The Climate and the Economy

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IIES

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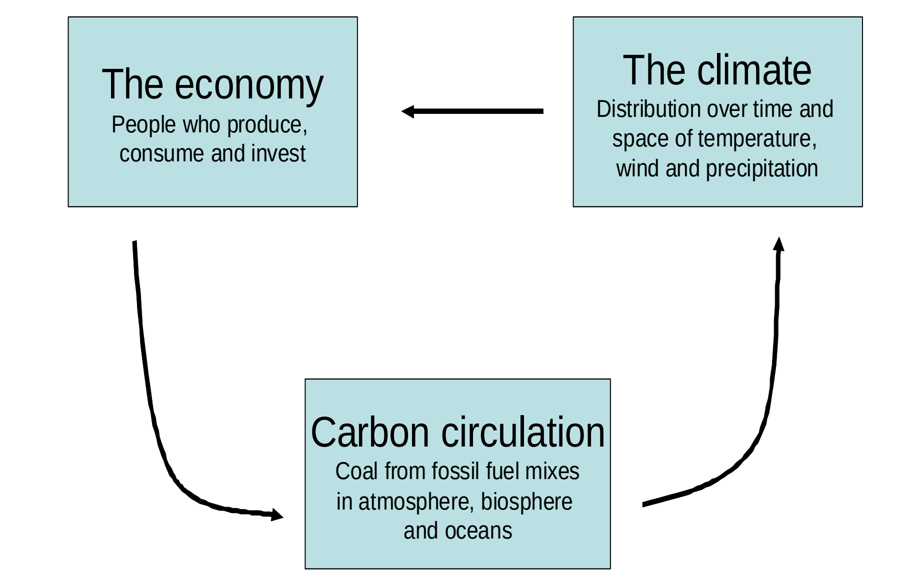
Overview

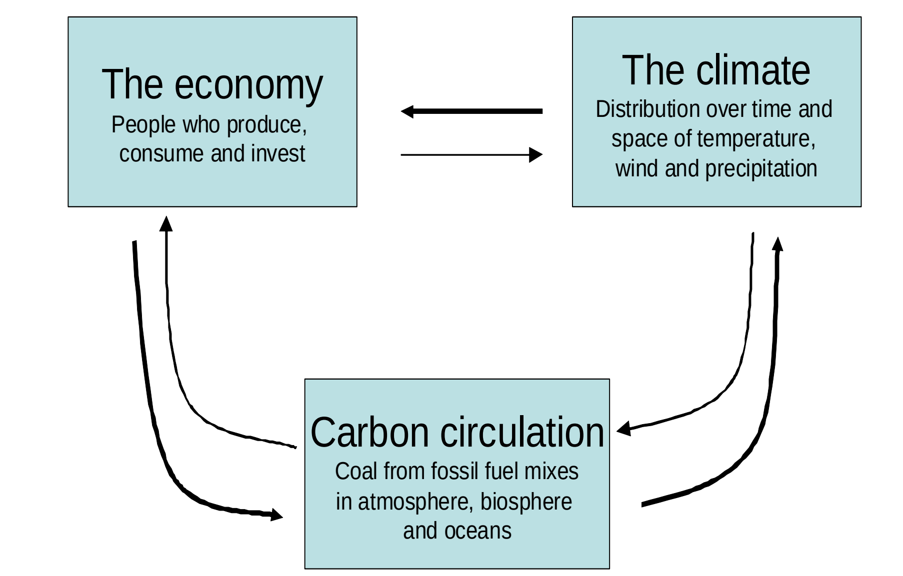
- Purpose: study models of the interaction between the global economy and the climate to
  - provide an understanding of important mechanisms and
  - analyze optimal policy.
- It involves results from both social and natural sciences.
- But we are economists – use our comparative advantage to contribute and critically analyze the economic side but take "conventional wisdom" from the natural sciences as given.
- Economics is key for analyzing effects of policy.

Purpose of economics here

- Emissions are caused by decisions taken by billions of people, firms and other agents acting on markets. Cannot be understood without economics.
- Economics is important for
  - analyzing effects of policy,
  - understanding endogenous adaptation and technical change,
  - and making forecasts.
- For all these purposes, develop IAMs: Integrated Assessment Models
- (first put together by Bill Nordhaus).

A schematic IAM: interactions




The economy
People who produce, consume and invest
The climate
Distribution over time and space of temperature, wind and precipitation
Carbon circulation
Coal from fossil fuel mixes in atmosphere, biosphere and oceans

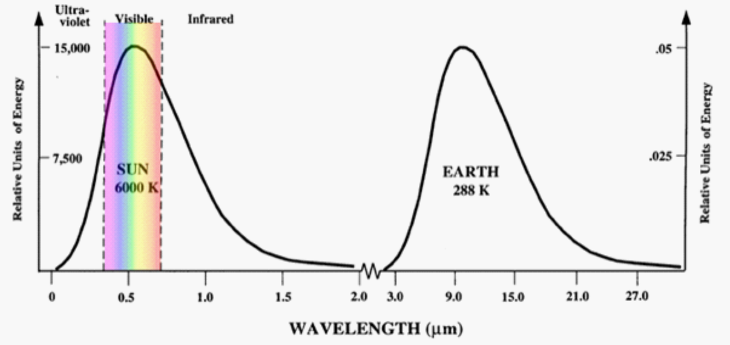
- Incoming *flow of energy*: short wave solar radiation (342 W/m^2 $= 2400 kW$ per football field), In a ▸ steady state, the incoming energy equals the outgoing energy. Then the earth's temperature will not change.
- Outgoing *energy flow*, consisting of
  - direct reflection (1/3)
  - long-wave (heat) radiation (2/3)
  - where the latter is a function of, in particular, temperature and greenhouse gas concentration in earth's atmosphere.
- What happens out of steady state? For example, without greenhouse gases and atmosphere, ground temperature would be -19. How can we understand this statement?

- A situation such that if you start there you remain there.
  - In economics we use *equilibrium to describe what the model predicts.*
  - In natural sciences they use equilibrium to describe what economists call *Steady States, a stationary situation.*
- A bath tub where the amount of water going in is the same as the amount of water going out (both evaporation and sink)/
- In an economy, a steady state involves investment equating the amount of capital depreciated and the distribution of income remaining constant.

Go Back

Radiation

- Visible sunlight and infrared heat waves are both electromagnetic radiation, but with different frequencies (freq=Speed of light/wave length).
- Frequency of radiation emitted depends on temperature. Compare with dimmer on halogen lights,

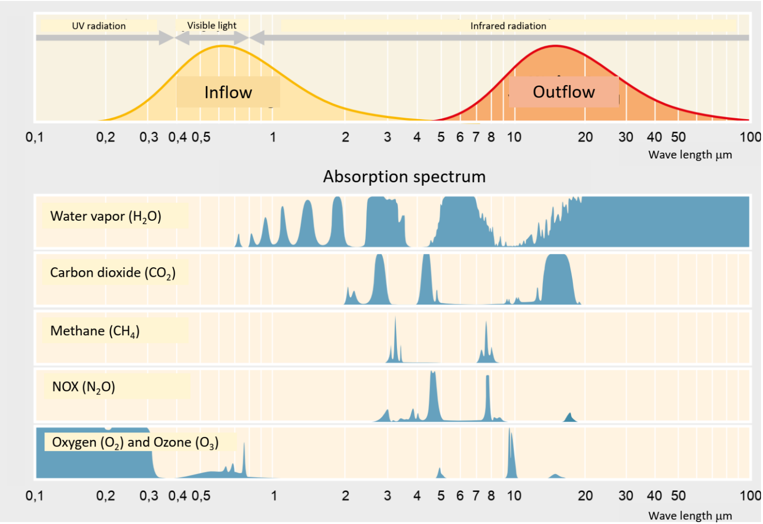


Comparison of the emission spectra of the sun and the earth. Note the huge disparity in the amount of energy emitted by the sun (left-hand scale) and the earth (right-hand scale).

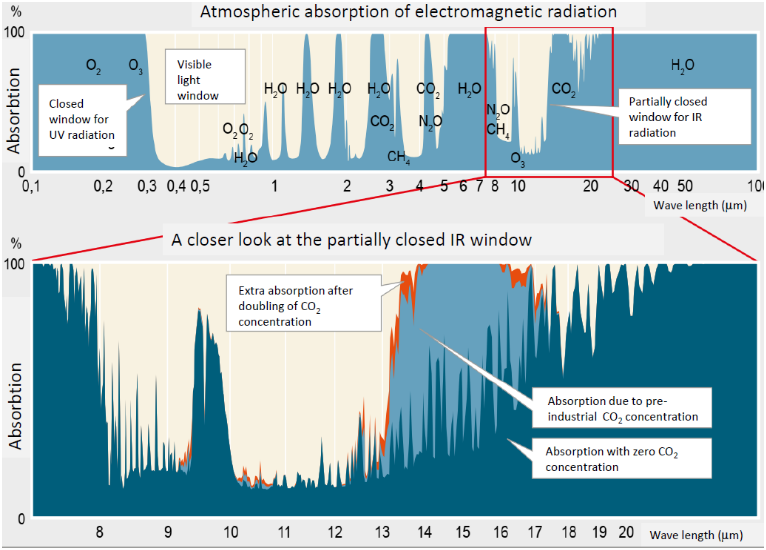
-

- When electromagnetic radiation passes through gases energy can be absorbed by the radiation making the molecules vibrate.
- For this to occur, the molecules resonance frequency (like the particular frequency a guitar string vibrates) must be aligned with the frequency of the radiation.
- CO_2 (and other molecules with three or more atoms) have resonance frequencies aligned with infrared radiation. Sunlight has a frequency much higher.
- Thus, CO_2 absorbs energy from heat radiation but not from sunlight.
- Gases with molecules with two atoms have much higher resonance frequencies but not as high as the frequency of visible light. Thus, oxygen (O_2) and nitrogen (N_2), making up 99% of the atmosphere are not greenhouse gases.
- Compare to a band playing in a bar. The bass guitar can make some objects, e.g. cups and cutlery vibrate, but a high pitched tone from the guitar has no effect.

Absorption of different radiation



More on absorption



Source: Bernes, Claes, (2016), "En varmare värld", Naturvårdsverket.

Energy transmission in atmosphere

- Even the small amount of CO_2 in the atmosphere (0.04%) makes it quite opaque for heat waves. One might think then that adding more does not have any effect (Ångström - Arrhenius controversy).

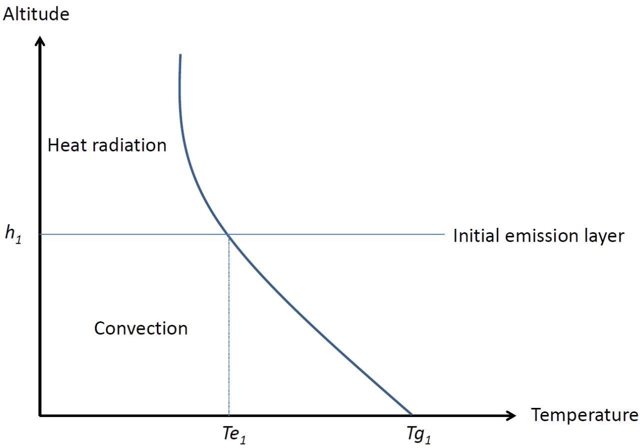
- Turns out to be wrong. Heat is transferred up in the atmosphere since it is colder the higher the altitude. Eventually, the radiation can escape into space. The altitude this occurs is called *emission level.*

- More CO_2 implies the emission level is moved up, where it *ceteris paribus* is colder.

- If the temperature at the emission level is colder, less energy is transmitted. This leads to a surplus – less energy escapes than comes in from the sun.

- The accumulation of energy increases the temperature in the atmosphere until the temperature at the emission level again is high enough to imply that the energy flow out in space is the same as the flow into earth.

Heat transfer and temperature gradient



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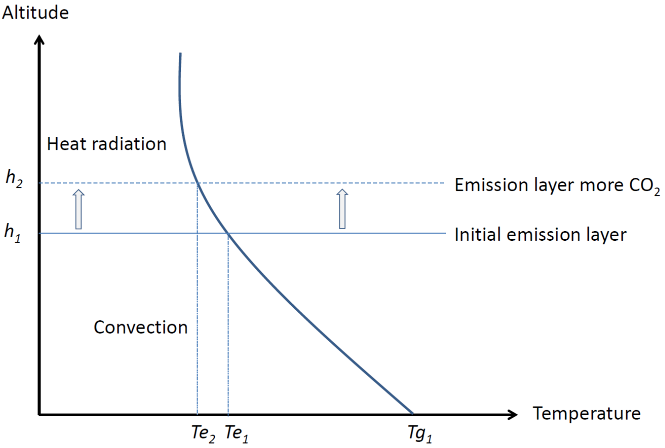


Figure: Lower temp at emission level -> less energy outflow. Surplus in energy budget.

Surplus leads to higher temperatures

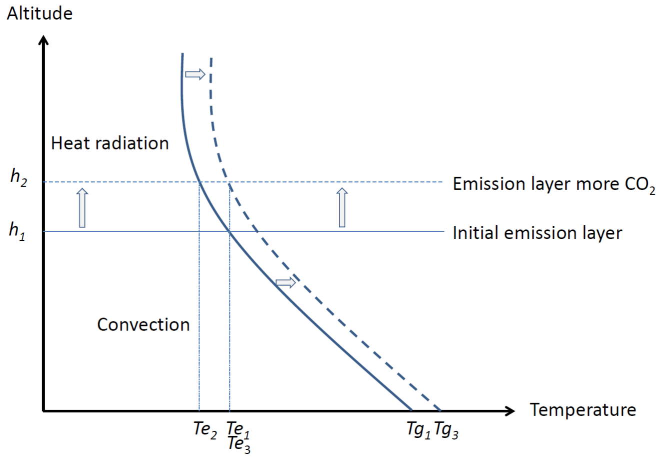


Figure: Heat accumulation gradually increases temperature. Gradient shifts rightwards until temp at h_2 has returned to Te_1 and ground temperature increased to Tg_3.

The Role of Greenhouse Gases

- What is the temperature at the emission level?
- In at steady state, it must equal inflow minus direct reflection, 342-100=242 $W/m2$.
- Use *Stefan-Boltzmann* law that says the heat radiation per square meter is a function of temperature satisfying

$$\text{energy flow} \quad = 5.67 \ * \ 10^{-8} \ * \ T^4 \ W/m2.$$

  where T is temperature in Kelvin degrees (centigrades above absolute zero). Solving $242 = 5.67 * 10^{-8} * T^4$ yields $T = 256$, which is $255 - 273 = -18^\circ C$. This would be the ground temperature without greenhouse gases.
- Greenhouse gases work like a blanket, heat is transported up implying a negative temperature gradient of $0.65^\circ C$ per $100m$. Emission layer at around 5000 meter. Thus, at ground level $50 * 0.65 = 32.5^\circ C$ warmer. A healthy ground temperature of $32.5 - 18 = 14.5^\circ C$.
- Without the Greenhouse Gas blanket, life as we know it could not have started

- Consider a system (e.g., the earth) in a situation of net energy flow = incoming flow - outgoing flow = 0.

- In such a case, the *energy budget is balanced,* defined as the difference between inflow and outflow of energy, and no heat is accumulated or lost (i.e. the temperature is in steady state).

- Suppose now that the energy budget is perturbed by a permanent positive amount f (inflow increased and/or outflow decreased).

- Now the budget is no longer balanced but in surplus and the system would no longer be in steady state.

- This leads to an accumulation of heat in the system, so the temperature rises in the system, faster the larger is the energy budget surplus.
- The speed of the temperature increase also depends on heat capacity of the system (mass and material).
    - E.g., compare a balloon filled with air and a balloon filled with water.
- A new balance (steady state) will be achieved.
    - It will require that there is an additional outflow that equals F (the forcing).
    - It will take time.
- As the temperature goes up, outgoing flow increases with higher temperature (sometimes called *Planck feedback*).

Energy outflow (planck Feedback)

- Outflow is an *increasing* function of temperature (Stefan-Boltzmann law). So a higher temperature *reduces* surplus.

- Define the increase in the outflow as $O\left(T_t\right)$, where T_t is the increase in temperature relative to pre-industrial times (now it is around 1.2°C). The marginal increase in outflow for a temperature increase is then the derivative of $O\left(T_t\right)$ with respect to T_t denoted $O'\left(T_t\right)$. How large is then $O\left(0\right)$?

- In reality, $O'\left(T_t\right)$ depends on T_t but if we consider small changes of temperature (a few degrees), we can approximate it to be constant, as evaluated at $T_t = 0$. Denote this constant κ_{Planck}.

- We can approximate the increase in outflow as $O\left(T_t\right) \approx \kappa_{Planck} T_t$ and the energy budget is then $f - O\left(T_t\right)$.

- The energy budget $f - O\left(T_t\right) \approx \kappa_{Planck} T_t$ affects the temperature. A surplus in the budget increases temperature and vice versa.

- Approximate the change in temperature per unit of time ($\frac{dT_t}{dt}$) as proportional (with constant σ) to surplus in budget. Then,

$$\frac{dT_t}{dt} = \sigma\left(f - \kappa_{Planck} T_t\right)$$

- What determines σ? Will there be a new equilibrium? Yes, when $T_t = \frac{f}{\kappa_{Planck}}$

- Using Stefan-Boltzmann law and temperature at the emision level of -18°C, $\kappa_{Planck} \approx 3.8 \frac{W/m^2}{^{\circ}C}$. Due to feedbacks, actual outflow will likely rise substantially less. A typical value imputed is $\kappa_{Planck} \approx 3.2 \frac{W/m^2}{^{\circ}C}$.

Forcing

- As discussed above, more greenhouse gases pushes emission level outwards which creates a surplus in energy budget relative to preindustrial situation.
- Surplus depends on greenhouse gas concentration.
- Most important is water vapor. Second is CO_2.
- Human activities has increased concentration of CO_2 and other greenhouse gases, e.g., methane. We also emit particles and aerosols that have a direct negative effect on reflection and a quite uncertain negative effect on the energy balance via changed cloud formation.
- Surplus (forcing) 1.7 and 1 W/m^2, respectively, yielding a total value for forcing of 2.7 W/m^2,.
- Disregarding the feedbacks, only considering the Planck feedback, we can calculate the long run effect of that on Earth's temperature as

$$\frac{2.7}{3.8} \approx 0.7^\circ C$$

Forcing in 2019 relative to 1750

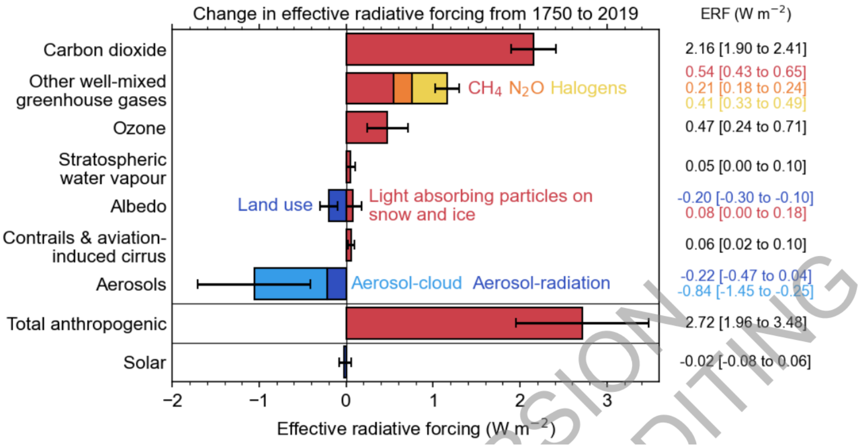
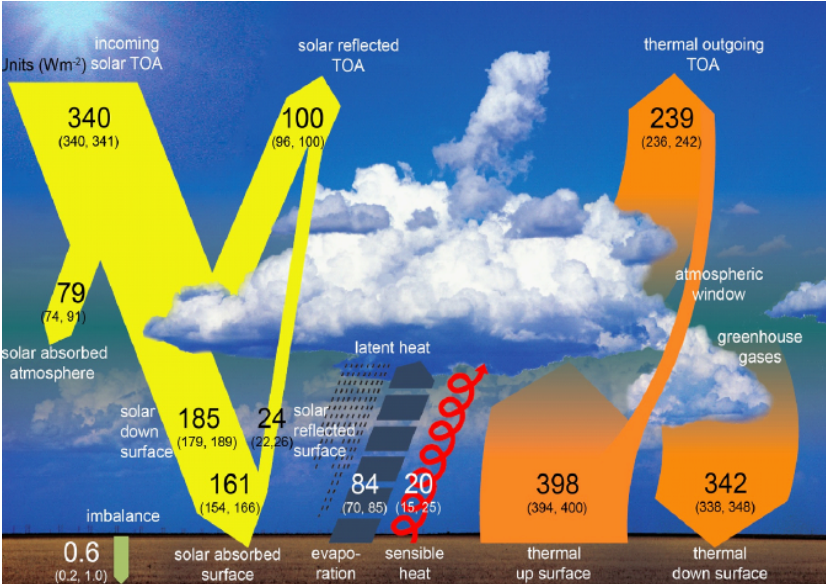


Figure: Fig 7.6 IPCC 6th report page 7-182.

Energy Flows



Orders of magnitude

- Area of Earth's surface is 510 million km^2. This is 510*$10^6 * 1000^2 = 5.1 \times 10^{14} m^2$. Thus, the inflow net of reflection is 240*$5.1 \times 10^{14} = 1.22 \times 10^{17} W$.
- A nuclear power plant is around 1000 MW, i.e., $10^9 W$. Thus, the inflow of solar energy is equivalent to $1.22 \times 10^8 = 122$ million nuclear power plants (NPP). We currently have around 440 in operation.
- The human induced forcing of 2.7W/m^2 is equivalent to $2.7 * 5.1 \times 10^{14} / 10^9 = 1.4$ million NPP.
- Global yearly energy use is around 600 million TJ, i.e., $6 * 10^{2+6+12} = 6 * 10^{20} J$. Dividing by the number of seconds per year, we get the average power use. 6*$10^{20} / (365 * 24 * 3600) \approx 1.9 \times 10^{13} W$ or 19000 NPP.
- Thus, solar inflow is $\frac{1.22 \times 10^{17}}{1.9 \times 10^{13}} \approx 6400$ times global energy use. If we could harness 0.1%, it would allow 6 times current energy use.

Feedback effects

- Gross flows as very large relative to direct greenhouse effect.
- Changed climate affects outflow indirectly. Example, more CO_2, leads to
  - higher concentration of water vapor, increase greenhouse effect.
  - changed cloud formation, change back radiation.
- We approximate these as reduction in outflow being linear in temp deviation, i.e., $\kappa_{other} T_t$.
- Additionally, the reflection of incoming sunlight may be changed
  - changes in ice-cover (albedo) and (again) changed cloud formation.
- Approximate also these as reductions in inflow being linear, $\kappa_{refl} T_t$.

- Let us include feedbacks in energy budget:

$$\begin{aligned}\frac{dT_t}{dt} &= \sigma\left(f + \kappa_{other}T_t - \kappa_{Planck}T_t + \kappa_{refl}T\right)\\ &= \sigma\left(f - \left(\kappa_{Planck} - \kappa_{other} - \kappa_{refl}\right)T_t\right).\end{aligned}$$

- The steady state for a given forcing f now becomes

$$T(f) = \frac{f}{\kappa_{Planck} - \kappa_{other} - \kappa_{refl}}$$

- A realistic value of $\kappa_{Planck} - \kappa_{other} - \kappa_{refl}$ is around 1.2 while $\kappa_{Planck} = 3.3$, but with large uncertainty. It is 1.2, the current f of 2.7 yields an increase in temperature of $2.7/1.2 = 2.25^{\circ}C$ rather than $0.8^{\circ}C$

- Direct effect of CO_2 emission on f, (as well as of κ_{Planck}) fairly certain. Not the case for feedbacks.

Current Feedbacks

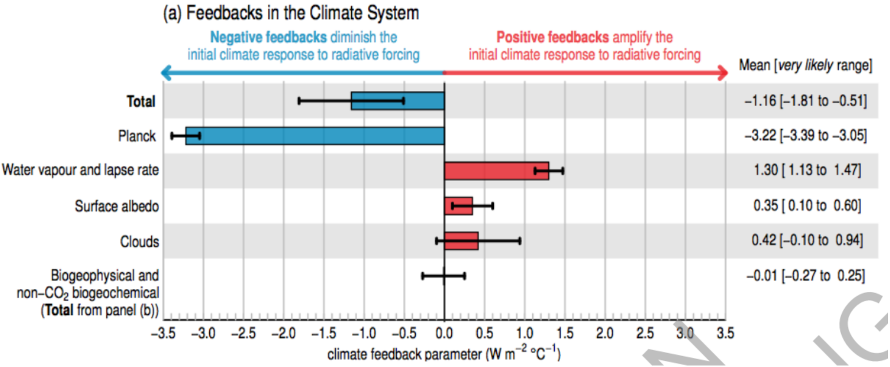


Figure: Figure TS.17 IPCC 6th report.

- Higher concentration of CO_2in atmosphere reduces outgoing (infra red) energy flow. Well approximated by a logarithmic function (Arrhenius greenhouse law, 1896). A concentration S of CO_2 in the atmosphere and the pre-industrial level S_0, yields

$$f_{CO_2}(S) = \frac{\eta}{\ln 2} \ln\left(\frac{S}{S_0}\right)$$

- $\eta \sim 3.7$. Combine with $T(f) = \frac{f}{\kappa_{Planck} - \kappa_{other} - \kappa_{refl}}$ gives

$$T(f(S)) = \frac{\eta}{\kappa_{Planck} - \kappa_{other} - \kappa_{refl}} \frac{1}{\ln 2} \ln\left(\frac{S}{S_0}\right).$$

- $\frac{\eta}{\kappa_{Planck} - \kappa_{other} - \kappa_{refl}}$ is labelled the *Equilibrium Climate Sensitivity (ECS),* measures long-run temperature impact of CO_2 doubling.
- IPCC 6th report: ECS is “likely" 2.5 to 4°C, with a "best estimate" of 3. Narrower than the 5th report's 1.5 to 4.5. "Likely" means a 2/3 confidence interval. A 90% interval is 2-5°C.

- Equation $\frac{dT_t}{dt} = \sigma\left(f - \left(\kappa_{Planck} - \kappa_{other} - \kappa_{refl}\right) T_t\right)$ does not take into account heating of oceans/atmosphere separately.
- Two other terms in energy budget for atmosphere, capturing energy flow from atmosphere to ocean and *vice versa.*
- These new terms *do not balance* if temperature is different (in an average sense).
- New law-of-motion for atmosphere

$$\frac{dT_t}{dt} = \sigma_1\left(f - \left(\kappa_{Planck} - \kappa_{other} - \kappa_{refl}\right) T_t - \sigma_2\left(T_t - T_t^L\right)\right)$$

where T_t and T_t^L, respectively, denote the atmospheric and ocean temperature in period t.

- Complete by setting

$$\frac{dT^L(t)}{dt} = \sigma_3 \left[T(t) - T^L(t) \right].$$

- Heating slows down, but is the same in the long run ($T(\infty) = T^L(\infty)$).

- One can (climate scientists do) use even more layers.

- Make a discrete time approximation. Yields a system of difference equations;

$$
\begin{aligned}
T_t &= T_{t-1} + \sigma_1 \left(\begin{array}{c} f_{t-1} - (\kappa_{Planck} - \kappa_{other} - \kappa_{refl})\, T_t \\ -\sigma_2\, (T_{t-1} - T^L_{t-1}) \end{array} \right) \\
T^L_t &= T^L_{t-1} + \sigma_3 \left(T_{t-1} - T^L_{t-1} \right)
\end{aligned}
$$

instead of

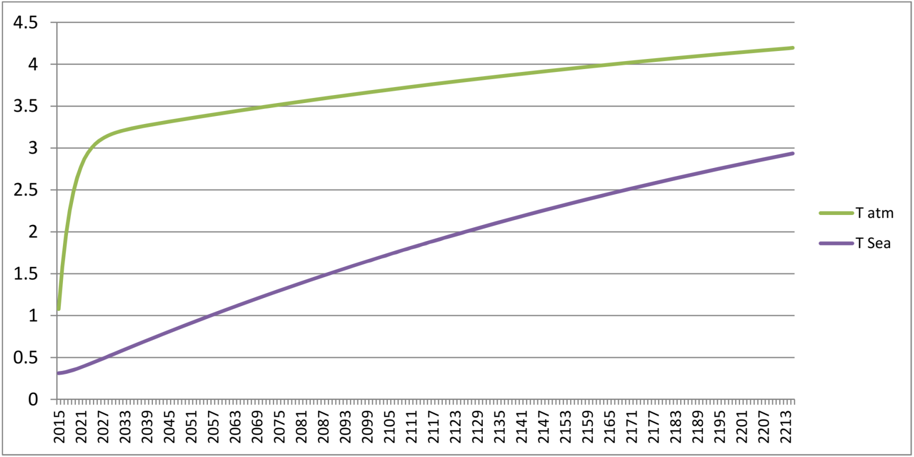
$$
\begin{aligned}
\frac{dT_t}{dt} &= \sigma_1 \left(f - (\kappa_{Planck} - \kappa_{other} - \kappa_{refl})\, T_t - \sigma_2 \left(T_t - T^L_t \right) \right) \\
\frac{dT^L_t}{dt} &= \sigma_3 \left(T_t - T^L_t \right)
\end{aligned}
$$

- Can easily be simulated in a spread-sheet program.

Calibration

- Folini et al. (2021), show that the carbon cycle model above closely replicates the mean behavior of the most advanced Earth System Models (CMIP5), if parameters are chosen appropriately.
- They choose, $\sigma_1 = 0.137$, $\sigma_2 = 0.73$, $\sigma_3 = 0.00689, \eta = 3.45$ and $\kappa = 1.06$ implying an ECS of 3.25. Note that σ's depend on time interval in discete approximation.
- Folini et al. choose initial temperatures $T_{2015} = 1.2778$, and $T^L_{2015} = 0.3132$ based on what the model predicts given historic emissions. An alternative would be to use the observed atmospheric ground temperature. The average temperature 2010-2019 over the average for the period 1880-1920 is 1.078, which would be a reasonable starting value.
- Note that σ_1 is much larger than σ_3. Atmospheres energy balance settles to a temporary steady state of 0 quickly.

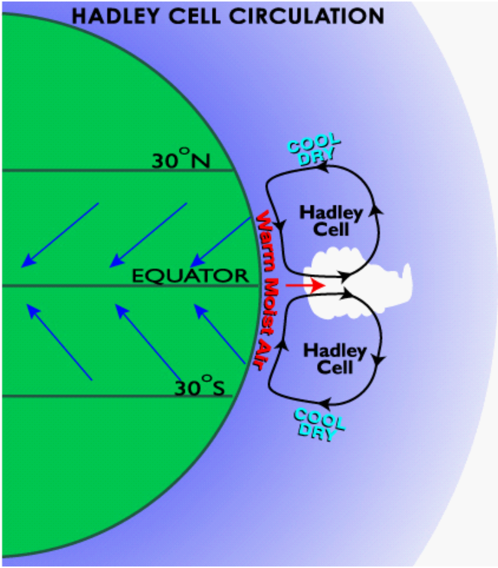
Simulation of a doubling of current forcing



More on climate models in practice

- Models of climate *around the world*.
- Circulation models (of water, air).
- Energy is not evenly radiated to the earth. Highest around equator.
- Creates systematic flows of air and water.
- Used to forecast weather, but also climate.

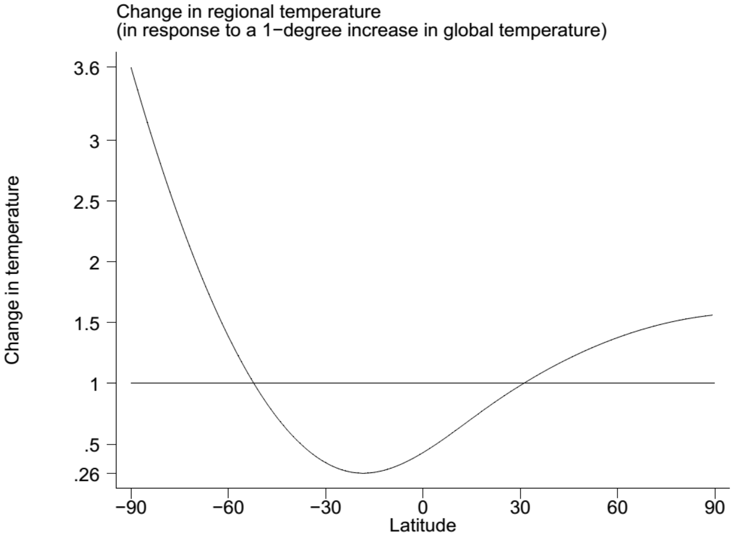
Circulation cells



- Ocean currents also transport heat from equator towards poles.
- More accurate descriptions need to model land masses and mountains.
- Climate models build on deterministic laws of physics but are chaotic in nature. This implies:
    - A "butterfly effect": small variation in initial state, e.g., its distribution of energy, leads to unsystematic and large differences in weather a few weeks later.
    - But unconditional distribution is stable, e.g., mean and variance of temperature and wind speeds.
    - Best forecast for forecasts beyond a few weeks is unconditional distribution.
- State-of-the-art climate models are build on these principles.

Downscaling (pattern scaling)

- Circulation models are (very) large and (very) time-consuming to run.
- Simplification: use a statistical representation of how a change in global mean temperature affects different locations.
- Turns out that global mean temperature is a quite good summary statistic for other aspects of climate –an approximate sufficient statistic.
- Relation between GMT and other aspects can be estimated using output from advanced Earth System Models.
- Simplest case: use latitude. Estimate a different sensitivity β_i for each latitude.
- $T_{i,t} = \bar{T}_i + \beta_i * T_t + z_{i,t}$

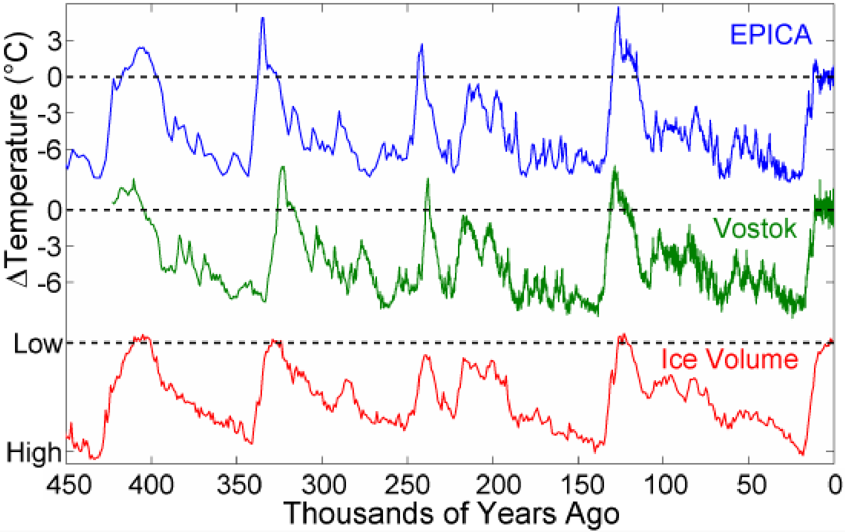

Change in regional temperature
(in response to a 1−degree increase in global temperature)
Change in temperature
3.6
3
2.5
2
1.5
1
.5
.26
−90
−60
−30
0
30
60
90
Latitude

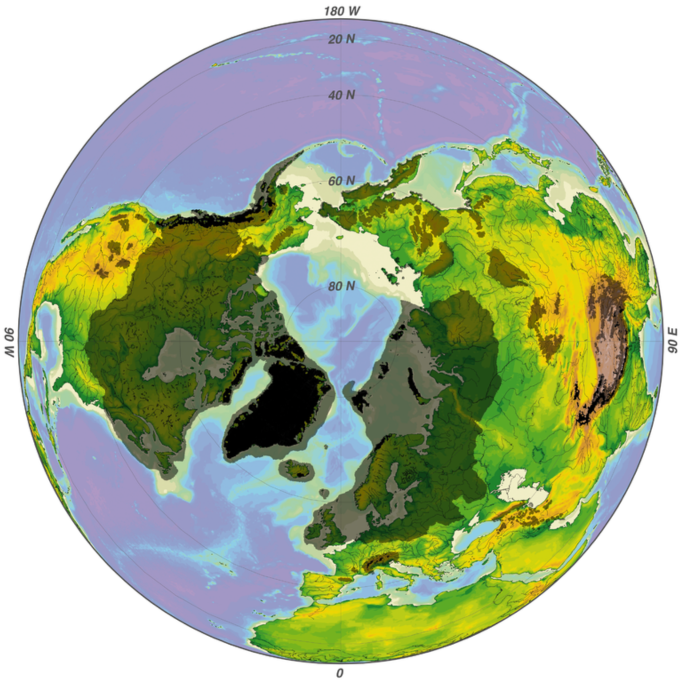
The historic climate

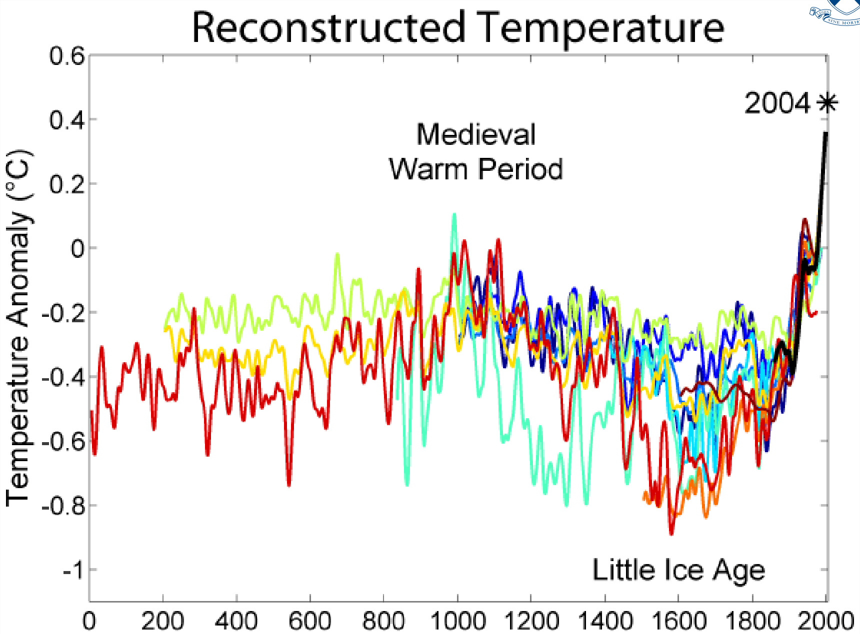
- Use various proxy data for temperature, e.g., tree rings, corals, plankton, pollen. . .
- Also data on historic greenhouse gas concentrations, trapped in ice.
- Positive correlation, with concentrations lagging temperature, suggesting positive feedback.

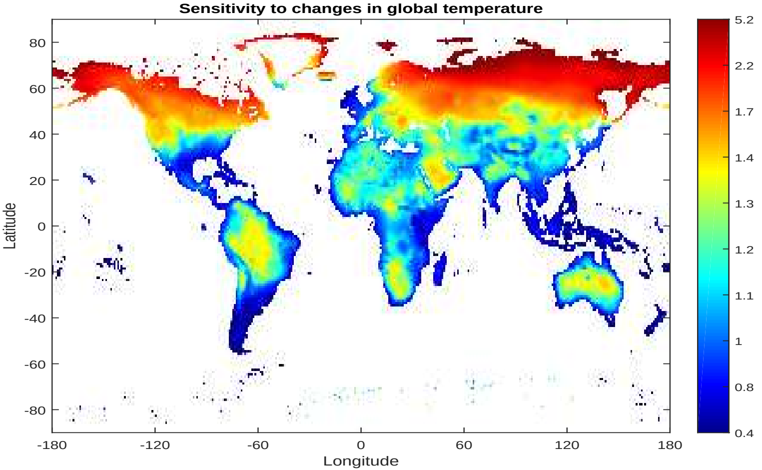
- Ice ages are much longer than interglacial periods.
- About 100,000 years between each interglacial period.
- Small changes in solar influx get amplified by feed-back.
- A key mechanism may be ice-albedo feedback (Arrhenius).
- A small negative F leads to a build-up of the icecap.
- An increased albedo of the earth amplifies the initial effect.
- Additional effects may come from greenhouse gases.
- For more recent climate info, see
  Link `https://youtu.be/gGOzHVUQCw0`

Ice Age Temperature Changes




180 W
20 N
40 N
60 N
80 N
90 W
90 E
0


Reconstructed Temperature
Temperature Anomaly (°C)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2004
Medieval
Warm Period
Little Ice Age



Sensitivity to changes in global temperature
80
60
40
20
0
-20
-40
-60
-80
Latitude
-180
-120
-60
0
60
120
180
Longitude
5.2
2.2
1.7
1.4
1.3
1.2
1.1
1
0.8
0.4

- Recall that the equilibrium climate sensitivity is affected by feedbacks

$$T(f) = \frac{\eta}{(\kappa_{Planck} - \kappa_{other} - \kappa_{refl})} \frac{1}{\ln 2} \ln\left(\frac{S}{\bar{S}}\right).$$

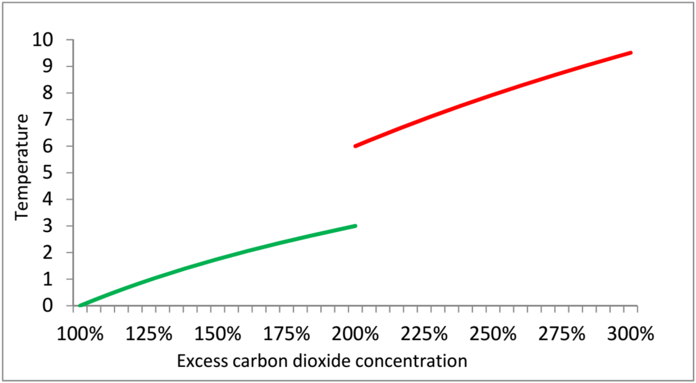
- We are quite uncertain about the value of the feedbacks.
- One thing that could happen is that it suddenly increases at some temperature. For example, suppose

$$\kappa_{other} + \kappa_{refl} = \left\{ \text{ 2.72 else2.1 if } T < 3^o C \right.$$

- This produces a jump in the relation between CO_2 and long-run temperature.
- Also simple to make this irreversible
- $\kappa_{other} + \kappa_{refl} = \begin{cases} 2.1 \text{ if } T \text{ ever was larger than } 3^o C \\ 2.72 \text{ else} \end{cases}$

Tipping points

- Suppose $\eta = 3.7$ and $\kappa = 3.3$. and $x = 2.1$ if $T < 3^o C$ and 2.72 else. Then, the relation between CO_2 concentration and long-run temperature looks like follows



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- Tipping points like then one described are possibilities and many of them are known to exist on local and regional scale, abrupt responses, tipping points and even reversals in the direction of change cannot be excluded (high conÖdence)." (IPCC AR6 WG1 Box TS 9).

- If they exist on a global scale and if so at which temperatures is much more debated and not likely unless global warmning goes much further than projected for the coming century also in quite pessimistic scenarios.

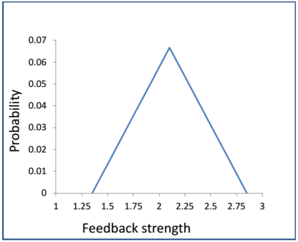
- IPCC 6th report claims “there is no evidence of such non-linear responses at the global scale in climate projections for the next century, which indicate a near-linear dependence of global temperature on cumulative GHG emissions. (IPCC AR6 WG1, chap. 1 p. 202).

- Uncertainty in the feedback produces a skewed distribution of the climate sensitivity.

- Since $\lambda \equiv \frac{\eta}{\kappa_{Planck} - \kappa_{other} - \kappa_{refl}}$ is a non-linear transformation of of κ_{other} and κ_{refl}, uncertainty about λ becomes very skewed with possibilities of very large values. is a non-linear transformation of x, uncertainty about λ becomes very skewed with possibilities of very large values.

- Suppose the uncertainty about $\kappa_{other} + \kappa_{refl}$ by a symmetric triangular density function with mode 2.1 and endpoints at 1.35 and 2.85. The mean, and most likely, value of $\kappa_{other} + \kappa_{refl}$ translates into a climate sensitivity of 3.

Feedback uncertainty



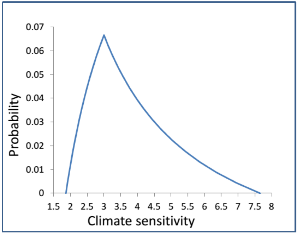


Figure: Example of symmetric uncertainty of feedbacks producing right skewed climate sensitivity.

The carbon cycle

The carbon cycle

- The burning of fossil fuel (coal, oil, natural gas) leads to CO_2 emissions into the atmosphere, which (as we have seen) leads to significant warming, at least in most of the probabilistic distribution of outcomes.

- Warming is potentially damaging to our economies—and we will look at damage measurements later—and if so the burning of fossil fuel is a negative externality. There is need for government action.

- For policy analysis as well as for forecasts, we need to know the dynamic mapping from path of emissions to path of CO_2 concentrations.

- We will mainly look at two approaches:

  1. Stock-flow approach. Idea: different interacting reservoirs of carbon, with a continuous flow between these. A stable system always tending towards a steady state.

  2. Non-structural (reduced form): define a depreciation function that specifies how much of an emitted unit remains in the atmosphere over time. Can be specified rather generally, but we look at a simple form.

     - We will also look at a formulation which blends the carbon cycle with a climate model directly: the CCR (carbon-climate response).

- Note difference between measuring emissions in CO_2 and C. A mole of carbon atoms weighs 12 grams and a mole of oxygen weighs 16. Then a kg of carbon produces $\frac{2*16+12}{12} \approx 3.67$ kg CO_2.

- Assume 2 reservoirs S and S^L. $S(t)$ represents the atmospheric carbon concentration in period t and $S^L(t)$ represents that in the deep oceans.
- Flows from S to S^L: proportional to S, with proportionality factor ϕ_1.
- Flow in other direction proportional to S^L, with proportionality factor ϕ_2.

Changes in stocks equal net flows (in minus out), apart from emission inflow E. This gives

$$\begin{aligned}\frac{dS(t)}{dt} &= -\phi_1 S(t) + \phi_2 S^L(t) + E(t)\\ \frac{dS^L(t)}{dt} &= -\phi_2 S^L(t) + \phi_1 S(t).\end{aligned}$$

With $E(t) = 0$, steady state satisfies

$$\begin{aligned}0 &= -\phi_1 S + \phi_2 S^L\\ 0 &= \phi_1 S - \phi_2 S^L\end{aligned}$$

which cannot be uniquely solved: all solutions satisfy $S = \frac{\phi_2}{\phi_1} S^L$. Why? Note that $S(t) + S^L(t) = \int_{s=0}^{t} E(s)ds$ at all times. The total amount of carbon increases as emissions continue.

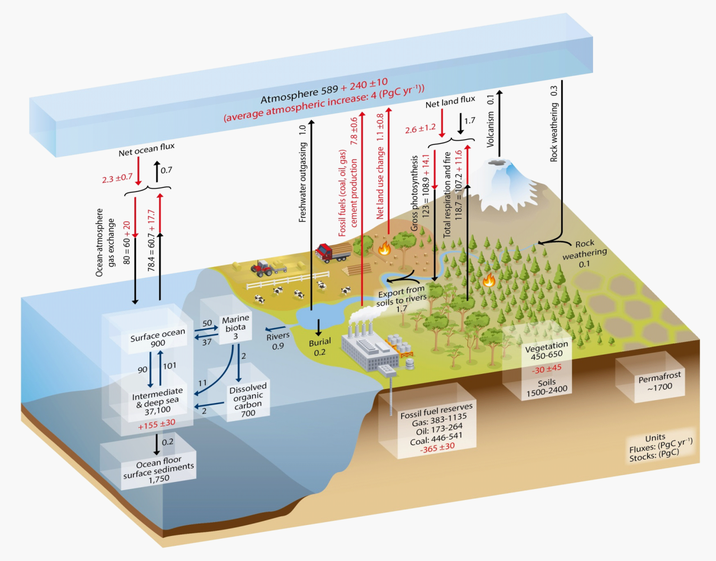
$$
\begin{aligned}
S_t - S_{t-1} &= -\phi_1 S_{t-1} + \phi_2 S^L_{t-1} + E_{t-1}. \\
S^L_t - S^L_{t-1} &= \phi_1 S_{t-1} - \phi_2 S^L_{t-1}
\end{aligned}
$$

- Same steady state and approximately the same dynamics.
- Linear systems (in discrete or continuous time) can be solved analytically.
- Suppose emissions stop at t. Then deviation from steady state $S_t = \frac{\phi_{21}}{\phi_{12}} S^L$ vanishes over time as determined by the factor

$$
(1 - \phi_1 - \phi_2)^{t+s}
$$

- The law of motion for the stocks follows ($s \geq 0$)

$$
\begin{aligned}
S_{t+s} &= \frac{\phi_2}{\phi_1+\phi_2}\left(S_t + S^L_t\right) - \frac{\phi_2 S^L_t - \phi_1 S_t}{\phi_1+\phi_2}\left(1 - \phi_1 - \phi_2\right)^s \\
S^L_{t+s} &= \frac{\phi_1}{\phi_1+\phi_2}\left(S_t + S^L_t\right) + \frac{\phi_2 S^L_t - \phi_1 S_t}{\phi_1+\phi_2}\left(1 - \phi_1 - \phi_2\right)^s.
\end{aligned}
$$



Global carbon cycle. Stocks in GtC (PgC) and flows GtC/year. Source: Intergovernmental Panel on Climate Change (IPCC) (2013), Figure 6.1.

A more general stock-flow model: three reservoirs

- S_t represents (carbon concentration in) the atmosphere in period t, S_t^U the surface ocean, and finally S_t^L the deep oceans.
- Flows still assumed to be proportional to stocks and change in a reservoir is equal to net flow into it.
- We then have

$$\begin{aligned}
S_t - S_{t-1} &= -\phi_{12} S_{t-1} + \phi_{21} S_{t-1}^U + E_{t-1} \\
S_t^U - S_{t-1}^U &= \phi_{12} S_{t-1} - (\phi_{21} + \phi_{23}) S_{t-1}^U + \phi_{32} S_{t-1}^L \\
S_t^L - S_{t-1}^L &= \phi_{23} S_{t-1}^U - \phi_{32} S_{t-1}^L .
\end{aligned}$$

- Two alternative ways to go.
  - Choose the parameters to make model dynamics match the dynamics of much more complicated carbon-cycle models as closely as possible.
  - Take linear model seriously and use measured flows.
- Let's use the pre-industrial flows and stocks for the calibration.
  - Before industrialization we had 589 GtC in atmosphere and a flow to surface ocean of 60 GtC, implying $\phi_{12} = \frac{60}{589} \approx 0.102$.
  - The flow from the surface ocean to the atmospere gives $\phi_{12} = \frac{60.7}{900} \approx 0.067$.
  - Use flow to deep ocean, giving $\phi_{23} = \frac{90}{900} = 0.100$.
  - Finally, the flow from the deep ocean to the surface ocean is the same, giving $\phi_{32} = \frac{90}{37100} \approx 0.00243$.

- Folini et al. (2021), show that the carbon cycle model above closely replicates the mean behavior of the most advanced Earth System Models (CMIP5), if parameters are chosen appropriately.

- They choose $\phi_{12} = 0.053, \phi_{21} = 0.0536$, $\phi_{23} = 0.0042$ and $\phi_{32} = 0.001422$ when the time step is a year. The initial values of the stocks are $S_{2015} = 850$, $S^U_{2015} = 765$ and $S^L_{2015} = 1799$. Note that in particular the deep oceans is much smaller than in reality. To model it that small makes the dynamics of the model more in line with the (much) more advanced models.

- If emissions stop, this system also approaches a steady state. Solve

$$\begin{aligned} 0 &= -\phi_{12} S + \phi_{21} S^{U} \\ 0 &= \phi_{12} S - (\phi_{21} + \phi_{23}) S^{U} + \phi_{32} S^{L} \\ 0 &= \phi_{23} S^{U} - \phi_{32} S^{L}. \end{aligned}$$

- Again no unique solution, but all solutions satisfy

$$\begin{aligned} S &= \frac{\phi_{21}}{\phi_{12}} \frac{\phi_{32}}{\phi_{23}} S^{L} \\ S^{U} &= \frac{\phi_{32}}{\phi_{23}} S^{L}. \end{aligned}$$

- i.e., proportions between stocks are always restored. Stocks sum to sum of past emissions.

- Structural model may be too simplified. Misses non-linearities, and other relevant variables.
- Could then instead try to match key characteristics directly (IPCC and Archer 2005).
  1. A share (ca 20-25%) stays very long (thousands of years) until CO_2 acidification has been buffered.
  2. The remainder decays with a half-life of a few centuries.
  3. A share (ca 50%) of carbon emissions is removed quite quickly (a few years to a few decades).
- These features can be modeled directly by a depreciation function (rather: "remainder function"), d_s that says how much remains of an emitted unit after s period.

$$d_s = \varphi_L + (1 - \varphi_L)\,\varphi_0\,(1 - \varphi)^s\,.$$

- Use decades.

$$d_s = \varphi_L + (1-\varphi_L)\,\varphi_0\,(1-\varphi)^s\,.$$

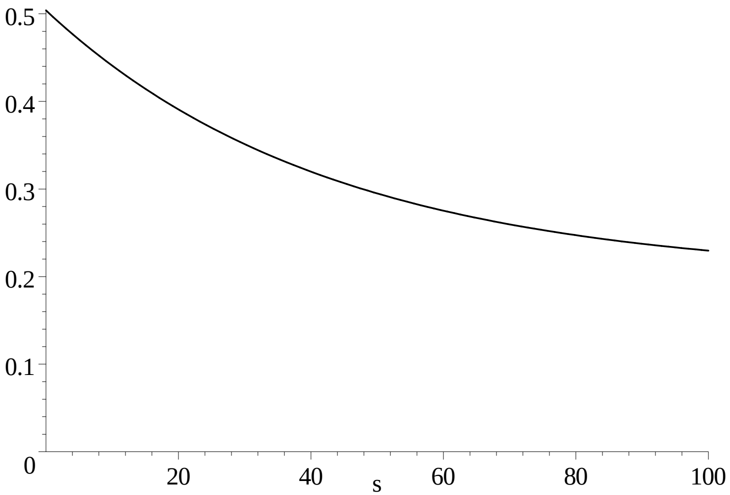
1. A share stays forever $d_\infty = .2$. So $\phi_L = .2$
2. The rest has a half life of 300 years: $(1-\varphi)^{30} = \frac{1}{2}$.
   Gives $\ln(1-\varphi) = \frac{\ln\frac{1}{2}}{30}$, so $-\varphi \approx= -0.023$.
3. Half of the emissions are gone fast: $d_1 = 0.5$,

- $d_1 = 0.5 = 0.2 + (1-0.2)\,\varphi_0\,(1-0.023)^1\,, \Rightarrow \varphi_0 = 0.38$.

Implies

$$d_s = \left[\varphi_L + (1-\varphi_L)\,\varphi_0\,(1-\varphi)^s\right]_{\varphi_L=0.2,\varphi=0.023,\varphi_{\mathbf{0}}=0.38}.$$

$$\left[\varphi_L + (1-\varphi_L)\,\varphi_0\,(1-\varphi)^s\right]_{\varphi_L=0.2,\varphi=0.023,\varphi_0=0.38}$$



Non-linearities

- The parameters in the models we have presented are likely to be affected by the emission scenario: the amount and timing of emissions.
- For example, more emissions reduce the capacity of oceans to store carbon (temperature and chemistry).
- Implies that more than 20-25% stays in atmosphere for thousands of years if cumulated emissions are large.
- With 10 times current cumulated emissions, twice as big a share is likely to remain.

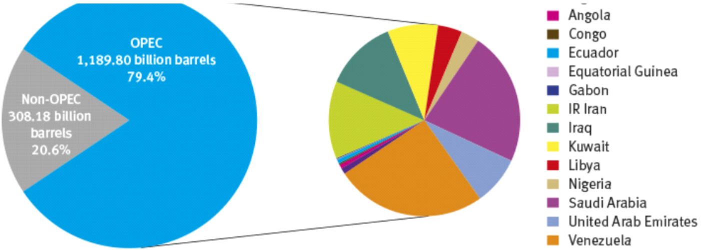
- The climate system (linking S to T) and the carbon cycle (linking E to S) are dynamic and non-linear.
- An increase in forcing has a delayed impact (increasing over time) on temperature and is concave (logarithmic).
- Emission of carbon has a decaying impact (decreasing over time) on atmospheric CO_2 concentration. The relation is convex since other sinks' storage capacities decrease when emissions have been large.
- Surprisingly, these non-linearities seem to cancel each other in most advanced climate models. The global mean temperature thus becomes approximately linear in cumulative emissions. $T_t = \sigma_{CCR} \sum_{s=0}^{t} M_s$
- According to the latest (6th) IPCC report, σ_{CCR} is "likely" (which should be interpreted as a 2/3 confidence interval) between 1.0 and 2.3 degrees Celsius per 1000 GtC (corresponding to 0.27-0.63° / $Tt$$CO_2$). This constant is called CCR (Carbon Climate Response, sometimes CRE or TCRE).

- Given a linear relation between accumulated emissions and temperature, a remaining carbon budget can be calculated.
- The large uncertainty about the CCR coefficient, makes this problematic.
- We have now emitted around 650 GtC. If CCR is 1, we have committed 0.6*1=0.65$^\circ C$ and can emit another 850 GtC before reaching 1.5$^\circ C$.
- This would take around 85 years with current emission rates.
- BUT, if CCR is 2.3, we have already passed 1.5 heating.
- This is genuine uncertainty. Probabilities are informed guesses.

How much carbon is there?

- Fossil fuels exists in many forms.
- Different costs of recovery.
- One classification is
  1. Reserves (recoverable under current economic and technological conditions)
  2. Resources (recoverable under possible future economic and technological conditions).
- Technological developments are and have been fast. Leading to continuos reclassifications.

OPEC's own estimates



OPEC proven crude oil reserves, at end 2018 (billion barrels, OPEC share)

Venezuela	302.81	25.5%	Kuwait	101.50	8.5%	Algeria	12.20	1.0%	Gabon	2.00	0.2%
Saudi Arabia	267.03	22.4%	UAE	97.80	8.2%	Ecuador	8.27	0.7%	Equatorial Guinea	1.10	0.1%
IR Iran	155.60	13.1%	Libya	48.36	4.1%	Angola	8.16	0.7%			
Iraq	145.02	12.2%	Nigeria	36.97	3.1%	Congo	2.98	0.3%			

Source: OPEC Annual Statistical Bulletin 2019.

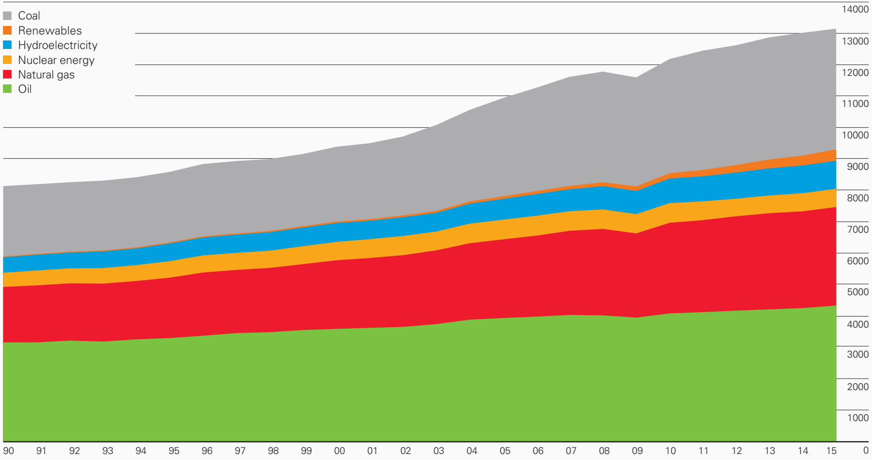
- Is 1190 billion brls a lot? A barrel is 1/7.33 tons and oil contains 85% carbon. So this is 138GtC. Likely gives 0.14-0.32$^{\circ}C$ warming using IPCC's likely CCR coefficient.

- Increase in GMT (global mean temperature T) is between 1 and 2.1 degrees Celsius per 1,000 GtC both in short and long run. This constant is called Carbon Climate Response (CCR).

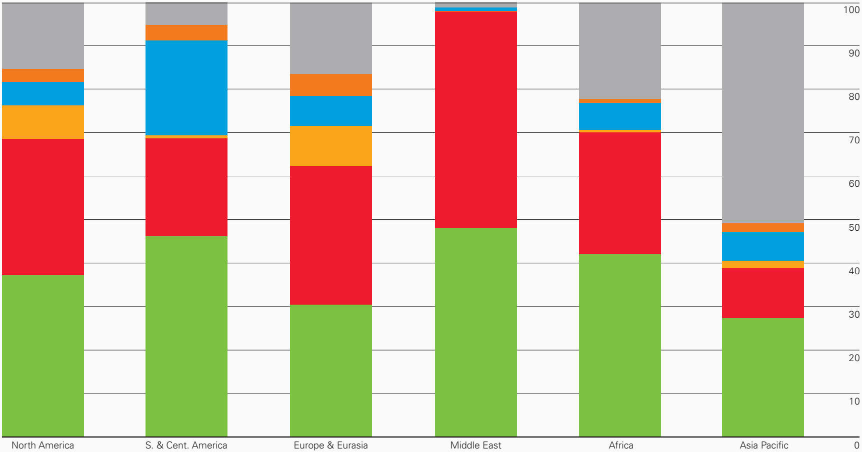
- Note that these are emissions not net contributions to the stock of CO_2.

World consumption

Million tonnes oil equivalent



Regional consumption by fuel 2015
Percentage

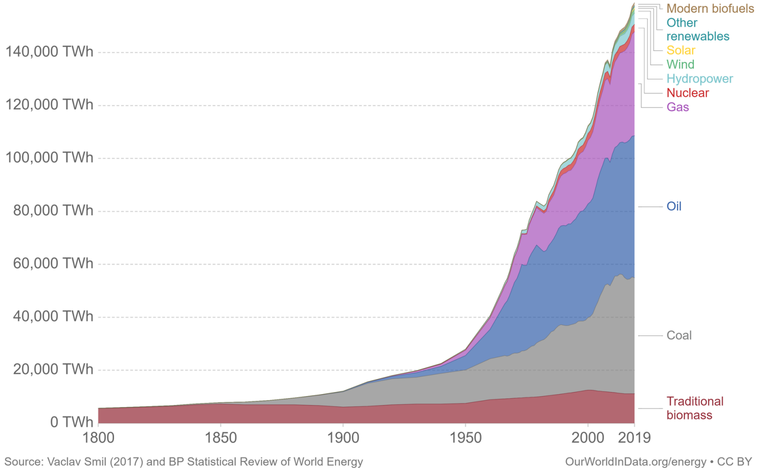


- What remains in the ground: (An assessment that depends on extracting technology, maybe more in the future)
  - Reserves of oil plus gas about 300 GtC (a bit more with fracking no more than 500).
  - Coal is more than 3000 GtC,
- Coal is what really matters.
- Emissions are (now) about 10Gtc/ per year which means that about half of that is the (permanent) net increase of CO_2.

Finite resources

- For the purposes of climate-economy modeling, we need
  - a long-run growth model consistent with data,
  - with a production function using energy as input, and
  - to also model supply of energy.
- Since industrial revolution, energy is largely about fossil fuel, a resource in finite supply.
- Today:
  - discuss supply and demand of finite resources, and
  - a primer on endogenous resource saving technical change.

Global primary energy supply (including conversion losses)



Global shares of different energy sources

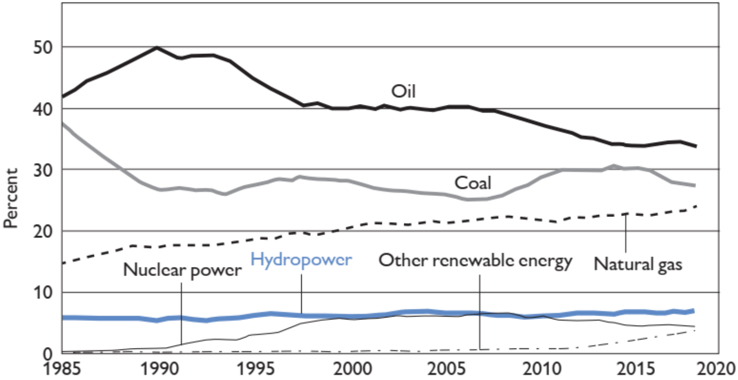


Figure: Source: BP Statistical Review of World Energy 2019

European shares of different energy sources

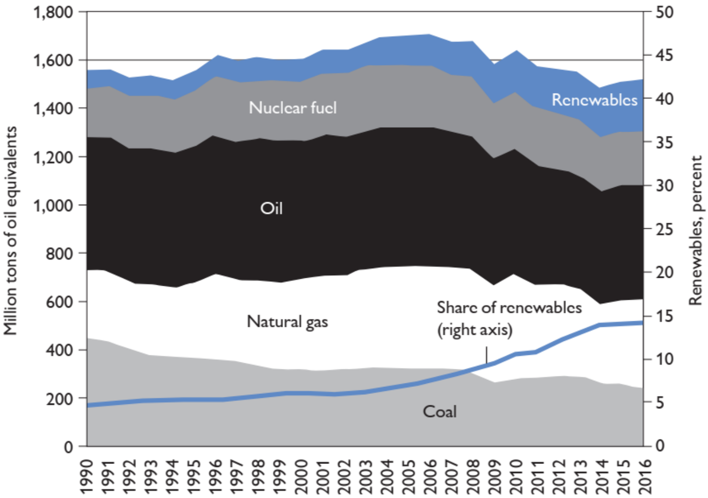
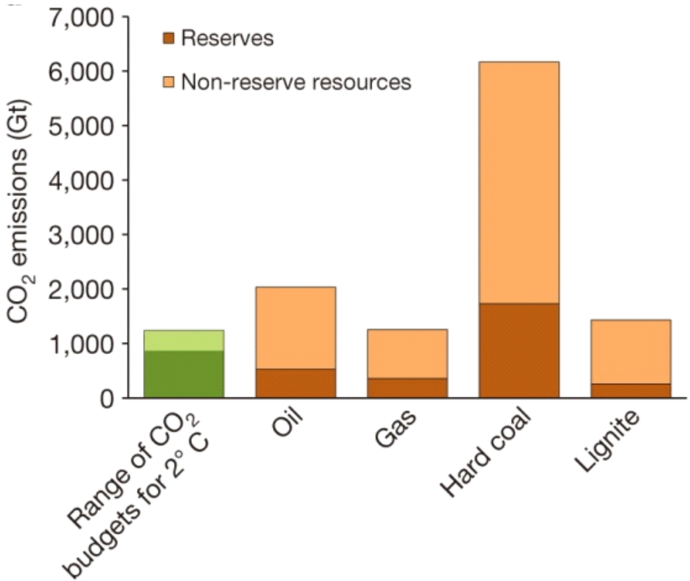


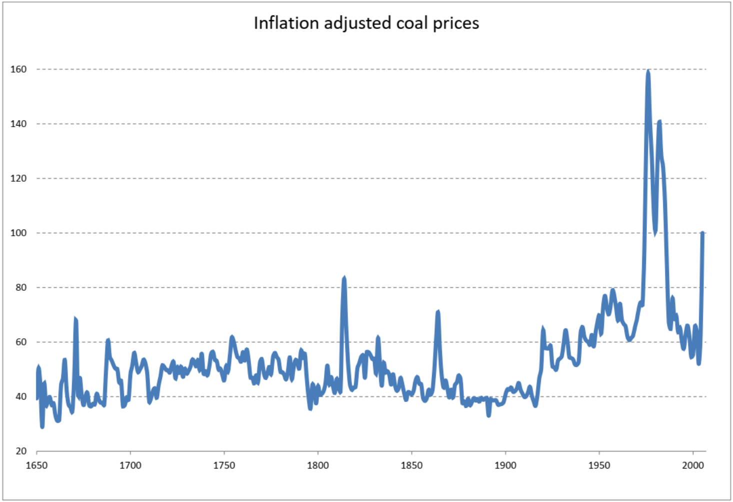
Figure: Source data: European Environment Agency

Oil price

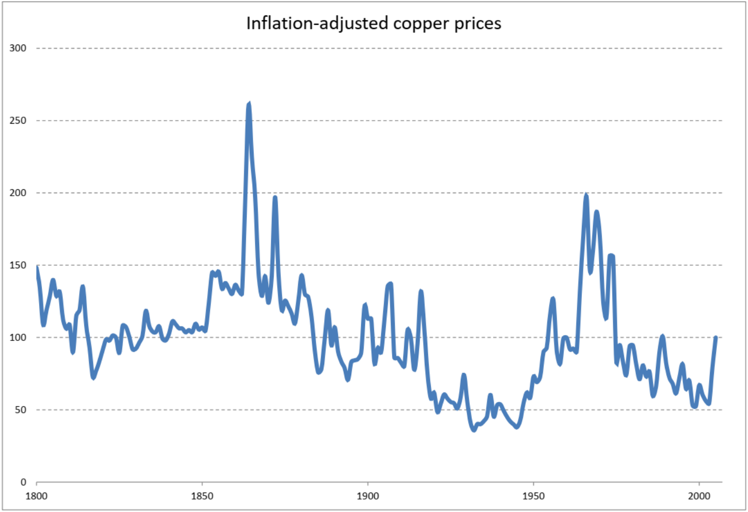


7,000
6,000
5,000
4,000
3,000
2,000
1,000
0
CO_2 emissions (Gt)
Reserves
Non-reserve resources
Range of CO_2 budgets for 2° C
Oil
Gas
Hard coal
Lignite

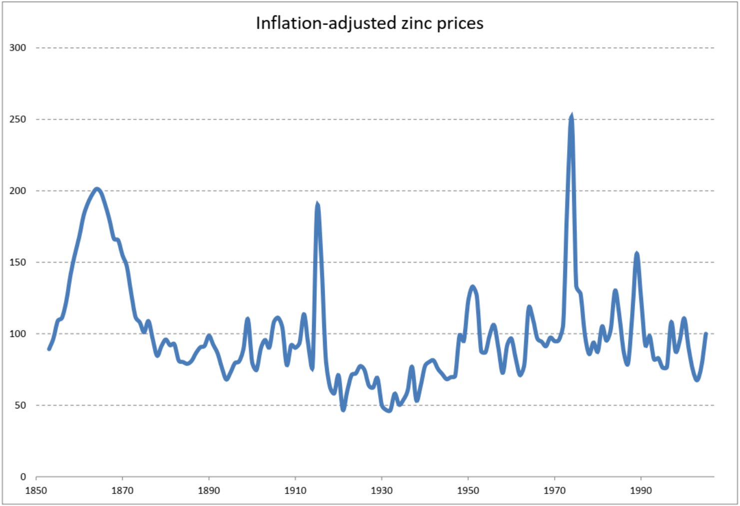
Coal price


Inflation adjusted coal prices
160
140
120
100
80
60
40
20
1650
1700
1750
1800
1850
1900
1950
2000

Copper price



Zinc price



Inflation-adjusted zinc prices
300
250
200
150
100
50
0
1850
1870
1890
1910
1930
1950
1970
1990

Real price composite of 57 minerals and energy sources

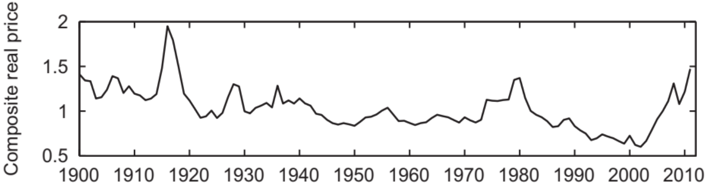
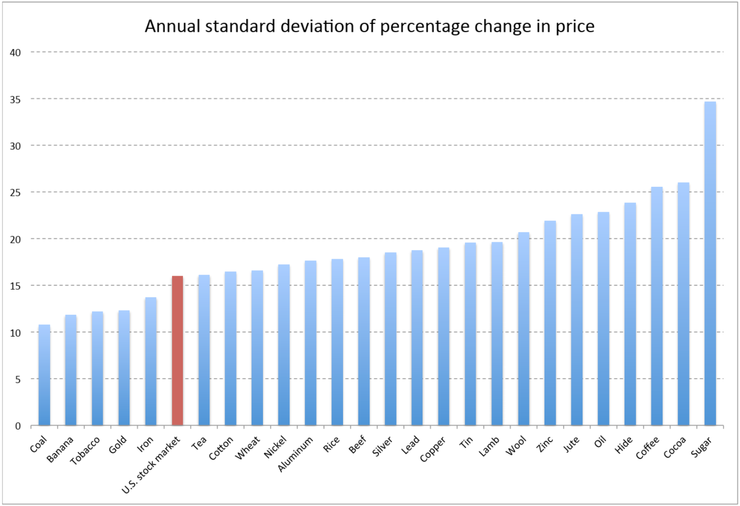


Figure: Source: Daniel Spiro, JEDC (2014).

Price volatility



A simple fossil fuel dichotomy

- Different fossil fuels have quite different supply characteristics. A general rule is that amount increases with cost of extraction.
- One end of spectrum – conventional oil. Exists in limited supply and is very cheap to extract relative to value. High profit margin.
- The other end – coal reserves. Very large quantities and price close to marginal cost. Low if any profit margin.
- But;
  - there are things in between, and
  - technological change shifts the boundaries.
- Still, even a small emission price makes coal unprofitable but this is not the case for conventional oil. (Current ETS price $\approx$ 5 cent per liter gasoline but kills coal power).

Finite Resource Theory: 1 Cake eating

- Consider planning problem under zero extraction costs.

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_t$$

subject to

$$\sum_{t=0}^{\infty} c_t \leq R_0$$

- Euler equation:

$$\begin{aligned} U'(c_t) &= \beta U'(c_{t+1}) \\ \frac{1}{c_t} &= \beta \frac{1}{c_{t+1}} \Rightarrow c_{t+1} = \beta c_t \end{aligned}$$

- Using resource constraint yields $c_t = (1-\beta) R_t$ where $R_{t+1} = R_t - c_t$. Implies $c_t = (1-\beta)\beta^t R_0$.

Finite Resource Theory: 2 Production

- Same problem, now with Cobb-Douglas production and full depreciation of capital Also cake-like.

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_t$$

$$\text{s.t.} \quad : \quad c_t + k_{t+1} = A k_t^{\alpha} e_t^{\nu} \text{ and } \sum_{t=0}^{\infty} e_t \leq R_0$$

- Saving rate is constant and equal to $\alpha\beta$ immediate to show from Euler equation. Now, two means of saving, the resource and capital. Must have equal return on equilibrium/optimum. Return on capital the marginal product and on the resource the price growth

$$\frac{\alpha A k_{t+1}^{\alpha} e_{t+1}^{\nu}}{k_{t+1}} = \frac{\alpha A k_{t+1}^{\alpha} e_{t+1}^{\nu}}{\alpha\beta A k_t^{\alpha} e_t^{\nu}} = \frac{\nu A k_{t+1}^{\alpha} e_{t+1}^{\nu} / e_{t+1}}{\nu A k_t^{\alpha} e_t^{\nu} / e_t} \Rightarrow e_{t+1} = \beta e_t$$

- Again: Solution: $e_t = (1-\beta) R_t$, where $R_{t+1} = R_t - e_t$. Hence $e_t = (1-\beta)\beta^t R_0$.

Finite Resource Theory: 3 Adding tech growth

- Now add technological growth at a gross rate of γ

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_t$$

$$\text{s.t.} \quad : \quad c_t + k_{t+1} = A\gamma^t k_t^{\alpha} e_t^{\nu} \text{ and } \sum_{t=0}^{\infty} e_t \leq R_0$$

- Again savings rate is $\alpha\beta$ and by arbitrage

$$\frac{\alpha\gamma^{t+1} A k_{t+1}^{\alpha} e_{t+1}^{\nu}}{\alpha\beta\gamma^t A k_t^{\alpha} e_t^{\nu}} = \frac{\nu\gamma^{t+1} A k_{t+1}^{\alpha} e_{t+1}^{\nu} / e_{t+1}}{\nu\gamma^t A k_t^{\alpha} e_t^{\nu} / e_t} \Rightarrow e_{t+1} = \beta e_t$$

- In balanced growth, capital and output grows at same rate and resource use fall at gross rate β.Thus $g = \gamma g^{\alpha} \beta^{\nu} = (\gamma\beta^{\nu})^{\frac{1}{1-\alpha}}$. For $\gamma > \beta^{-\nu}$ $g > 0$.
- From Euler equation, $g = \beta(1+r)$, so $1 + r = (\gamma\beta^{\nu})^{\frac{1}{1-\alpha}} / \beta$.Positive real interest rate if $g > \beta$, then resource price grows.

Finite Resource Theory: 3 Adding extraction costs

- Suppose there is a cost of extracting resources. The arbitrage intuition still works (Hotelling (1931)). Return on saving a unit of the resource with price p_t is now

$$\frac{p_{t+1} - mc_{t+1}}{p_t - mc_t}$$

which must equal return on saving in the form of capital. Yields

$$\frac{p_{t+1}}{p_t} = 1 + r_t + \frac{1}{p_t}\left(mc_{t+1} - (1 + r_t)mc_t\right)$$

- If $mc_{t+1} - (1 + r_t)mc_t < 0$. price growth is lower than the interest rate.

Challenges

- Key natural-resource “puzzles”:
  - Why no positive trend in prices?
  - Why so volatile? (And are natural resources different than other “commodities”?
  - Why upward trend in use?
- Related puzzle: why isn’t all the cheap fossil fuel extracted first?

Back to the production function

- With Cobb-Douglas production function, all income shares constant. Not too bad for capital and labor. What about energy?

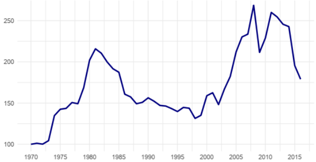


Figure: **The real price of a unit (Btu) of energy, U.S.**

Average real (using a GDP deflator) price of a Btu for the U.S., including all energy sources. **Source:** US Energy Information Administration.

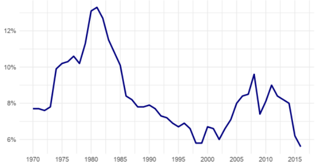


Figure: **The energy share in the U.S.**

A more reasonable production function

- Consider instead CES production function:

$$y \equiv F\left(Ak^{\alpha}l^{1-\alpha}, A_e e\right) = \left[(1-\gamma)\left(Ak^{\alpha}l^{1-\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left(A_e e\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

  with $\varepsilon < 1$. A is capital-labor augmenting technology and A_e is energy augmenting technology. A special case is Leontief ($\varepsilon = 0$): $y = \min\left\{Ak^{\alpha}l^{1-\alpha}, A_e e\right\}$
- This fits the short-run fluctuations in data really well. But non-explosive price paths is a knife-edge property (require $Ak^{\alpha}l^{1-\alpha}$ and $A_e e$ to grow at same rate). In general, income share of energy would go to zero or one depending on supply and growth rates of A and A_e.
- Need something that makes (relative) growth rates of A and A_e endogenous.

A static example of endogenous technology choice

- Assume

$$y = \left[(1-\gamma)(Ak)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left(A_e e\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$
$$s.t. G(A, A_e) = \bar{A}$$

so with $\bar{A}$ given but A and A_e endogenous: directed technical change.

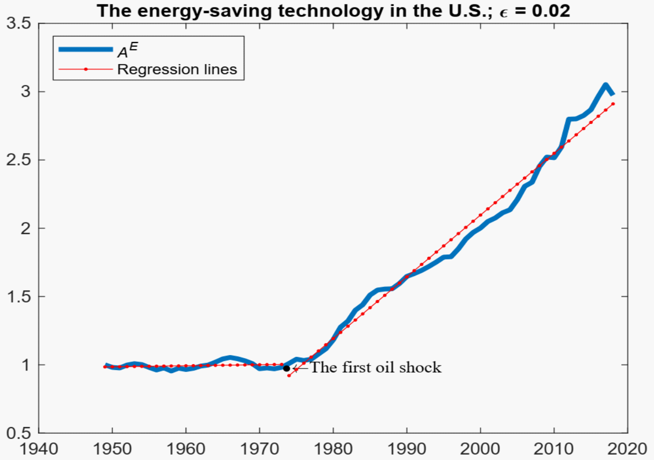
- Consider a simple case: suppose G is $\lambda \ln A + (1-\lambda)\ln A_e = \ln \bar{A}$
- Lagrangean with shadow value Λ. FOC:

$$A; \left[\frac{y}{Ak}\right]^{\frac{1}{\varepsilon}}(1-\gamma)k = \Lambda\frac{\lambda}{A} \text{ and } A_e; \left[\frac{y}{A_e e}\right]^{\frac{1}{\varepsilon}}\gamma e = \Lambda\frac{1-\lambda}{A_e}$$
$$\Rightarrow \frac{\left[\frac{y}{Ak}\right]^{\frac{1}{\varepsilon}}(1-\gamma)Ak}{\left[\frac{y}{A_e e}\right]^{\frac{1}{\varepsilon}}\gamma A_e e} = \frac{MP_k k}{MP_e e} = \frac{\lambda}{1-\lambda}$$

- Income shares are constant and independent of k and e. Income shares instead depend on the ratio $\frac{\lambda}{1-\lambda}$, how hard it is to improve capital efficiency relative to energy efficiency.

Dynamic directed technology choice

- We can now think of the choice of A and A_e as occurring over medium-run time. Higher energy prices, e.g., lead to higher growth rates of A_e (at the expense of A). On impact, energy income share increases but stabilizes over time.

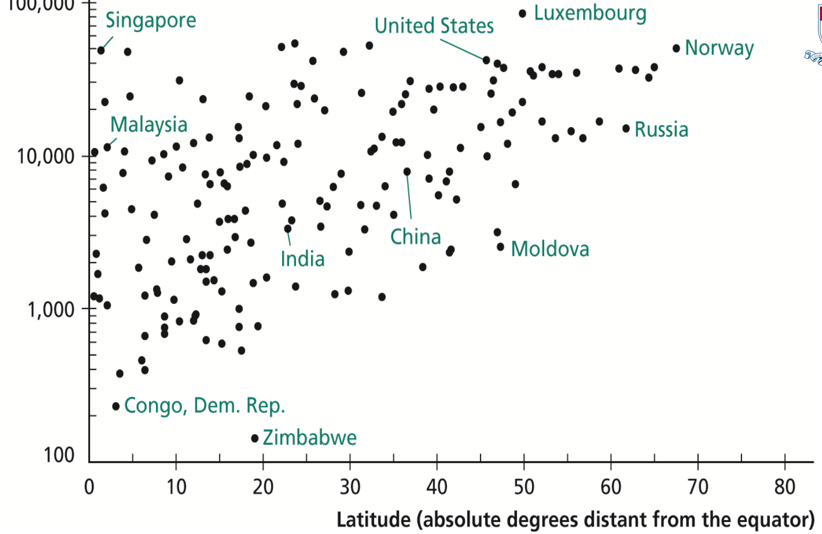


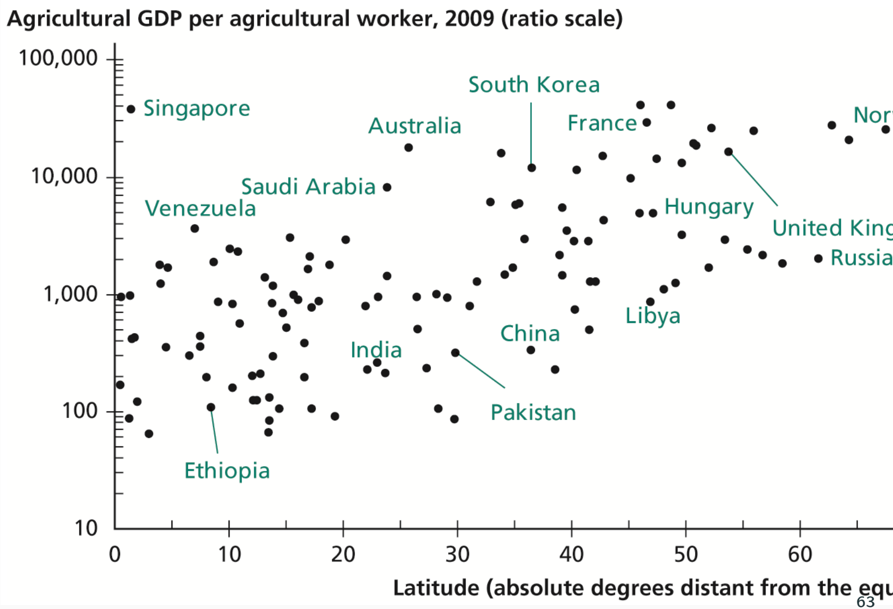
Summary

- There are puzzles in this area, and big quantitative challenges!
- Distinguish between oil and coal and remember that there are intermediates and technological change.
- Substantial medium run flexibility in production due to directed technical change. Energy and fossil fuel do not have to grow in parallel to output in medium and long-run.
- For long-run analysis, we are comfortable using C-D in energy, despite a very low short-run substitutability between energy and other inputs.
- Similar results can be derived for substitutability between different energy types.

Damages

- Give examples of different approaches to measuring and aggregating damages from climate change. Damages could be positive (bad) or negative (good).
- Climate change is a global phenomenon and affects the economy in a large number of ways.
- Two ways to estimate total effects:
    - bottom-up: quantify all potential effects and sum up
    - reduced form: directly use correlation between natural variation in climate and relevant aggregate outcomes (GDP, mortality, etc.).
- Approaches have different pros and cons. Complementary.


100,000
10,000
1,000
100
Singapore
Malaysia
United States
Luxembourg
Norway
Russia
China
India
Moldova
Congo, Dem. Rep.
Zimbabwe
0
10
20
30
40
50
60
70
80
Latitude (absolute degrees distant from the equator)


Agricultural GDP per agricultural worker, 2009 (ratio scale)
100,000
10,000
1,000
100
10
Singapore
Australia
South Korea
France
Nor
Saudi Arabia
Venezuela
Hungary
United King
Russia
Libya
China
India
Pakistan
Ethiopia
0
10
20
30
40
50
60
Latitude (absolute degrees distant from the equ

Nordhaus's DICE model (Dynamic Integrated model of Climate and the Economy) and the later RICE (also dynamic, but with R for Regional) do the bottom part as follows.

- Divide effects into: (i) agriculture, (ii) sea-level rise, (iii) other market sectors, (iv) health, (v) non-market amenity impacts, (vi) human settlements and eco-systems and (vii) catastrophes.

- Use 13 regions: U.S., OECD part of Europe, Eastern Europe, Japan, Russia, China, Africa, India, other high income, other middle, other low middle income, and low- and high-income OPEC.

- For each sector and region, a damage function, measuring the damage or willingness to pay for non-market items, is expressed as a % of GDP.
- Assuming damages are proportional to GDP amounts to TFP damages.
- For each region, sum over sectors.
- Add up to give a damage function per region.

- Most studied. Damage depends chiefly on CO_2, temperature, precipitation, and adaptation.
- Nordhaus summarizes various studies of effects:

Estimated Damages on Agriculture from CO2 Doubling

[Benefits are negative while damages are positive]

	Billions, 1990 US dollars	% of GDP
United States [a]	3.90	0.07
China [a, b]	-3.00	-0.51
Japan [a]	-17.20	-0.55
OECD Europe [a]	42.10	0.58
Russia [c]	-2.88	-0.87
India [d]	5.11	1.54
Other High Income [a, e]	-10.40	-1.14
High-Income OPEC [f]	0.00	0.00
Eastern Europe [g]	2.26	0.58
Middle Income [h]	19.51	1.43
Lower-Middle Income [I]	0.65	0.06
Africa [I]	0.10	0.06
Low Income [I]	0.30	0.06

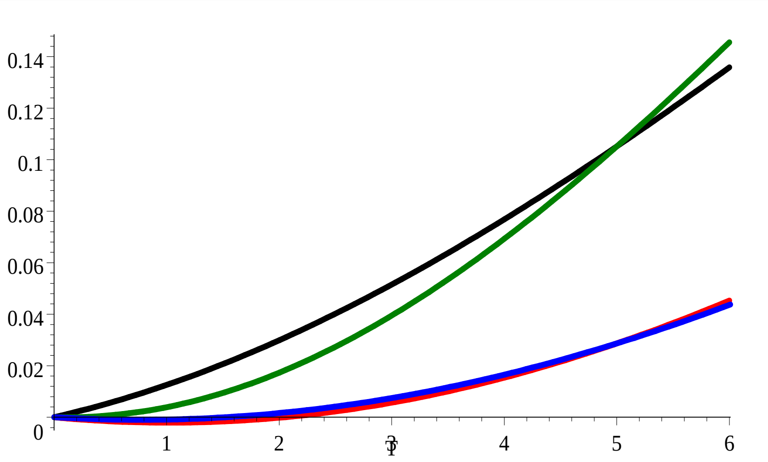
- Positive effects if initial temperature is below 11.5 degrees. Suggests quadratic damage $\alpha_{ag}^1 \left(T + T_0^j\right) + \alpha_{ag}^2 \left(T + T_0^j\right)^2 + \alpha_{ag}^j$.

Other sectors

- Similar approach but typically fewer studies to rely on.
- Does not add up to very much for a temperature increase of 2.5 degrees. Global population-weighted values for damages at 2.5 degrees: Ag =0.17%, other market =0.23%, coast =0.12%, health 0.56%, non-market -0.03, settlem. 0.1.
- Large heterogeneity. Over 1% loss in agriculture in India and Lower Middle Income (Brazil and others). 3% loss due to health in Africa.
- Total damage zero or negative in U.S. and China. Large (around 3%) in Africa and India.
- Catastrophic impacts added.

- Survey among (mostly natural-science) experts: “What is the probability of a permanent 25% loss in output if global warming is 3 and 6 degrees, respectively?”.
- Varied answers with mean 0.6 and 3.4% (median 0.5 and 2.0). Arbitrarily doubled and damage increased to 30% globally.
- Distributed over regions reflecting different vulnerability.
- Assuming risk aversion of 4 translated into willingness to pay to avoid risk.
- Leads to 1.02% and 6.94% WTP for 2.5 and 6 degrees warming globally, respectively.
- India twice as willing, the U.S. and China less than half.

- Damages as percent of GDP, described by $D(T) = 1 - \frac{1}{1+\theta_{j,1}T+\theta_{j,2}T^2}$ with region-specific $\theta_j s$, giving (Blue-USA, Red-Chi, Green-Eur, Black-LI).



Nordhaus (2013)

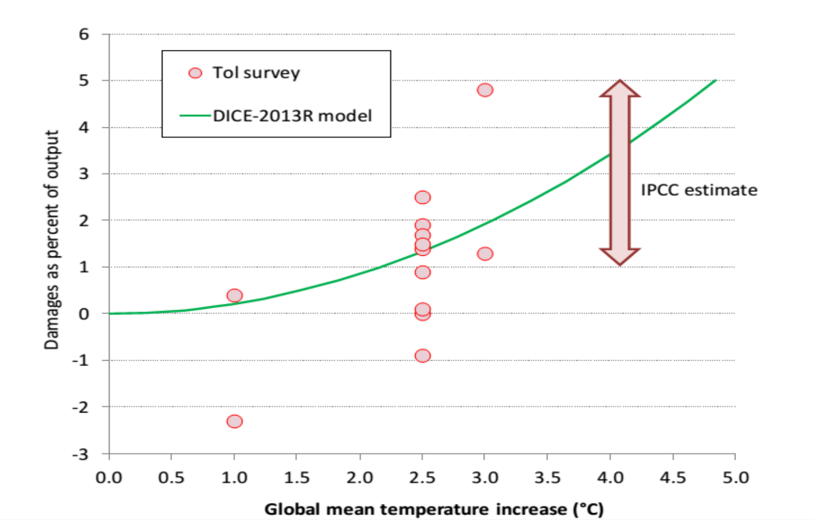
- Goes back to more ad-hoc description. Global damages

$$D(T) = 1 - \frac{1}{1 + 0.00267T^2} \approx 0.023 \left(\frac{T}{3}\right)^2 .$$

- Also allows a term in T^3 producing more convex damages.
- Other models have included even larger exponents on T, but without much of a motivation.
- The model FUND uses a *random exponent* from the interval 1.5-3.
- Nordhaus stresses that the damage function for high temperatures ($>$ 3 or 4 degrees?) should not be taken very seriously.

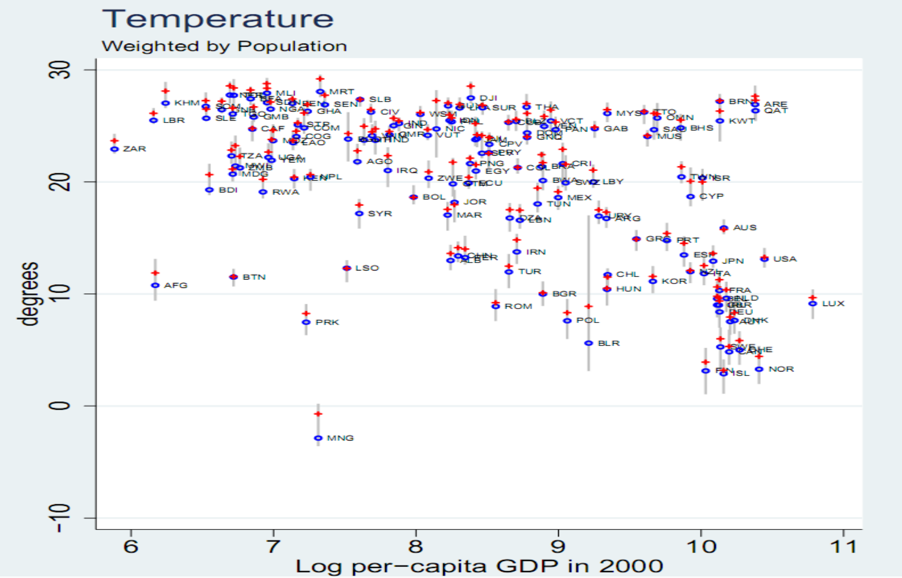
Ciscar et al. (PNAS Feb 2011)

- Another bottom-up studie, but for Europe only.
- Sums the impact for 5 types of damages: agricultural production, river floods, coastal effects, tourism (market), and health.
- Uses different high-resolution models 50x50 km and uses distribution of weather outcomes (not only temperature).
- Compares different scenarios for year 2080 to baseline of no climate change.
- For EU as a whole, yearly damages equivalent to 1% of consumption for 5.4 degree heating in EU. Small positive effects on tourism and substantial positive effects on Northern Europe.
- Relative to growth rate over 70 years ($1.02^{70} \approx 4$), these effects seem fairly small.



The solid line is the estimate from the DICE-2013
$D(T) = 1 - \frac{1}{1+0.00267T^2} \approx 0.023 \left(\frac{T}{3}\right)^2$.

- Idea is to use natural temporal variation in climate and correlate with economic outcomes—"natural experiments".
- Microstudies on agriculture, labor productivity, industrial output, health and mortality, conflicts and stability, crime, ... See Dell, Jones, and Olken, "What Do We Learn from the Weather? The New Climate-Economy Literature," NBER Working Papers 19578 (JEL).
- Microstudies yield credible identification, and perhaps yield insights on mechanisms, but often with limited external validity and no general-equilibrium effects taken into account.
- Aggregate reduced forms can complement in this sense. One of few: Dell, Jones, and Olken. NBER WP 14132.
- Monthly data on weather from 1900, 0.5 degree spatial resolution (interpolation) (use 50 last yearly obs). Economic data from Penn World Tables, 136 countries.
- Use Diff in Diff to obtain reliable estimates. Mostly across time, but also using within country variation.


Temperature
Weighted by Population
degrees
30
20
10
0
-10
6
7
8
9
10
11
Log per-capita GDP in 2000
ZAR
LBR
KHM
SLE
AFG
BDI
BTN
RWA
PRK
MNG
LSO
SYR
SLB
CIV
AGO
IRQ
BOL
ZWE
JOR
MAR
DJI
SUR
THA
MYS
GAB
MUS
BRN
ARE
QAT
KWT
CYP
AUS
USA
LUX
NOR
TUR
IRN
ROM
BGR
POL
BLR
CHL
HUN
KOR
MEX
TUN
LBY
JPN
FRA
ISL

- Assume

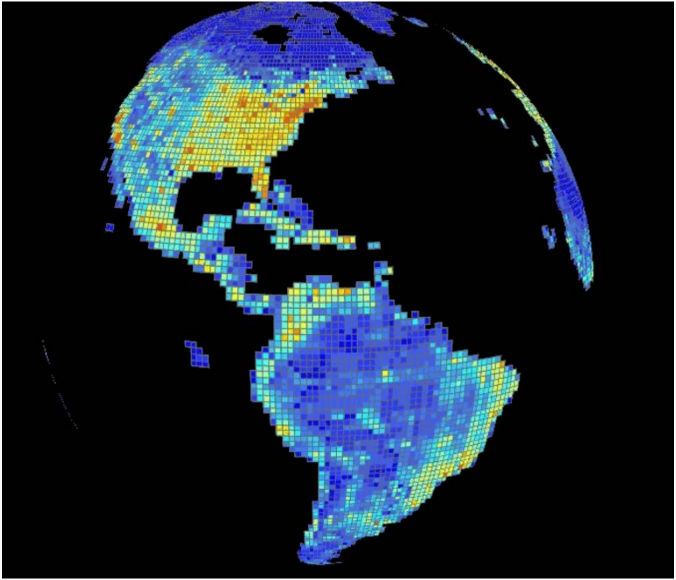
$$
\begin{aligned}
Y_{it} &= e^{\beta T_{it}} A_{it} L_{it};\ \beta \text{ captures level damage} \\
\frac{\Delta A_{it}}{A_{it}} &= g_i + \gamma T_{it};\ \gamma \text{ captures growth-rate damage}
\end{aligned}
$$

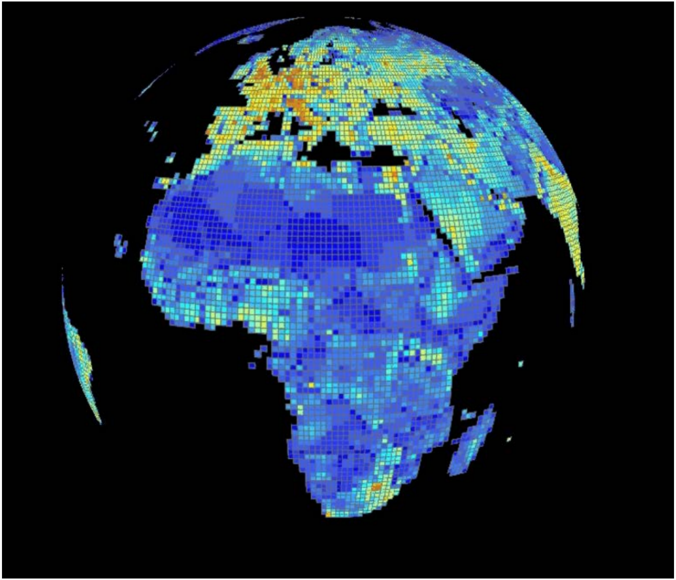
- Strong effects: one degree higher temperature leads to 1% less growth.
- But only in poor countries (below median at start).
- Persists for at least 10 years.
- Similar results for industrial output, aggregate investment, and political stability.
- Tentative conclusion: climate change is a big problem for sufficiently poor countries.
- Krusell and Smith (prel.) find other results: only level effects and no difference between poor and rich.
- Crucial feature is whether there are growth effects or not.

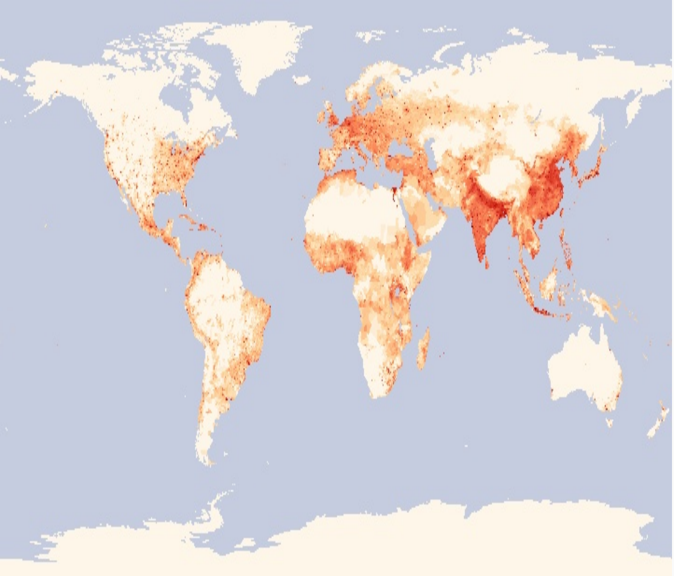
- Unit of analysis: $1^\circ \times 1^\circ$ global grid (land). 19,000 regions (cells).

- Nordhaus G-Econ database: GDP and population for all cells in 1990, 1995, 2000 and 2005.

- Produces nice charts!

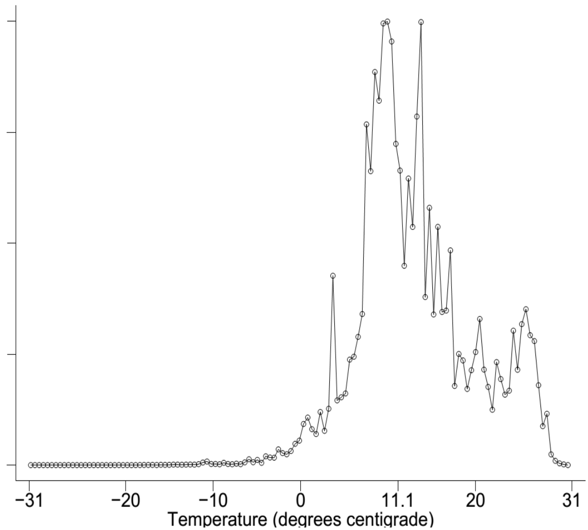




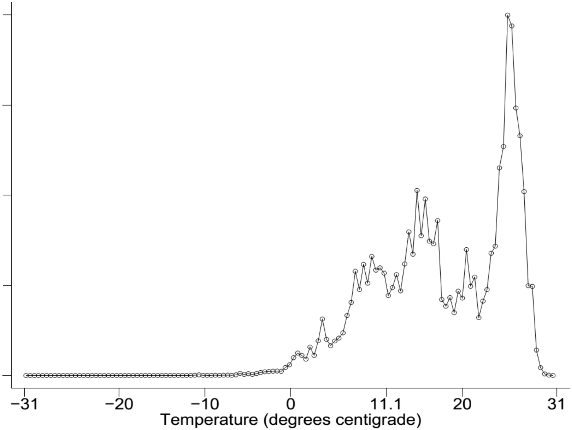


- Temperature data exists on same $1^\circ \times 1^\circ$ global grid.
- Assume relation between GDP and temperature is not random but reflects causal relationship. Use to assess consequences of changes in temperature.
- Obvious *pros* as well as *cons* with this methodology.

Share of global GDP vs. yearly mean temp



Population as function of local temperature



1. Assume potentially U-shaped damage function (damages output proportionally) in regional temperature T_j

$$D(T_j) = \begin{cases} 1 - \left(p + (1-p)e^{\gamma_h (T_j - T^\star)^2}\right) & \text{if } T_j < T^\star \\ 1 - \left(p + (1-p)e^{\gamma_l (T_j - T^\star)^2}\right) & \text{if } T_j > T^\star \end{cases}$$

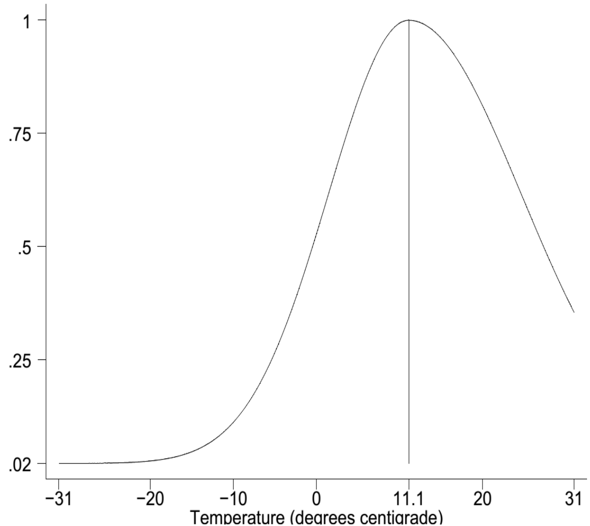
2. Apply same function to all regions (so there's an "ideal temperature").

3. Use climate predictions from ensemble of climate models to derive reduced-form relation global-regional temperature change (statistical downscaling): $T_j = f_j(T)$.

4. For each region, calculate damages at different global mean temperatures.

5. Aggregate damages and choose $(T^\star, \gamma_h, \gamma_l, p)$ to match aggregate damages implied by Nordhaus's DICE damage function.

Implied 1 – (damage function)



- Climate change affects regions very differently. Stakes big at regional level.
- In a recent paper, Krusell and Smith (2016) argue that although a tax on carbon would affect welfare positively in some average sense, there is a huge disparity of views on such a tax: 55% of the world's regions will be hurt and 45% benefit from climate change.
- Consequently, there are also strong indications that there will be significant migration pressures from climate change.

Conclusions of the Impact of Climate Change on the Economy

- Empirical support for substantial effects on the economy from climate change.
- Effects can be large in particular regions.
- Evidence does not point towards very large effects for moderate heating (< 4 degrees).
- Very little is known for more extreme scenarios.
- At least for moderate heating, percentage marginal damages per unit of extra ton in atmosphere may be approximately constant.
- Much to be learnt from further research.

What to do?

How to design a (common) optimal policy?

Main Goal

- Consider a world with a global externality: using fossil fuel for energy creates carbon dioxide.

- Energy is a required input for the production technology.

- Goal: Derive the optimal policy —here a tax on carbon— so that the externality is internalized.

Externality

- Higher levels of carbon dioxide in the atmosphere contributes to global warming, which in turn causes damages like production shortfalls, poor health or deaths, capital destruction and much more.

- Map carbon concentration to climate, and then map climate to damages.

- Expected sum of future damage elasticities: the percentage change in output resulting from a percentage change in the amount of carbon in the atmosphere, caused by emitting a unit of carbon today.

- Discounted because of time preferences and because of carbon depreciating.

Carbon circulation system: carbon is exchanged through various reservoirs such as the atmosphere, the terrestrial biosphere, and different layers of the ocean.

The representation of the carbon cycle in this paper is given by the equation:

$$(1 - d_s) = \phi_L + (1 - \phi_L)\phi_0(1 - \phi)^s$$

- ϕ_L: the share of carbon that stays in the atmosphere forever
- $(1 - \phi_0)$: of the carbon that does not stay in the atmosphere forever, this is the share that exits the atmosphere into the biosphere or ocean within a decade
- the remaining carbon in the atmosphere, $(1 - \phi_L)\phi_0$, decays at a geometric rate ϕ

1. A joint model of the climate and the economy.
2. Production Process (GDP) affected by Climate Change
3. Households with preferences (needed to evaluate outcomes)
4. Explicit use of energy that both contributes to GDP and emits CO_2
5. Inclusion of Exhaustible Resources that induces savvy economic behavior.

- Technology $Y_t = F_t(K_t, N_t, E_t, S_t)$
- There are many types of energy inputs $E_{j,t}$, $j = 1, \cdots, J$
- The first $J_g - 1$ sectors are "dirty" and the last one is "clean" energy
- For the dirty energy firms, $E_{j,t}$ is normalized so that one unit of $E_{j,t}$ produces one unit of carbon. Emissions are

$$M_t = \sum_{j=1}^{J_g-1} E_{j,t}$$

- $E_t = \sum_{j=1}^{J} E_{j,t} \alpha^j$, Actual amount of energy used
- Some energy resources have a finite stock, which is accounted for by the constraint $R_{j,t+1} = R_{j,t} - E_{j,t}^{j} \geq 0$
- Dirty energy has cost constant cost ξ_j. Clean energy has convex cost $\xi_J(E_{J,T})$.

- The climate variable S_t is the amount of carbon in the atmosphere.
- Depends on past emissions as in the reduced form way
- Define a function $\tilde{S}_t$ that maps the history of man made pollution into the current level of carbon dioxide.

$$S_t = \tilde{S}_t \left(\sum_{j=1}^{J_g - 1} E_{j,-T}, \sum_{j=1}^{J_g - 1} E_{j,-T+1}, \ldots, \sum_{j=1}^{J_g - 1} E_{j,t} \right) = \tilde{S}_t \left(M_{t-T+1}, M_{t-T+2}, M_t \right)$$

- Here, $-T$ is defined as the start of industrialization.

1. $U(C) = \log(C)$

2. $F_t(K_t, N_t, E_t, S_t) = [1 - D_t(S_t)]\, \widetilde{F}_t(K_t, N_t, E_t)$

   (has already subtracted the costs ξ_j of producing energy source j)

3. Damages: $[1 - D_t(S_t)] = \exp\{-\gamma_t(S_t - \bar{S})\}$

4. The function $\tilde{S}_t$ is linear and has the depreciation structure:

$$S_t - \bar{S} = \sum_{s=0}^{t+T} (1 - d_s) \sum_{j=1}^{J_g - 1} E_{j,t-s}$$

5. $(1 - d_s) = \phi_L + (1 - \phi_L)\phi_0(1 - \phi)^s$

What is the best that can be done?

- It is found by solving a social planner's problem
    - Representative household of the world
    - Technological, Climate and Exhaustability Constraints
- After that we worry about implementation

$$\max_{\substack{\{C_t, N_t, K_{t+\mathbf{1}}, R_{j,t+\mathbf{1}}, \\ E_{j,t}, S_t\}_{t=\mathbf{0}}^{\infty} \geq \mathbf{0}}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \qquad \text{s.t.}$$

$$C_t + K_{t+1} = F_t(K_t, N_t, E_t, S_t) + (1-\delta)K_t \qquad \text{FB}$$

$$E_t = \sum_j E_{j,t}\, \alpha^j \qquad \text{AGE}$$

$$R_{j,t+1} = R_{j,t} - E_{j,t} \geq 0 \qquad \text{for all } j \qquad ExE$$

$$S_t = \tilde{S}_t \left(\sum_{j=1}^{J_g - 1} E_{j,-T}, \sum_{j=1}^{J_g - 1} E_{j,-T+1}, \ldots, \sum_{j=1}^{J_g - 1} E_{j,t} \right) \qquad CC$$

Notation for the Planner's Problem

- $E_{j,t}$ is output of Energy of Sector (type) j measured in units of carbon emitted.

- α^j Conversion of units of energy of type j from being in terms of carbon emissions to units of energy.

- The marginal externality damage is the same for all j:

$$\Lambda_t^s = \mathbb{E}\sum_{i=0}^{\infty}\beta^i \frac{U'(C_{t+i})}{U'(C_t)}\frac{\partial F_{t+i}}{\partial S_{t+i}}\frac{\partial S_{t+i}}{\partial E_{j,t}}$$

- Under our specific assumptions, this expression simplifies to:

$$\Lambda_t^s = \mathbb{E}\sum_{i=0}^{\infty}\beta^i C_t \frac{Y_{t+i}}{C_{t+i}}\gamma_{t+i}(1-d_i)$$

- Further, if the planner's problem implies a constant savings rate, then the expression can be written as:

$$\Lambda_t^s = Y_t\left[\mathbb{E}\sum_{i=0}^{\infty}\beta^i \gamma_{t+i}(1-d_i)\right]$$

- The FOC of the planner says

$$\alpha_j \frac{\partial F_t}{\partial E_t} - \xi_j - \Lambda_t^s = 0$$

$$\max_{\{C_t, N_t, K_{t+1}\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$$

subject to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} q_t (C_t + K_{t+1})$$

$$= \mathbb{E}_0 \sum_{t=0}^{\infty} q_t ((1 + r_t - \delta) K_t + w_t N_t + T_t) + \Pi_t .$$

$$\Pi_0 = \max_{\{K_t, N_t, E_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} q_t \Bigg[F_t(K_t,$$

$$- r_t K_t - w_t N_t - \sum_{j=1}^{J} p_{j,t} E_{j,t} \Bigg]$$

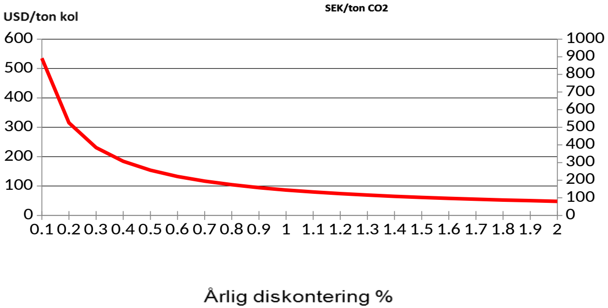
- $\tau_{j,t} = \Lambda_t^s$ for the "dirty" energy firms, and $\tau_{j,t} = 0$ for the "clean" energy firms.
- This is the optimal first best tax on carbon emissions.
- If there are multiple externalities (for instance an R&D component to the model) then a separate Pigouvian tax is required for each externality.

Comparing the Optimal Tax Rates

To understand the magnitude of the optimal tax rates given by this model, they can be compared with estimates from other models, and also with tax rates that are currently being used around the world.

- Nordhaus (2008) uses a discount rate of 1.5% and gets a tax of $30 per ton of coal. With the same discount rate, this paper gives a tax of $56.9 per ton of coal.
- Stern (2007) uses a discount rate of 0.1% and gets a tax of $250 per ton of coal. With the same discount rate, this paper gives a tax of $500 per ton of coal.
- In Sweden, the current tax on private consumption of carbon exceeds $600 per ton of carbon, which is larger than the estimates for the optimal tax in this paper. However, these taxes are significantly higher than many other countries, for instance the EU has a tax of around $77 per ton of carbon.

Sum damages over time => ”optimal” tax!



Sweden has carbon tax ~ 600 USD/tC!

What if we don't use the optimal tax?

- Let's use a recent (natural science-based) approximation of the effects on global temperature of fossil-fuel emissions.
- "Carbon Climate Response (CCR): for each 1,000GtC in cumulative historic emissions, global temperature rises by 1-2.1 degrees Celsius (1.8-3.8F).
- We have emitted about 550GtC so far (since industrial revolution).
- Remaining (conventional) oil+gas: about 300GtC. Limited warming if we use it up!
- Remaining coal: much more, possibly over 3,000GtC.
- => Coal is the main threat!

What would the optimal tax do?

- Wouldn't affect (conventional) oil and gas use.
  - A tax on oil and gas makes little difference: these fuels are so cheap to produce that markets will keep using them despite the tax.
  - It is indeed efficient from an economic perspective to use them up!
- A different story for coal:
  - Coal doesn't give a big profit per unit so a tax would make us stop using most of the coal.
  - Taking the climate damage into account, using coal simply isn't worth it.
- So: bad for the coal industry (the world over), no big deal otherwise

How costly is the optimal tax for us?

- Suppose we use "very cautious" discounting of 0.1%, implying a tax of $600/tC.
- Turns out Sweden has had that tax for over a decade. They did better than average during the Great Recession, no noticeable "leakage" of firms abroad.
- Significant scope for
  - Energy saving
  - Alternative technology

Policy instruments

- Baseline recommendation:
  - Tax carbon, world-wide
  - Required rate will not be a big blow to our global economy, but will (must) shake up coal industries
- What about alternatives, like cap-and-trade?
  - If managed so that the emission rights are as expensive as the carbon tax, ok!
  - In Europe, this is not the case —low world demand and high caps culprits.
- Do we need green subsidies?
  - Under an optimal carbon tax, maybe not; otherwise, yes.
- Should all countries mainly reduce emissions at home?
  - No: reduce them where they are least needed/least efficient (e.g., buy emission rights in EU trading system, pay to keep forests, ...)

Broad concluions so far

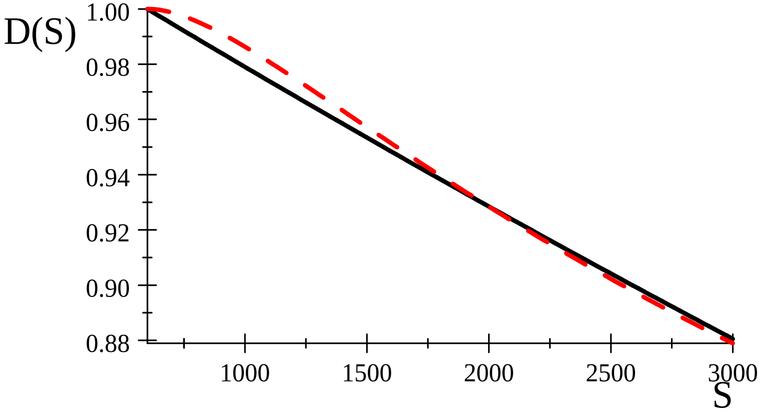
- climate change likely leads to non-negligible global damages
- very uneven effects across regions of world
- for world as a whole, costs likely not catastrophically large
- a robust result (in Golosov, et al., 2013): optimal policy involves rather modest tax on CO 2 and would not pose threat to economic well-being
- some elements of analysis subject to substantial uncertainty

- The burning of fossil fuel (oil, coal, natural gas) increases the CO_2 concentration in the atmosphere.
- CO_2 in the atmosphere is a greenhouse gas: it lets solar radiation pass through but blocks heat radiation.
- This leads to global warming. The logic is undisputed among scientists.
- The direct warming effect is significant, but not catastrophic.
- There are, however, feedback effects: creation of water vapor, melting of ice caps lowering solar reflection, cloud formation, ...
- The quantitative magnitudes of feedback are disputed. The "average" view seems to be that feedbacks strengthen the direct warming effect considerably, but there is much uncertainty.

- Global warming affects economic activity; in many places, the effect is to cause damages (to agriculture, human health, and so on).
- This is an externality: those emitting carbon into the atmosphere are not charged for the costs.
- Thus, in classical economic terms, we have a failure of markets. The prescription is government intervention: we need to artificially raise the cost of emissions to its proper societal value.
- Main recipe: use a tax. Well-known since Pigou (1920).
- The tax must be global: the externality is global.
- What is the appropriate level of the tax? For this, we use standard cost-benefit analysis.

- Nordhaus's aggregate damage function maps temperature into damages.
- Now consider collapsing the two steps, i.e.,
  1. the one from increased CO_2 concentration (S) to the change in global mean temperature (T)
  2. the one from T to damages, into one: from S to damages directly.
- For the first step use Arrhenius $T(S) = \frac{3}{\ln 2} \ln\left(\frac{S+600}{600}\right)$ where S is GtC over the pre-industrial level (600 GtC).
- For the second let $D(T)$ be Nordhaus's global damage function.
- Together, the two steps are $D(T(S))$ mapping additional atmospheric carbon to damages. Let's examine the mapping.

- It turns out that $1 - D\left(T\left(S\right)\right)$, i.e., how much is left after damages as a function of S, is well approximated by the function $e^{-\gamma S}$: for $\gamma = 5.3 * 10^{-5}$ (black), it is quite close to $1 - D\left(T\left(S\right)\right)$ (red dashed), as seen in the figure.



- Define Y_{net} as output net of damages and Y as gross output, implying $Y_{net} = (1 - D(T(S)))\,Y$.
- Using the approximation $(1 - D(T(S))) \approx e^{-\gamma S}$, we have $Y_{net} = e^{-\gamma S} Y$.
- Then, $\frac{\partial Y_{net}}{\partial S}\frac{1}{Y_{net}}$ is the marginal loss of net output from additional GtC in the atmosphere expressed as a share of net output.
- Using our approximation, we have $\frac{\partial Y_{net}}{\partial S}\frac{1}{Y_{net}} = \frac{\partial\left(e^{-\gamma S} Y\right)}{\partial S}\frac{1}{e^{-\gamma S} Y} = -\gamma$. I.e., the marginal losses are a constant proportion of GDP!
- This "elasticity" is thus independent of GDP and CO_2 concentration.
- With $\gamma = 5.3 * 10^{-5}$, one GtC extra in the atmosphere gives extra damages at 0.0053%. Recall the rate of accumulation of S_t.
- Robust?

2024 Update: Granular Model with various Suboptimal Policies

Climate Policy in the Wide World

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- Focus is on "the wide world".
  - This framework is a neoclassical growth model with carbon-cycle and climate blocks that builds on Nordhaus, but these blocks are updated to reflect the latest climate-science insights.
- We include very high geographic resolution: $1^\circ \times 1^\circ$ latitude-longitude cells, with each cell assigned to a country.
- Despite this complexity, our model is also highly accessible to others, i.e., no need for advanced numerical toolboxes.

- They conduct three kinds of suboptimal policy options that have been discussed or already implemented.

  1. How far can a modest uniform carbon tax go to limit global warming and damages around the world?
  2. How costly is it to deviate from tax uniformity and allow poor countries to not tax carbon?
  3. How successful is a policy that refrains from carbon taxation and instead focuses on promoting green energy (reminiscent of the Inflation Reduction Act)?

- The first policy is *successful* in mitigating global warming, the second is very *costly*, and the third is both *costly* and *unsuccessful*.

- The model consists of all countries in the world.
- Each country is divided into regions (grid cells) that use capital, labor, and energy to produce a final good that is identical across regions and countries.
- Most regions cannot produce oil: those that can only export oil in exchange for consumption goods (no further international trade).
  - Additional energy sources (coal, green etc.) are produced within each country.
- Capital, labor, and energy services can move freely within countries but not at all across.
  - Hence, factor prices are required to be the same within each country.
- They make assumptions such that saving rates are easy to solve for separately.

- Each country j contains a large number of identical consumers with preferences given by

$$\sum_{t=0}^{\infty} N_{j,t}\ \beta^t\ \log(c_t),$$

N_j is total population that follows an exogenous path; we define $x_{j,t+1} \equiv N_{j,t+1}/N_{j,t}$

- Consumers in country j are initially endowed with k_{j0} units of capital/capita and they can save by investing in capital in their own country. (No international capital markets!)

- Consumers in oil-producing regions extract and sell oil from a finite reserve, R_j, at zero cost (as in Golosov et al., 2014).

- Governments in each country tax emissions and rebate all the proceeds to consumers in the country.

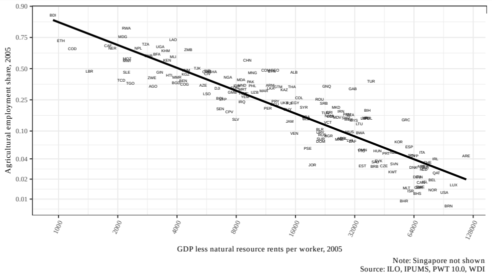
- One sector features firms that produce final output.
- The other sectors are the energy input producers.
- The production function is the same for these activities but the TFP components may differ.
    - The relative TFP factors will constitute the relative prices of the different energy inputs.
- Final-good producing firms in region i and country j solve

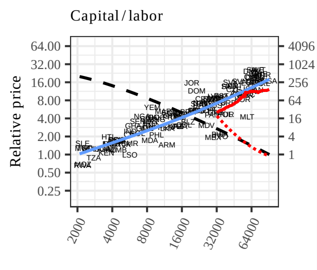
$$\pi_{ijt} = \max_{k_{ijt}, n_{ijt}, e_{ijt}} A_{ijt} \left(k_{ijt}^{\alpha} n_{ijt}^{1-\alpha-\nu_j} e_{ijt}^{\nu_j} \right)^{\varphi_j} - r_{jt} k_{ijt} - n_{jt} w_{ijt} - p_{jt} e_{ijt}.$$

- Implicit in the above expression is a fixed factor that can be thought of as land.

Poor countries are about agriculture, and capital/labor shares in agriculture increase with development.

Left: ag. empl. shares on GDP/worker; right: k/l, r/w (dashed).





Call for a richer production structure, or at the very least α_j.

They use log, one sector, and Cobb-Douglas to get easy-to-solve-for saving rates. An alternative is elasticity ϵ and CRRA ϵ.

- All energy inputs (except oil) can be produced within each country with p_κ units of the final good are required to produce e_κ units of energy source, $\kappa \in \{c, g, f\}$.

- A region that has access to the fracking input produces an "oil composite" o as

$$o = \left(\lambda e_o^{\rho_o} + (1 - \lambda) e_f^{\rho_o}\right)^{\frac{1}{\rho_o}}$$

  – ρ_o determines the EOS between the e_o and e_f.

- The supply of energy services is then a CES aggregate

$$e = \left(\lambda_o o^\rho + \lambda_c e_c^\rho + \lambda_g e_g^\rho\right)^{\frac{1}{\rho}}$$

  – ρ determines the EOS between the energy inputs.

- "A" indicates TFP and it has several components. Formally:

$$A_{ijt} = \exp(z_{ijt}) D_{ijt}$$
$$z_{ijt} = z_{ij} + \sum_{s=0}^{\infty} g_{js}$$

- g is the exogenous growth rate. For $t > T$, $g_j = g$.

- Note that TFP
  - has one *region-specific* component that is constant over time (z_{ij}),
  - one *country-specific* component that is changing over time (g_{jt}), and
  - one *endogenous* component that responds to climate change (D_{ijt}).

- TFP damages are described by a U-shape in local temperature:

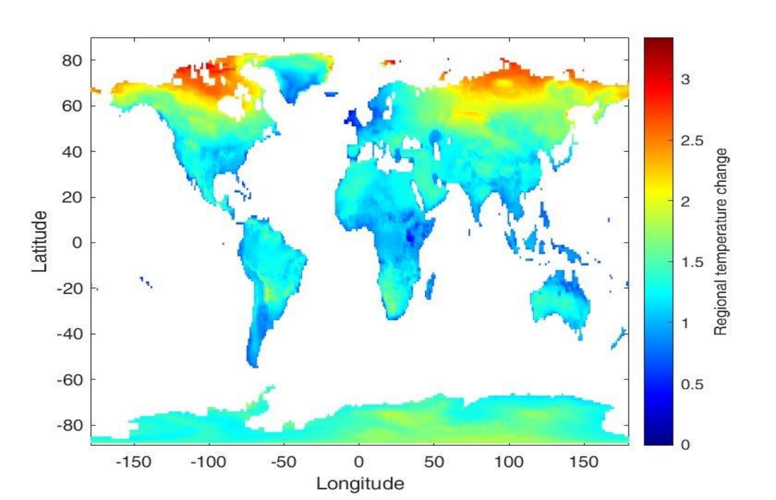
$$D_{ijt} = D(T_{ijt}) = \exp(\kappa_1 T_{ijt})(1 + \exp(\kappa_2(T_{ijt} - \kappa_3)))^{\frac{-\kappa_4}{\kappa_2}},$$

  – T_{ijt} is the local temperature in region i and country j at time t (inspired by Krusell and Smith (2022), Burke, Hsiang, and Miguel (2015) and Cruz and Rossi-Hansberg (2023).

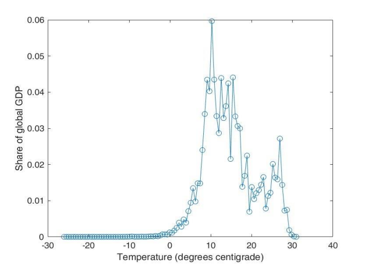
- Compute T_{ijt} with "statistical downscaling" where the global temperature is a sufficient statistic for the temperature in each region:

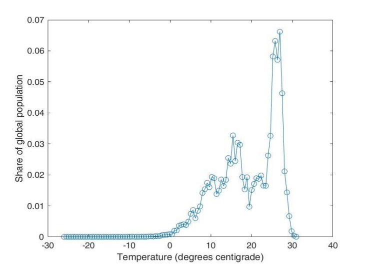
$$T_{ijt} = \hat{T}ij + \gamma_{ij}(T_t - T_0).$$

Regional T as a function of global T



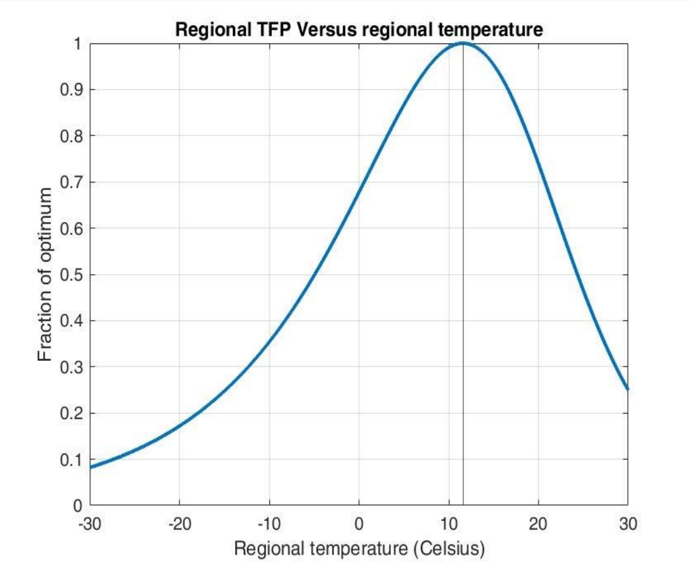
Temperatures and GDP and population shares (2005)





- Most of the output are produced in regions where $\mathbb{E}[T] \approx 11.6^\circ$C.
- A lot of people live where $\mathbb{E}[T] > 11.6^\circ$C.

The implied (inverse) U-shape for TFP



- The carbon cycle and temperature dynamics, respectively:

$$\begin{aligned} S_t - S_{t-1} &= \phi_{12} S_{t-1} + \phi_{21} S^U_{t-1} + E_{t-1} \\ S^U_t - S^U_{t-1} &= \phi_{12} S_{t-1} - (\phi_{21} + \phi_{23}) S^U_{t-1} + \phi_{32} S^L_{t-1} \\ S^L_t - S^L_{t-1} &= \phi_{23} S^U_{t-1} + \phi_{32} S^L_{t-1}, \end{aligned}$$

- E: global (sum over (i,j)) emissions; S, S^U, and S^L: CO_2 stocks in the atmosphere, surface oceans and biosphere, and deep oceans.

$$\begin{aligned} T_t &= T_{t-1} + \sigma_1 \left(F_t - \kappa T_{t-1} - \sigma_2 \left(T_{t-1} - T^L_{t-1} \right) \right) \\ T^L_t &= T^L_{t-1} + \sigma_3 \left(T_{t-1} - T^L_{t-1} \right) \end{aligned}$$

- $F_t = \chi \frac{\eta}{\ln 2} \ln \left(\frac{S_t}{S_0} \right)$ ($\chi > 1$ captures non-CO2 forcing) and T^L is ocean temperature. System replicates temperature graphs above.

- For given policy sequences $\{\tau_{ijt}\}_{t=0}^{\infty}$ across the world, the model can be solved forward.
  - The only forward-looking decisions involve the consumers' problems that delivers solutions for saving rates that only depend on exogenous parameters.
  - The saving rates themselves are time-dependent and forward-looking, but satisfy a simple recursion.
- Given (k_j, n_j), one can compute country y_j and input demands, assuming a value for the world price of oil.
  - TFP levels are endogenous but predetermined at each point in time.
- Use a simple fixed-point algorithm for finding the oil price that clears the world markets period by period.

$$
\begin{aligned}
p_j &= \left(\lambda_o^{\frac{1}{1-\rho}}\hat{p}_{oj}^{\frac{\rho}{\rho-1}} + \lambda_c^{\frac{1}{1-\rho}} p_{cj}^{\frac{\rho}{\rho-1}} + \lambda_g^{\frac{1}{1-\rho}} p_{gj}^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}} \\
\hat{p}_{oj} &= \left(\lambda^{\frac{1}{1-\rho_h}} p_o^{\frac{\rho_h}{\rho_h-1}} + (1-\lambda)^{\frac{1}{1-\rho_h}} p_{fj}^{\frac{\rho_h}{\rho_h-1}}\right)^{\frac{\rho_h-1}{\rho_h}} \\
e_{oij} &= e_{ij}\left(\frac{\lambda_o p_j}{\hat{p}_{oj}}\right)^{\frac{1}{1-\rho}}\left(\frac{\lambda \hat{p}_{oj}}{p_o}\right)^{\frac{1}{1-\rho_h}} \\
e_{fij} &= e_{ij}\left(\frac{\lambda_o p_j}{\hat{p}_{oj}}\right)^{\frac{1}{1-\rho}}\left(\frac{(1-\lambda)\hat{p}_{oj}}{p_f}\right)^{\frac{1}{1-\rho_h}} \\
e_{mij} &= e_{ij}\left(\frac{\lambda_m p_j}{p_{mj}}\right)^{\frac{1}{1-\rho}}, \; m = c, g.
\end{aligned}
$$

All underlying prices are exogenously given except p_o,

- The production function for a regional firms (omitting time subscripts) can be written as

$$y_i = \left(\frac{\nu_j \varphi_j}{p_j}\right)^{\frac{\nu_j \varphi_j}{1-\nu_j-\varphi_j}} A_{ij}^{\frac{1}{1-\nu_j-\varphi_j}} \left(k_i^{\alpha} n_i^{1-\alpha-\nu_j}\right)^{\frac{\varphi_j}{1-\nu_j \varphi_j}}.$$

- Summing over regions: $y_j = \sum_{i=1}^{I} y_i$, $k_j = \sum_{i=1}^{I} k_i$, $n_j = \sum_{i=1}^{I} n_i$ we get per-capita output in a country j

$$y_j = \left(\frac{\nu_j \varphi_j}{p_j}\right)^{\frac{\nu_j \varphi_j}{1-\nu_j-\varphi_j}} \left(\sum_{i=1}^{I} A_{ij}^{\frac{1}{(1-\nu_j \varphi_j)(1-\varphi_j)}}\right)^{1-\varphi_j} k_j^{\alpha_k \varphi_j} n_j^{\alpha_n \varphi_j}.$$

- The only remaining endogenous variable is p_j, which is determined on the world market.

Above, we have an expression for e_{oijt}. Summing over regions and countries we get, after manipulation, oil demand

$$\text{Oil demand}_t = \sum_{ij} \Pi_{jt}(p_{ot})\ \nu_j\ \varphi_j\ y_{ijt}\ N_{jt},$$

where Π_{jt} is a known function of p_{ot}.

Turning to supply, the oil producer's maximization problem delivers

$$R_{t+1} = \beta \frac{1 - s_t}{1 - s_{t+1}} R_t\ ,\ s_t = \frac{\beta x_{t+1}}{1 - s_{t+1} + \beta x_{t+1}}.$$

Given an exogenous sequence $\{x_{t+1}\}_{t=0}^{\infty}$, we can solve backwards:

$$\text{Oil supply}_t = \beta \sum_j \frac{1 - s_{jt}}{1 - s_{jt+1}} N_{jt} R_{jt}.$$

Solve for p_{ot} by setting $\text{Oil supply}_t = \text{Oil demand}_t$.

They can also derive a forward-looking equation in saving rates in oil-consuming regions, and write per-capita savings of country j as

$$k_{j,t+1} = \frac{s_{jt}(1+\hat{\tau}_{jt})}{x_{j,t+1}} A_{jt} k_{jt}^{\frac{\alpha\varphi_j}{1-\nu_j\varphi_j}},$$

with

$$s_{jt} = \frac{\frac{\alpha\beta\varphi}{1-\nu\varphi} x_{j,t+1}}{1 - s_{j,t+1} + \frac{\alpha\beta\varphi}{1-\nu\varphi} x_{j,t+1}}.$$

The heterogeneity across economies appear in multiple places:

- saving rates
- population growth rates
- taxes, φ_j, ν_j
- TFP, costs of producing energy services.

1. Solve for the saving rates $\forall j$ (no endogenous variables).

2. Compute the equilibrium forward, starting at time 0. The endogenous state variables at $t = 0$ are K_j, T_j, oil resources by j; state variables in the carbon cycle and climate system.

    - Compute all TFP levels around the world and solve for the oil price in the period, which requires a numerical solution but only involves one equation in one unknown.
    - $p_{o,0}$ and τ_j gives the demand for all fuels and thus total emissions, so temperatures can be updated to next-period values.
    - The government BC is used to compute the carbon-revenue transfer rates $\hat{\tau}_j$.
    - Update the capital stocks and oil resources to their next-period values.

3. This completes the procedure for going from period 0 to period 1. Proceed to all future periods.

- They make use of the G-Econ database, version 4.0, which provides data on GDP and population (N) for every $1^\circ \times 1^\circ$ cell that contains land for the model's base year, 2005.
  - The database contains GDP and N data for 16,443 cells in 2005, and these cells that comprise the basic unit of analysis in the model.
- Estimated and projected N growth rates from 1990 to 2100 by country taken from the United Nations. Between 2100 and 2200, assume a linear progression from the 2100 rate to 0.
- The exogenous part of TFP grows at a rate of 1.5%/year, but developing countries are allowed to catch up.
- MPK:s are equalized in period 0 (following Caselli & Feyrer, 2007), which can be used to pin down the φ's (are found in the range $0.8 \leq \varphi_j \leq 1$).

- The elasticity of substitution between oil, coal, and the green energy source is set to 2 (for all countries).

- The elasticity of substitution between conventional oil and fracking is set to 10 (only the U.S. is assumed to have fracking).

- $E[p_o]_{2005-2009}$=\$70 per barrel or \$606.5/ton of carbon.

- $E[p_c]_{2005-2009}$=\$74/ton or \$103.35/ton of carbon.

- They set p_g based on the current relative price between green energy and oil.

- With these prices and observed quantities, the λs in the energy aggregator can be computed (Golosov, Hassler, Krusell, and Tsyvinski (2014)).

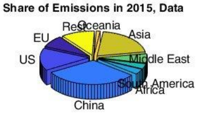
They incorporate heterogeneity in the energy income share with data on national energy use from the World Bank by computing

$$\nu_i = \frac{e_i^{int}}{\hat{e}_i^{int}} \nu,$$

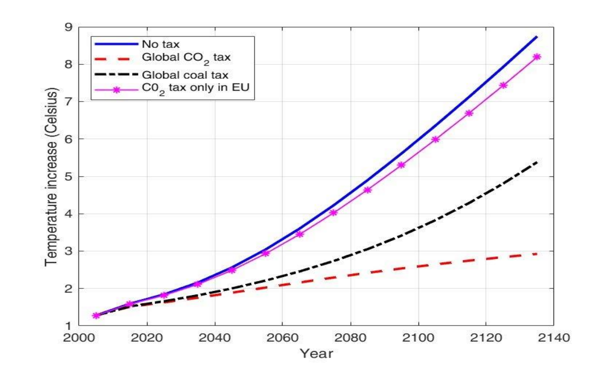
where e_i^{int} is national energy intensity (energy use in oil equivalents divided by PPP-adjusted GDP in year 2000), $\hat{e}_i^{int}$ is average energy intensity, and ν=0.035.

(Relies on the price of energy being equal across countries.)

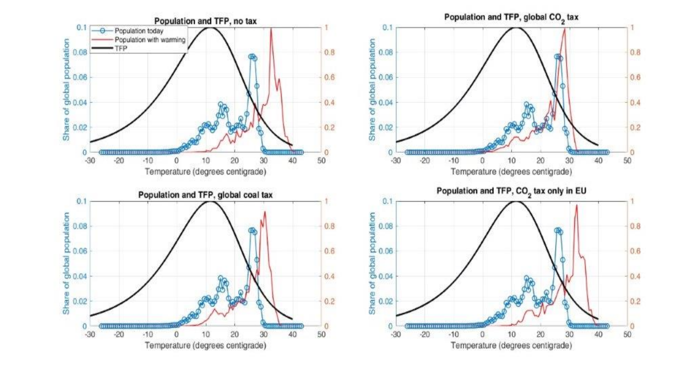
Note that the model is quite successful in matching observed CO_2 emissions, even though these were not directly targeted (China subsidizes fossil fuel use.).



Consider a tax of USD $20/ton CO_2 at the initial date; it then grows at the rate of world GDP.

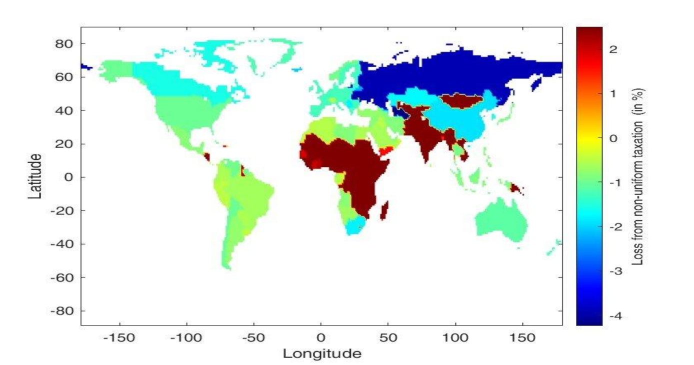


The difference between no tax and the modest tax is striking: about 6°C by 2140!



- Distributional consequences: populations being moved significantly to the right for the no-tax policy.
- As if hit by several Great Depressions at once for the some of the poorest regions.

- The Pigou principle: the tax should equal the negative externality caused. Since the negative externality is global, the tax should be the same everywhere: it should be uniform.
- However, we often hear arguments that for the sake of fairness, some poor regions should be "let off the hook".
- They here quantify exactly how costly deviations from a uniform taxation policy are in dollar terms.
- They again start with a τ of about US $20/ton.
  - Compare the results to a setting where the poorest countries, defined to be below 25% of global GDP/capita, have a zero/very low tax;
  - The tax in the ROW is then increased so that the increase in T is the same as with uniform taxation (3.1°C) at $t = 15$.
  - They consider a τ that is $20 \times \tau_{modest}$ in the participating countries, and $0.06 \times \tau_{modest}$ for the poor.



80
60
40
20
0
-20
-40
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-80
Latitude
-150
-100
-50
0
50
100
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Longitude
2
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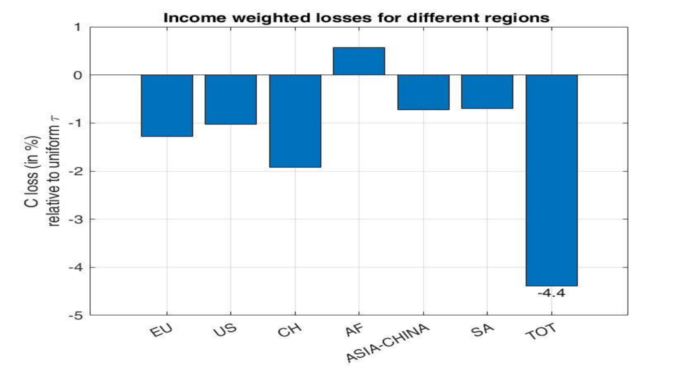
0
-1
-2
-3
-4
Loss from non-uniform taxation (in %)

Aggregate effects of non-uniform taxation



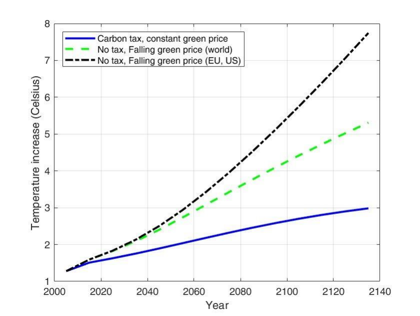
Policy experiment III: faster technical change in green

The climate-change aspects of the IRA boil down to the idea that cheap green technology will compete out fossil fuel.

They evaluate this idea by considering a scenario where the relative price of green energy falls by 2%/year (at zero cost), while the relative price of fossil fuel production is unchanged.

Two cases:

1. The U.S.-like policy generates fast green technology growth everywhere in the whole world.
2. The sped-up green technology growth will occur only in the U.S. and the EU.



- With green growth only in the EU and the U.S., global warming becomes almost as high as with no policy at all. Also insufficient when the technology spreads around the whole world.
- Key issue: green technology increases overall energy consumption but is ineffective in competing out fossil fuel.

Conclusions

- They offer a model of economics and climate change with very high regional resolution.
  - The model rests on standard microeconomic foundations that allows for cost-benefit analysis.
  - The spatial dispersion of the welfare effects of global warming are found to swamp the average effects.
  - Proof of concept: lots of room for improvements regarding heterogeneity in energy supplies and technologies, production structures.
- They also find that
  - Even a modest, globally uniform carbon tax would be extremely valuable.
  - A non-uniform tax on carbon is quite inefficient.
  - Relying on a push for green technical change only tax appears like a risky policy.

References

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