Economics 4230: Macro Modeling
Dynamic Fiscal Policy

José Víctor Ríos Rull

Spring Semester 2024

Most material developed by Dirk Krueger

University of Pennsylvania

Organizational Details (Material also in Canvas)

- **Time of Class:** Mon., Wed., 1:45 - 3:15pm
- **Class Web Page:** http://www.sas.upenn.edu/~vr0j/4230-24/
- **Class Syllabus:** http://www.sas.upenn.edu/~vr0j/4230-24/syl4230.pdf
- **Lecture notes:** Available at: http://www.sas.upenn.edu/~vr0j/4230-24/PennFiscalNew.pdf
- **Class slides:** Available at: http://www.sas.upenn.edu/~vr0j/4230-24/index.html
- **Diary of what we did in class:** Available at: http://www.sas.upenn.edu/~vr0j/4230-24/diary.html

People

- Instructor: José Víctor Ríos Rull
- Time of Class: Monday, Wednesday, 1:45 - 3:15pm. PCPSE 100
- Office Hours: Mon 3:30-4:30 and by appointment. vr0j@upenn.edu

Course Outline and Overview

- Advanced undergraduate class
- Prerequisites: Econ 101 and 102 and math background required to pass these classes (i.e. Math 114, 115 or equivalent, we use calculus)
- Study the impact of fiscal policy (taxation, government spending, government deficit and debt, social security) on individual household decisions and the macro economy as a whole
- Economics and Climate Change. We will look at the classic problem of an externality and study it in the context of climate change.
- Class consists of model-based analysis, motivated by real world data and policy reforms

- 3 Homeworks and 3 midterms.

Homeworks, Midterms, Worth and Dates

	Fraction	Points	Date
Homework 1	8.33%	25	Due February 19
Midterm 1	25%	75	February 21
Homework 2	8.33%	25	Due April 1
Midterm 2	25%	75	April 3
Homework 3	8.33%	25	Due April 29
Midterm 3	25%	75	May 1
Total	100%	300	

Homeworks

- Due date stated on homework. Due in class or in my mailbox by the end of class of the specified date. Late homework is **not** accepted.

- Grading complaints: **within one week of return** of homework written statement specifying complaint in detail. I will regrade entire assignment. No guarantee that revised score higher than original score (and may be lower).

- Work in groups on homeworks permitted, but everybody needs to hand in **own** assignment. Please state whom you worked with.

Exams

- Three midterms each make up 25% of total grade.
- Not cumulative.
- Dates: Dates: February 19, April 1, April 29.

Points Achieved	Letter Grade
285 - 300	A +
270 - 284.5	A
255 - 269.5	A -
240 - 245.5	B +
225 - 239.5	B
210 - 224.5	B -
195 - 209.5	C +
180 - 194.5	C
165 - 179.5	C -
150 - 164.5	D +
135 - 149.5	D
less than 135	F

Content of Course

- Some Basic Empirical Facts about the Size of the Government (Part I)
- A Simple Model of Intertemporal Choice (Part II)
- The Full Life Cycle Model (Part III)
- Positive Analysis of Fiscal Policy (Part IV)
- Pigou Taxation (Part V)
- Climate Change and the Economy (Part VI)
- Optimal Policy (Part VII)

Part I

Introduction and Main Facts

$$
\begin{aligned}
C &= \text{Consumption} \\
I &= \text{(Gross) Investment} \\
G &= \text{Government Purchases} \\
X &= \text{Exports} \\
M &= \text{Imports} \\
Y &= \text{Nominal GDP} \\
Y &= C + I + G + (X - M)
\end{aligned}
$$

US 2018 Main Macro Aggregates Bureau of Economic Analysis

In 2019 increased 2.3% in 2020 -2.8%, 2021 5.9%, 2021 2.1%, 2023 3.0%

	Billions of dollars	Perc of GDP
Gross domestic product	20,500.6	100.00
Personal consumption expenditures	13,951.6	68.05
Goods	4,342.1	21.18
Services	9,609.4	46.87
Gross private domestic investment	3,652.2	17.82
Fixed investment	3,595.6	17.54
Nonresidential	2,800.4	13.66
Structures	637.1	3.11
Equipment	1,236.3	6.03
Intellectual property products	927.0	4.52
Residential	795.3	3.88
Change in private inventories	56.5	0.28
Net exports of goods and services	–625.6	-3.05
Exports	2,530.9	12.35
Imports	3,156.5	15.40
Government expenditures	3,522.5	17.18
Federal	1,319.9	6.44
National defense	779.0	3.80
Nondefense	540.9	2.64
State and local	2,202.6	10.74

Two Deficits

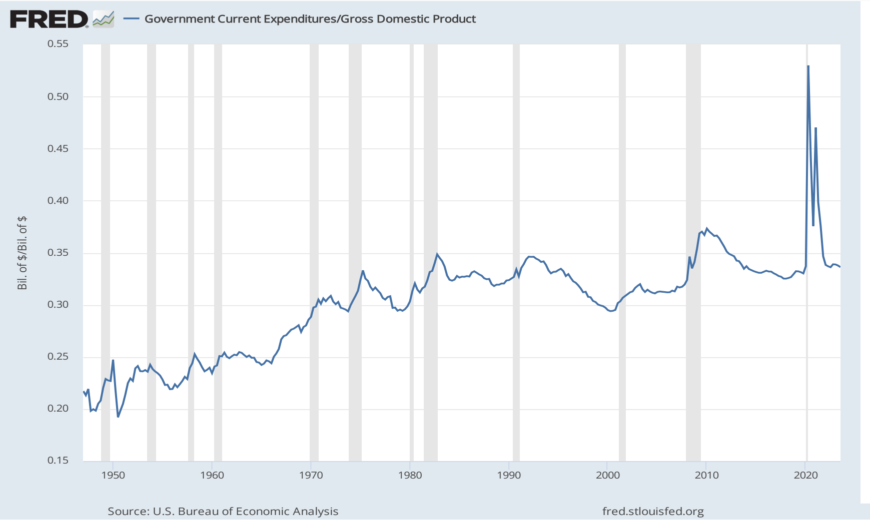
- Federal Government Budget Deficit (more below)
- Trade Deficit (or Current Account Deficit): Trade Balance (TB)

$$TB = X - M$$

$$\begin{aligned}
\text{Current Account Balance} &= \text{Trade Balance}+\text{Net Unilateral Transfers} \\
\text{Capital Account Balance this year} &= \text{Net wealth position at end of this year} \\
&\quad -\text{Net wealth position at end of last year} \\
\text{Current Account Balance this year} &= \text{Capital Account Balance this year}
\end{aligned}$$

Trade Balance as Share of GDP, 1970-2022

Government Outlays as Fraction of GDP, 1970-2022



The Government Budget

- Budget Deficit/Surplus

$$\begin{aligned}\text{Budget Surplus} &= \text{Total Federal Tax Receipts}\\ &\quad -\text{Total Federal Outlays}\end{aligned}$$

- Federal outlays

$$\begin{aligned}\text{Total Federal Outlays} &= \text{Federal Purchases of Goods and Services}\\ &\quad +\text{Transfers}\\ &\quad +\text{Interest Payments on Fed. Debt}\\ &\quad +\text{Other (small) Items}\end{aligned}$$

- Federal government deficits ever since 1969 (short interruption in late 90's)
- Federal debt and deficit are related by

$$\begin{aligned}\text{Fed. debt at end of this year} &= \text{Fed. debt at end of last year}\\ &\quad +\text{Fed. budget deficit this year}\end{aligned}$$

2015 Federal Budget (in billion $)

Receipts	3,453.3
Individual Income Taxes	1,532.7
Social Insurance Receipts	1,189.5
Corporate Income Taxes	344.7
Seignorage	110.4
Excise taxes	101.3
Customs duties	38.1
Other	136.6
Outlays	4,022.9
National Defense	705.6
International Affairs	45.7
Health	372.5
Medicare	485.7
Income Security	597.4
Social Security	730.8
Net Interest	230.0
Other	435.5
Surplus	–1,299.6

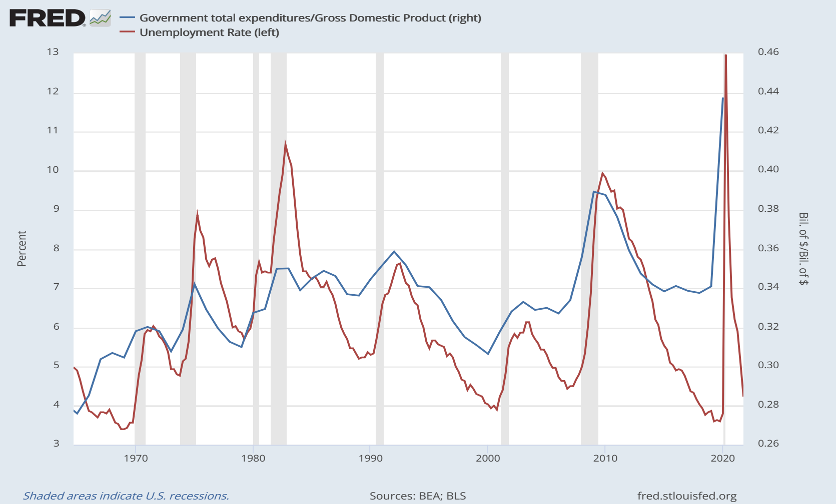
State and Local Budgets (in billion $)		
	2011	2013
Total Revenue	2,618	2,690
Property Taxes	445.8	445.4
Taxes on Production and Sales	464.0	496.4
Individual Income Taxes	285.3	338.5
Corporation Net Income Tax	48.4	53.0
Transfers from Federal Gov.	647.6	584.7
All Other	722.9	762.4
Total Expenditures	2,583.8	2,643.1
Education	862.27	876.6
Highways	153.9	158.7
Public Welfare	494.7	516.4
All Other	1,072.9	1,091.4
Surplus	34.2	47.3

Fiscal Variables and the Business Cycle

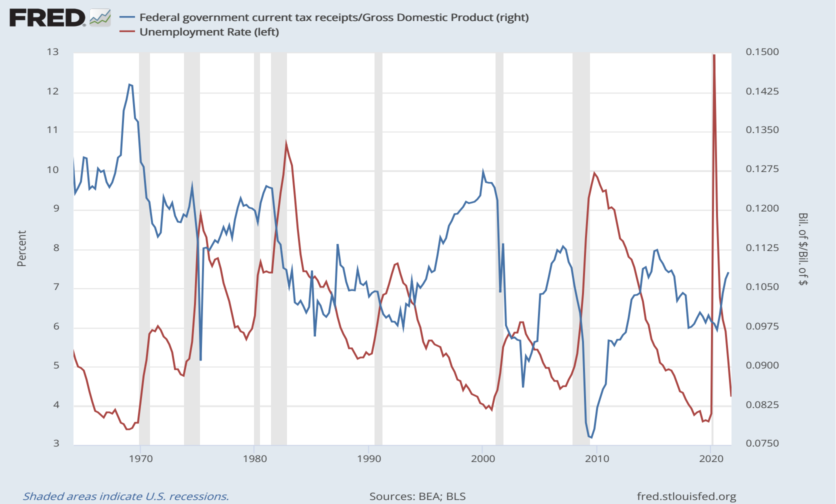
- Use the unemployment rate as indicator for the business cycle: high unemployment rates indicate recessions, low unemployment rates indicate expansions

- Does fiscal policy (government spending, taxes collected, government deficit) vary systematically over the business cycle?

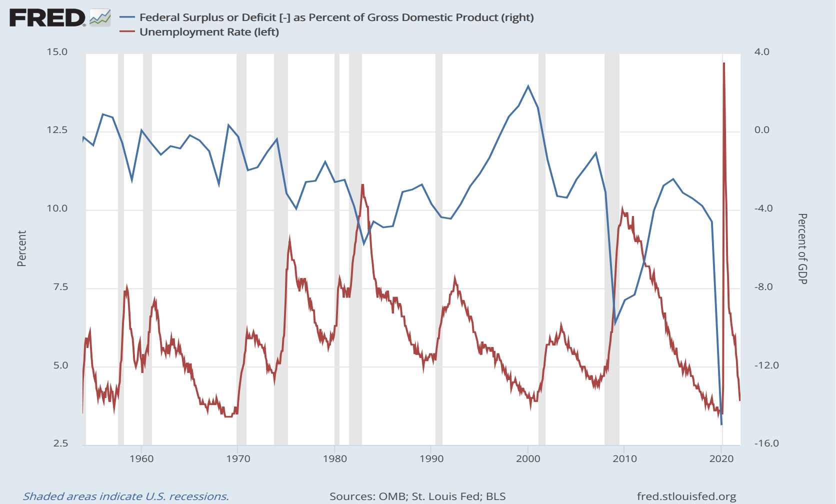
Government Outlays and Unemployment Rate, 1965-2021



Gov Taxes and Unemployment Rate, 1965-2021



Deficit and Unemployment Rate, 1965–2021



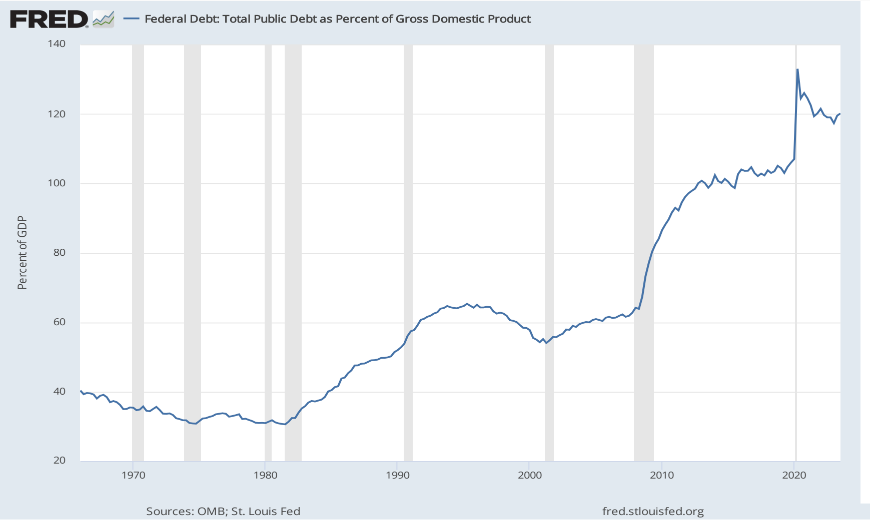
- $$\begin{aligned} \text{Government Outlays to GDP ratio} &= \frac{\mathit{Outlays}}{\mathit{GDP}} \\ \text{Deficit-GDP ratio} &= \frac{\mathit{Deficit}}{\mathit{GDP}} \\ \text{Debt-GDP ratio} &= \frac{\mathit{Debt}}{\mathit{GDP}} \end{aligned}$$

- $$\begin{aligned} \text{Debt at end of this year} &= \text{Debt at end of last year} \\ &\quad + \text{Budget deficit this year} \end{aligned}$$

Government Outlays to GDP ratio, 2006

- US: 36.4%
- Canada: 39.3%
- Japan: 36.0%
- Sweden: 54.3%, France: 52.7%, Germany: 45.3%

Debt to GDP Ratio, 1965-2023



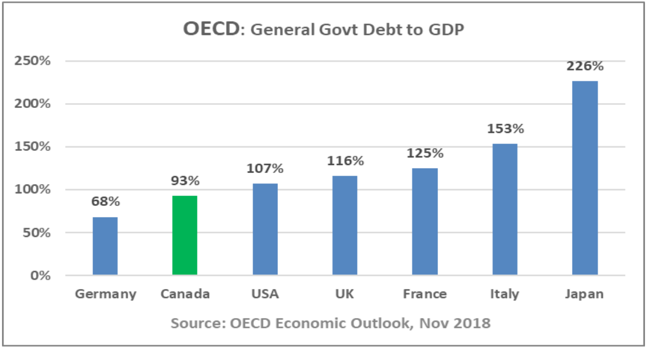
International Debt to GDP Ratios (OECD)

Includes Currency and Deposits (Overzealous Measure)

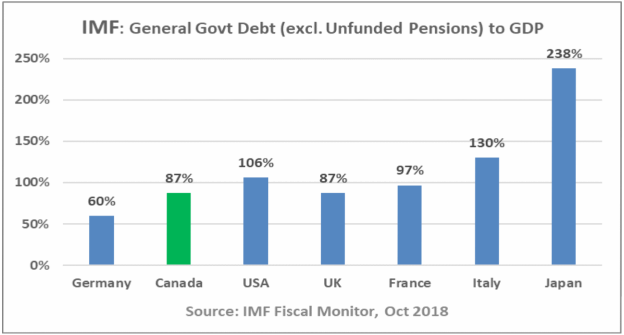
Country	2010	2011	2012	2013	2014	2015	2016	2017
Estonia	11.93	9.54	13.15	13.62	13.85	12.75	12.73	12.55
Chile	15.27	17.85	18.37	18.99	22.39	24.41	28.08	29.65
Denmark	53.44	60.11	60.62	56.73	59.14	53.79	52.60	49.96
Sweden	52.59	53.28	54.40	57.15	63.40	61.56	60.33	57.95
Australia	41.92	46.31	59.25	55.77	61.63	64.28	68.64	65.72
Germany	84.45	84.18	88.11	83.27	83.35	78.96	76.01	71.52
Ireland	83.50	111.46	129.36	131.73	121.20	88.52	84.14	77.24
Canada	105.22	107.88	111.54	107.51	108.54	114.75	114.13	109.10
Spain	66.56	77.69	92.53	105.73	118.41	116.31	116.52	114.66
United Kingdom	86.56	100.31	104.11	99.92	109.92	109.45	119.38	116.91
Belgium	107.98	110.60	120.47	118.48	131.11	127.67	128.44	121.90
France	101.00	103.81	111.94	112.47	120.16	120.83	125.46	124.25
United States	125.85	130.98	132.69	136.28	135.60	136.60	138.51	135.66
Portugal	104.07	107.85	137.10	141.43	151.40	149.15	145.32	145.38
Italy	124.88	117.94	136.24	143.69	156.06	157.03	154.90	152.61
Greece	128.97	110.91	164.11	179.69	180.82	182.94	185.79	188.73
Japan	207.52	222.31	230.39	233.22	238.46	237.39	234.55	
Mexico	31.15	37.14	41.13	47.11	50.06	53.33	51.79	
Switzerland	42.62	43.03	43.81	43.08	43.14	43.18	42.46	

Public Debt Including Some Unfunded Public Sector Liabilities

OECD Nov 2018



IMF Nov 2018


IMF: General Govt Debt (excl. Unfunded Pensions) to GDP
250%
200%
150%
100%
50%
0%
60%
87%
106%
87%
97%
130%
238%
Germany
Canada
USA
UK
France
Italy
Japan
Source: IMF Fiscal Monitor, Oct 2018

Part II

The Benchmark Model

- Why a model? Because now we want to *understand* the effects of government activity (not just simply describe them).

- Why a two period (dynamic) model? Because the government choice of policies today affect what it can do tomorrow (a tax cut today, together with a budget deficit, requires higher taxes or lower spending tomorrow). Therefore need a model where choices today affect choices tomorrow. Simplest such model is a two-period model.

- Model is due to Irving Fisher (1867-1947), extension due to Albert Ando (1929-2003) and Franco Modigliani (1919-2003) and Milton Friedman (1912-2006).

- Single household, lives for two periods (working life, retired life)
- Cares about consumption in first period, c_1, and second period, c_2.
- Utility function

$$U(c_1, c_2) = u(c_1) + \beta u(c_2)$$

  where $\beta \in (0, 1)$ measures household's impatience.
- Function u satisfies $u'(c) > 0$ (more is better) and $u''(c) < 0$ (but at a decreasing rate).
- Income $y_1 > 0$ in the first period and $y_2 \geq 0$ in the second period. Income is measured in units of the consumption good, not in terms of money.
- Starts life with initial wealth $A \geq 0$, due to bequests; measured in terms of the consumption good.
- Can save or borrow at real interest rate r

- Nominal and real interest rates

$$1 + r = \frac{1 + i}{1 + \pi}$$

- Approximately (as long as $r\pi$ is small)

$$\begin{aligned} i &= r + \pi \\ r &= i - \pi \end{aligned}$$

- Budget constraint in period 1

$$c_1 + s = y_1 + A$$

where s is household's saving (borrowing if $s < 0$).

- Second period budget constraint

$$c_2 = y_2 + (1 + r)s$$

- Decision problem of household:
  Choose (c_1, c_2, s) to maximize lifetime utility, subject to the budget constraints.

- Simplify: consolidate two budget constraints into intertemporal budget constraint by substituting out saving: solve second budget constraint for s to obtain

$$s = \frac{c_2 - y_2}{1 + r}$$

- Substitute into first budget constraint:

$$c_1 + \frac{c_2 - y_2}{1 + r} = y_1 + A$$

  or

$$c_1 + \frac{c_2}{1 + r} = y_1 + \frac{y_2}{1 + r} + A$$

- Interpretation: price of consumption in first period is 1. Price of consumption in period 2 is $\frac{1}{1+r}$, equal to relative price of consumption in period 2, relative to consumption in period 1.

- Intertemporal budget constraint says that total expenditures on consumption goods $c_1 + \frac{c_2}{1+r}$, measured in prices of the period 1 consumption good, equal total income $y_1 + \frac{y_2}{1+r}$, measured in units of the period 1 consumption good, plus initial wealth. Sum of labor income $y_1 + \frac{y_2}{1+r}$ also referred to as human capital.

- Let $I = y_1 + \frac{y_2}{1+r} + A$ denote total lifetime income, consisting of human capital and initial wealth.

- Maximization problem

$$\max_{c_1, c_2} \quad \{u(c_1) + \beta u(c_2)\}$$

$$s.t. \quad c_1 + \frac{c_2}{1+r} = I$$

- Lagrangian method or substitution method

- Lagrangian

$$\mathcal{L} = u(c_1) + \beta u(c_2) + \lambda \left[I - c_1 - \frac{c_2}{1+r} \right]$$

- Taking first order conditions with respect to c_1 and c_2 yields

$$\begin{aligned} u'(c_1) - \lambda &= 0 \\ \beta u'(c_2) - \frac{\lambda}{1+r} &= 0 \end{aligned}$$

- We can rewrite both equations as

$$\begin{aligned} u'(c_1) &= \lambda \\ \beta(1+r)u'(c_2) &= \lambda \end{aligned}$$

- Combining yields

$$u'(c_1) = \beta(1+r)u'(c_2)$$

or

$$u'\left(I - \frac{c_2}{1+r}\right) = (1+r)\beta u'(c_2)$$

- Existence of unique solution? Assume Inada condition

$$\lim_{c \to 0} u'(c) = \infty$$

define

$$f(c_2) = u'\left(I - \frac{c_2}{1+r}\right) - (1+r)\beta u'(c_2)$$

and use the Intermediate Value Theorem to show that there is a value for c_2 that makes $f(c_2) = 0$.

- Optimality condition

$$u'(c_1) = \beta(1+r)\ u'(c_2)$$

- Equalize marginal rate of substitution between consumption tomorrow and consumption today, $\frac{\beta u'(c_2)}{u'(c_1)}$, with relative price of consumption tomorrow to consumption today, $\frac{\frac{1}{1+r}}{1} = \frac{1}{1+r}$.

- This condition, together with the intertemporal budget constraint, uniquely determines the optimal consumption choices (c_1, c_2), as a function of incomes (y_1, y_2), initial wealth A and the interest rate r.

What is next:

- Explicit solution for a simply example
- Graphic representation of general case
- Changes in income (y_1, y_2, A) and the interest rate r

An Example: Period utility is $u(c) = \log(c)$; $u'(c) = \frac{1}{c}$

- Optimality condition becomes

$$\begin{aligned} \frac{\beta * \frac{1}{c_2}}{\frac{1}{c_1}} &= \frac{1}{1+r} \\ \frac{\beta c_1}{c_2} &= \frac{1}{1+r} \\ c_2 &= \beta(1+r)c_1 \end{aligned}$$

- Inserting this into the lifetime budget constraint yields

$$\begin{aligned} c_1 + \frac{\beta(1+r)c_1}{1+r} &= I \\ c_1(1+\beta) &= I \\ c_1 &= \frac{I}{1+\beta} \\ c_1(y_1, y_2, A, r) &= \frac{1}{1+\beta}\left(y_1 + \frac{y_2}{1+r} + A\right) \end{aligned}$$

- Since $c_2 = \beta(1+r)c_1$ we find

$$c_2 = \frac{\beta(1+r)}{1+\beta} I = \frac{\beta(1+r)}{1+\beta} \left(y_1 + \frac{y_2}{1+r} + A \right)$$

- Finally, since savings $s = y_1 + A - c_1$

$$\begin{aligned} s &= y_1 + A - \frac{1}{1+\beta} \left(y_1 + \frac{y_2}{1+r} + A \right) \\ &= \frac{\beta}{1+\beta} (y_1 + A) - \frac{y_2}{(1+r)(1+\beta)} \end{aligned}$$

  which may be positive or negative, depending on how high first period income and initial wealth is compared to second period income.

- Optimal consumption choice today is simple: eat a fraction $\frac{1}{1+\beta}$ of total lifetime income I today and save the rest.

- Note: the higher is income y_1 relative to y_2, the higher is saving s.

- For general utility functions $u(.)$, we cannot solve for the optimal consumption and savings choices analytically.
- But we can do graphical analysis. Idea: make a plot with c_1 on x-axis and c_2 on y-axis.
- Plot budget line and indifference curve and derive tangency point, which is the optimal choice.
- The computer can always be used.

- Combination of all (c_1, c_2) that can be exactly afforded.

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} + A$$

- Suppose $c_2 = 0$. Then can afford $c_1 = y_1 + A + \frac{y_2}{1+r}$ in the first period.
- Suppose $c_1 = 0$. Then can afford $c_2 = (1+r)(y_1 + A) + y_2$ in the second period.
- Slope of the budget line is

$$\begin{aligned} \text{slope} &= \frac{c_2^b - c_2^a}{c_1^b - c_1^a} \\ &= \frac{(1+r)(y_1 + A) + y_2}{-\left(y_1 + A + \frac{y_2}{1+r}\right)} \\ &= -(1+r) \end{aligned}$$

- Utility function tells us how the household values consumption today and consumption tomorrow.
- Indifference curve is a collection of bundles (c_1, c_2) that yield the same utility:

$$v = u(c_1) + \beta u(c_2)$$

- Slope: totally differentiate with respect to (c_1, c_2) :

$$dc_1 * u'(c_1) + dc_2 * \beta u'(c_2) = 0$$

- Rewriting

$$\frac{dc_2}{dc_1} = -\frac{u'(c_1)}{\beta u'(c_2)} = \text{MRS}$$

- For example $u(c) = \log(c)$ we find

$$\frac{dc_2}{dc_1} = -\frac{c_2}{\beta c_1}$$

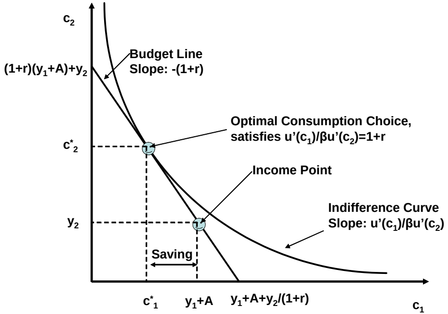
- Optimality condition

$$-\frac{u'(c_1)}{\beta u'(c_2)} = -(1+r) = \text{slope}$$

or

$$\text{MRS} = \frac{\beta u'(c_2)}{u'(c_1)} = \frac{1}{1+r}$$

- Interpretation: at the optimal consumption choice the cost, in terms of utility, of saving one more unit equals the benefit of saving that unit.
  The cost of saving one more unit, i.e. consume one unit less in first period, in terms of utility equals $u'(c_1)$. Saving one more unit yields $(1+r)$ more units of consumption tomorrow. In terms of utility, this is worth $(1+r)\beta u'(c_2)$.



Optimal Consumption Choice

- Analyze how changes in income and the interest rate affect household consumption and savings decisions

- Why? Fiscal policy changes level and timing of after-tax income. Government deficits and monetary policy may change real interest rates.

Income Changes again for $u(c) = \log(c)$

$$
\begin{aligned}
I &= y_1 + \frac{y_2}{1+r} + A \\
c_1 &= \frac{I}{1+\beta} \\
c_2 &= \frac{\beta(1+r)}{1+\beta} I \\
s &= \frac{\beta}{1+\beta}(y_1 + A) - \frac{y_2}{(1+r)(1+\beta)}
\end{aligned}
$$

We have
$$
\begin{aligned}
\frac{dc_1}{dI} &= \frac{1}{1+\beta} > 0 \\
\frac{dc_1}{dI} &= \frac{\beta(1+r)}{1+\beta} > 0
\end{aligned}
$$
and thus

$$
\begin{aligned}
\frac{dc_1}{dA} &= \frac{dc_1}{dy_1} = \frac{1}{1+\beta} > 0 \text{ and } \frac{dc_1}{dy_2} = \frac{1}{(1+\beta)(1+r)} > 0 \\
\frac{dc_2}{dA} &= \frac{dc_2}{dy_1} = \frac{\beta(1+r)}{1+\beta} > 0 \text{ and } \frac{dc_2}{dy_2} = \frac{\beta}{1+\beta} > 0 \\
\frac{ds}{dA} &= \frac{ds}{dy_1} = \frac{\beta}{1+\beta} > 0 \text{ and } \frac{ds}{dy_2} = -\frac{1}{(1+\beta)(1+r)} < 0
\end{aligned}
$$

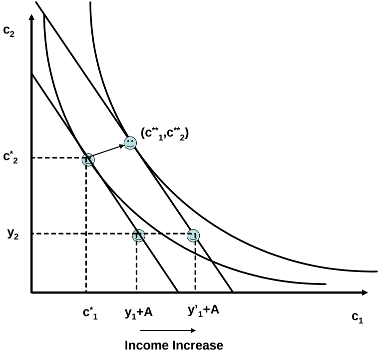
Income Changes: General Case

- Suppose income in the first period y_1 increases to $y_1' > y_1$.

- Budget line shifts out in a parallel fashion (since interest rate does not change).

- Consumption in both periods increases: positive income effect.

- Similar analysis for change in A or y_2.



A Change in Income

- Three effects, stemming from the budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} + A \equiv I(r)$$

1. The present value of resources shrinks
2. The present value of expenditures shrinks
3. Consumption in the second period becomes relatively cheaper than consumption in the first period.

- Whether the reduction of the present value of resources is larger than the reduction of the present value of expenditures, this is whether the wealth effect is positive or negative depends on whether the agent is a saver (the wealth or income effect is positive) or a borrower (the wealth effect is negative).

- Example $u(c) = \log(c)$. Optimal choices

$$\begin{aligned} c_1 &= \frac{1}{1+\beta} * I(r) \\ c_2 &= \frac{\beta(1+r)}{1+\beta} * I(r) \end{aligned}$$

- An increase in r reduces lifetime income $I(r)$, unless $y_2 = 0$. This is the negative wealth effect, reducing consumption in both periods.

- For c_1 this is the only effect: absent a change in $I(r)$, c_1 does not change. For this special example income and substitution effect exactly cancel out.

- For c_2 both income and substitution effects are positive. Remembering that $I(r) = A + y_1 + \frac{y_2}{1+r}$, we see that

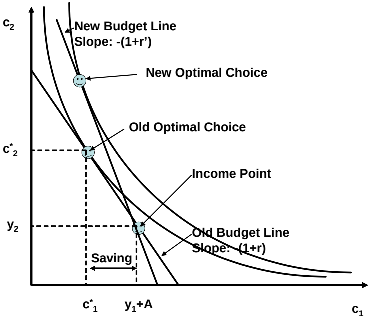
$$c_2 = \frac{\beta(1+r)}{1+\beta}(A + y_1) + \frac{\beta}{1+\beta}y_2$$

which is increasing in r.

- Increase in the interest rate from r to $r' > r$. Indifference curves do not change. Budget line gets steeper.

- Income point $c_1 = y_1 + A, c_2 = y_2$ remains affordable.

- Budget line tilts around the autarky point and gets steeper.



An Increase in the Interest Rate

Proposition

Let (c_1^*, c_2^*, s^*) denote the optimal consumption and saving choices associated with interest rate r. Furthermore denote by $(\hat{c}_1^*, \hat{c}_2^*, \hat{s}^*)$ the optimal consumption-savings choice associated with interest $\hat{r} > r$

1. *If $s^* > 0$ (that is $c_1^* < A + y_1$ and the agent is a saver at interest rate r), then $U(c_1^*, c_2^*) < U(\hat{c}_1^*, \hat{c}_2^*)$ and either $c_1^* < \hat{c}_1^*$ or $c_2^* < \hat{c}_2^*$ (or both).*
2. *Conversely, if $\hat{s}^* < 0$ (that is $\hat{c}_1^* > A + y_1$ and the agent is a borrower at interest rate $\hat{r}$), then $U(c_1^*, c_2^*) > U(\hat{c}_1^*, \hat{c}_2^*)$ and either $c_1^* > \hat{c}_1^*$ or $c_2^* > \hat{c}_2^*$ (or both).*

Proof ($s^* > 0$) I

- Budget constraints read as

$$\begin{aligned} c_1 + s &= y_1 + A \\ c_2 &= y_2 + (1+r)s \end{aligned}$$

- (c_1^*, c_2^*, s^*) is optimal for r. If $\widehat{r} > r$, the agent can choose

$$\begin{aligned} \tilde{c}_1 &= c_1^* > 0 \\ \tilde{s} &= s^* > 0 \end{aligned}$$

  and

$$\begin{aligned} \tilde{c}_2 &= y_2 + (1+\widehat{r})\tilde{s} \\ &= y_2 + (1+\widehat{r})s^* \\ &> y_2 + (1+r)s^* = c_2^* \end{aligned}$$

Proof ($s^* > 0$) II

- Since $\tilde{c}_1 \geq c_1^*$ and $\tilde{c}_2 > c_2^*$ we have

$$U(c_1^*, c_2^*) < U(\tilde{c}_1, \tilde{c}_2)$$

- The optimal choice at $\widehat{r}$ is obviously no worse, and thus

$$U(c_1^*, c_2^*) < U(\tilde{c}_1, \tilde{c}_2) \leq U(\widehat{c}_1^*, \widehat{c}_2^*)$$

- But

$$U(c_1^*, c_2^*) < U(\widehat{c}_1^*, \widehat{c}_2^*)$$

requires either $c_1^* < \widehat{c}_1^*$ or $c_2^* < \widehat{c}_2^*$ (or both).

QED.

- So far assumed that household can borrow freely at interest rate r. Now suppose that household cannot borrow at all, that is, let us impose the additional constraint on the consumer maximization problem that

$$s \geq 0.$$

Let (c_1^*, c_2^*, s^*) denote the optimal consumption choice the household would choose *in the absence* of the borrowing constraint.

- If optimal unconstrained choice satisfies $s^* \geq 0$, then it remains optimal.
- If optimal unconstrained choice satisfies $s^* < 0$, then it is optimal to set

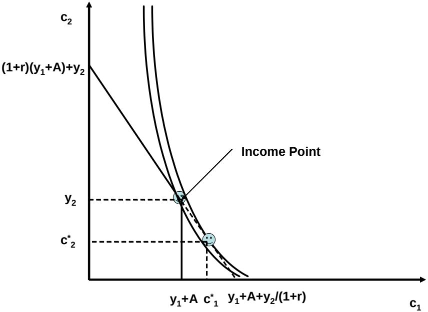
$$\begin{aligned} c_1 &= y_1 + A \\ c_2 &= y_2 \\ s &= 0 \end{aligned}$$

- Welfare loss from inability to borrow.

- In the presence of borrowing constraints has a kink at $(y_1 + A, y_2)$.

- For $c_1 < y_1 + A$ we have the usual budget constraint, as here $s > 0$ and the borrowing constraint is not binding.

- But with borrowing constraint any consumption $c_1 > y_1 + A$ is unaffordable, so the budget constraint has a vertical segment at $y_1 + A$



Borrowing Constraints

Borrowing Constraints and Income Changes

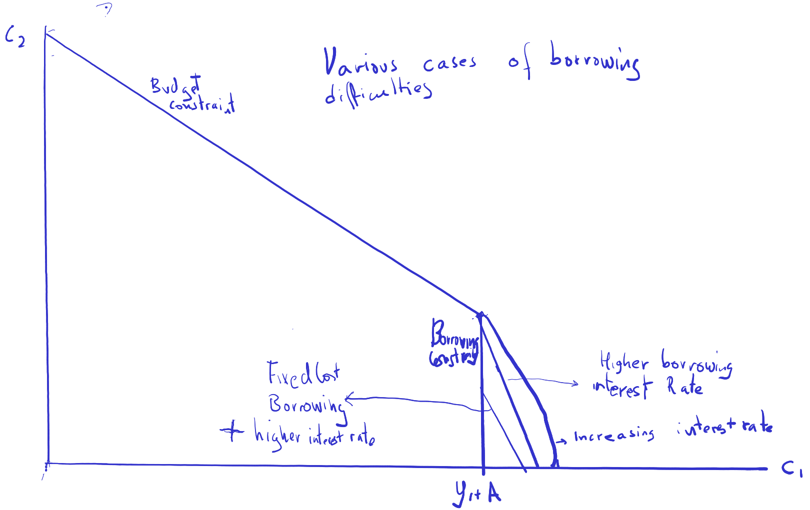
- Effects of income changes on consumption choices are potentially more extreme in the presence of borrowing constraints, which may give the government's fiscal policy extra power.
- Change in second period income y_2. With borrowing constraints optimal choice satisfies

$$\begin{aligned} c_1 &= y_1 + A \\ c_2 &= y_2 \\ s &= 0 \end{aligned}$$

- Increase in y_2 does not affect consumption in the first period of her life and increases consumption in the second period of his life one-for-one with income.
- Increase in y_1 on the other hand, has strong effects on c_1. If, after the increase it is still optimal to set $s = 0$ (which will be the case if the increase in y_1 is small), then c_1 increases one-for-one with the increase in current income and c_2 remains unchanged.

Types of Borrowing Constraints

- Cannot borrow at all
- Can borrow at a higher interest rate than the rate at which can save
- Can borrow at an ever increasing interest rate (due to increased rate of default)
- There is a fixed cost to start borrowing


C_2
Budget constraint
Various cases of borrowing difficulties
Borrowing Constraint
Fixed Cost Borrowing
+ higher interest rate
Higher borrowing interest Rate
Increasing interest rate
C_1
$y_1 + A$

- Objective: endogenize income (y_1, y_2, A) and interest rate r. Landmark paper by Peter Diamond (1965).

- 2 period-lived overlapping generations world.

- Households maximize

$$u(c_1, c_2) = \log(c_1) + \log(c_2)$$

- Budget constraint: $A = y_2 = 0$ (retired when old). Income when young equals wage: $y_1 = w$. Thus

$$c_1 + \frac{c_2}{1+r} = w$$

- Optimal consumption and savings decisions

$$
\begin{aligned}
c_1 &= \frac{1}{2} w \\
c_2 &= \frac{1}{2} w(1+r) \\
s &= \frac{1}{2} w
\end{aligned}
$$

- Firms hire l workers, pay wages w, lease capital k at rate ρ, produce consumption goods according to production function $y = k^{\alpha} l^{1-\alpha}$.

- Takes (w, ρ) as given, and chooses (l, k) to maximize profits

$$\max_{(k,l)} k^{\alpha} l^{1-\alpha} - wl - \rho k$$

- First order conditions

$$\begin{aligned} (1-\alpha) k^{\alpha} l^{-\alpha} &= w \\ \alpha k^{\alpha-1} l^{1-\alpha} &= \rho. \end{aligned}$$

- Capital stock k_1 in period 1 given.

- Labor market clearing:

$$l_1 = 1$$

- Thus wages given by

$$w = (1 - \alpha)k_1^\alpha$$

- Only asset is physical capital stock. Thus savings have to equal k_2. Asset market clearing condition

$$s = k_2$$

- Plugging in for $s = \frac{1}{2}w$ and using equilibrium wage function gives:

$$\frac{1}{2}(1-\alpha)k_1^{\alpha} = k_2.$$

- Steady state: level of capital that remains constant over time, $k_1 = k_2 = k$.

- Steady state satisfies

$$
\begin{aligned}
\frac{1}{2}(1-\alpha)k^{\alpha} &= k \\
k^{*} &= \left[\frac{1}{2}(1-\alpha)\right]^{\frac{1}{1-\alpha}}
\end{aligned}
$$

- Steady state wages are given by

$$w = (1-\alpha)\left(k^*\right)^{\alpha} = (1-\alpha)\left[\frac{1}{2}(1-\alpha)\right]^{\frac{\alpha}{1-\alpha}}$$

- Steady state interest rate r? When households save in period 1, they purchase capital k_2 which is used in production and earns rental rate ρ.

- Rental rate given by:

$$\rho = \alpha k^{\alpha-1} l^{1-\alpha} = \alpha \left(\left[\frac{1}{2}(1-\alpha) \right]^{\frac{1}{1-\alpha}} \right)^{\alpha-1} = \frac{2\alpha}{1-\alpha}$$

- If we assume that capital completely depreciates after production, then

$$1 + r = \rho = \frac{2\alpha}{1-\alpha}$$

- Time extends from $t = 0$ forever.

- Each period t a total number N_t of new young households are born that live for two periods.

- Assume population grows at a constant rate n:

$$N_t = (1+n)^t N_0 = (1+n)^t$$

- Household problem:

$$\max_{c_{1t}, c_{2t+1}, s_t} \{\log(c_{1t}) + \beta \log(c_{2t+1})\}$$

$$\begin{aligned} c_{1t} + s_t &= w_t \\ c_{2t+1} &= (1 + r_{t+1}) s_t. \end{aligned}$$

with solution:

$$\begin{aligned} c_{1t} &= \frac{1}{1+\beta} w_t \\ s_t &= \frac{\beta}{1+\beta} w_t \end{aligned}$$

- Aggregate output Y_t given by

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

- Wages

$$w_t = (1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha}$$

- Labor market clearing condition:

$$L_t = N_t$$

- Thus (with $k_t = \frac{K_t}{N_t}$)

$$w_t = (1-\alpha)\left(\frac{K_t}{N_t}\right)^{\alpha} = (1-\alpha)k_t^{\alpha}$$

- Capital market

$$s_t N_t = K_{t+1}$$

- Rewriting:

$$s_t = \frac{K_{t+1}}{N_t} = \frac{K_{t+1}}{N_{t+1}} * \frac{N_{t+1}}{N_t} = k_{t+1}(1+n)$$

- Plugging in from the saving function

$$s_t = \frac{\beta}{1+\beta} w_t = \frac{\beta}{1+\beta}(1-\alpha)k_t^{\alpha} = k_{t+1}(1+n)$$

- Thus

$$k_{t+1} = \frac{\beta(1-\alpha)}{(1+\beta)(1+n)} k_t^{\alpha}$$

- Aggregate population in period t is $N_{t-1} + N_t$.

- Per capita output is

$$y_t = \frac{Y_t}{N_{t-1} + N_t} = \frac{K_t^{\alpha} N_t^{1-\alpha}}{N_{t-1} + N_t}$$

- Steady state: situation in which the per capita capital stock k_t is constant over time thus and $k_{t+1} = k_t$

- Steady state satisfies

$$k = \frac{\beta(1-\alpha)}{(1+\beta)(1+n)} k^{\alpha}$$

or

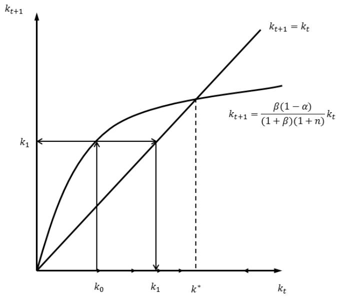
$$k^* = \left[\frac{\beta(1-\alpha)}{(1+\beta)(1+n)} \right]^{\frac{1}{1-\alpha}}$$

- Plotting k_{t+1} against k_t (together with 45^0-line) we can determine steady states, entire dynamics of model.

$$k_{t+1} = \frac{\beta(1-\alpha)}{(1+\beta)(1+n)} k_t^{\alpha}$$

- If $k_t = 0$, then $k_{t+1} = 0$. Since $\alpha < 1$, the curve $\frac{\beta(1-\alpha)}{(1+\beta)(1+n)} k_t^{\alpha}$ is strictly concave, initially above 45^0-line, but eventually intersects it.

- Unique positive steady state k^*. This steady state is globally asymptotically stable.



k_{t+1}
$k_{t+1} = k_t$
$k_{t+1} = \frac{\beta(1-\alpha)}{(1+\beta)(1+n)} k_t$
k_1
k_0
k_1
k^*
k_t

Special Topic

Income, Wealth and Health Inequality

Properties of Inequality

- We talk about some statistics of income and wealth inequality
    - The small correlation between income and wealth (different people)
    - Large concentration of both
    - Preponderance of business income.
    - Low (negative) correlation of transfers and income
- In any case they are dwarfed by the enormous inequality in longevity and health (40 times larger)

Special Topic

Bankruptcy

Bankruptcy Filing is Legal in the U.S.

- People and firms can file a process to discharge their debts.
- It poses a limit on assets kept that varies by state.
- It cannot be repeated within 8 years (Chapter 7, discharge of debts)
- It is a protection order from creditors.
- It affects negatively the credit score. Something that we think says something about people even if we are not sure exactly what.

Special Topic

Measurement of GDP

What is GDP?

- Three ways to Measure it (Uses, What is earned from it and sum (weigthed by relative prices) of all things produced in a place in a year)
- It is not welfare (inequality, other things matter)
- Issue with how it changes over time. Traditionally Mismeaured
    - Measurement of Quality of goods.
    - We measure expenditures not quantities and prices (especially for services).
    - Free goods (via advertising), Google? TVE?

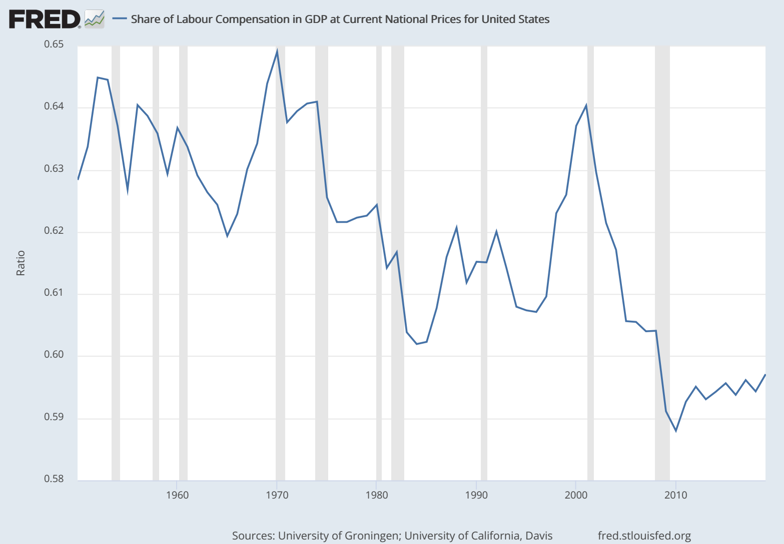
Special Topic

Labor Share

Labor Share: Theory

- Under Competition in Factor Markets
- Cobb-Douglas Technology
- Labor Sare is Constant
- Over the last 200 years rates of return have been constant
- Over the last 200 years Wages have been growing
- Labor Share Has been reasonably Constant for a long time
- But ....it has been shrinking in the last 20 years

Labor Share: DATA



Explanations I: Non Competition

- There is Labor, Capital and PROFIT shares
- Capital and Labor are not changing
- Profit Share is increasing.
- Some Evidence of this

Explanation II: labor Saving technological Change

- Technical progress substitutes Capital for Labor
- And capital is getting Cheaper
- AND Elasticity of Substitution is not Coming from Cobb-Douglas
- So Labor compensation is shrinking

Explanation III: Artifact of Changes in Accounting

- Things that Before were intermediate goods are now investment
  - R& D
  - Software
- Consequently, there is more investment and more payments to Capital

Alternative View of Elsby, Hobijn, and Sahin on the Decline of the U.S. Labor Share

They make 5 points

1. About a third of the decline in the published labor share appears to be an artifact of statistical procedures used to impute the labor income of the self-employed that underlies the headline measure.
2. Movements in labor's share are not solely a feature of recent U.S. history: The relative stability of the aggregate labor share prior to the 1980s in fact veiled substantial, though offsetting, movements in labor shares within industries. By contrast, the recent decline has been dominated by the trade and manufacturing sectors.
3. U.S. data provide limited support for neoclassical explanations based on the substitution of capital for (unskilled) labor to exploit technical change embodied in new capital goods.
4. Prima facie evidence for institutional explanations based on the decline in unionization is inconclusive.
5. Offshoring of the labor-intensive component of the U.S. supply chain as a leading potential explanation of the decline in the U.S. labor share over the past 25 years.

Back To the Core

Fiscal Policy in the Two Period Model

With Lump Sum Taxes

- It Depends.
  - YES: If government expenditures are not perfect substitutes of private consumption.
    - Paid by lump sum Taxes. The Budget Constrant becomes

$$\widehat{C}^y + \widehat{S} = \widehat{W} = W - \widehat{T}^y,$$

    - So Consumption is $C^y = \frac{W - T^y}{2}$ and $G = T^y + T^o$
  - NO: If Government Expenditures are perfect substitutes of consumption of the old who are the only ones taxed, we get.
    - Now while consumption is $\widehat{C}^o = (1+r)S - T^o$, the utility function would be $u(C^y, \widehat{C}^o + T^o)$
  - NO: if taxes that are rebated in the same period:

$$\widehat{C}^y + S = W - T^y + Tr^y, \qquad \widehat{C}^o = (1+r)S - T^o + Tr^o$$

- In general, with
    - utility functions not log, or
    - income in the second period,
    - or leisure choice, or
    - not lump sum taxes,
- Fiscal policy matters!!!

Labor income Taxes and first period transfers when $u(c_1) + \beta u(c_2)$

- Consider the budget constraint to be

$$\begin{aligned} c_1 + s &= w(1-\tau) + T \\ c_2 &= (1+r)s \end{aligned}$$

- The first order condition (after substituting c_2 and s) is

$$u'(c_1) = (1+r)\,\beta\, u'\left[(w(1-\tau) + T - c_1)\,(1+r)\right]$$

- But if there is no net collection by the government of any revenue, i.e. if $\tau w = T$ we have the same allocation as if there were no taxes

$$u'(c_1) = (1+r)\beta u'\left[(w - c_1)(1+r)\right]$$

- No net wealth-income or substitution effects

- Consider the budget constraint to be

$$
\begin{aligned}
(1+\tau^c)c_1 + s &= w + T \\
c_2 &= (1+r)s
\end{aligned}
$$

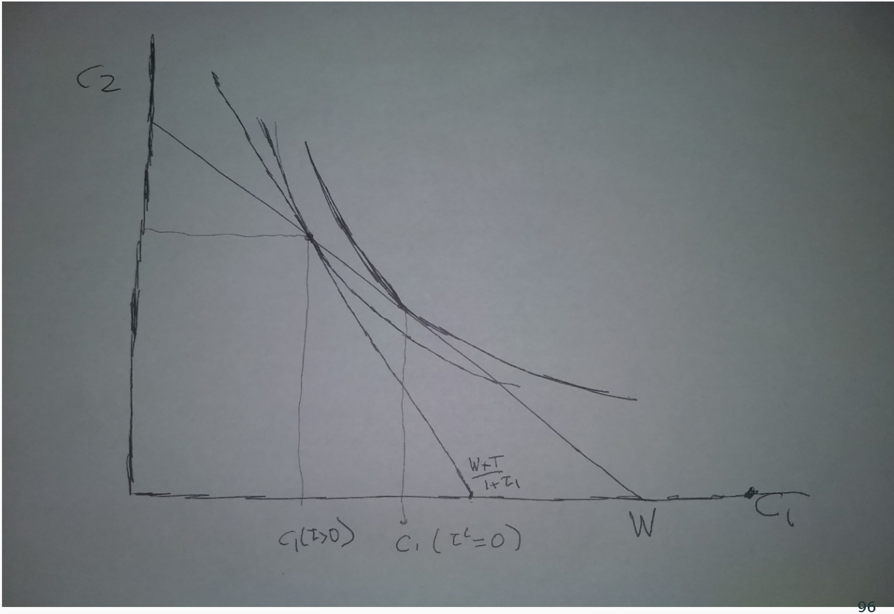
- The first order condition (after substituting c_2 and s) is

$$u'(c_1) = (1+r)(1+\tau^c)\,\beta\,u'\left[\left(w + T - c_1(1+\tau^c)\right)(1+r)\right]$$

- If there is no collection by the government of any revenue, i.e. if $\tau^c c_1 = T$ (note that the household cannot take this into account) things ARE different

$$u'(c_1) = (1+r)(1+\tau^c)\,\beta\,u'\left[\left(w - c_1\right)(1+r)\right]$$

- No net wealth-income effect but a substitution effect. Now c_1 is lower.


c_2
$\frac{W+T}{1+\tau_1}$
W
c_1
$c_1(\tau>0)$
$c_1(\tau^c=0)$