. GDP)**

| Part of the Budget | 2012 | 2017 | 2022 |
|---|---|---|---|
| $FI$ in Social Insurance (SS+Med.) | 6.5% | 6.5% | 6.8% |
| $FI$ in Rest of Federal Government | 2.5% | 2.7% | 3.0% |
| **Total** $FI$ | 9.0% | 9.1% | 9.8% |

Table 4.4: Fiscal and Generational Imbalance

### 4.3.4 Interpretation

The key findings from the preceding table can be summarized as follows:

1. The total fiscal imbalance of the government is huge; it requires the confiscation of 9% of GDP in perpetuity to close this imbalance from the perspective of 2012. Expressed in terms of a required increase in labor income taxes, this would come to about a 20% point increase (over and above the taxes already in place, and assuming no negative effects on labor supply induced by the tax hike).

2. If no policy changes are taken, the measure $FI$ grows over time at a gross rate of $(1 + r) = 1.0368$ per year. It is a debt that, without any action of repayment, simply accumulates at the gross interest rate.

3. By far the largest part of the fiscal imbalance is due to Medicare. The fiscal imbalance form this government program accounts for about half of the entire fiscal imbalance (under the Alternative policy scenario), making it clear that further reform of Medicare entitlements are necessary to bring the fiscal situation of the U.S. closer to balance. The large imbalance in this program stems from two elements: per-capita expenditures grow faster than GDP per capita (mainly due to faster increases of medical goods prices, relative to the CPI) and the population rapidly ages, and thus the number of people eligible for medicare increases.

4. The rest of the government programs contribute only moderately to total fiscal imbalance (and under the Baseline scenario actually help to reduce it). This result, however, is subject to quite conservative estimates of the increase in government discretionary spending and

**Generational Imbalance, Alternative (Bill of 2012 Dollars)**

| Part of the Budget | 2012 | 2017 | 2022 |
|---|---|---|---|
| $FI$ in Social Insurance | $65,934$ | $72,036$ | $83,606$ |
| $FI$ in Social Security | $20,077$ | $22,272$ | $26,660$ |
| $GI$ in Social Security (incl. Trust Fund) | $19,586$ | $21,726$ | $26,032$ |
| $FI-GI$ in Social Security | $491$ | $546$ | $628$ |
| $FI$ in Medicare | $45,857$ | $49,764$ | $56,946$ |
| $GI$ in Medicare | $34,487$ | $38,311$ | $44,693$ |
| $FI-GI$ in Medicare (incl. Trust Fund) | $11,370$ | $11,453$ | $12,253$ |

Table 4.5: Fiscal and Generational Imbalance

optimistic estimates of government revenues as the Alternative policy scenario shows.

5. The total fiscal imbalance measured by Gokhale and Smetters dwarfs the official most commonly reported measure of government indebtedness, namely the outstanding government debt, roughly by a factor of 5 (and recall, government debt rose very steeply in the years prior to 2012).

Now we turn to the generational imbalance, splitting the fiscal imbalance between generations living today and future generations. Of particular interest is the division of the entire imbalance for the two programs that transfers across generations, namely Social Security and Medicare.

1. The table reinforces the conclusion that the majority of the fiscal imbalance, and certainly the share of it due to social insurance programs, is due to Medicare. What the table also shows is that about 3/4 of the Medicare fiscal imbalance is accounted for by benefits for generations currently alive exceeding taxes paid by these generations.[7] However, for Medicare even future generations, under current law, would see benefits exceeding contributions (in present discounted value terms), mostly

[7] Of course part of this is due to the fact that of the people currently alive a significant share are retirees that will not contribute to Medicare tax payments but receive benefits, whereas future generations will still fully go through the life cycle of first paying taxes and then receiving benefits.

due to the very costly prescription drug benefits afforded to Medicare recipients.

2. In contrast, the fiscal imbalance in social security is due *entirely* to past and current generations. The contribution of future generations $FI - GI$ is approximately zero, indicating that future generations, under current law, are set to receive the same benefits, in present value terms as they pay in the form of payroll taxes. Of course, to the extent that future policy changes will have to deal with bringing the fiscal imbalance back to balance, and to the extent that these future policy changes (reductions in benefits or increases in taxes) impact future generations, this conclusion about future benefits and taxes roughly balancing could substantially be altered.

3. As one might expect, the exact magnitude of the numbers presented in this section are somewhat sensitive to the assumptions made, especially with respect to the growth rate of wages, the discount rate applied to future revenues and outlays, and the temporary differential between expenditure growth in Medicare and the economy as a whole. However, the general conclusion from Gokhale's report persists: large spending cuts in government programs or substantial tax increases are required to restore fiscal balance, especially in the to biggest entitlement programs, Medicare and Social Security. What are the effects on the economy? In order to answer that question we first need to study what effects tax increases have, if these taxes are distortionary, as all real world taxes are.

## 4.4 Consumption, Labor and Capital Income Taxation

### 4.4.1 The U.S. Federal Personal Income Tax

**Brief History**[8]

In early U.S. history the country relied on few taxes, on alcohol, tobacco and snuff, real estate sold at auctions, corporate bonds and slaves.[9] To finance the British-American war in 1812 sales taxes on gold, silverware and other jewelry were added. All internal taxes (that is taxes on residents of the U.S.) were abolished in 1817, with the government relying exclusively on tariffs on imported goods to fund its operations.

The civil war from 1861-1865 demanded increased funds for the federal government. In 1862 the office of Commissioner of Internal Revenue (the predecessor of the modern IRS) was established, with the rights to assess, levy and collect taxes, and the right to enforce the tax laws though seizure of property and income and through prosecution. During the civil war individuals earning between $\$600 - \$10,000$ had to pay an income tax of 3%, with higher rates for people with income above $\$10,000$. Note however, that \$600 was well beyond the average income of a person at that time, so that a vast majority of the U.S. population was exempted from the income tax altogether. In addition to income taxes, additional sales and excise taxes were introduced (an excise tax is a sales tax levied on a particular set of commodities; alcohol, tobacco and gambling are the most common goods on which excise taxes are applied). Furthermore, for the first time an inheritance tax was introduced. Total tax collections reached \$310 million in 1866, the highest amount in U.S. history to that point, and an amount not reached again until 1911.

The general income tax was scrapped again in 1872, alongside other taxes besides excise taxes on alcohol and tobacco. It was briefly re-introduced in 1894, but challenged in court and declared unconstitutional in 1895, because it did not levy taxes and distribute the funds among states in accordance

[8] The subsection is partially based on "History of the Income Tax in the United States" by Scott Moody, senior economist at the *Tax Foundation*.

[9] The Independence war was, to significant degree, also financed by contributions from private wealthy individuals as well as, later during the war, by loans from European countries, notably France, Spain and the Netherlands.

with the constitution. The modern federal income tax was permanently introduced in the U.S. in 1913 through the 16-th Amendment to the Constitution. The amendment gave Congress legal authority to tax income and pathed the way for a revenue law that taxed incomes of both individuals and corporations. By 1920 IRS revenue collections totaled \$5.4 billion dollars, rising to \$7.3 billion dollars at the eve of WWII. However, the income tax was still largely a tax on corporations and very high income individuals, since exemption levels were high. Consequently the majority of U.S. citizens were not subject to federal income taxes. However, in 1943 the government introduced a withholding tax on wages (before taxes were paid once a year, in one installment) that covered most working Americans. Consequently, by 1945 the number of income taxpayers increased to 60 million (out of a population of 140 million) and tax revenues from the federal income tax increased to \$43 billion, a six-fold increase from the revenues in 1939. Thus the universal federal income tax in the U.S. was born during WWII.

The post-war period saw a series of changes in tax laws. The most far-reaching tax reforms in recent history have been the tax reforms by President Reagan of 1981 and 1986, President Clinton's tax reform of 1993 and the recent tax reforms of President George W. Bush in 2001-2003. The Reagan tax reforms reduced income tax rates by individuals drastically (with a total reduction amounting to the order of $\$500-600$ billion), partially offset by an increase in tax rates for corporations and moderate increases of taxes for the very wealthy.. Under the pressure of mounting budget deficits President Clinton partially reversed Reagan's tax cuts in 1993, in order to avoid extensive budget deficits and thus the expansion of government debt in the future. Further tax reforms under the Clinton presidency included tax cuts for capital gains, the introduction of a \$500 tax credit per child and tax incentives for education expenses. Finally, the recent large tax cuts in 2003 by President Bush temporarily (the package was originally scheduled to expire in 2012, and did partially expire in 2013) reduced dividend and capital gains taxes as well as marginal income taxes, and increased child tax credits for most American tax payers.

In order to analyze the economic impacts of these different changes in the tax code on private decisions, the distribution of income, wealth and welfare as well as to discuss the normative rationale (are these reforms "good"?) we need to go beyond simply describing the reforms and return to our theoretical analysis of what taxes and tax reforms do to private household decisions, and, once aggregated across the entire population, to the macro economy as a whole.

### Concepts and Definitions

Let by $y$ denote total household income from all sources that is subject to the personal income tax. In case the tax code includes provisions for tax deductions and exemptions we will denote these by $d$. In the presence of deductions and exemptions taxable income is given by $y - d$, and we assume that $d$ is equal to zero unless noted explicitly. If $d = 0$ household income and taxable income coincide (and we won't distinguish the two unless we have to for specific example tax codes). A tax code is defined by a function $T(y)$ that specifies for each possible (taxable) income $y$ gives the amount of taxes that are due to be paid. In both the political as well as the academic discussion of income taxation two important concepts of tax *rates* emerge.

**Definition 9** *For a given tax code $T$ we define as*

1. *the average tax rate of an individual with taxable income $y$ as*

$$t(y) = \frac{T(y)}{y}$$

   *for all $y > 0$.*

2. *the marginal tax rate of an individual with taxable income $y$ as*

$$\tau(y) = T'(y)$$

   *whenever $T'(y)$ is well-defined (that is, whenever $T'(y)$ is differentiable at income $y$).*

The average tax rate $t(y)$ indicates what *fraction* of her taxable income a person with income $y$ has to deliver to the government as tax. The marginal tax rate $\tau(y)$ measures how high the tax rate is on the last dollar earned, for a total taxable income of $y$. It also answers the question how many cents for an additional dollar of income a person that already has income $y$ needs to pay in taxes.

Note that rather than defining a tax code by the function $T(y)$ we can equivalently define it by specifying the the average tax rate schedule $t(y)$since

$$T(y) = y * t(y)$$

or by the marginal tax rate schedule $\tau(y)$ since

$$T(y) = T(0) + \int_0^y T'(y)dy \tag{4.8}$$

where the equality follows from the fundamental theorem of calculus.[10]. In fact, the current U.S. federal personal income tax code is defined by a collection of marginal tax rates; the tax code $T(y)$ can be recovered[11] using equation (4.8).

So far we have made no assumption on how the tax code looks like. Broadly speaking, tax codes can be classified into three categories, based on the following definitions.

**Definition 10** *A tax code is called progressive if the function $t(y)$ is strictly increasing in $y$ for all income levels $y$, that is, if the share of income due to be paid in taxes strictly increases with the level of income.*[12] *A tax system is called progressive over an income interval $(y_l, y_h)$ if $t(y)$ is strictly increasing for all income levels $y \in (y_l, y_h)$.*

**Definition 11** *A tax code is called regressive if the function $t(y)$ is strictly decreasing in $y$ for all income levels $y$, that is, if the share of income due to be paid in taxes strictly decreases with the level of income.*[13] *A tax system is called regressive over an income interval $(y_l, y_h)$ if $t(y)$ is strictly decreasing for all income levels $y \in (y_l, y_h)$.*

**Definition 12** *A tax code is called proportional if the function $t(y)$ is constant $y$ for all income levels $y$, that is, if the share of income due to be paid in taxes is constant in the level of income. A tax system is called proportional over an income interval $(y_l, y_h)$ if $t(y)$ is constant for all income levels $y \in (y_l, y_h)$.*

[10] Strictly speaking, we also need to specify the tax paid by households with zero income, $T(0)$, but it is often implicitly assumed that $T(0) = 0$

[11] Gladly, the IRS provides tables that specify how much taxes a household with given taxable income $y$ has to pay, so households do not have to solve the integral in equation (4.8) themselves.

[12] Formally, a tax code is progressive if $t'(y) > 0$ whenever the average tax function has well-defined derivative. Note that the definition does not require the function $t(y)$ to be differentiable since we can define what it means for a function to be strictly increasing mathematically without requiring that the function is differentiable: a function $t$ is strictly increasing if for all $(y_1, y_2)$ with $y_1 < y_2$ we have $t(y_1) < t(y_2)$.

[13] Formally, a tax code is regressive if $t'(y) < 0$ whenever the average tax function has well-defined derivative.

Let us look first at several examples, and then at some general results concerning tax codes.

**Example 13** *A head tax or poll tax*

$$T(y) = T$$

*where* $T > 0$ *is a number. That is, all people pay the tax* $T$*, independent of their income. Obviously this tax is regressive since*

$$t(y) = \frac{T}{y}$$

*is a strictly decreasing function of* $y$*. Also note that the marginal tax is* $\tau(y) = 0$ *for all income levels, since the tax that a person pays is independent of her income*

**Example 14** *A flat tax or proportional tax*

$$T(y) = \tau * y$$

*where* $\tau \in [0, 1)$ *is a parameter. In particular,*

$$t(y) = \tau(y) = \tau$$

*that is, average and marginal tax rates are constant in income and equal to the tax rate* $\tau$*. Clearly this tax system is proportional.*

**Example 15** *A flat tax with deduction*

$$T(y) = \begin{cases} 0 & \text{if } y < d \\ \tau(y-d) & \text{if } y \geq d \end{cases}$$

*where* $d, \tau \geq 0$ *are parameters. Here the household pays no taxes if her income does not exceed the exemption level* $d$*, and then pays a fraction* $\tau$ *in taxes on every dollar earned above* $d$*. For this tax system we have to distinguish between income* $y$ *and taxable income* $y - d$*; as in the previous example taxes are proportional to taxable income, but now a part* $d$ *of income is not taxable (previously that part* $d$ *was equal to zero). For this tax code one can compute average and marginal tax rates as*

$$t(y) = \begin{cases} 0 & \text{if } y < d \\ \tau\left(1 - \frac{d}{y}\right) & \text{if } y \geq d \end{cases}$$

*and*

$$\tau(y) = \begin{cases} 0 & \text{if } y < d \\ \tau & \text{if } y \geq d \end{cases}$$

*Thus this tax system is progressive for all income levels above* $d$; *for all income levels below it is trivially proportional.*

**Example 16** *A tax code with step-wise increasing marginal tax rates. Such a tax code is defined by its marginal tax rates and the income brackets for which these taxes apply. I constrain myself to three brackets, but one could consider as many brackets as you wish.*

$$\tau(y) = \begin{cases} \tau_1 & \text{if } 0 \leq y < b_1 \\ \tau_2 & \text{if } b_1 \leq y < b_2 \\ \tau_3 & \text{if } b_2 \leq y < \infty \end{cases}$$

*The tax code is characterized by the three marginal rates* $(\tau_1, \tau_2, \tau_3)$ *and income cutoffs* $(b_1, b_2)$ *that define the income tax brackets. It is somewhat burdensome*[14] *to derive the tax function* $T(y)$ *and the average tax* $t(y)$; *here we simply state without proof that if* $\tau_1 < \tau_2 < \tau_3$ *then this tax system is*

[14] In fact, it is not so hard if you know how to integrate a function. For $0 \leq y < b_1$ we have

$$T(y) = \int_0^y \tau(y)dy = \int_0^y \tau_1 dy = \tau_1 \int_0^y dy = \tau_1 y,$$

for $b_1 \leq y < b_2$ we have

$$T(y) = \int_0^y \tau(y)dy = \int_0^{b_1} \tau_1 dy + \int_{b_1}^y \tau_2 dy = \tau_1 b_1 + \tau_2(y - b_1)$$

and finally for $y \geq b_2$ we have

$$T(y) = \int_0^{b_1} \tau_1 dy + \int_{b_1}^{b_2} \tau_2 dy + \int_{b_2}^y \tau_3 dy = \tau_1 b_1 + \tau_2(b_2 - b_1) + \tau_3(y - b_2)$$

Consequently average tax rates are given by

$$t(y) = \begin{cases} \tau_1 & \text{if } 0 \leq y < b_1 \\ \frac{\tau_1 b_1}{y} + \tau_2\left(1 - \frac{b_1}{y}\right) & \text{if } b_1 \leq y < b_2 \\ \frac{\tau_1 b_1 + \tau_2(b_2 - b_1)}{y} + \tau_3\left(1 - \frac{b_2}{y}\right) & \text{if } b_2 \leq y < \infty \end{cases}$$

It is tedious but straightforward to show that $t(y)$ is increasing in $y$, strictly so if $y \geq b_1$.

*proportional for $y \in [0, b_1]$ and progressive for $y > b_1$. Obviously, with just two brackets we get back a flat tax with deduction, if we choose $\tau_1 = 0$.*

The reason we looked at the last example is that the current U.S. tax code resembles the example closely, but currently consists of seven marginal tax rates and six income cut-offs that define the income tax brackets. The income cut-offs vary with family structure, that is, depend on whether an individual is filing a tax return as single, as married filing jointly with her spouse or as married but filing separately.

Now let us briefly derive an important result for progressive tax systems. Since it is easiest to do the proof of the result if the tax schedule is differentiable (that is $T'(y)$ is well-defined for all income levels), we will assume this here.

**Theorem 17** *A tax system characterized by the tax code $T(y)$ is progressive, that is, $t(y)$ is strictly increasing in $y$ (i.e. $t'(y) > 0$ for all $y$) if and only if the marginal tax rate $T'(y)$ is higher than the average tax rate $t(y)$ for all income levels $y > 0$, that is*

$$T'(y) > t(y)$$

**Proof.** Average taxes are defined as

$$t(y) = \frac{T(y)}{y}$$

But using the rule for differentiating a ratio of two functions we obtain

$$t'(y) = \frac{yT'(y) - T(y)}{y^2}$$

But this expression is positive if and only if

$$yT'(y) - T(y) > 0$$

or

$$T'(y) > \frac{T(y)}{y} = t(y)$$

■

Intuitively, for average tax rates to increase with income requires that the tax rate you pay on the last dollar earned is higher than the average tax rate you paid on all previous dollars. Another way of saying this: one can only increase the average of a bunch of numbers if one adds a number that is bigger than the previous average. This result provides us with another, completely equivalent, way to characterize a progressive tax system. Obviously a similar result can be stated and proved for a regressive or proportional tax system.

### The Current Tax Code

Now let us look at the current U.S. federal personal income tax code. The tax rates an individual faces depends on whether the individual is single or married, and if married, if she files a tax return jointly with her spouse or not. Before looking at the tax rates applying to different levels of income, we first have to discuss what income is subject to income taxes.

The income to which the tax function is applied is called *taxable income*; this is the entity we have previously denoted by $y$ or $y - d$, depending on the context. In order to derive taxable income we start with the concept of *gross income.* This gross income consists of

$$\begin{aligned} \text{Gross Income} &= \text{Wages and Salaries} \\ &\quad + \text{Interest Income and Dividends} \\ &\quad + \text{Net Business Income} \\ &\quad + \text{Net Rental Income} \\ &\quad + \text{Other Income} \end{aligned}$$

Most of these categories are self-explanatory; the "net" in net business income and net rental income refers to income net of business expenses or expenses for the rental property on which income is earned (e.g. expenditures for repairs in the case of rental income, certain expenses such as business dinners for the case of business income). Other income includes unemployment insurance benefits, alimony, income from gambling, income from illegal activities (which is evidently often not reported on the tax returns filed by those engaging in these activities). There are, however, important sources of income that are not part of *gross income* and thus not taxable. Examples include child support, gifts below a certain threshold, interest income from state and local bonds (so-called Muni's), welfare and veterans benefits. Also, certain parts of employee compensation, such as employer contributions for health insurance and retirement accounts, are not part of *gross income* and thus not taxable.

From Gross Income one arrives at *Adjusted Gross Income* (AGI) by subtracting contributions to Individual Retirement Accounts (IRA's), alimony, and health insurance payments by self-employed for themselves and their families. Finally, taxable income is derived from AGI by subtracting deductions and exemptions. Personal exemptions are amounts by which AGI is reduced that depend on the number of family members the tax payer supports. Everybody is entitled to claim him- or herself as an exemption; in addition, one can claim his/her spouse and children, unless the spouse files for taxes separately. In 2013, per exemption, the tax payer is entitled to deduct \$3,900 from AGI. With respect to deductions, each tax payer has the choice to claim the standard deduction, or to claim itemized deductions. The standard deduction for 2013 amounts to \$12,200 for married households filing jointly and \$6,100 for single households. If one opts to use itemized deductions, the include mortgage interest payments state and local income and property taxes, medical expenses above a certain threshold, charitable contributions, moving expenses related to relocation for employment and other small items. Whether to claim the standardized deduction or to use itemized deductions obviously depends on the amount the itemized deductions add up to for a given tax payer. In general, households with large mortgages or huge medical bills tend to opt for the itemized deduction option.[15]

After these adjustments to AGI one finally arrives at taxable income $y$, to which the tax code is applied to calculate the tax liability of a household. Comparing this liability to the withholdings of the tax year, and subtracting tax credits (such as child care expenses, taxes paid in foreign countries, the earned income tax credit and tax credits for college tuition) finally yields the taxes that are due upon filing your income tax on April 15 (or the rebate owed to the tax payer should tax liabilities minus credits fall short of withholdings). Now we want to discuss the current U.S. tax code, that is, the schedule that for each taxable income determines how large the tax payers' liabilities are.

[15] The discussion in these notes ignores the Alternative Minimum Tax (AMT) as well as other important details of the tax code (e.g. the Earned Income Tax Credit, EITC). The AMT is a second, altogether different tax code, and the actual taxes a household has to pay is the maximum of the taxes owed under the standard code and under the AMT. The AMT was originally introduced to avoid having high income households pay little or no taxes because some of these households could claim very large deductions. Whether a household is subject to the AMT depends on many things, but broadly it is relevant for high income households (with incomes exceeding \$75,000) that claim high deductions under the standard tax code).

**Tax Rates for 2013, Singles**

| Income | $T'(y)$ | $T(y)$ |
|---:|---:|---:|
| $0 \leq y < \$8,925$ | 10% | $0.1y$ |
| $\$8,925 \leq y < \$36,250$ | 15% | $\$892 + 0.15(y - 8,925)$ |
| $\$36,250 \leq y < \$87,850$ | 25% | $\$4,991 + 0.25(y - 36,250)$ |
| $\$87,850 \leq y < \$183,250$ | 28% | $\$17,891 + 0.28(y - 87,850)$ |
| $\$183,250 \leq y < \$398,350$ | 33% | $\$44,603 + 0.33(y - 183,250)$ |
| $\$398,350 \leq y < \$400,000$ | 35% | $\$115,586 + 0.35(y - 398,350)$ |
| $\$400,000 \leq y < \infty$ | 39.6% | $\$116,164 + 0.396(y - 400,000)$ |

Table 4.6: Marginal Tax Rates in 2003, Households Filing Single

Table 4.6 summarizes marginal tax rates and the total tax code for individuals filing single. As we can see from the tax schedule, for the first $\$8,925$ the tax code is proportional, with a marginal and average tax rate of 10%. After that, the tax code becomes progressive, since marginal tax rates are increasing (strictly so at the income bracket points) in income $y$. The last column shows total taxes owed; the function $T(y)$ is derived by using $T(0) = 0$ and integrating the marginal tax schedule with respect to income.

The tax code is depicted graphically in figures 4.1 and 4.2. We see that the tax code is defined by seven marginal tax rates and seven income brackets (determined by six thresholds, or bend points) for which the marginal tax rates apply. Since marginal tax rates are increasing with income, average tax rates are increasing with income as well, strictly so after the first income bracket.

This is exactly what figure 4.2 documents. Average taxes are flat at 10% for the first $\$8,925$ and then strictly increasing. A comparison of the two graphs also shows that average taxes are always lower than marginal taxes, strictly so for all income levels above $\$8,925$. Note that if we were to continue the average tax plot for higher and higher income, average taxes would approach the 39.6% mark, the highest marginal tax rate, as income becomes large.

In Table 4.6 we document the tax code applying for a married couple that files a joint tax return. Encoded in these two tax schedules is the so-called marriage penalty. Consider the following hypothetical situation: Angelina and Brad are madly in love and think about getting married. Each of them is making $\$100,000$ as taxable income. Simply living together without being

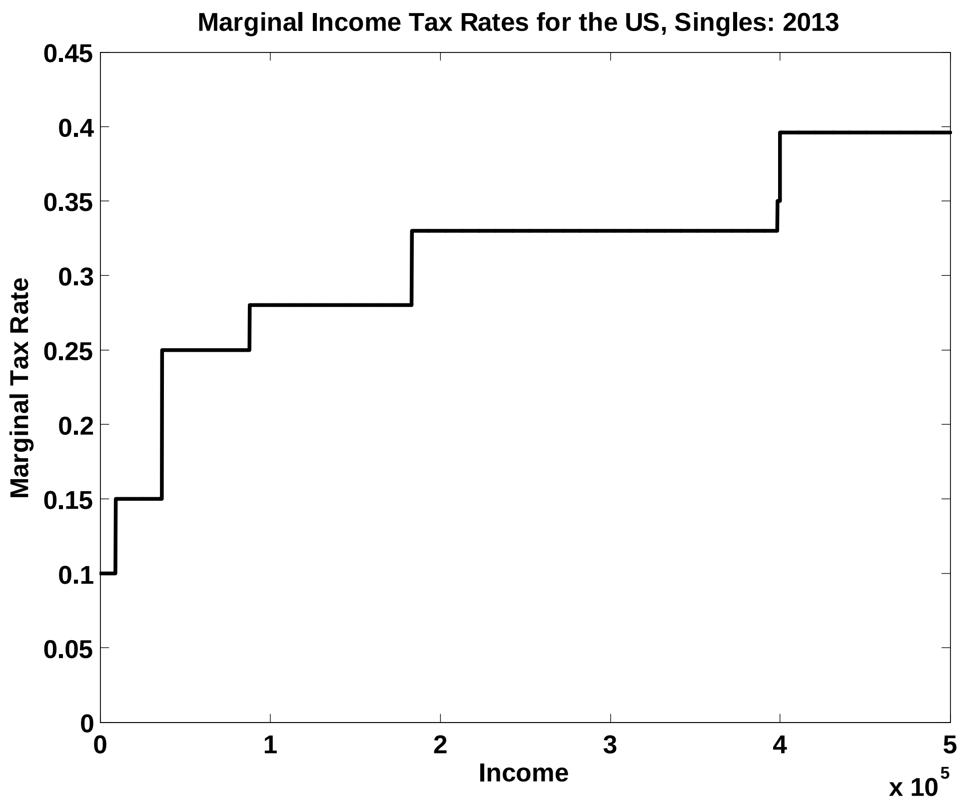


Figure 4.1: U.S. Marginal Income Taxes, Individuals Filing Single

married, Angelina pays taxes

$$T^A = T(100,000) = \$17,891 + 0.28(100,000 - 87,850) = \$21,2932 = T^B$$

and Brad pays the same amount. So the joint tax liability of the couple is $42,586. If they marry (even ignoring the $100,000 cost for the wedding), they now pay taxes of

$$T^{A+D} = T(200,000) = \$28,457 + 0.28(200,000 - 146,400) = \$43,465$$

that is, the mere act of marriage increases their joint tax liability by close to $1,000.

Note that one can also construct a reverse example. Now suppose that Angelina makes $180,000 and Brad makes $36,250. Getting married and

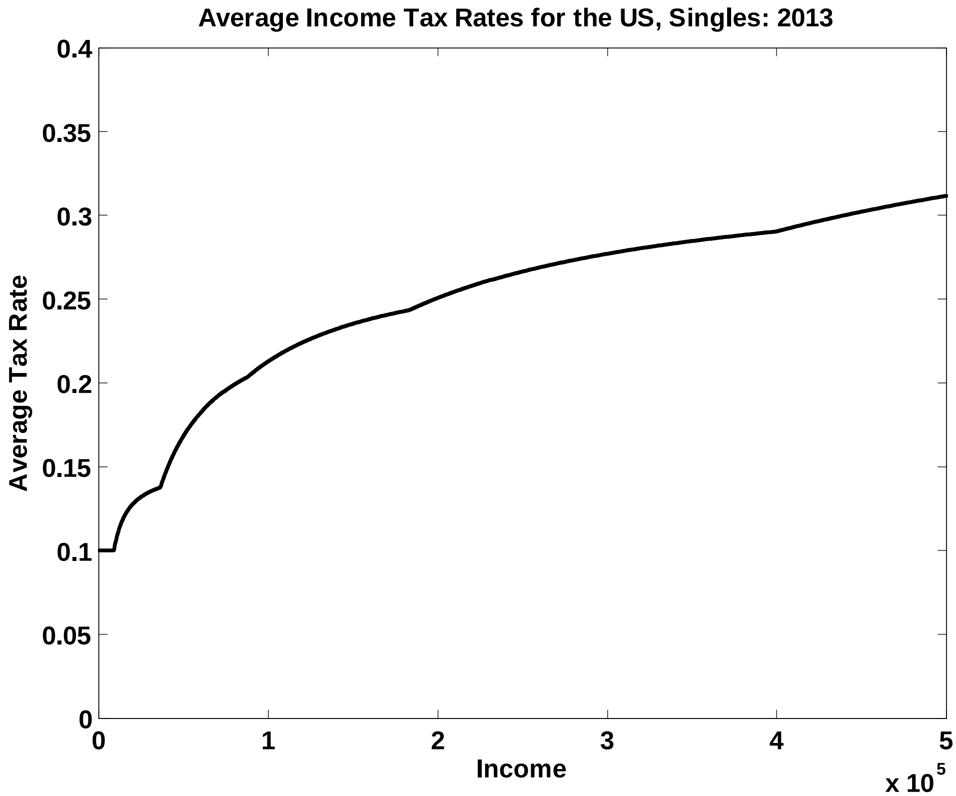


Figure 4.2: U.S. Average Tax Rate for Individuals Filing Single

filing a single tax return yields taxes for the family of

$$T^{A+D} = T(216,215) = \$28,457 + 0.28(216,215 - 146,400) = 48,005$$

whereas pre-marriage taxes are given by

$$\begin{aligned} T^A &= T(180,000) = 17,891 + 0.28(180,000 - 87,850) = 43,693 \\ T^B &= T(36,250) = \$4,991 \end{aligned}$$

and thus total tax liabilities without getting married, total $48,684, and thus getting married saves them about $679 in taxes. Thus, whether it pays to get married for tax reasons depends on how incomes within the couple are distributed. In general, with fairly equal incomes it does not pay, whereas with incomes substantially different between the two partners it pays to get married and file taxes jointly.

**Tax Rates for 2013, Married Filing Jointly**

| Income | $T'(y)$ | $T(y)$ |
|---|---|---|
| $0 \leq y < \$17,850$ | 10% | $0.1y$ |
| $\$17,850 \leq y < \$72,500$ | 15% | $\$1,785 + 0.15(y - 17,850)$ |
| $\$72,500 \leq y < \$146,400$ | 25% | $\$9,982 + 0.25(y - 72,500)$ |
| $\$146,400 \leq y < \$223,050$ | 28% | $\$28,457 + 0.28(y - 146,400)$ |
| $\$223,050 \leq y < \$398,350$ | 33% | $\$49,919 + 0.33(y - 223,050)$ |
| $\$398,350 \leq y < \$450,000$ | 35% | $\$107,768 + 0.35(y - 398,350)$ |
| $\$450,000 \leq y < \infty$ | 39.6% | $\$125,846 + 0.396(y - 450,000)$ |

Table 4.7: Marginal Tax Rates in 2003, Married Households Filing Jointly

In fact, these examples are not just artefacts, but part of a general problem. One can show that it is *impossible* to design a tax system that simultaneously is:

1. Progressive, as defined in the previous section

2. Satisfies across family equity: families with equal household incomes pay equal taxes (independent of how much of that income is earned by different members of each household)

3. Marriage-neutral: a given family pays the same taxes independent of whether the partners of the family are married or not.

The only tax system that satisfies both 2. and 3. is a proportional income tax (you may check this for the income examples above), but it is evidently not progressive. This raises the question why most tax systems we observe in the real world (certainly including the U.S. personal income tax system) are indeed progressive. Perhaps the most important answer is that under certain assumptions it is socially optimal (in a sense defined precisely below) to implement a progressive tax system.

**Normative Arguments for Progressive Taxation**

For simplicity assume that there are only two households in the economy, household 1 with taxable income of $\$100,000$ and household 2 with taxable income of $\$20,000$. Again for simplicity assume that their lifetime utility $u(c)$ only depends on their current after-tax income $c = y - T(y)$, which we assume to be equal to consumption (implicitly we assume that households only live for one period). Finally assume that the lifetime utility function $u(c)$ is of log-form.[16]

We want to compare social welfare under two tax systems, a hypothetical proportional tax system and a system of the form in the last example. For concreteness, let the second tax system be given by

$$\tau(y) = \begin{cases} 0\% & \text{if } 0 \leq y < 15000 \\ 10\% & \text{if } 15000 \leq y < 50000 \\ 20\% & \text{if } 50000 \leq y < \infty \end{cases}$$

Under this tax system total tax revenues from the two agents are

$$\begin{aligned} & T(15,000) + T(100,000) \\ = \; & 0.1 * (20000 - 15000) \\ & +0.1 * 35000 + 0.2(100000 - 50000) \\ = \; & \$500 + \$13500 \\ = \; & \$14000 \end{aligned}$$

and consumption for the households are

$$\begin{aligned} c_1 &= 20000 - 500 = 19500 \\ c_2 &= 100000 - 13500 = 86500 \end{aligned}$$

In order to enable the appropriate comparison, we first have to determine the proportional tax rate $\tau$ such that total tax revenues are the same under the hypothetical proportional tax system and the progressive tax system above system. We target total tax revenues of \$14000. But then

$$\begin{aligned} 14000 &= \tau * 20,000 + \tau * 100,000 = \tau * 120,000 \\ \tau &= \frac{14,000}{120,000} = 11.67\% \end{aligned}$$

[16] For the argument to follow it is only important that $u$ is strictly concave. The log-formulation is chosen for simplicity. Also, as long as current high income makes future high income more likely, the restriction to lifetime utility being defined over current after-tax income does not distort our argument. If we define the function $V(c)$ as the lifetime utility of a person with current after tax labor income $c$, as long as this function is increasing and strictly concave in $y$ (which it will be if after-tax income is positively correlated over time and the period utility function is strictly concave), the argument below gues through unchanged.

is the proportional tax rate required to collect the same revenues as under our progressive tax system. Under the proportional tax system consumption of both households equals

$$\begin{aligned} c_1 &= (1-0.1167)*20000 = 17667 \\ c_2 &= (1-0.1167)*100000 = 88333 \end{aligned}$$

Which tax system is better? This is a hard question to answer in general, because under the progressive tax system the person with $20,000$ of taxable income is better off, whereas the person with $100,000$ is worse off than under a pure proportional system. So without an ethical judgement about how important the well-being of both households is we cannot determine which tax system is to be preferred. In the language of economic theory, neither policy Pareto-dominates the other.

Such judgements are often made in the form of a social welfare function

$$W(u(c_1),\ldots,u(c_N))$$

where $N$ is the number of households in the society and $W$ is an arbitrary function, that tells us, given the lifetime utilities of all households, $u(c_1),\ldots,u(c_N)$, how happy the society as a whole is. So far we have not made any progress, since we have not said anything about how the social welfare function $W$ looks like. Here are some commonly studied examples:

**Example 18** *Household $i$ is a "dictator"*

$$W(u(c_1),\ldots,u(c_N)) = u(c_i)$$

This means that only household $i$ counts when calculating how well-off a society is. Obviously, under such a social welfare function the best thing a society can do is to maximize household $i$'s lifetime utility. For the example above, if the dictator is household 1, then the progressive tax system is preferred by society to the proportional tax system, and if household 2 is the dictator, the proportional tax system beats the progressive system. Note that even though dictatorial social welfare functions seem somehow undesirable, there are plenty of examples in history in which such a social welfare function was implemented (you pick your favorite dictator).

Clearly the previous social welfare functions seem unfair or undesirable (although there is nothing logically wrong with them). Two other types of social welfare functions have enjoyed popularity among philosophers, sociologists and economists are:

**Example 19** *Utilitarian social welfare function*

$$W(u(c_1), \ldots, u(c_N)) = u(c_1) + \ldots + u(c_N)$$

*that is, all household's lifetime utilities are weighted equally.*

This social welfare function posits that everybody's utility should be counted equally. The intellectual basis for this function is found in John Stuart Mill's (1806-1873) important work "Utilitarism" (published in 1863). In the book he states as highest normative principle

> Actions are right in proportion as they tend to promote happiness; wrong as they tend to produce the reverse of happiness

He refers to this as the "Principle of Utility". Since everybody is equal according to his views, society should then adopt policies that maximize the sum of utility of all citizens. For our simple example the Utilitarian social welfare function would rank the progressive tax code and the proportional tax code as follows

$$\begin{aligned} W^{\text{prog}}(u(c_1), u(c_2)) &= \log(19500) + \log(86500) = 21.2461 \\ W^{\text{prop}}(u(c_1), u(c_2)) &= \log(17667) + \log(88333) = 21.1683 \end{aligned}$$

and thus the progressive tax code dominates a purely proportional tax code, according to the Utilitarian social welfare function.

**Example 20** *Rawlsian social welfare function*

$$W(u(c_1), \ldots, u(c_N)) = \min_i \{u(c_1), \ldots, u(c_N)\}$$

*that is, social welfare equals to the lifetime utility of that member of society that is worst off.*

The idea behind this function is John Rawls idea of the *veil of ignorance.* Suppose you don't know whether you are going to be born as a household that will have low or high income. Then, if, prenatally, you are risk-averse you would like to live in a society that makes you live a decent life even in the worst possible realization of your income prospects. That is exactly what the Rawlsian social welfare function posits. For our simple example it is easy to see that the progressive tax system is preferred to a proportional tax system since

$$\begin{aligned} W^{\text{prog}}(u(c_1), u(c_2)) &= \min\{\log(c_1), \log(c_2)\} = \log(c_1) = \log(19500) \\ W^{\text{prop}}(u(c_1), u(c_2)) &= \min\{\log(c_1), \log(c_2)\} = \log(c_1) = \log(17667) < W^{\text{prog}}(u(c_1), u(c_2)) \end{aligned}$$

In fact, under the assumption that taxable incomes are not affected by the tax code (i.e. people work and save the same amount regardless of the tax code - it may still differ across people, though-) then one can establish a very strong result.

**Theorem 21** *Suppose that $u$ is strictly concave and the same for every household. Then under both the Rawlsian and the Utilitarian social welfare function it is optimal to have complete income redistribution, that is*

$$c_1 = c_2 = \ldots = c_N = \frac{y_1 + y_2 + \ldots + y_N - G}{N} = \frac{Y - G}{N}$$

*where $G$ is the total required tax revenue and $Y = y_1 + y_2 + \ldots + y_N$ is total income (GDP) in the economy. The tax code that achieves this is given by*

$$T(y_i) = y_i - \frac{Y - G}{N}$$

*i.e. to tax income at a 100% and then rebate $\frac{Y-G}{N}$ back to everybody.*

We will omit the proof of this result here (and come back to it once we talk about social insurance). But the intuition is simple: suppose tax policy leaves different consumption to different households, for concreteness suppose that $N = 2$ and $c_2 > c_1$. Now consider taking way a little from household 2 and giving it to household 1 (but not too much, so that afterwards still household 2 has weakly more consumption than household 1). Obviously under the Rawlsian social welfare function this improves societal welfare since the poorest person has been made better off. Under the Utilitarian social welfare function, since the utility function of each agent is concave and the same for every household, the loss of agent 2, $u'(c_2)$ is smaller than the gain of agent 1, $u'(c_1)$, since by concavity $c_2 > c_1$ implies

$$u'(c_1) > u'(c_2).$$

Evidently the assumption that changes in the tax system do not change a households' incentive to work, save and thus generate income is a strong one. Just imagine what household would do under the optimal policy of complete income redistribution (or take your favorite ex-Communist country and read a history book of that country). Therefore we now want to analyze how income and consumption taxes change the economic incentives of households to work, consume and save.

### 4.4.2 Theoretical Analysis of Consumption Taxes, Labor Income Taxes and Capital Income Taxes

In order to meaningfully talk about the trade-offs between consumption taxes, labor income taxes and capital income taxes we need a model in which households decide on consumption, labor supply and saving. We therefore extend our simple model and allow households to choose how much to work. Let $l$ denote the total fraction of time devoted to work in the first period of a household's life; consequently $1-l$ is the fraction of total time in the first period devoted to leisure. Furthermore let by $w$ denote the real wage. We assume that in the second period of a person's life the household retires and doesn't work. Also, we will save our discussion of a social security system for the next chapter and abstract from it here. Finally we assume that households may receive social security benefits $b \geq 0$ in the second period of life. The household maximization problem becomes

$$\max_{c_1 c_2, s, l} \log(c_1) + \theta \log(1-l) + \beta \log(c_2) \tag{4.9}$$

$$s.t.$$

$$(1+\tau_{c_1})c_1 + s = (1-\tau_l)wl \tag{4.10}$$

$$(1+\tau_{c_2})c_2 = (1+r(1-\tau_s))s + b \tag{4.11}$$

where $\theta$ and $\beta$ are preference parameters, $\tau_{c_1}, \tau_{c_2}$ are proportional tax rates on consumption, $\tau_l$ is the tax rate on labor income, $r$ is the return on saving, and $\tau_s$ is the tax rate on that return. The parameter $\beta$ has the usual interpretation, and the parameter $\theta$ measures how much households value leisure, relative to consumption. Obviously there are a lot of different tax rates in this household's problem, but then there are a lot of different taxes actual U.S. households are subject to.

To solve this household problem we first consolidate the budget constraints into a single, intertemporal budget constraint. Solving equation (4.11) for $s$ yields

$$s = \frac{(1+\tau_{c_2})c_2 - b}{(1+r(1-\tau_s))}$$

and thus the intertemporal budget constraint (by substituting for $s$ in (4.10))

$$(1+\tau_{c_1})c_1 + \frac{(1+\tau_{c_2})c_2}{(1+r(1-\tau_s))} = (1-\tau_l)wl + \frac{b}{(1+r(1-\tau_s))}$$

In order to solve this problem, as always, we write down the Lagrangian, take first order conditions and set them to zero. Before doing so let us rewrite the budget constraint a little bit, in order to provide a better interpretation of it. Since $l = 1-(1-l)$ the budget constraint can be written as

$$\begin{aligned}(1+\tau_{c_1})c_1 + \frac{(1+\tau_{c_2})c_2}{(1+r(1-\tau_s))} &= (1-\tau_l)w*(1-(1-l)) + \frac{b}{(1+r(1-\tau_s))} \\ (1+\tau_{c_1})c_1 + \frac{(1+\tau_{c_2})c_2}{(1+r(1-\tau_s))} + (1-l)(1-\tau_l)w &= (1-\tau_l)w + \frac{b}{(1+r(1-\tau_s))}\end{aligned}$$

The interpretation is as follows: the household has potential income from social security $\frac{b}{(1+r(1-\tau_s))}$ and from supplying *all* her time to the labor market. With this she buys three goods: consumption $c_1$ in the first period, at an effective (including taxes) price $(1+\tau_{c_1})$, consumption $c_2$ in the second period, at an effective price $\frac{(1+\tau_{c_2})}{(1+r(1-\tau_s))}$ and leisure $1-l$ at an effective price $(1-\tau_l)w$, equal to the opportunity cost of not working, which is equal to the after-tax wage.

The Lagrangian reads as

$$\begin{aligned} L &= \log(c_1) + \theta\log(1-l) + \beta\log(c_2) \\ &\quad +\lambda\left((1-\tau_l)w + \frac{b}{(1+r(1-\tau_s))} - (1+\tau_{c_1})c_1 - \frac{(1+\tau_{c_2})c_2}{(1+r(1-\tau_s))} - (1-l)(1-\tau_l)w\right)\end{aligned}$$

and we have to take first order conditions with respect to the three choice variables $c_1, c_2$ and $l$ (or $1-l$, which would give exactly the same results).

These first order conditions, equated to 0, are

$$\begin{aligned}\frac{1}{c_1} - \lambda(1+\tau_{c_1}) &= 0 \\ \frac{\beta}{c_2} - \lambda\frac{(1+\tau_{c_2})}{(1+r(1-\tau_s))} &= 0 \\ \frac{-\theta}{1-l} + \lambda(1-\tau_l)w &= 0\end{aligned}$$

or

$$\frac{1}{c_1} = \lambda(1+\tau_{c_1}) \tag{4.12}$$

$$\frac{\beta}{c_2} = \lambda\frac{(1+\tau_{c_2})}{(1+r(1-\tau_s))} \tag{4.13}$$

$$\frac{\theta}{1-l} = \lambda(1-\tau_l)w \tag{4.14}$$

Now we can, as always, substitute out the Lagrange multiplier $\lambda$. Dividing equation (4.13) by equation (4.12) one obtains the standard intertemporal Euler equation, now including taxes:

$$\frac{\beta c_1}{c_2} = \frac{(1+\tau_{c_2})}{(1+\tau_{c_1})} * \frac{1}{(1+r(1-\tau_s))} \tag{4.15}$$

and dividing equation (4.14) by equation (4.12) yields the crucial intratemporal optimality condition of how to choose consumption, relative to leisure, in the first period:

$$\frac{\theta c_1}{1-l} = \frac{(1-\tau_l)w}{(1+\tau_{c_1})}. \tag{4.16}$$

These two equations, together with the intertemporal budget constraint, can be used to solve explicitly for the optimal consumption and labor (leisure) choices $c_1, c_2, l$ (and, of course, equation (4.10) can be used to determine the optimal savings choice $s$). Before doing this we want to interpret the optimality conditions (4.15) and (4.16) further. Equation ((4.15)) is familiar: if consumption taxes are uniform across periods (that is, $\tau_{c_1} = \tau_{c_2}$) then it says that the marginal rate of substitution between consumption in the second and consumption in the first period

$$\frac{\beta u'(c_2)}{u'(c_1)} = \frac{\beta c_1}{c_2}$$

should equals to the relative price between consumption in the second to consumption in the first period, $\frac{1}{(1+r(1-\tau_s))}$, the inverse of the gross after tax interest rate. With differential consumption taxes, the relative price has to be adjusted by relative taxes $\frac{(1+\tau_{c_2})}{(1+\tau_{c_1})}$. The intertemporal optimality condition has the following intuitive comparative statics properties

**Proposition 22** *1. An increase in the capital income tax rate $\tau_s$ reduces the after-tax interest rate $1+r(1-\tau_s)$ and induces households to consume more in the first period, relative to the second period (that is, the ratio $\frac{c_1}{c_2}$ increases).*

*2. An increase in consumption taxes in the first period $\tau_{c_1}$ induces households to consume less in the first period, relative to consumption in the second period (that is, the ratio $\frac{c_1}{c_2}$ decreases).*

*3. An increase in consumption taxes in the second period $\tau_{c_2}$ induces households to consume more in the first period, relative to consumption in the second period (that is, the ratio $\frac{c_1}{c_2}$ increases).*

**Proof.** Obvious, simply look at the intertemporal optimality condition.
■

The intra-temporal optimality condition is new, but equally intuitive. It says that the marginal rate of substitution between current period leisure and current period consumption,

$$\frac{\theta u'(1-l)}{u'(c_1)} = \frac{\theta c_1}{1-l}$$

should equal to the after-tax wage, adjusted by first period consumption taxes (that is, the relative price between the two goods) $\frac{(1-\tau_l)w}{(1+\tau_{c_1})}$. Again we obtain the following comparative statics results

**Proposition 23** *1. An increase in labor income taxes $\tau_l$ reduces the after-tax wage and reduces consumption, relative to leisure, that is $\frac{c_1}{1-l}$ falls. This substitution effect suggests (we still have to worry about the income effect) that an increase in $\tau_l$ reduces both current period consumption and current period*

2. *An increase in consumption taxes* $\tau_{c_1}$ *reduces consumption, relative to leisure, that is* $\frac{c_1}{1-l}$ *falls. Again, this substitution effect suggests that an increase in* $\tau_{c_1}$ *reduces both current period consumption and current period labor supply.*

**Proof.** Obvious, again simply look at the intratemporal optimality condition. ■

According to Edward Prescott, this years' Nobel price winner in economics (and, incidentally, my advisor) this proposition is the key to understanding recent cross-country differences in the amount of hours worked per person.[17] Before developing his arguments and the empirical facts that support them in more detail we state and prove the important, useful and surprising result that uniform proportional consumption taxes are equivalent to a proportional labor income tax.

**Proposition 24** *Suppose we start with a tax system with no labor income taxes,* $\tau_l = 0$ *and uniform consumption taxes* $\tau_{c_1} = \tau_{c_2} = \tau_c$ *(the level of capital income taxes is irrelevant for this result). Denote by* $c_1, c_2, l, s$ *the optimal consumption, savings and labor supply decision. Then there exists a labor income tax* $\tau_l$ *and a lump sum tax* $T$ *such that for* $\tau_c = 0$ *households find it optimal to make exactly the same consumption choices as before.*

***Proof.*** *Under the assumption that the consumption tax is uniform, it drops out of the intertemporal optimality condition* (4.15) *and only enters the optimality condition* (4.15). *Rewrite that optimality condition as*

$$\frac{\theta c_1}{(1-l)w} = \frac{(1-\tau_l)}{(1+\tau_c)}$$

*The right hand side, for* $\tau_l = 0$, *is equal to*

$$\frac{1}{(1+\tau_c)}$$

*But if we set* $\hat{\tau}_l = \frac{\tau_c}{1+\tau_c}$ *and* $\hat{\tau}_c = 0$, *then*

$$\frac{(1-\hat{\tau}_l)}{(1+\hat{\tau}_c)} = 1 - \frac{\tau_c}{1+\tau_c} = \frac{1}{(1+\tau_c)},$$

[17] See Edward C. Prescott (2004), "Why do Americans Work so much more than Europeans?," *NBER Working Paper 10316.*

*that is, the household faces the same intratemporal optimality condition as before. This, together with the unchanged intertemporal optimality condition, leads to the same consumption, savings and labor supply choices, if the budget constraint remains the same. But this is easy to guarantee with the lump-sum tax $T$, which is set exactly to the difference of tax receipts under consumption and under labor taxes.* ■

Before making good use of this proposition in explaining cross-country differences in hours worked we now want to give the explicit solution of the household decision problem. From the intratemporal optimality condition we obtain

$$c_1 = \frac{(1-\tau_l)(1-l)w}{(1+\tau_{c_1})\theta} \tag{4.17}$$

intertemporal optimality condition we obtain

$$\begin{aligned} c_2 &= \beta c_1(1+r(1-\tau_s))\frac{(1+\tau_{c_1})}{(1+\tau_{c_2})} \\ &= \frac{(1-\tau_l)(1-l)w}{(1+\tau_{c_1})\theta}\beta(1+r(1-\tau_s))\frac{(1+\tau_{c_1})}{(1+\tau_{c_2})} \\ &= \frac{(1-\tau_l)(1-l)w}{\theta}\frac{\beta(1+r(1-\tau_s))}{(1+\tau_{c_2})} \end{aligned} \tag{4.18}$$

Plugging all this mess into the budget constraint

$$\begin{aligned} \frac{(1-\tau_l)(1-l)w}{\theta} + \beta\frac{(1-\tau_l)(1-l)w}{\theta} &= (1-\tau_l)wl + \frac{b}{(1+r(1-\tau_s))} \\ (1+\beta)\frac{(1-\tau_l)(1-l)w}{\theta} &= (1-\tau_l)wl + \frac{b}{(1+r(1-\tau_s))} \end{aligned}$$

which is one equation in the unknown $l$. Sparing you the details of the algebra, the optimal solution for labor supply $l$ is

$$l^* = \frac{1+\beta}{1+\beta+\theta} - \frac{b}{(1+r(1-\tau_s))\theta w(1-\tau_l)(1+\beta+\theta)} \tag{4.19}$$

In particular, if there are no social security benefits (i.e. $b=0$), then the optimal labor supply is given by

$$l^* = \frac{1+\beta}{1+\beta+\theta} \in (0,1)$$

The more the household values leisure (that is, the higher is $\theta$), the less she finds it optimal to work. In this case labor supply is independent of the after-tax wage (and thus the labor tax rate), since with log-utility income and substitution effect cancel each other out. With $b > 0$, note that higher social security benefits in retirement reduce labor supply in the working period (partly we have implicitly assumed that current labor income does not determine future retirement benefits). Finally note that if $b$ gets really big, then the optimal $l^* = 0$ (the solution in (4.19) does not apply anymore).

Obviously one can now compute optimal consumption and savings choices. Here we only give the solution for $b = 0$; it is not particularly hard, but algebraically messy to give the solution for $b > 0$. From (4.17) we have

$$\begin{aligned} c_1 &= \frac{(1-\tau_l)(1-l^*)w}{(1+\tau_{c_1})\theta} \\ &= \frac{(1-\tau_l)}{(1+\tau_{c_1})(1+\beta+\theta)} w \end{aligned}$$

and from (4.18) we have

$$c_2 = \frac{\beta(1-\tau_l)(1+r(1-\tau_s))}{(1+\beta+\theta)(1+\tau_{c_2})} w$$

and finally from the first budget constraint (4.10) we find

$$\begin{aligned} s &= (1-\tau_l)wl - (1+\tau_{c_1})c_1 \\ &= \frac{(1+\beta)(1-\tau_l)w}{1+\beta+\theta} - \frac{(1-\tau_l)w}{1+\beta+\theta} \\ &= \frac{\beta(1-\tau_l)w}{1+\beta+\theta} \end{aligned}$$

### An Application: International Differences in Labor Income Taxation and Hours Worked

In this section we apply the general theoretical results derived in the previous subsection to an important macroeconomic questions. In the very influential and quite controversial paper Edward Prescott first documented that Europeans work significantly fewer hours than Americans, and argued that higher taxes on market work and higher taxes on market consumption goods could explain this finding. In this section we use our general theory to revisit this argument.

The last proposition in the previous section shows that what really matters for household consumption and labor supply decisions is the tax wedge $\frac{(1-\tau_l)}{(1+\tau_c)}$ in the intratemporal optimality condition

$$\frac{\theta c}{1-l} = \frac{(1-\tau_l)}{(1+\tau_c)} w \tag{4.20}$$

where we have dropped the period subscript on consumption. Clearly both labor and consumption taxes are crucial determinants of labor supply. In order to make this equation useful for data work we need to specify where wages come from. Recall than in section 2.5 and we derived wages from the optimal demand of firms operating the neoclassical production function:

$$Y = K^\alpha L^{1-\alpha}.$$

The firm takes wages and rental rates of capital $(w, \rho)$ as given and chooses how much total labor and capital $(L, K)$ to hire to maximize profits

$$\max_{(K,L)} K^\alpha L^{1-\alpha} - wL - \rho K$$

where $K$ is the capital stock used by the firm, $\rho$ is the rental rate of capital (equal to the gross interest rate $1 + r$), and $L$ is the amount of labor hired at wages $w$. The parameter $\alpha$ is telling us how important capital is, relative to labor, in the production of output. It also turns out to be equal to the capital share (the fraction of income accruing to capital income).

Taking the first order condition with respect to $L$ and setting it equal to 0 yields

$$\begin{aligned} (1-\alpha)K^\alpha L^{-\alpha} &= w \\ \frac{(1-\alpha)K^\alpha L^{1-\alpha}}{L} &= w \\ (1-\alpha)\frac{Y}{L} &= w \\ (1-\alpha)Y &= wL \end{aligned} \tag{4.21}$$

The last expression demonstrates that the labor share $\frac{\text{Labor Income}}{\text{GDP}} = \frac{wl}{Y}$ equals $1-\alpha$, so that the capital share equals $\alpha$. Now we use equation (4.21) to substitute out the wage $w$ in equation (4.20) to obtain

$$\frac{\theta c}{1-l} = \frac{(1-\tau_l)}{(1+\tau_c)}(1-\alpha)\frac{Y}{L} \tag{4.22}$$

But in equilibrium the labor demand of firms $L$ has to equal to labor supply $l$ of households, and thus

$$L = l.$$

Exploiting this equality in (4.22) and solving this equation for labor supply $l$ yields, after some tedious algebra

$$l = \frac{1-\alpha}{1-\alpha+\frac{\theta(1+\tau_c)}{(1-\tau_l)}\frac{c}{Y}} \in (0,1) \tag{4.23}$$

Letting $i$ denote the name of a country, the total amount of hours worked (as a fraction of total time available in a year $t$) is thus given by

$$l_{it} = \frac{1-\alpha}{1-\alpha+\frac{\theta(1+\tau_{cit})}{(1-\tau_{lit})}\frac{c_{it}}{Y_{it}}} \tag{4.24}$$

Equation (4.24) is the starting point of our empirical analysis of differences in labor supply across countries. You may think that I just rewrote equation (4.23) and indexed it by country, but this is not quite true. Equation (4.24) makes very precise what we allow to vary across countries and what not. We take the view that production technologies and utility functions are the same across countries and time (thus $\theta$ and $\alpha$ are not indexed by $i$ or $t$) and then want to ask to what extent differences in taxes *alone* can account for differences in hours worked. Obviously we do not expect an answer such as 100%, since countries differ by more than just tax rates, but we are curious how important differences in taxes are. The name of the game now is to choose parameter values $\alpha, \theta$, measure tax rates $\tau_{cit}, \tau_{lit}$ and hours worked $l_{it}$ and consumption-output ratios $\frac{c_{it}}{Y_{it}}$ from the data for different countries $i$ and see to what extent the $l_{it}$ predicted by the model coincide with those from the data.

First let us look at the data. Table 4.8 presents data for GDP per person (between 15 and 64), total hours worked per person and labor productivity (GDP per hours worked) for the major industrialized countries (the so-called G7 countries) in the mid-90's (before the boom and bust of the IT bubble). All data normalize the U.S. to 100 for comparison.

The first column shows GDP per person of working age. We observe that GDP per capita is by $25-40\%$ lower in Europe than in the U.S. The third column, labor productivity, shows that this large difference is not mainly due to differences in productivity (in fact, productivity is higher in France

| Country | GDP p.p. | Hours | GDP p.h. |
|---|---|---|---|
| Germany | 74 | 75 | 99 |
| France | 74 | 68 | 110 |
| Italy | 57 | 64 | 90 |
| Canada | 79 | 88 | 89 |
| United Kingdom | 67 | 88 | 76 |
| Japan | 78 | 104 | 74 |
| United States | 100 | 100 | 100 |

Table 4.8: Labor Supply, Productivity and GDP, 1993-96

| Country | GDP p.p. | Hours | GDP p.h. |
|---|---|---|---|
| Germany | 75 | 105 | 72 |
| France | 77 | 105 | 74 |
| Italy | 53 | 82 | 65 |
| Canada | 86 | 94 | 91 |
| United Kingdom | 68 | 110 | 62 |
| Japan | 62 | 127 | 49 |
| United States | 100 | 100 | 100 |

Table 4.9: Labor Supply, Productivity and GDP, 1970-74

than in the U.S. and similar between the U.S. and Germany and Italy. The main differences in GDP per capita stem from vastly lower hours worked in these countries, compared to the U.S. The differences here are staggering; they mean that if in the U.S. everybody is working for 8 hours on average, in Germany it is 6 hours, in France $5\frac{1}{2}$ hours and in Italy a little more than 5 hours. In reality most of the differences come from the fact that Europeans work less days per year (i.e. have more vacation) and that fewer working age persons are working. In particular, the labor force participation rates of woman in this European countries is much smaller than in the U.S.

Maybe Europeans simple have a bigger taste for leisure, and Americans a bigger taste for consumption. But then we would expect these numbers to be constant over time (unless somehow magically preferences have changed over time in these countries). Table 4.9 shows that this is not the case. Here we summarize the same data as in the previous table, but now for the early 70's.

The difference across time is striking. GDP per capita, relative to the

U.S. in Germany, France and Italy is roughly at the same level as in the mid-90's, lagging the U.S. by $25 - 40\%$. But in the early 70's this was *not* due to fewer hours worked, but rather due to lower productivity. In fact, in the early 70's Germans and French worked more than Americans, and Italians only a little less. So the last 30 years saw a substantial catch-up in productivity in Europe, relative to the U.S., and a shocking decline of relative hours worked in Europe, relative to the U.S. The question is: why?

Equation (4.24) gives a potential answer: big changes in labor income and consumption tax rates. In order to see whether this explanation holds water, in a quantitative sense, one needs to measure $\frac{c_{it}}{Y_{it}}, \tau_{cit}, \tau_{lit}$ for the different countries and the different time periods. That's what the paper by Prescott does. Without going into the specifics, here are the main principles:

- The ratio of consumption and GDP, $\frac{c_{it}}{Y_{it}}$ is easily determined from NIPA accounts. Some assumption has to be made for government spending, since it partially provides consumption services, and thus should be counted as part of $c$, according to the model. Prescott assumes that all but military government spending is yielding private consumption. Another issue is how to deal with indirect consumption taxes. In NIPA they are part of consumption expenditures, but in the model clearly not part of $c$. Prescott adjusts the data accordingly.

- The consumption tax $\tau_{cit}$ is set to the ratio between total indirect consumption taxes and total consumption expenditures in the data. Since sales taxes tend to be proportional to the sales price of goods, this is probably a good approximation

- With respect to labor income taxes things are a bit more problematic because of the progressive nature of the tax code. Labor income taxes are composed of two parts, the proportional payroll tax for social security and then the general income tax. Thus Prescott takes

  $$\tau_l = \tau_{ss} + \tau_{inc}$$

  For $\tau_{ss}$ he basically takes the payroll tax rates (currently 15.3%, shared equally by employers and employees). In order to compute an appropriate marginal income tax rate $\tau_{inc}$ he first computes average income taxes by dividing total direct taxes paid in the data by total national income. Then he multiplies the resulting average tax rate by 1.6, in order to capture the fact that with a progressive tax code marginal taxes are higher than average taxes (and empirical studies of taxes paid by individuals find that, when comparing average and marginal tax rates, the factor of 1.6 seems the best approximation).

Finally we need to specify two parameter values, $\theta$ and $\alpha$. Since $\alpha$ equals the capital share, Prescott takes it to equal $\alpha = 0.3224$, the average across countries and time in the period under consideration. We saw above that the parameter $\theta$ determines the fraction of time worked. Prescott chooses $\theta$ in such a way that in the model the number of hours spent working equals the average hours (across countries) in the data, which requires 1.54. Note that he does not, in this way, rig the results in his favor, since he wants to explain cross-country differences in hours worked, and not the average level of hours worked.

Let us look at the result of this exercise. Table 4.10 summarizes them for the 1993-96 period. Note that

$$\frac{(1-\tau_l)}{(1+\tau_c)} = 1-\tau$$

where $\tau = \frac{\tau_l+\tau_c}{1+\tau_c}$ is the combined labor income and consumption tax rate relevant for the labor supply decision. The tax rate $\tau$ gives the fraction of each dollar earned that can not be consumed, but needs to be paid in taxes, either as direct labor income taxes or consumption taxes. Another way of saying this, a person wanting to spend one dollar on consumption needs to earn $x$ dollars as labor income, where $x$ solves

$$\begin{aligned} x(1-\tau) &= 1 \text{ or} \\ x &= \frac{1}{1-\tau} \end{aligned}$$

But now for the numbers.

We observe that measured effective tax rates differ substantially by countries. Whereas in the U.S. for one dollar of consumption $\frac{1}{0.6} = 1.667$ dollars of income need to be earned, the corresponding number in Germany and France is 2.44 Euro per Euro of consumption, and for Italy that number rises to a whopping 2.78 Euro. Without major differences in the consumption-output ratio these differences translate into substantial differences in hours worked, of about 5 hours per week between the U.S. and Germany/France. In the data, that difference is 6.4 hours for the U.S. versus Germany and 8.4 hours for the U.S. versus France. Similar numbers are obtained for Italy.

| Country | Tax Rate $\tau$ | $\frac{c}{Y}$ | Hours per Person per Week | |
|---|---|---|---|---|
| | | | Actual | Predicted |
| Germany | 0.59 | 0.74 | 19.3 | 19.5 |
| France | 0.59 | 0.74 | 17.5 | 19.5 |
| Italy | 0.64 | 0.69 | 16.5 | 18.8 |
| Canada | 0.52 | 0.77 | 22.9 | 21.3 |
| United Kingdom | 0.44 | 0.83 | 22.8 | 22.8 |
| Japan | 0.37 | 0.68 | 27.0 | 29.0 |
| United States | 0.40 | 0.81 | 25.9 | 24.6 |

Table 4.10: Actual and Predicted Labor Supply, 1993-96

| Country | Tax Rate $\tau$ | $\frac{c}{y}$ | Hours per Person per Week | |
|---|---|---|---|---|
| | | | Actual | Predicted |
| Germany | 0.52 | 0.66 | 24.6 | 24.6 |
| France | 0.49 | 0.66 | 24.4 | 25.4 |
| Italy | 0.41 | 0.66 | 19.2 | 28.3 |
| Canada | 0.44 | 0.72 | 22.2 | 25.6 |
| United Kingdom | 0.45 | 0.77 | 25.9 | 24.0 |
| Japan | 0.25 | 0.60 | 29.8 | 35.8 |
| United States | 0.40 | 0.74 | 23.5 | 26.4 |

Table 4.11: Actual and Predicted Labor Supply, 1970-74

Overall, the model does very well in explaining the cross-country differences in hours worked, with the average difference between actual and predicted weekly hours worked amounting to 1.14 hours. Furthermore a large part of the difference in hours worked between the U.S. and Europe (but not all of it) is explained by tax differences, the only element of the model that we allow to vary across countries.

The ultimate test for the model is whether it can also explain the fact that in the early 70's Europeans did not work less than Americans. Obviously, for the model to get this observation right it needs to be the case that in that time period taxes very not that different between the U.S. and Europe. Table 4.11 summarizes the results for the early 1970's.

We observe that the model is not quite as successful matching all countries, but it does predict that in the early 70's Germans and French did not work so much less than Americans, precisely because tax rates on labor were lower then than in the 90's in these countries. Quantitatively, the two big failures of the model are Japan and Italy, where actual hours worked severely lag behind those predicted by the model. What explains this? Something else but taxes must have depressed labor supply in these countries in this time period. But rather than speculating about this other sources, let us summarize our analysis by noting that differences in tax rates and their change over time can explain a large part of the fact that in the last 30 years Europeans started working significantly less, compared to their American brethren.

# Chapter 5

# Unfunded Social Security Systems

All industrialized countries and many developing nations run a largely unfunded social security system. In such a system workers' wages and salaries are taxed, and the tax revenues are used immediately to pay pension benefits to currently retired workers. A system where tax receipts are not saved at all, but immediately paid out to retirees is called a pay-as-you-go or unfunded system. Before analyzing such a system theoretically I first want to give you a short account of the history of the German system, because it was the first of its kind, and then a more detailed account of the history and the current structure of the US system.

## 5.1 History of the German Social Security System

The current public social security system was introduced in Germany by then Reichskanzler Otto von Bismark (and Kaiser Wilhelm I, of course) in 1889, in conjunction with other social insurance programs. While a social security system may be justified on normative grounds, as we will see below, as important at the time were political economy considerations. The social democrats and the labor movement in general gained popularity with their call for social reform. In order to prevent the further rise of the Social Democrats Bismark followed two strategies: he restricted access of the Social Democrats to political representation (let alone office), but, second, adopted part of their social agenda to curb their popularity and revolutionary potential.

At the time of the introduction social security benefits started at the age of seventy (which was beyond the life expectancy at the time). Most of old-age consumption was still provided by the older people themselves, as most people worked until they died, or by their families. Social security benefits were financed by a lump sum tax (that is, by contributions that were independent of income). The average benefits were about 120 Marks per year, and the system initially only applied to workers (not to Angestellte, farmers etc.). To get a sense of how big these benefits are, note that in 1889 Germany (the Deutsche Reich) had a population of about 48,7 million people and a nominal NNP of about 22,2 Mrd Mark. Thus nominal NNP per capita per year amounted to 500 Marks per year. For its inception the German system was basically a pay-as-you-go system, with some capital accumulation within the system in the early years of the system, devised to have a buffer for demographic shocks.

After its introduction, the social security system in Germany was augmented and reformed many times. The list below contains only the most significant reforms[1]

- In 1891 the Invalidenrente was introduced, providing people permanently unable to work with a basic public pension (a maximum of 150 Marks per year).

- In 1911 the Hinterbliebenenrente was introduced, granting public pensions to families of dead workers, in the event the other family members (that is, commonly the wife) were unable to work.

- In 1916 the retirement age was reduced to 65 years, effectively doubling the number of recipients of public pension benefits.

- Somewhat surprisingly, the system remained fairly unchanged through both world wars and the Nazi regime.

- In 1957 benefits were linked to wages. Instead of a fixed contribution social security taxes were now proportional to labor income. The tax rate was fixed at 14%. This meant that higher wages lead to higher

[1] The discussion here is a summary of the information provided here: http://www.ihr-rentenplan.de/html/geschichte_rente_1.html

contributions and thus higher benefits, in a pay-as-you-go system. Effectively, from this date no substantial capital was accumulated within the system.

- In 1968 the system was also formally declared a pure pay-as-you-go system, legally sanctioning the already existing practice. Since this time the social security system went through periodic financing crises that were dealt with the small reforms and adjustment (mostly increases of the tax rate, which now stands at approximately 20%). In the 1990's the situation and especially the future outlook deteriorated, due to demographic changes. Life expectancy increased and fertility rates decreased, leading to a higher (predicted) dependency ratio (the ratio of people above 65 to the population aged 16-65) and thus to the imminent need for reform. This reform could take several forms
  - Increase social security tax rates
  - Reduce benefits (e.g. increase the retirement age)
  - Limit the scope of the program by reducing benefits and giving incentives to complement public pensions by private retirement accounts (the Riester Rente).

Now let us turn to the history of social security in the United states.

## 5.2 History of the US Social Security System

The current national social security system in the U.S. was introduced rather late (in international comparison) through the Social Security Act in 1935.[2] While several U.S. states introduced some public pension systems for needy elders, it was not until 1935 that a national old-age social insurance system was put into place. Schieber and Shoven point to three major forces responsible for the introduction of social security at that time.

1. Changing Economic and Social Structures: like in other industrialized countries the U.S. economy had undergone a dramatic transition for an agrarian to an industrialized economy. The share of employment

[2] This section draws heavily on the book by S. Schieber and J. Shoven (1999), *The Real Deal: The History and Future of Social Security.*

in agriculture dropped from more than 50% in 1880 to less than 20% in 1935. Why was a life on the farm less likely to leave the elders impoverished? First, farms were largely family businesses where tasks could be allocated among family members according to their physical abilities; older people still could contribute by doing physically less demanding work. In addition, older people tended to be the owners of the farms, thus guaranteeing them a stream of income even after retirement from agricultural work. More than often ownership of the farm was transferred to children in exchange for the (promise of) provision of consumption until death. Second, employment opportunities in agriculture were less volatile than in the rest of the economy, where unemployment rates were higher and varied substantially in a matter of a few years.

2. The great depression in 1929-1932, the most severe recession in U.S. economic history, severely diminished unemployment opportunities of the elderly (but not only those) and destroyed most of the wealth that they had accumulated for retirement. On September 1, 1929, the value of all stocks listed at the New York Stock Exchange amounted to \$89.7 billion; in the middle of 1932 that value had fallen to \$15.6 billion, a decline of over 80%. In 1930 and 1931 alone over $3,000$ banks suspended operations or permanently closed, with total deposits being lost amount to more than \$2 billion. With the decline of prices for agricultural products (between 1924 and 1931 the price of wheat dropped by 66%, that of cotton by 75%) incomes and asset values in the agricultural sector declined severely and mortgage values soared. As a consequence, the great depression left an entire generation impoverished.

3. Franklin D. Roosevelt had campaigned with his proposal for a "New Deal" economic policy. According to Francis Perkins, Roosevelt's labor secretary the new deal was "a general concept meant to be psychologically soothing to people who were victims of the crashing markets. It was an idea that all the political and practical forces of the community should and could be directed to making life better for ordinary people. Out of this general idea several public programs, one of which was social security, arose, to deal with the specific problems of different segments of the population. In the case of social security, this group was evidently the impoverished elders.

The social security act of 1935 originally intended to use the 2% payroll tax (1% on employers and employees each) for the accumulation of financial assets for retirement. Roosevelt explained why a special tax was introduced to finance old age pension benefits (rather than general tax revenues):

> These taxes were never a problem of economics. They are politics all the way through. We put those payroll contributions there so as to give the contributors a legal, moral, and political right to collect their pensions. With these taxes in there, no damn politician can ever scrap my social security program. [Franklin D. Roosevelt]

By 1939 it had become clear, however, that the problem of widespread poverty of the old could not be tackled appropriately (since the old are already old, made few contributions and hence qualified for minuscule pension benefits), and the system was effectively changed to its current pay-as-you go character. While taxes and benefits have changed dramatically over time, the basic principles of the system remain unchanged from 1939.

## 5.3 The Current US System

The current social security system is basically as pay-as-you-go system. This means that taxes paid by current workers are immediately used for paying benefits of current retirees. A fully funded system, on the contrary, would save taxes of current workers, invest them in some assets (bonds, stocks, real estate) and uses the returns, including principal, to pay benefits when these current workers are old. While it is true that the U.S. social security system has accumulated some assets (the so-called trust fund), this trust fund has been accumulated with the expressed purpose of handling the retirement of the massive baby boom generation without having to increase payroll taxes.

The current system is defined by three things: a payroll tax rate $\tau$, a maximum amount of earnings $\bar{y}$ for which this payroll tax applies and a benefit formula that calculates social security benefits as a function of the labor earnings over your lifetime. On the tax side, both employers and employees currently pay a proportional tax on labor income of $\tau = 6.2\%$, for a total of 12.4% of wages and salaries. This tax rate applies to all income below a threshold (for 2007) of $\$97,500$. For wages and salaries above this threshold,

| Year | Max. Taxable Ear. | Tax Rate[3] |
|---|---|---|
| 1937 | $3,000 | 2.00% |
| 1950 | $3,000 | 3.00% |
| 1960 | $4,800 | 6.00% |
| 1970 | $7,800 | 8.40% |
| 1980 | $29,700 | 10.16% |
| 1990 | $51,300 | 12.40% |
| 1998 | $68,400 | 12.40% |
| 2007 | $97,500 | 12.40% |
| 2012 | | |
| 2014 | | |

Table 5.1: Social Security Tax Rates

no additional taxes have to be paid, so that the total maximum amount an employee has to pay in 2007 is

$$0.062 * 97,500 = \$6,045$$

Table 5.1 summarizes the tax rates and income thresholds for which these tax rates apply, for different years in history. Note that these tax rates do not include medicare taxes. What we observe from the tax rates in the table is a rapid expansion of the scope of the system in the 1950́'s through the 1970's, when both tax rates and promised benefits increased sharply.

Now that we understand how much in social security taxes households have to pay we want to discuss how social security benefits are calculated, under current law. Note that this law can be changed at any time, as can be the social security tax rates. Let us consider a person that just turned 65 and retires in 2007, and let us compute her social security benefits. This is done in two steps. In the first step one computes her average indexed monthly earnings (AIME). This is basically the average monthly salary that the person made during her working life, where salaries early in her life are adjusted by inflation and average wage growth in the economy. After we computed AIME, we apply a benefit formula $f$ to compute his benefits:

$$b = f(AIME)$$

We now discuss both steps in detail.

1. Computation of average indexed monthly earnings. Suppose that our person worked for 45 years, from age 20 to age 64. Thus she started working in the year 1963; let his income in year $t$ be denoted by $y_t$, for $t = 1963, 1964, \ldots 2007$. Furthermore denote maximal taxable earnings in year $t$ by $\bar{y}_t$; for selected years table 5.1 gives the values of these maximal taxable earnings. Now we proceed in four steps

   (a) Define qualified earnings as

   $$\hat{y}_t = \min\{y_t, \bar{y}_t\}$$

   for each year $t$. That is, if in a given year the person's salary has not exceeded the maximum taxable threshold, then $\hat{y}_t = y_t$ and qualified earnings equal actual earnings. If they do exceed the threshold, then $\hat{y}_t = \bar{y}_t$ and qualified earnings equal maximal taxable earnings for that year.

   (b) Now we have qualified earnings for each year 1963 to 2007, $\hat{y}_t$. Now we have to account for the fact that in real terms a salary was worth more in 1963 then it is today because there was inflation between 1963 and 2007. In order to express 1963 salaries in 2007 prices we use the Consumer Price Index (CPI). Let $P_{1963}$ denote the CPI in 1963 and $P_{2007}$ the CPI in 2007. Then $\frac{P_{2007}}{P_{1963}}$ is the relative price of a typical basket of consumption goods in 2007, relative to 1963. This relative price will be bigger than 1 because there was inflation between the two years. Thus to express our person's 1963 salary in terms of 2007, we take

   $$\tilde{y}_{1963} = \hat{y}_{1963} * \frac{P_{2007}}{P_{1963}}$$

   and in general

   $$\tilde{y}_t = \hat{y}_t * \frac{P_{2007}}{P_t}$$

   (c) Finally we adjust wages and salaries by average wage growth. Even real wages (that is, nominal wages, divided by the price level) tend to grow over time because of technological progress. On average this growth rate was about 1.7% per annum for the

U.S. Define as the gross growth rate of average wages between 1963 and 2007

$$G_{1963,2007} = \frac{\bar{w}_{2007}}{\bar{w}_{1963}}$$

and in general

$$G_{t,2007} = \frac{\bar{w}_{2007}}{\bar{w}_t}$$

where $\bar{w}_t$ is the average wage (per hour) at time $t$. As indicated before, in general we expect $G_{t,2007} > 1$. Since real wages grow over time, and thus the tax base for social security payroll taxes, the current system lets retired households benefit from this growth in real wages. In addition to inflation earnings in early years of our person's life are therefore adjusted in the following fashion

$$Y_t = \tilde{y}_t * G_{t,2007}$$

where $Y_t$ is called the indexed earnings from year $t$.

(d) After this ordeal we arrive at 45 numbers, $\{Y_{1963}, Y_{1961}, \ldots, Y_{2007}\}$. We compute average indexed monthly earnings by selecting the 35 highest entries from the list $\{Y_{1963}, Y_{1961}, \ldots, Y_{2007}\}$, summing them up and dividing by 35 (that is, taking the average of the 35 best earnings years. This yields the person's $AIME$.

2. The second step of computing social security benefits is considerably easier. We simply insert $AIME$ into the following benefit formula[4]

$$b = \begin{cases} 0.9AIME & \text{if } AIME \leq \$680 \\ 612 + 0.32(AIME - 680) & \text{if } \$680 < AIME \leq \$4,100 \\ 1706.4 + 0.15(AIME - 4,100) & \text{if } \$4,100 < AIME \end{cases} \tag{5.1}$$

This looks messy, but simply states that for each of the first 680 dollars 90 cents of benefits are earned. For each additional dollar earned between \$606 and \$4,100 an additional 32 cents in benefits are obtained, and for each dollar above \$4,100 another 15 cents are added to benefits. Equation (6.3) gives our person's benefits in 2007. For that point on every year his so-computed benefits are simply indexed by inflation,

[4] This is the benefit formula for 2006. The formula for 2007 will slightly differ from the one given in the text.

that is, if the inflation rate between 2007 and 2008 is 3%, then his benefits increase by 3% between 2007 and 2007. Benefits are paid until our person dies.

From the previous discussion we see that social security benefits are perfectly determined by average indexed monthly earnings, that is, by the best 35 working years. Since benefits depend positively on $AIME$, rational forward-looking household understand that working more today will increase social security benefits, although the link becomes weaker the higher is income. Define the replacement rate as

$$rr(AIME) = \frac{b(AIME)}{AIME}$$

that is, as the ratio between social security benefits and $AIME$. Obviously the replacement rate depends on $AIME$. Figure 5.1 plots the replacement rate, as a function of $AIME$. It is first constant at 90% and then strictly declining. This means that the higher your average indexed monthly earnings are, the lower is the fraction of these earnings that you receive as benefits. Remember that payroll taxes are proportional to earnings. Thus the social security system contains a redistributive component: households with low earnings receive more in benefits than they contributed in taxes, whereas for high income earners the situation is reversed.

One can also interpret this redistribution as insurance: if you don't know whether you are born as a person with high abilities and thus high income or a person with low earnings abilities, then ex ante (pre-birth) you like a system that redistributes between low-and high income earners, if you are risk-averse. So what is redistribution ex post is insurance ex ante. But also note that the extent of redistribution is limited since there is a cap on the social security taxes, as discussed before. In addition capital income, by construction, is not subject to social security taxes, so no redistribution between workers and capitalists takes place.

After this discussion of the actual system we will now use our theoretical model to analyze the positive and normative effects of a pay-as-you go social security system. We will first show that such a system decreases private savings rates, and then discuss under what condition the introduction of a social security system is, in fact, a good idea.

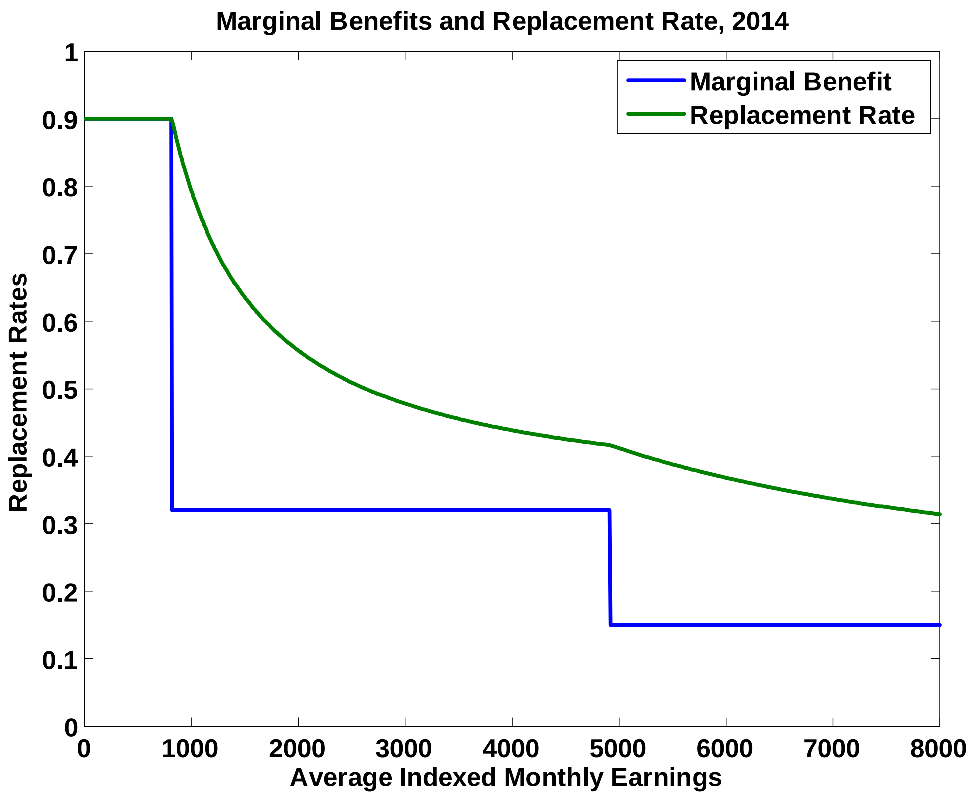


Figure 5.1: Social Security Replacement Rate and Marginal Benefits

## 5.4 Theoretical Analysis

### 5.4.1 Pay-As-You-Go Social Security and Savings Rates

Now we use the model to analyze a policy issue that has drawn large attention in the public debate. The personal saving rate -the fraction of disposable income that private households save- has declined from about 7-10% in the 60's and 70's to close to 0 right now. Since saving provides the funds for investment a lower saving rate, so a lot of people argue, harms growth be reducing investment.[5] Some economists argue that the expansion of the social security system has led to a decline in personal saving. We want to analyze this claim using our simple model. We look at a pay-as-you go social security

[5] This argument obviously ignores the increased inflow of foreign funds into the US.

system, in which the currently working generation pays payroll taxes, whose proceeds are used to pay the pensions of the currently retired generation. The key is that current taxes are paid out immediately, and not invested. We make the following simplifications to our model. We interpret the second period of a person's life as his retirement, so in the absence of social security he has no income apart from his savings, i.e. $y_2 = 0$. Let $y$ denote the income in the first period.

The household maximizes

$$\begin{aligned}
& \max_{c_1,c_2,s} \log(c_1) + \beta \log(c_2) \qquad (5.2) \\
& \text{s.t.} \\
c_1 + s &= (1-\tau)y \\
c_2 &= (1+r)s + b
\end{aligned}$$

Let us assume that the population grows at rate $n$, so when the household is old there are $(1+n)$ as many young guys around compared when he was young. Also assume that incomes grow at rate $g$ (because of technical progress) making younger generations having higher incomes. Finally assume that the social security system balances its budget, so that total social security payments equal total payroll taxes. This implies that

$$b = (1+n)(1+g)\tau y \qquad (5.3)$$

The household benefits from the fact that population grows over time since when he is old there are more people around to pay his pension. In addition these people paying for pensions have higher incomes because of technical progress. Using the social security budget constraint (5.3) we can rewrite the budget constraints of the household as

$$\begin{aligned}
c_1 + s &= (1-\tau)y \\
c_2 &= (1+r)s + (1+n)(1+g)\tau y_1
\end{aligned}$$

Again we can write this as a single intertemporal budget constraint

$$c_1 + \frac{c_2}{1+r} = (1-\tau)y + \frac{(1+n)(1+g)\tau y}{1+r} = I(\tau) \qquad (5.4)$$

where we emphasize that now discounted lifetime income depends on the size of the social security system, as measured by the tax rate $\tau$. Maximizing

(2.1) subject to (5.4) yields, as always

$$\begin{aligned} c_1 &= \frac{I}{1+\beta} \\ c_2 &= \frac{\beta}{1+\beta}(1+r)I \\ s &= (1-\tau)y - \frac{I}{1+\beta} \end{aligned} \tag{5.5}$$

So what does pay-as-you go social security do to saving? Using the definition of $I(\tau)$ in (5.5) we find

$$\begin{aligned} s &= (1-\tau)y - \frac{I}{1+\beta} \\ &= (1-\tau)y - \frac{(1-\tau)y}{1+\beta} - \frac{(1+n)(1+g)\tau y}{(1+r)(1+\beta)} \\ &= \frac{\beta(1-\tau)y}{1+\beta} - \frac{(1+n)(1+g)\tau y}{(1+r)(1+\beta)} \\ &= \frac{\beta y}{1+\beta} - \frac{(1+n)(1+g)\tau y + \beta\tau y(1+r)}{(1+r)(1+\beta)} \\ &= \frac{\beta y}{1+\beta} - \frac{(1+n)(1+g) + \beta(1+r)}{(1+r)(1+\beta)} * \tau y \end{aligned}$$

which is obviously decreasing in $\tau$. So indeed the bigger the public pay-as-you-go system, the smaller are private savings. Note that due to the pay-as-you go nature of the system the social security system itself does not save, so total savings in the economy unambiguously decline with an increase in the size of the system as measured by $\tau$. To the extent that this harms investment, capital accumulation and growth the pay-as-you-go social security system may have substantial negative long-run effects.

### 5.4.2 Welfare Consequences of Social Security

Second, we use the model to analyze a policy issue that has drawn large attention in the public debate. From a normative perspective, should the government run a pay-as-you go social security system or should it leave the financing of old-age consumption to private households (which is equivalent, under fairly weak conditions, to a fully funded government-run pension system). In a pure pay-as-you go social security system currently working generation pays payroll taxes, whose proceeds are used to pay the pensions of the currently retired generation. The key is that current taxes are paid out immediately, and not invested. In a fully funded system the contributions of the current young are saved (either by the households themselves in private accounts akin to the Riester Rente, or by the government). Future pension benefits are then financed by these savings, including the accumulated interest. The key difference is that with in a pay-as-you go system current contributions are used for current consumption of the old (as long as these generations do not save), whereas with a funded system these contributions augment savings (equal to investment in a closed economy). One can show that under fairly general conditions the physical capital stock in an economy with pay-as-you go social security system is lower than in an identical economy with a fully funded system.

But rather than studying capital accumulation directly, we restrict our analysis to a partial equilibrium analysis, asking whether individual households are better off in a pay-as-you-go system relative to a fully funded system, keeping the interest rate fixed (in a closed production economy the interest rate equals the marginal product of capital and thus is lower in an economy with more capital).

So under which condition is the introduction of social security good for the welfare of the household in the model? This has a simple and intuitive answer in the current model. When maximizing (5.2), subject to (5.4), we see that the social security tax rate only appears in $I(\tau)$, which is given as

$$I(\tau) = (1-\tau)y + \frac{(1+g)(1+n)\tau y}{1+r}. \tag{5.6}$$

So the question of whether social security is beneficial boils down to giving conditions under which $I(\tau)$ is strictly increasing in $\tau$. Rewriting (5.6) yields

$$\begin{aligned} I(\tau) &= y_1 - \tau y + \frac{(1+g)(1+n)\tau y}{1+r} \\ &= y + \left[\frac{(1+g)(1+n)}{1+r} - 1\right]\tau y \end{aligned}$$

and thus the pay-as-you go social security system is welfare improving if and only if $(1+n)(1+g) > 1+r$. Since, empirically speaking, $n * g$ is small relative to $n, g$ or $r$ (on an annual level $g$ is somewhere between $1-2\%$

for most industrialized countries, $n$ is even smaller and in some countries, including Germany, negative), the condition is well approximated by

$$n + g > r$$

That is, if the population growth rate plus income growth exceeds the private returns on the households's saving, then a given household benefits from pay-as-you-go social security This condition makes perfect sense. If people save by themselves for their retirement, the return on their savings equals $1 + r$. If they save via a social security system (are forced to do so), their return to this forced saving consists of $(1+n)(1+g)$ (more people with higher incomes will pay for the old guys). This result makes clear why a pay-as-you-go social security system may make sense in some countries (those with high population growth), but not in others, and that it may have made sense in Germany in the 60's and 70's, but not in the 90's. Just some numbers: the current population growth rate in Germany is, say about $n = 0\%$ (including immigration), productivity growth is about $g = 1\%$ and the average return on the stock market for the last 100 years is about $r = 7\%$. This is the basis for many economists to call for a reform of the social security system in many countries. Part of the debate is about how one could (partially) privatize the social security system, i.e. create individual retirement funds so that basically each individual would save for her own retirement, with return $1 + r > (1 + n)(1 + g)$. Abstracting from the fact that saving in the stock market is fairly risky even over longer time horizons (and the return on saver financial assets is not that much higher than $n + g$), the biggest problem for the transition is one missing generation. At the introduction of the system there was one old generation that received social security but never paid taxes for it. Now we face the dilemma: if we abolish the pay-as-you go system, either the currently young pay double, for the currently old and for themselves, or we just default on the promises for the old. Both alternatives seem to be difficult to implement politically and problematically from an ethical point of view. The government could pay out the old by increasing government debt, but this has to be financed by higher taxes in the future, i.e. by currently young and future generations. Hence this is problematic, too. The issue is very much open, and since I did research on this issue in my own dissertation I am happy to talk to whoever is interested in more details.

### 5.4.3 The Insurance Aspect of a Social Security System

Modern social security systems provide some form of insurance to individuals, namely insurance against the risk of living longer than expected. In other words, social security benefits are paid as long as the person lives, so that people that live (unexpectedly) longer receive more over their lifetime than those that die prematurely. Note, however, that such insurance need not be provided by the government via social security, but could also be provided by private insurance contract. In fact, private annuities are designed to exactly provide the same insurance. We will briefly discuss below why the government may be in a better position to provide this insurance. Before doing so I first want to demonstrate that providing such insurance, privately or via the social security system is indeed beneficial for private households.

First we consider a household in the absence of private or public insurance markets. The household lives up to two periods, but may die after the first period. Let $p$ denote the probability of surviving. We normalize the utility of being dead to 0 (this is innocuous because our households can do nothing to affect the probability of dying) and for simplicity abstract from time discounting. The agent solves

$$\max_{c_1,c_2,s} \log(c_1) + p\log(c_2)$$

$$\text{s.t.}$$

$$\begin{aligned} c_1 + s &= y \\ c_2 &= (1+r)s \end{aligned}$$

Note that we have implicitly assumed that the household is not altruistic, so that the savings of the household, should she die, are lost without generating any utility. As always, we can consolidate the budget constraint, to yield

$$c_1 + \frac{c_2}{1+r} = y$$

and the solution to the problem takes the familiar form

$$\begin{aligned} c_1 &= \frac{1}{1+p}y \\ c_2 &= \frac{p(1+r)}{1+p}y \end{aligned}$$

where $p$ takes the place of the time discount factor $\beta$.

Now consider the same household with a social security system in place. The budget constraints reads as usual

$$\begin{aligned} c_1 + s &= (1-\tau)y \\ c_2 &= (1+r)s + b \end{aligned}$$

But now the budget constraint of the social security administration becomes

$$pb = (1+n)(1+g)\tau y$$

The new feature is that social security benefits only need to be paid to a fraction $p$ of the old cohort (because the rest has died). Consolidating the budget constraints an substituting for $b$ yields

$$c_1 + \frac{c_2}{1+r} = y + \tau y \left( \frac{(1+n)(1+g)}{p(1+r)} - 1 \right)$$

The household may benefit from a pay-as-you-go social security system for two reasons. First, as we saw above, if $(1+n)(1+g) > 1+r$, the implicit return on social security is higher than the return on private assets. This argument had nothing to do with insurance at all. But now, as long as $p < 1$, even if $(1+n)(1+g) \leq 1+r$ social security may be good, since the surviving individuals are implicitly insured by their dead brethren: the implicit return on social security is $\frac{(1+n)(1+g)}{p} > (1+n)(1+g)$. If you survive you get higher benefits, if you die you don't care about receiving nothing.

Now suppose that $(1+n)(1+g) = 1+r$, that is, the first reason for social security is absent by assumption, because we want to focus on the insurance aspect. The implicit return on social security is then $\frac{(1+n)(1+g)}{p} = \frac{1+r}{p}$. Now consider the other alternative of providing insurance, via the purchase of private annuities. An annuity is a contract where the household pays 1 Euro today, for the promise of the insurance company to pay you $1 + r_a$ Euros as long as you live, from tomorrow on (and in the simple model, you live only one more period). But what is the equilibrium return $1 + r_a$ on this annuity. Suppose there is perfect competition among insurance companies, resulting in zero profits. The insurance company takes 1 Euro today (which it can invest at the market interest rate $1 + r$). Tomorrow it has to pay out with probability $p$ (or, if the company has many customers, it has to pay out to a fraction $p$ of its customers), and it has to pay out $1+r_a$ per Euro of insurance contract. Thus zero profits imply

$$1 + r = p(1 + r_a)$$

or

$$1 + r_a = \frac{1 + r}{p}.$$

This is the return on the annuity, conditional on surviving, which coincides exactly with the expected return via social security, as long as $(1+n)(1+g) = 1+r$. That is, insurance against longevity can equally be provided by a social security system or by private annuity markets. The only difference is that the size of the insurance is fixed by the government in the case of social security, and freely chosen in the case of private annuities.

In practice in the majority of the countries it is the government, via some sort of social security, that provides this insurance. Private annuity markets do exist, but seem to be quite thin (that is, not many people purchase these private annuities).

There are at least two reasons that I can think of

- If there is already a public system in place (for whatever reason), there are no strong incentives to purchase additional private insurance, unless the public insurance does not extent to some members of society.

- In the presence of adverse selection private insurance markets may not function well. If individuals have better information about their life expectancy than insurance companies, then insurance companies will offer rates that are favorable for households with high life expectancy and bad for people with low life expectancy. The latter group will not buy the insurance, leaving only the people with bad risk (for the insurance companies) in the markets. Rates have to go up further. In the end, the private market for annuities may break down (nobody but the very worst risks purchase the insurance, at very high premium). The government, on the other hand, can force all people into the insurance scheme, thus avoiding the adverse selection problem.

Another problem with insurance, so called moral hazard, will emerge in the next section where we discuss social insurance, especially unemployment insurance.

# Chapter 6

# Social Insurance

The term "Social Insurance" stands for a variety of public insurance programs, all with the aim of insuring citizens of a rich, modern society against the major risks of life: unemployment (unemployment insurance), becoming poor at young and middle ages (welfare, food stamps), becoming poor in old age because of unexpected long life (social security), becoming sick in old age (medicare). These risks and policies to insure the risks vary in their details, but their basic features are similar. Therefore, rather than describing all of them in detail, we will focus on the main risk during a person's working life: unemployment.[1]

## 6.1 International Comparisons of Unemployment Insurance

Before providing a theoretical rationale for publicly provided unemployment insurance (and the limits thereof) we first want to document and discuss the astounding international differences in the generosity and length of unemployment insurance benefits. Before doing so, let us first briefly discuss when people tend to get unemployed in modern economies

The unemployment rate is very counter-cyclical. It increases in recessions and increases in expansions. Figure 6.1 plots the unemployment rate for the U.S. for the last 40 years. We clearly see that the unemployment rate

[1] In fact, in the previous chapter we already discussed the important social insurance function social security plays, by insuring longevity risk.

increased during all recessions in the last 40 years. Formally a recession may be defined as two consecutive quarters of declining real GDP, but since output is produced using capital and labor, a decline in output almost automatically means a reduction in labor being used in production. Ignoring the important possibility that laid-off workers leave the labor force, this means that the unemployment rate necessarily increases in economic downturns, and many economists think of a significant increase in the unemployment rate as one of the *defining features* of recessions.

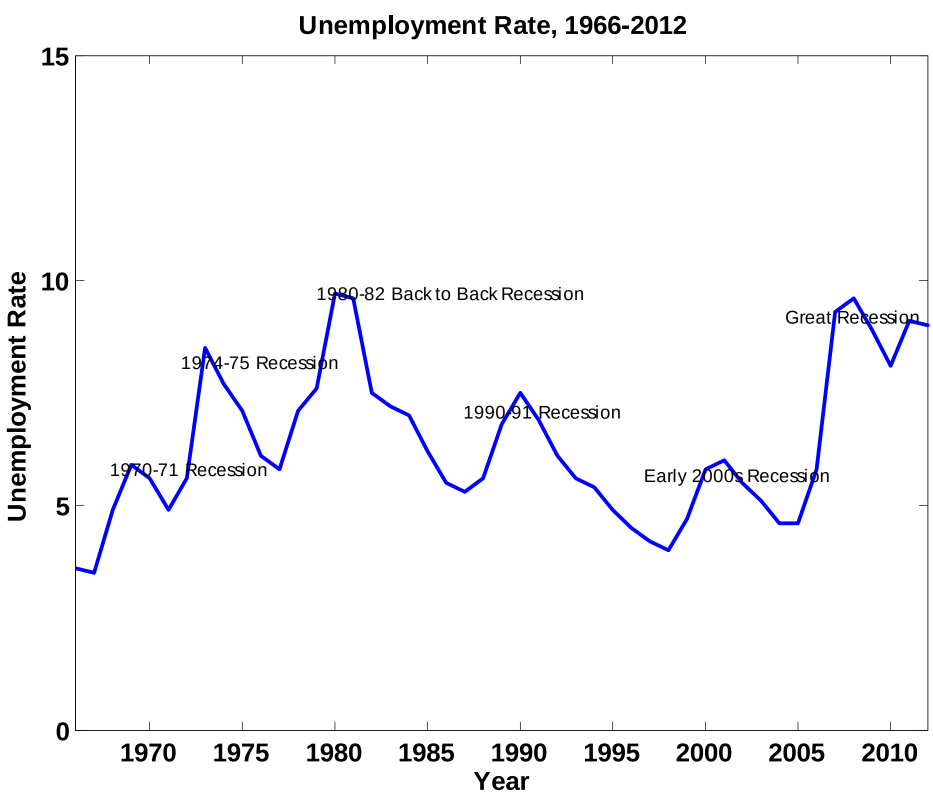


Figure 6.1: The U.S. Unemployment Rate

But why exactly does the unemployment rate go up in recessions? Is it because more people than normal get fired, or less people than normal get hired. Here are the basic facts on job creation and job destruction for the U.S. manufacturing sector:[2]

[2]These facts come from the great book by Davis, Haltiwanger and Schuh *Job Creation*

- Job turnover is large. In a typical year 1 out of every ten jobs in manufacturing is destroyed and a comparable number of jobs is created at different plants.

- Most of the job creation and destruction over a twelve-month interval reflects highly persistent plant-level employment changes. This persistence implies that most jobs that vanish at a particular plant in a given twelve-month period fail to reopen at the same location within the next two years.

- Job creation and destruction are concentrated at plants that experience large percentage employment changes. Two-thirds of job creation and destruction takes place at plants that expand or contract by 25% or more within a twelve-month period. About one quarter of job destruction takes place at plants that shut down.

- Job destruction exhibits greater cyclical variation than job creation. In particular, recessions are characterized by a sharp increase in job destruction accompanied by a mild slowdown in job creation.

- Gross job creation is relatively stable over the business cycle, whereas gross job destruction moves strongly countercyclical: it is high in recessions and low in booms.

- In severe recessions such as the 74-75 recession or the 80-82 back to back recessions and the great recession up to 25% of all manufacturing jobs are destroyed within one year, whereas in booms the number is below 5%.

- Time a worker spends being unemployed also varies over the business cycle, with unemployment spells being longer on average in recession years than in years before a recession.

This last fact is made concise in table 6.1. It shows the average length of unemployment spells in two years, 2006, the last full year of the expansion of the 2000's and 2010, the first year after the official end of the great recession.

We observe that in a recession year many more unemployment spells last for more than half a year than in an expansion, where most workers that are

*and Destruction* which collects and describes these facts for the manufacturing sector.

| Unemployment Spell | 2006 | 2010 |
|---|---|---|
| < 5 weeks | 37% | 19% |
| 5 - 14 weeks | 30% | 22% |
| 15 - 26 weeks | 15% | 16% |
| > 26 weeks | 18% | 43% |

Table 6.1: Length of Unemployment Spells

| | Unemployment (%) | | | ≥ 1 Year | | |
|---|---|---|---|---|---|---|
| | 2000 | 2008 | 2011 | 1999 | 2006 | 2011 |
| Belgium | 6.9 | 7.0 | 7.2 | 60.5 | 51.2 | 48.3 |
| France | 9.0 | 7.8 | 9.7 | 38.7 | 41.9 | 41.4 |
| Germany | 8.0 | 7.5 | 5.9 | 51.7 | 56.4 | 48.0 |
| Netherlands | 3.1 | 3.1 | 4.5 | 43.5 | 40.0 | 33.6 |
| Spain | 11.7 | 11.3 | 21.6 | 46.3 | 21.7 | 41.6 |
| Italy | 10.1 | 6.7 | 8.4 | 61.4 | 49.6 | 51.9 |
| Greece | 11.2 | 7.7 | 17.7 | 55.3 | 54.3 | 49.6 |
| Portugal | 4.0 | 7.7 | 12.9 | 41.2 | 50.2 | 48.2 |
| Sweden | 5.6 | 6.2 | 7.5 | 30.1 | 13.0 | 17.2 |
| UK | 5.4 | 5.7 | 8.0 | 29.6 | 22.3 | 33.4 |
| US | 4.0 | 5.8 | 9.7 | 6.8 | 10.0 | 31.7 |
| Tot. OECD | 6.1 | 6.0 | 8.0 | 32.2 | 31.4 | 33.6 |

Table 6.2: Unemployment Rates, OECD

laid off find a new job within a matter of 5 weeks, and certainly within 14 weeks. By international standards the fraction of households in long term unemployment (longer than six months or longer than one year) in the U.S. remains relatively small, but is strongly on the rise, as the next table suggests.

From table 6.2 we observe several other things. First, unemployment rates in Europe were significantly higher prior to the great recession, relative to the unemployment rate in the U.S. Note that this had not always been the case. In fact, in the 1970's it was the U.S. that had higher unemployment rates than Europe, but then the situation reversed. Second, abstracting from South Europe (Portugal, Spain, Italy and Greece) unemployment rates in 2011, right after the great recession, were actually higher in the U.S. than in Europe. Third, from the second half of the table we see that long-term

| | Single | | | With Dependent Spouse | | |
|---|---|---|---|---|---|---|
| | 1. Y. | 2.-3. Y. | 4.-5. Y. | 1. Y. | 2.-3. Y. | 4.-5. Y. |
| Belgium | 79 | 55 | 55 | 70 | 64 | 64 |
| France | 79 | 63 | 61 | 80 | 62 | 60 |
| Germany | 66 | 63 | 63 | 74 | 72 | 72 |
| Netherlands | 79 | 78 | 73 | 90 | 88 | 85 |
| Spain | 69 | 54 | 32 | 70 | 55 | 39 |
| Sweden | 81 | 76 | 75 | 81 | 100 | 101 |
| UK | 64 | 64 | 64 | 75 | 74 | 74 |
| US | 34 | 9 | 9 | 38 | 14 | 14 |

Table 6.3: Unemployment Benefit Replacement Rates

unemployment used to be almost nonexistent in the U.S. prior to the great recession, whereas it accounts for a very significant fraction of all unemployed in most European countries. Finally, after the Great Recession this finding has diminished significantly, on account of the very substantial rise in long term unemployment in the U.S. The European unemployment dilemma has for long time been a dilemma of long-term unemployment, and it appears that for the time being this phenomenon has reached the U.S. as well.

How can the dramatic differences in unemployment rates between the U.S. and Europe prior to the Great Recession, and in particular the large difference in long-term unemployed during the 1980's through 2007, be explained. This is a very complex problem. In a very influential paper Lars Ljungqvist and Tom Sargent relate long-term unemployment rates to the generosity of the European unemployment benefits. Table 6.3 summarizes unemployment benefit replacement rates for various countries, as a function of the length of unemployment, for the mid-90's. The table has to be read as follows. A 79 for Belgium in year 1 means that a typical worker in Belgium that is unemployed for no more than one year receives 79% of her last wage as unemployment compensation.

This table tells us the following.[3] First, replacement rates used to be much lower in the U.S. than in Europe. Second, and possibly more important, while in the U.S. benefits used to drop sharply after 13 weeks, in many

[3]This table is still fairly accurate, apart from the fact that the duration of unemployment rates in the U.S. has been extended significantly in the aftermath of the great recession

European countries the replacement rate remains over 60% three years into an unemployment spell. Imagine what this may do to incentives to find a new job.

As we will show below, publicly provided unemployment benefits may provide very valuable social insurance. On the other hand, it may reduce incentive to keep job or find new ones. What is puzzling, however, is why, basically with unchanged benefit schemes over time, Europe did very well in the 60's and 70's, but fell behind (in the performance of their labor markets) in the 80's and 90's. Prescott's taxation story, discussed, may be part of the story. Ljungqvist and Sargent offer the following explanation. The 60's and 70's were a period of tranquil economic times, in the sense that a laid-off worker did not suffer large skill losses when being laid off. In the 80's the situation changed and laid-off workers faced a higher risk of losing their skills when becoming unemployed (they call this increased turbulence). Thus in earlier times the European benefit system was not too distortive; it provided insurance and didn't induce laid-off households not to look for new jobs (because they had good skills and thus could find new, well-paid jobs easily). In the 80's, with higher chances of skill losses upon lay-off the benefit system becomes problematic. A newly laid off worker in Europe has access to high and long-lasting unemployment compensation; on the other hand, he may have lost his skill and thus is not offered new jobs that are attractive enough. Now he decides to stay unemployed, rather than accept a bad job. Higher turbulence plus generous benefits create the European unemployment dilemma.

Note that this idea also provides a potential explanation for why U.S. unemployment rate have been as high or higher than those of European In fact, in a recent paper two (ex-) Penn graduate students, Kurt Mitman and Stan Rabinovich argue that fitting the time varying duration of unemployment benefits into a standard model of equilibrium unemployment[4] one can account for essentially the entire unemployment fluctuations over the business cycle, including the very slow decline of unemployment rates after the Great Recession. Viewed through the lens of their work, the unemployment rate did not decline more sharply in 2010-2012 because the extension of unemployment rates reduced the incentives of unemployed workers to search

[4]The current standard model of unemployment is the celebrated Mortensen-Pissarides model for which the authors, together with Peter Diamond, received the Nobel Prize in Economics.

for a new job, and crucially, reduced the incentives of firms to search for new workers.

After having discussed what all may be wrong with generous unemployment benefits, let us provide a theoretical rationale for its existence in the first place, before coming back to the incentive problems such a system may create.

## 6.2 Social Insurance: Theory

In this section we will study a simple insurance problem, first in the absence, then in the presence of a government-run public insurance system. We will focus on unemployment as the risk the household faces and on unemployment insurance as the government policy enacted to deal with it. Exactly the same analysis can be carried out for health risk and public health insurance, and death risk and social security.

### 6.2.1 A Simple Intertemporal Insurance Model

Our agent lives for two periods. In the first period he has a job for sure and earns a wage of $y_1$. In the second period he may have a job and earn a wage of $y_2^e$ or be unemployed and earn $y_2^u$, which may or may not be equal to zero. Let $p$ denote the probability that he has a job and $1-p$ denote the probability that he is unemployed. For simplicity assume that the interest rate $r=0$. The utility function is given by

$$u(c_1) + pu(c_2^e) + (1-p)u(c_2^u)$$

where $u$ is the period utility function, $c_2^e$ is his consumption if he is employed in the second period and $c_2^u$ is his consumption if he is unemployed in the second period. His budget constraints are

$$\begin{aligned} c_1 + s &= y_1 \\ c_2^e &= y_2^e + s \\ c_2^u &= y_2^u + s \end{aligned}$$

### 6.2.2 Solution without Government Policy

Let us start solving the model without government intervention. For now there is no public unemployment insurance. For concreteness suppose that income in the first period is given by $y_1$ and income in the second period is $y_2$. First let's assume that there is no uncertainty in the second period about income, so that $y_2^e = y_2^u = y_2$, i.e. the household has the same income no matter whether employed or not. For simplicity assume that $y_1 = y_2 = y$ (that is, the safe income in the second period equals that in the first period.

**Log-Utility**

Then the maximization problem reads as

$$
\begin{aligned}
& \max \log(c_1) + p\log(c_2^e) + (1-p)\log(c_2^u) \\
& \text{s.t.} \\
c_1 + s &= y \\
c_2^e &= y + s \\
c_2^u &= y + s
\end{aligned}
$$

Obviously in this situation the household does not face any uncertainty, and his choice problem is the standard one studied many times before in this class. First we recognize that

$$c_2^e = y + s = c_2^u = c_2$$

that is, the household consumes the same whether employed or unemployed (not surprisingly, since his income is not affected by his unemployment status). Thus the problem reduces to

$$
\begin{aligned}
& \max \log(c_1) + \log(c_2) \\
& \text{s.t.} \\
c_1 + s &= y \\
c_2 &= y + s
\end{aligned}
$$

The optimal solution to this problem is given by

$$
\begin{aligned}
c_1 &= c_2 = c_2^e = c_2^u = y \\
s &= 0
\end{aligned}
$$

The household has safe and perfectly smooth income and thus simply consumes his income in every period.

Now let us introduce uncertainty: let $y_1 = y$, and $p = 0.5$ and $y_2^e = 2y_1 = 2y$ and $y_2^u = 0$. That is, the household's expected income in the second period is

$$0.5 * 2y + 0.5 * 0 = y$$

as before. But now the household does face uncertainty and we are interested in how his behavior changes in the light of this uncertainty. His maximization problem now becomes

$$\begin{aligned}
& \max \log(c_1) + 0.5\log(c_2^e) + 0.5\log(c_2^u) \\
& \text{s.t.} \\
c_1 + s &= y \qquad (6.1) \\
c_2^e &= 2y + s \qquad (6.2) \\
c_2^u &= s \qquad (6.3)
\end{aligned}$$

This is a somewhat more complicated problem, so let us tackle it carefully. There are 4 choice variables, $(c_1, c_2^e, c_2^u, s)$. One could get rid of one by consolidating two of the three budget constraints, but that makes the problem more complicated than easy.

Let us simply write down the Lagrangian and take first order conditions. Since there are three constraints, we need three Lagrange multipliers, $\lambda_1, \lambda_2, \lambda_3$. The Lagrangian reads as

$$L = \log(c_1) + 0.5\log(c_2^e) + 0.5\log(c_2^u) + \lambda_1(y - c_1 - s) + \lambda_2(2y + s - c_2^g) + \lambda_3(s - c_2^u)$$

Taking first order conditions with respect to $(c_1, c_2^e, c_2^u, s)$ yields

$$\begin{aligned}
\frac{1}{c_1} - \lambda_1 &= 0 \\
\frac{0.5}{c_2^e} - \lambda_2 &= 0 \\
\frac{0.5}{c_2^u} - \lambda_3 &= 0 \\
-\lambda_1 + \lambda_2 + \lambda_3 &= 0
\end{aligned}$$

or

$$
\begin{aligned}
\frac{1}{c_1} &= \lambda_1 \\
\frac{0.5}{c_2^e} &= \lambda_2 \\
\frac{0.5}{c_2^u} &= \lambda_3 \\
\lambda_2 + \lambda_3 &= \lambda_1
\end{aligned}
$$

Substituting the first three equations into the last yields

$$\frac{0.5}{c_2^e} + \frac{0.5}{c_2^u} = \frac{1}{c_1} \tag{6.4}$$

Now we use the three budget constraints (6.1)-(6.3) to express consumption in (6.4) in terms of saving:

$$\frac{0.5}{2y+s} + \frac{0.5}{s} = \frac{1}{(y-s)}$$

which is one equation in one unknown, namely $s$. Unfortunately this equation is not linear in $s$, so it is a bit more difficult to solve than usual. Let us bring the equation to one common denominator, $s * (2y+s) * (y-s)$, to obtain

$$\frac{0.5s(y-s)}{s\,(2y+s)\,(y-s)} + \frac{0.5\,(2y+s)\,(y-s)}{s\,(2y+s)\,(y-s)} = \frac{s\,(2y+s)}{s\,(2y+s)\,(y-s)}$$

or

$$\frac{0.5s(y-s) + 0.5\,(2y+s)\,(y-s) - s\,(2y+s)}{s\,(2y+s)\,(y-s)} = 0$$

But this can only be 0 if the numerator is 0, or

$$0.5s(y-s) + 0.5\,(2y+s)\,(y-s) - s\,(2y+s) = 0$$

Multiplying things out and simplifying a bit yields

$$s^2 + ys - \frac{1}{2}y^2 = 0$$

This is a quadratic equation, which has in general two solutions.[5] They are

$$
\begin{aligned}
s_1 &= -\frac{y}{2} - \sqrt{\frac{3}{4}y^2} = -\frac{1}{2}y\left(1+\sqrt{3}\right) < 0 \\
s_2 &= -\frac{y}{2} + \sqrt{\frac{3}{4}y^2} = \frac{1}{2}y\left(\sqrt{3}-1\right) > 0
\end{aligned}
$$

The first solution can be discarded on economic grounds, since it leads to negative consumption $c_2^u = s = -\frac{1}{2}y\left(1+\sqrt{3}\right)$. Thus the optimal consumption and savings choices with uncertainty satisfy

$$
\begin{aligned}
\hat{s} &= \frac{1}{2}y\left(\sqrt{3}-1\right) > 0 \\
\hat{c}_1 &= y - \frac{1}{2}y\left(\sqrt{3}-1\right) = \frac{1}{2}y\left(3-\sqrt{3}\right) < y \\
\hat{c}_2^e &= 2y + \hat{s} = \frac{1}{2}y\left(3+\sqrt{3}\right) \\
\hat{c}_2^u &= \frac{1}{2}y\left(\sqrt{3}-1\right)
\end{aligned}
$$

We make the following important observation. Even though income in the first period and expected income in the second period has not changed at all, compared to the situation without uncertainty, now households increase their savings and reduce their first period consumption level:

$$
\begin{aligned}
\hat{c}_1 &= \frac{1}{2}y\left(3-\sqrt{3}\right) < y = c_1 \\
\hat{s} &= \frac{1}{2}y\left(\sqrt{3}-1\right) > 0 = s
\end{aligned}
$$

[5]Remember that if you have an equation

$$x^2 + ax + b = 0$$

where $a, b$ are parameters, then the two solutions are given by

$$
\begin{aligned}
x_1 &= -\frac{a}{2} - \sqrt{\frac{a^2}{4} - b} \\
x_2 &= -\frac{a}{2} + \sqrt{\frac{a^2}{4} - b}
\end{aligned}
$$

For these solutions to be well-defined real numbers we require $\frac{a^2}{4} - b > 0$.

This effect of increasing savings in the light of increased uncertainty (again: expected income in the second period remains the same, but has become more risky) is called precautionary savings. Households, as precaution against income uncertainty in the second period, save more with increased uncertainty, in order to assure decent consumption even when times turn out to be bad.

### General Condition for Precuationary Saving

We assumed that households have log-utility. But our result that households increase savings in response to increased uncertainty holds for arbitrary strictly concave utility functions that have a positive third derivative, or $u'''(c) > 0$ (one can easily check that log-utility satisfies this).

To show this now assume that $y_2^e = y + \epsilon$ and $y_2^u = y - \epsilon$, with equal probability, where the parameter $\epsilon$ measures the extent of income risk the household faces. We seek a general condition on the period utility function $u(.)$ such that optimal saving $s$ of the household increases with income risk $\epsilon$. Repeating the same steps as for log-utility above delivers the general Euler equation in the presence of risk

$$u'(c_1) = 0.5u'(c_2^e) + 0.5u'(c_2^u).$$

Interpretation of RHS [TBC] Plugging in from the three budget constraints and exploiting the specific form of income in the second period yields

$$u'(y - s) = 0.5u'(y + \epsilon + s) + 0.5u'(y - \epsilon + s).$$

In this equation the degree of income risk $\epsilon$ enters directly, and indirectly since optimal saving $s = s(\epsilon)$ is a function of income risk $\epsilon$. The goal is to sign the derivative $s'(\epsilon)$ of saving with respect to $\epsilon$. To do so we first rewrite the previous equation more explicitly as

$$u'(y - s(\epsilon)) = 0.5u'(y + \epsilon + s(\epsilon)) + 0.5u'(y - \epsilon + s(\epsilon)).$$

and then totally differentiate with respect to $\epsilon$, that is, take the derivative of both sides of the previous equation, capturing both the direct effect and the indirect effect of $\epsilon$ through $s$. Doing so (and assuming that $s$ is indeed differentiable in $\epsilon$) yields

$$-u''(y-s(\epsilon))*s'(\epsilon) = 0.5u''(y+\epsilon+s(\epsilon))*(1+s'(\epsilon))+0.5u''(y-\epsilon+s(\epsilon))*(-1+s'(\epsilon))$$

Solving this arguably messy equation for $s'(\epsilon)$ yields

$$s'(\epsilon) = \frac{0.5[u''(y+\epsilon+s(\epsilon)) - u''(y-\epsilon+s(\epsilon))]}{-[u''(y-s(\epsilon)) + 0.5u''(y+\epsilon+s(\epsilon)) + 0.5u''(y-\epsilon+s(\epsilon))]}$$

Interpret (TBC)

**Quadratic Utility and Certainty Equivalence (Again)**

Note that strict concavity alone (that is, risk-aversion) is not enough for this result. In fact, if utility is $u(c) = -\frac{1}{2}(c - 100,000)^2$ (with $100,000$ being the bliss point of consumption) then the household would choose exactly the same first period consumption and savings choice with or without uncertainty. In this case the first order conditions become

$$\begin{aligned}
-(c_1 - 100,000) &= \lambda_1 \\
-0.5(c_2^e - 100,000) &= \lambda_2 \\
-0.5(c_2^u - 100,000) &= \lambda_3 \\
\lambda_2 + \lambda_3 &= \lambda_1
\end{aligned}$$

Inserting the first three equations into the fourth yields

$$-(c_1 - 100,000) = -0.5(c_2^e - 100,000) - 0.5(c_2^u - 100,000)$$

or

$$c_1 = 0.5(c_2^e + c_2^u)$$

Now using the budget constraints one obtains

$$\begin{aligned}
y - s &= 0.5(2y + s + s) \\
y - s &= y + s \\
2s &= 0
\end{aligned}$$

and thus the optimal savings choice with quadratic utility is $s = 0$, as in the case with no uncertainty. Economists often say that under quadratic utility optimal consumption choices exhibit "certainty equivalence", that is, even with risk households make exactly the same choices as without uncertainty. Note that obviously realized consumption in period differs with and without uncertainty. With uncertainty one consumes $2y$ with probability 0.5 and 0 with probability 0.5, whereas under certainty one consumes $y$ for sure. So while expected consumption remains the same, realized consumption (and thus welfare) does not. Finally note that with quadratic utility households are risk-averse and thus dislike risk, but they optimally don't change their saving behavior to hedge against it. It is easy to verify that with quadratic utility $u''' = 0$, thus providing no contradiction to our previous claim about precautionary savings.

### 6.2.3 Public Unemployment Insurance

Rather than to dwell on this point, let us introduce a public unemployment insurance program and determine how it changes household decisions and individual welfare. The government levies unemployment insurance taxes on employed people in the second period at rate $\tau$ and pays benefits $b$ to unemployed people, so that the budget of the unemployment insurance system is balanced. There are many people in the economy, so that the fraction of employed in the second period is $p = 0.5$ and the fraction of unemployed is $1 - p = 0.5$ Thus the budget constraint of the unemployment administration reads as

$$0.5\tau y_2 = 0.5b$$

or

$$\tau y_2 = b$$

and the budget constraints in the second period become

$$\begin{aligned} c_2^e &= (1-\tau)y_2 + s \\ c_2^u &= b + s \\ &= \tau y_2 + s \end{aligned}$$

For concreteness suppose that $\tau = 0.5$ and $y_2 = 2y_1 = 2y$ as before, so that

$$c_2^e = y + s \tag{6.5}$$

$$c_2^u = y + s \tag{6.6}$$

That is, the unemployment system perfectly insures the unemployed: unemployment benefits are exactly as large as after tax income when being employed. We can again solve for optimal consumption and savings choices. One could set up a Lagrangian and proceed as always, but in this case a little bit of clever thinking gives us the solution much easier. From (6.5) and (6.6) it immediately follows that

$$c_2^e = c_2^u = c_2$$

no matter what $s$ is. But then the maximization problem of the household boils down to

$$\begin{aligned} & \max \log(c_1) + 0.5\log(c_2) + 0.5\log(c_2) \\ = & \max \log(c_1) + \log(c_2) \\ & \text{s.t.} \\ c_1 + s = & \; y \\ c_2 = & \; y + s \end{aligned}$$

with obvious solution

$$\begin{aligned} c_1 &= c_2 = y \\ s &= 0 \end{aligned}$$

exactly as in the case without income uncertainty. That is, when the government completely insures unemployment risk, private households make exactly the same choices as if there was no income uncertainty.

Three final remarks:

1. In terms on welfare, would individuals rather live in a world with or without unemployment insurance? With perfect unemployment insurance their lifetime utility equals

$$V^{ins} = \log(y) + \log(y)$$

which exactly equals the lifetime utility without income uncertainty. Without unemployment insurance lifetime utility is

$$V^{no} = \log\left(\frac{1}{2}y\left(3-\sqrt{3}\right)\right) + 0.5\log\left(\frac{1}{2}y\left(3+\sqrt{3}\right)\right) + 0.5\log\left(\frac{1}{2}y\left(\sqrt{3}-1\right)\right)$$

and it is easy to calculate that $V^{ins} > V^{no}$.

2. Even if the unemployment insurance would only provide partial insurance, that is $0 < \tau < 0.5$, the household would still be better of with that partial insurance than without any insurance (although it becomes

more messy to show this). Risk-averse individuals always benefit from public (or private) provision of actuarially fair insurance; but they prefer more insurance to less, absent any adverse selection or moral hazard problem.

3. Above we have made a strong case for the public provision of complete unemployment insurance. No country provides full insurance against being unemployed, not even the European welfare states. Why not? In contrast to the model, where getting unemployed is nothing households can do something about, in the real world with perfect insurance a strong moral hazard problem arises. Why work if one get's the same money by not working. Consider the following simple extension of the model in which the household maximizes

$$u(c_1) + p(e)u(c_2^e) + (1 - p(e))u(c_2^u) - 0.5e^2$$

where the new variable $e$ measures the effort of keeping or finding a job. We assume that larger is $e$ the larger is $p(e)$, and and to obtain the strongest result we also assume that $p(e = 0) = 0$, that is, without effort the probability of finding a job is zero. Now suppose the government implements a public unemployment insurance that fully insures away the unemployment risk, so that $c_2^e = c_2^u = c_2$. Then the objective function of the household reads as

$$u(c_1) + p(e)u(c_2^e) + (1 - p(e))u(c_2^u) - 0.5e^2 = u(c_1) + u(c_2) - 0.5e^2$$

and the optimal effort choice is obviously $e = 0$. But this in turn implies $p(e) = 0$) and thus everyone is unemployed, no taxes are collected, no benefits can be paid out and the system breaks down. Thus because of the moral hazard problem the full insurance policy is not sustainable.

As always, the policy maker faces an important and difficult trade-off between insurance and economic incentives. If the government could perfectly monitor individuals and thus observe whether they became unemployed because of bad luck or own fault and also monitor their intensity in looking for a new job, then things would be easy: simply condition payment of benefits on good behavior. But if these things are private information of the households, then the complicated trade-off between efficiency and insurance arises, and the optimal design on an optimal unemployment insurance system becomes a difficult theoretical problem, one that has seen very many interesting research papers in the last 5 years. These, however, are well beyond the scope of this class.

# Part III

# Optimal Fiscal Policy

# Chapter 7

# Optimal Fiscal Policy with Commitment

## 7.1 The Ramsey Problem

## 7.2 Main Results in Optimal Taxation

# Chapter 8

# The Time Consistency Problem

# Chapter 9

# Optimal Fiscal Policy without Commitment

# Part IV

# The Political Economics of Fiscal Policy

# Chapter 10

# Intergenerational Conflict: The Case of Social Security

# Chapter 11

# Intragenerational Conflict: The Mix of Capital and Labor Income Taxes

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