Economics 4230: Macro Modeling
Dynamic Fiscal Policy

José Víctor Ríos Rull

Spring Semester 2024

Most material developed by Dirk Krueger

University of Pennsylvania

Organizational Details (Material also in Canvas)

- **Time of Class:** Mon., Wed., 1:45 - 3:15pm

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  http://www.sas.upenn.edu/~vr0j/4230-24/

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- **Diary of what we did in class:** Available at:
  http://www.sas.upenn.edu/~vr0j/4230-24/diary.html

People

- Instructor: José Víctor Ríos Rull

- Time of Class: Monday, Wednesday, 1:45 - 3:15pm. PCPSE 100

- Office Hours: Mon 3:30-4:30 and by appointment. vr0j@upenn.edu

Course Outline and Overview

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- Class consists of model-based analysis, motivated by real world data and policy reforms

- 3 Homeworks and 3 midterms.

Homeworks, Midterms, Worth and Dates

	Fraction	Points	Date
Homework 1	8.33%	25	Due February 19
Midterm 1	25%	75	February 21
Homework 2	8.33%	25	Due April 1
Midterm 2	25%	75	April 3
Homework 3	8.33%	25	Due April 29
Midterm 3	25%	75	May 1
Total	100%	300	

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- Due date stated on homework. Due in class or in my mailbox by the end of class of the specified date. Late homework is **not** accepted.

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- Work in groups on homeworks permitted, but everybody needs to hand in **own** assignment. Please state whom you worked with.

Exams

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Grades

Points Achieved	Letter Grade
285 - 300	A +
270 - 284.5	A
255 - 269.5	A -
240 - 245.5	B +
225 - 239.5	B
210 - 224.5	B -
195 - 209.5	C +
180 - 194.5	C
165 - 179.5	C -
150 - 164.5	D +
135 - 149.5	D
less than 135	F

Content of Course

- Some Basic Empirical Facts about the Size of the Government (Part I)

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- Optimal Policy (Part VII)

Part I

Introduction and Main Facts

$$C = \text{Consumption}$$

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Y &= C + I + G + (X - M)
\end{aligned}
$$

US 2018 Main Macro Aggregates Bureau of Economic Analysis

In 2019 increased 2.3% in 2020 -2.8%, 2021 5.9%, 2021 2.1%, 2023 3.0%

	Billions of dollars	Perc of GDP
Gross domestic product	20,500.6	100.00
Personal consumption expenditures	13,951.6	68.05
Goods	4,342.1	21.18
Services	9,609.4	46.87
Gross private domestic investment	3,652.2	17.82
Fixed investment	3,595.6	17.54
Nonresidential	2,800.4	13.66
Structures	637.1	3.11
Equipment	1,236.3	6.03
Intellectual property products	927.0	4.52
Residential	795.3	3.88
Change in private inventories	56.5	0.28
Net exports of goods and services	–625.6	-3.05
Exports	2,530.9	12.35
Imports	3,156.5	15.40
Government expenditures	3,522.5	17.18
Federal	1,319.9	6.44
National defense	779.0	3.80
Nondefense	540.9	2.64
State and local	2,202.6	10.74

Two Deficits

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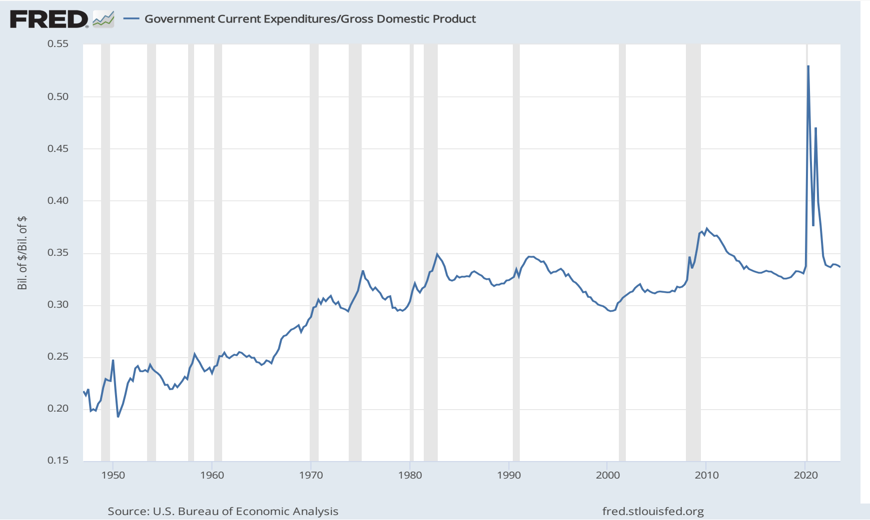
Trade Balance as Share of GDP, 1970-2022

Government Outlays as Fraction of GDP, 1970-2022



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- Federal debt and deficit are related by

$$\begin{aligned}\text{Fed. debt at end of this year} &= \text{Fed. debt at end of last year}\\ &\quad +\text{Fed. budget deficit this year}\end{aligned}$$

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Other	136.6

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Outlays	4,022.9
National Defense	705.6
International Affairs	45.7
Health	372.5
Medicare	485.7
Income Security	597.4
Social Security	730.8
Net Interest	230.0
Other	435.5

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Surplus	–1,299.6

State and Local Budgets (in billion $)		
	2011	2013
Total Revenue	2,618	2,690
Property Taxes	445.8	445.4
Taxes on Production and Sales	464.0	496.4
Individual Income Taxes	285.3	338.5
Corporation Net Income Tax	48.4	53.0
Transfers from Federal Gov.	647.6	584.7
All Other	722.9	762.4
Total Expenditures	2,583.8	2,643.1
Education	862.27	876.6
Highways	153.9	158.7
Public Welfare	494.7	516.4
All Other	1,072.9	1,091.4
Surplus	34.2	47.3

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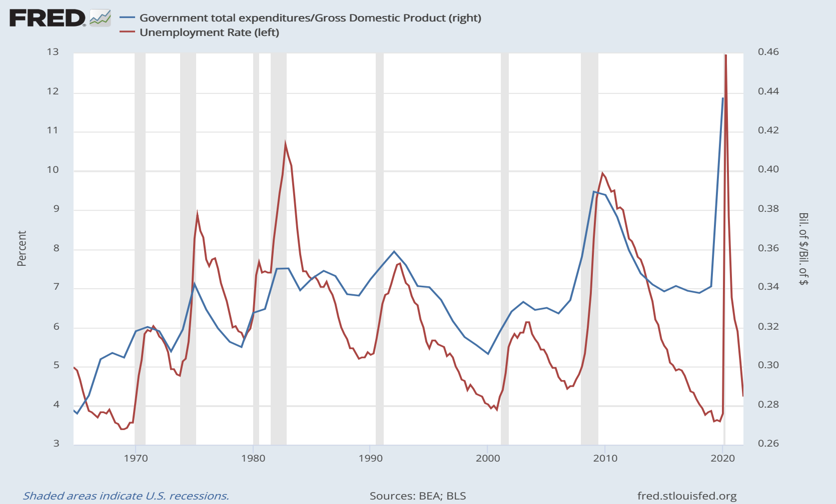
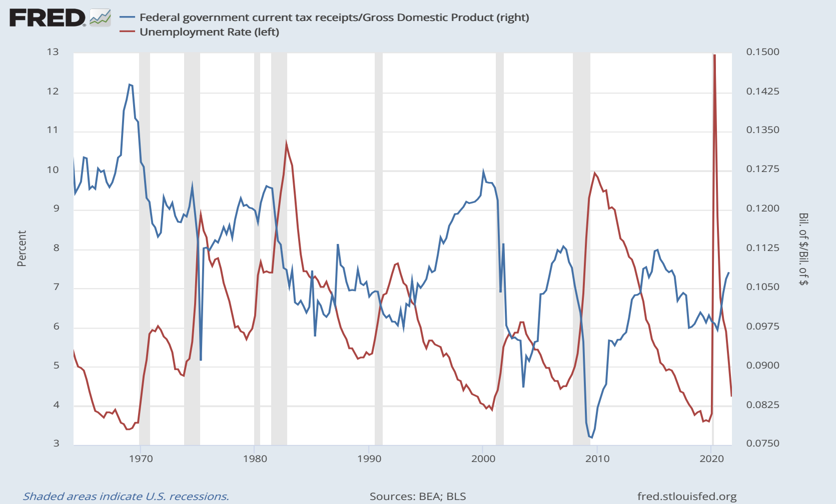
- Does fiscal policy (government spending, taxes collected, government deficit) vary systematically over the business cycle?

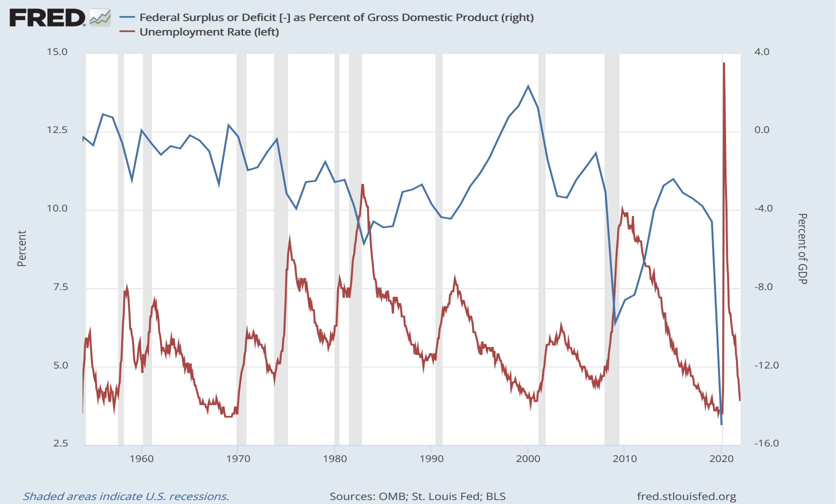
Government Outlays and Unemployment Rate, 1965-2021



Gov Taxes and Unemployment Rate, 1965-2021



Deficit and Unemployment Rate, 1965-2021



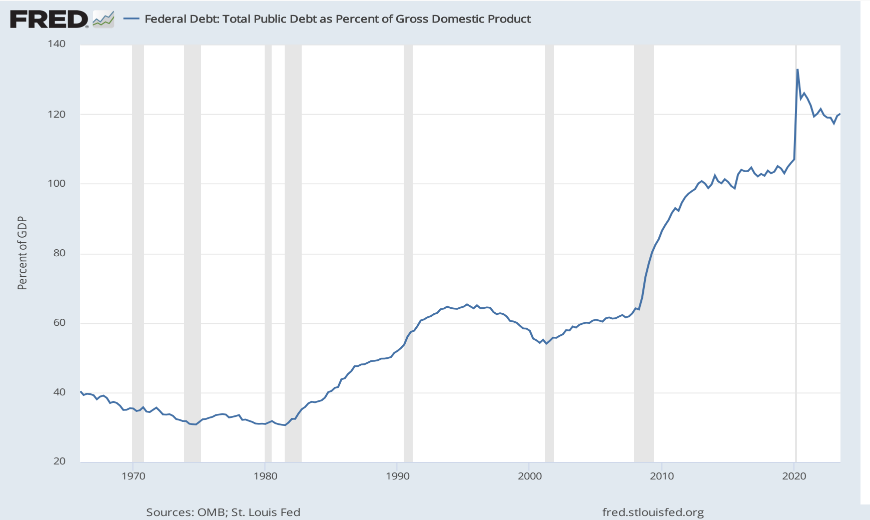
- $$\begin{aligned}\text{Government Outlays to GDP ratio} &= \frac{Outlays}{GDP}\\ \text{Deficit-GDP ratio} &= \frac{Deficit}{GDP}\\ \text{Debt-GDP ratio} &= \frac{Debt}{GDP}\end{aligned}$$

- $$\begin{aligned}\text{Debt at end of this year} &= \text{Debt at end of last year}\\ &\quad + \text{Budget deficit this year}\end{aligned}$$

Government Outlays to GDP ratio, 2006

- US: 36.4%
- Canada: 39.3%
- Japan: 36.0%
- Sweden: 54.3%, France: 52.7%, Germany: 45.3%

Debt to GDP Ratio, 1965-2023



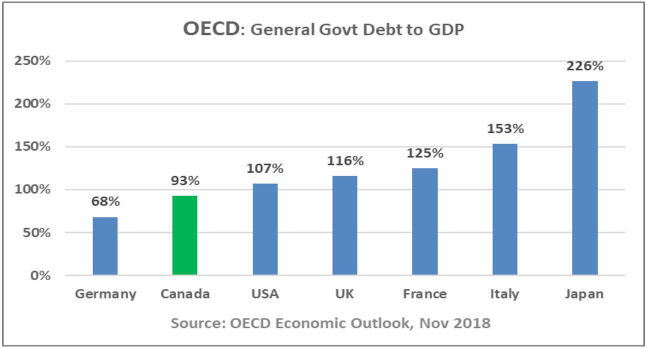
International Debt to GDP Ratios (OECD)

Includes Currency and Deposits (Overzealous Measure)

Country	2010	2011	2012	2013	2014	2015	2016	2017
Estonia	11.93	9.54	13.15	13.62	13.85	12.75	12.73	12.55
Chile	15.27	17.85	18.37	18.99	22.39	24.41	28.08	29.65
Denmark	53.44	60.11	60.62	56.73	59.14	53.79	52.60	49.96
Sweden	52.59	53.28	54.40	57.15	63.40	61.56	60.33	57.95
Australia	41.92	46.31	59.25	55.77	61.63	64.28	68.64	65.72
Germany	84.45	84.18	88.11	83.27	83.35	78.96	76.01	71.52
Ireland	83.50	111.46	129.36	131.73	121.20	88.52	84.14	77.24
Canada	105.22	107.88	111.54	107.51	108.54	114.75	114.13	109.10
Spain	66.56	77.69	92.53	105.73	118.41	116.31	116.52	114.66
United Kingdom	86.56	100.31	104.11	99.92	109.92	109.45	119.38	116.91
Belgium	107.98	110.60	120.47	118.48	131.11	127.67	128.44	121.90
France	101.00	103.81	111.94	112.47	120.16	120.83	125.46	124.25
United States	125.85	130.98	132.69	136.28	135.60	136.60	138.51	135.66
Portugal	104.07	107.85	137.10	141.43	151.40	149.15	145.32	145.38
Italy	124.88	117.94	136.24	143.69	156.06	157.03	154.90	152.61
Greece	128.97	110.91	164.11	179.69	180.82	182.94	185.79	188.73
Japan	207.52	222.31	230.39	233.22	238.46	237.39	234.55	
Mexico	31.15	37.14	41.13	47.11	50.06	53.33	51.79	
Switzerland	42.62	43.03	43.81	43.08	43.14	43.18	42.46	

Public Debt Including Some Unfunded Public Sector Liabilities

OECD Nov 2018



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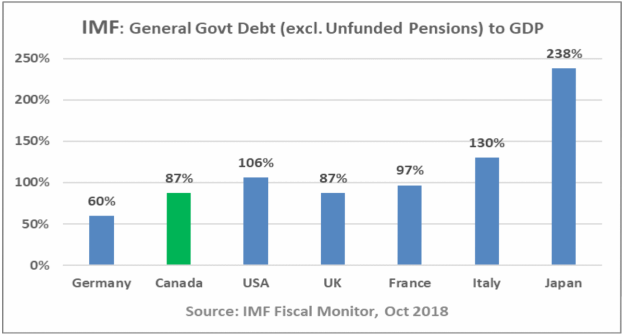

Part II

The Benchmark Model

Intro: Intertemporal Choice Model

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- Why a two period (dynamic) model? Because the government choice of policies today affect what it can do tomorrow (a tax cut today, together with a budget deficit, requires higher taxes or lower spending tomorrow). Therefore need a model where choices today affect choices tomorrow. Simplest such model is a two-period model.

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- Model is due to Irving Fisher (1867-1947), extension due to Albert Ando (1929-2003) and Franco Modigliani (1919-2003) and Milton Friedman (1912-2006).

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A Simple Two Period Model

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where $\beta \in (0, 1)$ measures household's impatience.

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- Can save or borrow at real interest rate r

- Nominal and real interest rates

$$1 + r = \frac{1 + i}{1 + \pi}$$

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- Second period budget constraint

$$c_2 = y_2 + (1 + r)s$$

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- Simplify: consolidate two budget constraints into intertemporal budget constraint by substituting out saving: solve second budget constraint for s to obtain

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- Decision problem of household:
  Choose (c_1, c_2, s) to maximize lifetime utility, subject to the budget constraints.

- Simplify: consolidate two budget constraints into intertemporal budget constraint by substituting out saving: solve second budget constraint for s to obtain

$$s = \frac{c_2 - y_2}{1 + r}$$

- Substitute into first budget constraint:

$$c_1 + \frac{c_2 - y_2}{1 + r} = y_1 + A$$

  or

$$c_1 + \frac{c_2}{1 + r} = y_1 + \frac{y_2}{1 + r} + A$$

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- Let $I = y_1 + \frac{y_2}{1+r} + A$ denote total lifetime income, consisting of human capital and initial wealth.

- Maximization problem

$$\max_{c_1, c_2} \quad \{u(c_1) + \beta u(c_2)\}$$

$$s.t. \quad c_1 + \frac{c_2}{1+r} = I$$

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- Lagrangian method or substitution method

- Lagrangian

$$\mathcal{L} = u(c_1) + \beta u(c_2) + \lambda \left[I - c_1 - \frac{c_2}{1+r} \right]$$

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- Combining yields

$$u'(c_1) = \beta(1+r)u'(c_2)$$

or

$$u'\left(I - \frac{c_2}{1+r} \right) = (1+r)\beta u'(c_2)$$

- Existence of unique solution? Assume Inada condition

$$\lim_{c \to 0} u'(c) = \infty$$

define

$$f(c_2) = u'\left(I - \frac{c_2}{1+r}\right) - (1+r)\beta u'(c_2)$$

and use the Intermediate Value Theorem to show that there is a value for c_2 that makes $f(c_2) = 0$.

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- This condition, together with the intertemporal budget constraint, uniquely determines the optimal consumption choices (c_1, c_2), as a function of incomes (y_1, y_2), initial wealth A and the interest rate r.

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- Inserting this into the lifetime budget constraint yields

$$\begin{aligned} c_1 + \frac{\beta(1+r)c_1}{1+r} &= I \\ c_1(1+\beta) &= I \\ c_1 &= \frac{I}{1+\beta} \\ c_1(y_1, y_2, A, r) &= \frac{1}{1+\beta}\left(y_1 + \frac{y_2}{1+r} + A\right) \end{aligned}$$

- Since $c_2 = \beta(1+r)c_1$ we find

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- Finally, since savings $s = y_1 + A - c_1$

$$\begin{aligned} s &= y_1 + A - \frac{1}{1+\beta} \left(y_1 + \frac{y_2}{1+r} + A \right) \\ &= \frac{\beta}{1+\beta} (y_1 + A) - \frac{y_2}{(1+r)(1+\beta)} \end{aligned}$$

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- Note: the higher is income y_1 relative to y_2, the higher is saving s.

Graphic Solution of the Model

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- The computer can always be used.

- Combination of all (c_1, c_2) that can be exactly afforded.

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The Budget Line

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- Suppose $c_1 = 0$. Then can afford $c_2 = (1+r)(y_1 + A) + y_2$ in the second period.
- Slope of the budget line is

$$\begin{aligned} \text{slope} &= \frac{c_2^b - c_2^a}{c_1^b - c_1^a} \\ &= \frac{(1+r)(y_1 + A) + y_2}{-\left(y_1 + A + \frac{y_2}{1+r}\right)} \\ &= -(1+r) \end{aligned}$$

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Indifference Curves

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- For example $u(c) = \log(c)$ we find

$$\frac{dc_2}{dc_1} = -\frac{c_2}{\beta c_1}$$

- Optimality condition

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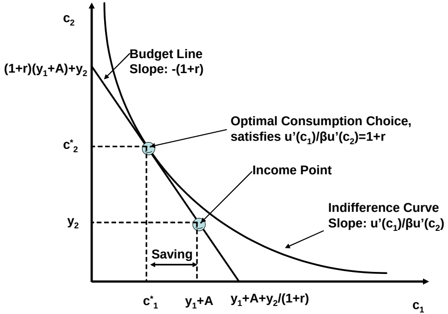
- Interpretation: at the optimal consumption choice the cost, in terms of utility, of saving one more unit equals the benefit of saving that unit.
  The cost of saving one more unit, i.e. consume one unit less in first period, in terms of utility equals $u'(c_1)$. Saving one more unit yields $(1+r)$ more units of consumption tomorrow. In terms of utility, this is worth $(1+r)\beta u'(c_2)$.



Optimal Consumption Choice

- Analyze how changes in income and the interest rate affect household consumption and savings decisions

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- Why? Fiscal policy changes level and timing of after-tax income. Government deficits and monetary policy may change real interest rates.

Income Changes again for $u(c) = \log(c)$

$$
\begin{aligned}
I &= y_1 + \frac{y_2}{1+r} + A \\
c_1 &= \frac{I}{1+\beta} \\
c_2 &= \frac{\beta(1+r)}{1+\beta} I \\
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We have
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\begin{aligned}
\frac{dc_1}{dI} &= \frac{1}{1+\beta} > 0 \\
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and thus

$$
\begin{aligned}
\frac{dc_1}{dA} &= \frac{dc_1}{dy_1} = \frac{1}{1+\beta} > 0 \text{ and } \frac{dc_1}{dy_2} = \frac{1}{(1+\beta)(1+r)} > 0 \\
\frac{dc_2}{dA} &= \frac{dc_2}{dy_1} = \frac{\beta(1+r)}{1+\beta} > 0 \text{ and } \frac{dc_2}{dy_2} = \frac{\beta}{1+\beta} > 0 \\
\frac{ds}{dA} &= \frac{ds}{dy_1} = \frac{\beta}{1+\beta} > 0 \text{ and } \frac{ds}{dy_2} = -\frac{1}{(1+\beta)(1+r)} < 0
\end{aligned}
$$

- Suppose income in the first period y_1 increases to $y_1' > y_1$.

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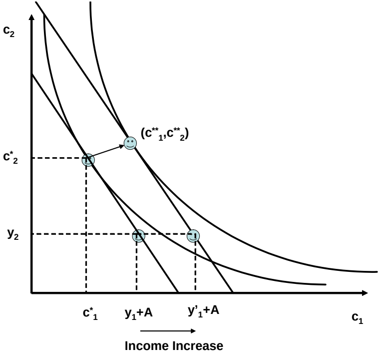
- Budget line shifts out in a parallel fashion (since interest rate does not change).
- Consumption in both periods increases: positive income effect.
- Similar analysis for change in A or y_2.



A Change in Income

Interest Rate Changes

- Three effects, stemming from the budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} + A \equiv I(r)$$

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3. Consumption in the second period becomes relatively cheaper than consumption in the first period.

- Three effects, stemming from the budget constraint

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1. The present value of resources shrinks
2. The present value of expenditures shrinks
3. Consumption in the second period becomes relatively cheaper than consumption in the first period.

- Whether the reduction of the present value of resources is larger than the reduction of the present value of expenditures, this is whether the wealth effect is positive or negative depends on whether the agent is a saver (the wealth or income effect is positive) or a borrower (the wealth effect is negative).

Interest Rate Changes: Example

- Example $u(c) = \log(c)$. Optimal choices

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\begin{aligned}
c_1 &= \frac{1}{1+\beta} * I(r) \\
c_2 &= \frac{\beta(1+r)}{1+\beta} * I(r)
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- Example $u(c) = \log(c)$. Optimal choices

$$\begin{aligned} c_1 &= \frac{1}{1+\beta} * I(r) \\ c_2 &= \frac{\beta(1+r)}{1+\beta} * I(r) \end{aligned}$$

- An increase in r reduces lifetime income $I(r)$, unless $y_2 = 0$. This is the negative wealth effect, reducing consumption in both periods.

- For c_1 this is the only effect: absent a change in $I(r)$, c_1 does not change. For this special example income and substitution effect exactly cancel out.

- For c_2 both income and substitution effects are positive. Remembering that $I(r) = A + y_1 + \frac{y_2}{1+r}$, we see that

$$c_2 = \frac{\beta(1+r)}{1+\beta}(A + y_1) + \frac{\beta}{1+\beta}y_2$$

which is increasing in r.

- Increase in the interest rate from r to $r' > r$. Indifference curves do not change. Budget line gets steeper.

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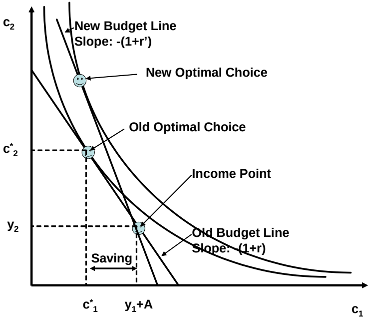
- Income point $c_1 = y_1 + A, c_2 = y_2$ remains affordable.

- Increase in the interest rate from r to $r' > r$. Indifference curves do not change. Budget line gets steeper.

- Income point $c_1 = y_1 + A, c_2 = y_2$ remains affordable.

- Budget line tilts around the autarky point and gets steeper.



An Increase in the Interest Rate

Proposition

Let (c_1^*, c_2^*, s^*) denote the optimal consumption and saving choices associated with interest rate r. Furthermore denote by $(\widehat{c}_1^*, \widehat{c}_2^*, \widehat{s}^*)$ the optimal consumption-savings choice associated with interest $\widehat{r} > r$

1. *If $s^* > 0$ (that is $c_1^* < A + y_1$ and the agent is a saver at interest rate r), then $U(c_1^*, c_2^*) < U(\widehat{c}_1^*, \widehat{c}_2^*)$ and either $c_1^* < \widehat{c}_1^*$ or $c_2^* < \widehat{c}_2^*$ (or both).*
2. *Conversely, if $\widehat{s}^* < 0$ (that is $\widehat{c}_1^* > A + y_1$ and the agent is a borrower at interest rate $\widehat{r}$), then $U(c_1^*, c_2^*) > U(\widehat{c}_1^*, \widehat{c}_2^*)$ and either $c_1^* > \widehat{c}_1^*$ or $c_2^* > \widehat{c}_2^*$ (or both).*

Proof ($s^* > 0$) I

- Budget constraints read as

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- (c_1^*, c_2^*, s^*) is optimal for r. If $\widehat{r} > r$, the agent can choose

$$\begin{aligned} \tilde{c}_1 &= c_1^* > 0 \\ \tilde{s} &= s^* > 0 \end{aligned}$$

and

$$\begin{aligned} \tilde{c}_2 &= y_2 + (1+\widehat{r})\tilde{s} \\ &= y_2 + (1+\widehat{r})s^* \\ &> y_2 + (1+r)s^* = c_2^* \end{aligned}$$

Proof ($s^* > 0$) II

- Since $\tilde{c}_1 \geq c_1^*$ and $\tilde{c}_2 > c_2^*$ we have

$$U(c_1^*, c_2^*) < U(\tilde{c}_1, \tilde{c}_2)$$

Proof ($s^* > 0$) II

- Since $\tilde{c}_1 \geq c_1^*$ and $\tilde{c}_2 > c_2^*$ we have

$$U(c_1^*, c_2^*) < U(\tilde{c}_1, \tilde{c}_2)$$

- The optimal choice at $\widehat{r}$ is obviously no worse, and thus

$$U(c_1^*, c_2^*) < U(\tilde{c}_1, \tilde{c}_2) \leq U(\widehat{c}_1^*, \widehat{c}_2^*)$$

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$$U(c_1^*, c_2^*) < U(\tilde{c}_1, \tilde{c}_2) \leq U(\widehat{c}_1^*, \widehat{c}_2^*)$$

- But

$$U(c_1^*, c_2^*) < U(\widehat{c}_1^*, \widehat{c}_2^*)$$

  requires either $c_1^* < \widehat{c}_1^*$ or $c_2^* < \widehat{c}_2^*$ (or both).

QED.

- So far assumed that household can borrow freely at interest rate r. Now suppose that household cannot borrow at all, that is, let us impose the additional constraint on the consumer maximization problem that

$$s \geq 0.$$

Let (c_1^*, c_2^*, s^*) denote the optimal consumption choice the household would choose *in the absence* of the borrowing constraint.

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- Welfare loss from inability to borrow.

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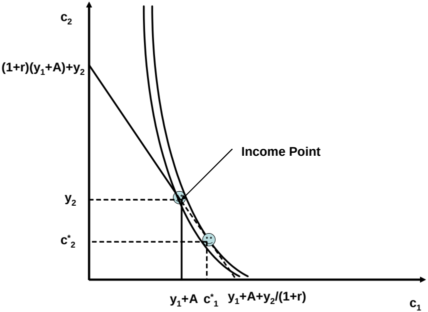
- But with borrowing constraint any consumption $c_1 > y_1 + A$ is unaffordable, so the budget constraint has a vertical segment at $y_1 + A$



Borrowing Constraints

Borrowing Constraints and Income Changes

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- Increase in y_2 does not affect consumption in the first period of her life and increases consumption in the second period of his life one-for-one with income.
- Increase in y_1 on the other hand, has strong effects on c_1. If, after the increase it is still optimal to set $s = 0$ (which will be the case if the increase in y_1 is small), then c_1 increases one-for-one with the increase in current income and c_2 remains unchanged.

Types of Borrowing Constraints

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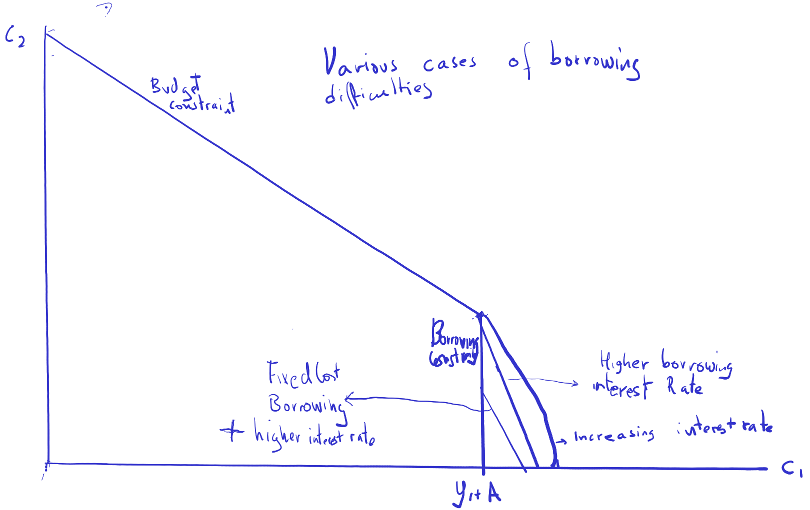
- There is a fixed cost to start borrowing



c_2
Budget constraint
Various cases of borrowing difficulties
Borrowing Constraint
Fixed Cost Borrowing + higher interest rate
Higher borrowing interest Rate
Increasing interest rate
$y_1 + A$
c_1

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$$u(c_1, c_2) = \log(c_1) + \log(c_2)$$

- Budget constraint: $A = y_2 = 0$ (retired when old). Income when young equals wage: $y_1 = w$. Thus

$$c_1 + \frac{c_2}{1+r} = w$$

- Optimal consumption and savings decisions

$$
\begin{aligned}
c_1 &= \frac{1}{2} w \\
c_2 &= \frac{1}{2} w(1+r) \\
s &= \frac{1}{2} w
\end{aligned}
$$

- Firms hire l workers, pay wages w, lease capital k at rate ρ, produce consumption goods according to production function $y = k^{\alpha} l^{1-\alpha}$.

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- First order conditions

$$\begin{aligned} (1-\alpha) k^{\alpha} l^{-\alpha} &= w \\ \alpha k^{\alpha-1} l^{1-\alpha} &= \rho. \end{aligned}$$

Equilibrium

- Capital stock k_1 in period 1 given.

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- Labor market clearing:

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- Thus wages given by

$$w = (1 - \alpha)k_1^{\alpha}$$

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- Plugging in for $s = \frac{1}{2}w$ and using equilibrium wage function gives:

$$\frac{1}{2}(1-\alpha)k_1^{\alpha} = k_2.$$

Equilibrium: Steady State

- Steady state: level of capital that remains constant over time, $k_1 = k_2 = k$.

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- Steady state satisfies

$$\begin{aligned} \frac{1}{2}(1-\alpha)k^{\alpha} &= k \\ k^* &= \left[\frac{1}{2}(1-\alpha)\right]^{\frac{1}{1-\alpha}} \end{aligned}$$

- Steady state wages are given by

$$w = (1-\alpha)\left(k^*\right)^{\alpha} = (1-\alpha)\left[\frac{1}{2}(1-\alpha)\right]^{\frac{\alpha}{1-\alpha}}$$

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$$w = (1-\alpha)(k^*)^\alpha = (1-\alpha)\left[\frac{1}{2}(1-\alpha)\right]^{\frac{\alpha}{1-\alpha}}$$

- Steady state interest rate r? When households save in period 1, they purchase capital k_2 which is used in production and earns rental rate ρ.

- Rental rate given by:

$$\rho = \alpha k^{\alpha-1} l^{1-\alpha} = \alpha \left(\left[\frac{1}{2}(1-\alpha) \right]^{\frac{1}{1-\alpha}} \right)^{\alpha-1} = \frac{2\alpha}{1-\alpha}$$

- If we assume that capital completely depreciates after production, then

$$1 + r = \rho = \frac{2\alpha}{1-\alpha}$$

- Time extends from $t = 0$ forever.

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- Assume population grows at a constant rate n:

$$N_t = (1+n)^t N_0 = (1+n)^t$$

- Household problem:

$$\max_{c_{1t}, c_{2t+1}, s_t} \{\log(c_{1t}) + \beta \log(c_{2t+1})\}$$

$$\begin{aligned} c_{1t} + s_t &= w_t \\ c_{2t+1} &= (1 + r_{t+1}) s_t. \end{aligned}$$

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with solution:

$$
\begin{aligned}
c_{1t} &= \frac{1}{1+\beta} w_t \\
s_t &= \frac{\beta}{1+\beta} w_t
\end{aligned}
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- Aggregate output Y_t given by

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- Wages

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- Thus (with $k_t = \frac{K_t}{N_t}$)

$$w_t = (1-\alpha)\left(\frac{K_t}{N_t}\right)^{\alpha} = (1-\alpha)k_t^{\alpha}$$

- Capital market

$$s_t N_t = K_{t+1}$$

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- Rewriting:

$$s_t = \frac{K_{t+1}}{N_t} = \frac{K_{t+1}}{N_{t+1}} * \frac{N_{t+1}}{N_t} = k_{t+1}(1+n)$$

- Plugging in from the saving function

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- Thus

$$k_{t+1} = \frac{\beta(1-\alpha)}{(1+\beta)(1+n)} k_t^{\alpha}$$

- Aggregate population in period t is $N_{t-1} + N_t$.

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- Per capita output is

$$y_t = \frac{Y_t}{N_{t-1} + N_t} = \frac{K_t^{\alpha} N_t^{1-\alpha}}{N_{t-1} + N_t}$$

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- Steady state satisfies

$$k = \frac{\beta(1-\alpha)}{(1+\beta)(1+n)} k^{\alpha}$$

or

$$k^* = \left[\frac{\beta(1-\alpha)}{(1+\beta)(1+n)}\right]^{\frac{1}{1-\alpha}}$$

- Plotting k_{t+1} against k_t (together with 45^0-line) we can determine steady states, entire dynamics of model.

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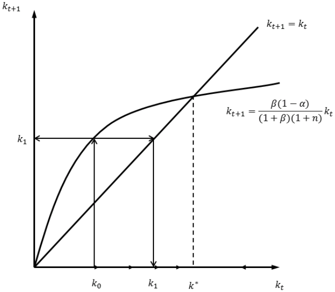
- Unique positive steady state k^*. This steady state is globally asymptotically stable.



k_{t+1}
$k_{t+1} = k_t$
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k_1
k_0
k_1
k^*
k_t

Special Topic

Income, Wealth and Health Inequality

- We talk about some statistics of income and wealth inequality

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    - The small correlation between income and wealth (different people)

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    - Preponderance of business income.
    - Low (negative) correlation of transfers and income
- In any case they are dwarfed by the enormous inequality in longevity and health (40 times larger)

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Bankruptcy

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- It affects negatively the credit score. Something that we think says something about people even if we are not sure exactly what.

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Measurement of GDP

What is GDP?

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  - Free goods (via advertising), Google? TVE?

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- Under Competition in Factor Markets

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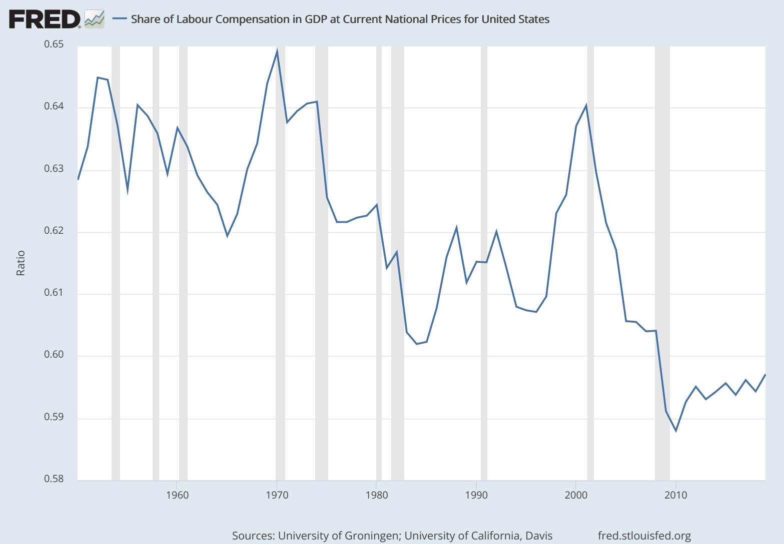
- But ....it has been shrinking in the last 20 years

Labor Share: DATA



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Explanation III: Artifact of Changes in Accounting

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- Consequently, there is more investment and more payments to Capital

Alternative View of Elsby, Hobijn, and Sahin on the Decline of the U.S. Labor Share

They make 5 points

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5. Offshoring of the labor-intensive component of the U.S. supply chain as a leading potential explanation of the decline in the U.S. labor share over the past 25 years.

Back To the Core

Fiscal Policy in the Two Period Model

Will Fiscal Policy Affect Path of The Economy? I

With Lump Sum Taxes

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  - NO: if taxes that are rebated in the same period:

$$\widehat{C}^y + S = W - T^y + Tr^y, \qquad \widehat{C}^o = (1+r)S - T^o + Tr^o$$

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- Fiscal policy matters!!!

Labor income Taxes and first period transfers when $u(c_1) + \beta u(c_2)$

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- No net wealth-income or substitution effects

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- No net wealth-income effect but a substitution effect. Now c_1 is lower.

Distortionary tax returned as lump sum