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Biased Technological Change and Poverty Traps

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Abstract

This paper presents a model in which technological change increases the share of reproducible factors at the expense of nonreproducible ones. When reproducible factors are abundant, firms have incentives to adopt technologies that are intensive in such resources, and this increases the incentives to invest more in them. This feedback process may generate growth or also stagnation: when reproducible factors are not abundant, firms do not have incentives to adopt technologies intensive in those resources and technological change does not take place. The paper also analyzes how biased technological change affects interpersonal distribution of income: nonreproducible factors are more equally distributed than reproducible ones, thus biased technological change increases inequality.

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I. Introduction

Empirical evidence shows that technological change is biased in favor of human capital (see Leavi and Murnane, 1992; and Acemoglu 2002), whether the share of physical capital is more or less constant. This means that technological change increases the share of reproducible factors such as human and physical capital, at the expense of nonreproductive ones such as raw labor. This paper analyzes the consequences that this biased technological change has for growth and development, focusing in explaining the stagnation that many developing countries suffer (see Easterly 1994, 2001) and the increasing polarization of countries (see Quah 1996, 1997).

Table 1 shows the portion of full-time male workers' total earnings for two educational levels in U.S.A, the more highly educated group's share of earnings in 1987 was 44% higher than in 1971. These data seem to suggest that technological change has been biased rather than neutral: the share of human capital has risen.

Table 1: Full-Time Male Workers

	1971	1979	1987
Share of 12 years of schooling	0.72	0.68	0.61
Share of 16 years of schooling	0.27	0.32	0.39

Source: Leavi and Murnane's tabulation of Current Population Survey (table 5 and 6 in Leavi and Murnane 1992).

Table 2 shows a striking difference between low-paid workers' earnings growth rates and the growth rates of per capita GDP: the average growth rate of low-paid workers' earnings was -0.8% against a 2% average growth rate of per capita GDP. This mean that low-paid workers in 1989 earned only 89% of their counterparts' earnings 14 years before. By contrast, the per capita GDP in 1989 was 32% higher than 14 years before. This empirical evidence also supports the idea that the share of raw labor has been reduced over time and that technological change is biased. If the share of labor were constant (as in most growth models), wages and per capita output would grow at the same rate.

Growth models were built using Kaldor's six stylized facts as a base (see Kaldor, 1961), the fifth of such stylized facts is that labor and capital receive constant shares of total income. The main idea of this paper is that even though the share of physical capital and labor has been more or less constant, technology is becoming more intensive in human capital. As a consequence, the share of reproducible factors increases at the expense of the nonreproducible ones.

This paper presents an endogenous growth model in which technological change does not increase the total factor productivity, but increases the share of reproducible factors at the expense of the nonreproducible factors. More precisely, firms choose among technologies of Cobb-Douglas type with different levels of sophistication. More sophisticated technologies are more intensive in reproducible factors, that is, the share of reproducible factors increases with the sophistication of the technology. There are also learning costs, which increase with the sophistication of the technology and decreases with the technological experience of society. Firms in countries in which technological experience and reproducible factors are abundant have incentives to adopt sophisticated technologies, this technological change increases technological experience and the share of reproducible factors and incentivates the accumulation of even more reproducible factors, this feed-back process generates permanent growth. However, when the initial levels of technological experience and reproducible factors are low, firms have no incentive to adopt sophisticated technologies and technological change does not take place. In this case countries converge to a steady state in which there is neither technological change nor growth. These results are consistent with the empirical findings by Easterly (1994, 2001) who showed that many developing countries are stagnated, and with (Quah 1996, 1997), who observed an increasing polarization among countries.

Table 2: First Decile Male Workers' Labor Earnings and per capita GDP in U.S.A.

Year	D1 Lab. earnings Growth %	Per Capita GDP Growth %	D1 Lab. earnings Index	Per Capita GDP Index
1975	——	——	100	100
1976	0.0	4.3	100	104
1977	1.0	3.9	101	108
1978	3.0	4.1	104	113
1979	-1.9	1.3	102	114
1980	-2	-2.2	100	112
1981	-2	1.3	98	113
1982	-3	-3.9	95	109
1983	-2.1	2.6	93	112
1984	-4.3	6.3	89	119
1985	0.0	1.9	89	121
1986	2.2	1.7	91	123
1987	-3.2	2.0	88	126
1988	2.2	3.0	90	129
1989	-1.1	2.2	89	132
Average	-0.8	2.0	——	——

Source D1: Table 5.3 OECD' Employment Outlook, July 1993. GDP: Penn. World Table.

Another empirical fact that this paper pays attention is the increasing inequality among workers (see Leavi and Murnane, 1992; and Acemoglu 2002 for excellent surveys). The preferences presented in the paper are of "keeping up with the Joneses" type, in which the elasticity of substitution is higher for richer agents, who therefore have a higher propensity to save. This preference implies that the stationary property rights distribution is such that nonreproducible factors (such as raw labor) are more equally distributed than reproducible factors (such as human and physical capital). Thus biased technological change rises the inequality of income distribution since the share of the less equality distributed factors (reproducible factors) increases at the expenses of the more equally distributed production factors (nonreproducible ones).

This paper is fundamentally related to the poverty trap literature (see among others Azariadis and Drazen, 1990; Azariadis, 1996; Deardorff, 2001; and Galor, 1996). The former papers do not analyze biased technological change and poverty traps are generated by non-convexities in most of them.

The paper is also related with endogenous growth literature, (Romer, 1986; Lucas, 1988; Rebelo, 1992). Especially relevant is the literature on endogenous technological change (see among others Aghion and Howitt, 1992; Grossman and Helpman, 1991; Romer, 1990) and the literature on learning by doing (see among others Lucas, 1988; Stokey, 1988; and Matsuyama, 1992). In the above-mentioned papers the share of reproducible factors is constant over time.

There is a recent and important literature about technological change and wage inequality (see among others Acemoglu, 1998, 2000, 2002; Caselli, 1999; Krusell, Ohanian, Rios-Rull and Violante, 2000; Galor and Tsiddon, 1997; Galor and Moav, 2000; Greenwood and Yorukoglu, 1997; Violante, 2002). In such literature faster technological change increases the demand for skilled workers and causes a rise in wage inequality. There are substantial differences between those papers and the present one, the most important is that they do not focus on the consequences that biased technological change has on developing countries, which is the main topic here.

The structure of the paper is as follows: Section II presents the basic model. Section III analyzes the long-run behavior of the model, that section analyses the steady states of the model and finds conditions under which long-run growth is possible. Section IV analyzes the dynamic behavior of the model. Section V characterizes the permanent growth paths. Section VI presents some extensions of the model. Section VII reaches the conclusions. All the proofs are in Appendix I. Appendix II shows that if preferences are conventional, biased technological change does not affect income distribution.

II. The Model

Time is discrete with an infinite horizon. There are two types of production factors: reproducible factors, which are accumulable over time, and nonreproducible factors, which are not accumulable. Reproducible factors are denoted by H, the nonreproducible ones by L. While H may be viewed as a composite of physical and human capital, L may be viewed as a composite of raw labor and other nonreproducible factors. The most intuitive way to interpret this model is that the technological change reduces the share of raw labor (L) in favor of the share of human capital (H).

There is a single good in the economy that can be used for consumption and investment:

$$Y_t = C_t + H_{t+1} - (1-\delta)H_t \tag{1}$$

where Y_t denotes production, C_t denotes consumption, and $\delta \in (0,1)$ denotes the depreciation rate.

A. Technology:

There is a continuum of technologies differentiated by their sophistication level. The sophisticated level of a technology is indexed by $z \in \left[\frac{1}{(1-\underline{\alpha})}, +\infty\right)$, where $\underline{\alpha} \in [0,1)$. A higher index z means a higher level of sophistication. The technology z is represented by the production function $F^z(.)$:

$$F^z(H,L) = Ae^{\Psi\left(1-Max\left\{\frac{z}{X},1\right\}\right)}(H)^{1-\frac{1}{z}}L^{\frac{1}{z}} \tag{2}$$

where A and Ψ are positive constants and $X \in \left[\frac{1}{(1-\underline{\alpha})}, +\infty\right)$ denotes the "technological experience" that is defined as a weighted geometrical average of the technologies used in the past:

$$X_{t+1} \equiv \prod_{i=0}^{\infty} z_{t-i}^{\eta(1-\eta)^i} = z_t^{\eta} X_t^{(1-\eta)} \tag{3}$$

where $\eta \in (0,1]$. The share of reproducible factors is denoted by α_t: $\alpha_t \equiv 1 - \frac{1}{z_t}$ and belongs to the set $[\underline{\alpha}, 1)$. Thus, $\underline{\alpha}$ is the lower boundary of the share of reproducible factors.

The meaning of sophistication in this model is that more sophisticated technologies are more intensive in reproducible factors. The intuition is that a more sophisticated technology requires more knowledge in order to use it and as a consequence more sophisticated technologies are more intensive in human capital which is a reproducible factor.

The function $e^{\Psi\left(1-Max\left\{\frac{z}{X},1\right\}\right)}$ may be interpreted as the state of "know-how". If a technology is more sophisticated than the technologies used in the past, then agents cannot use this technology at its maximum potential productivity, that is, the state of know-how is smaller than one.

The function $e^{-\Psi Max\left\{\frac{z}{X},1\right\}}$ may be interpreted as the portion of the output that is required as the cost of learning to use a new technology. The learning cost increases with the parameter Ψ and with the difference between the sophistication of the technology and the technological experience.

B. Initial Property Rights Distribution:

There is a continuum of types of agents indexed by $i \in [0,1]$ distributed according to the measure μ with support on $[0,1]$. Agents' life is infinite and population is constant. Agent of type i has $s^L(i)$ units of the nonreproducible factor in each period and $s_0^H(i)h_0$ units of the reproducible factor in period 0 (the first period), where h_0 is the per capita amount of the reproducible factor in period 0. Thus the average $s_0^H(i)$ should be one: $\int_0^1 s_0^H(i)d\mu = 1$. The amount of nonreproducible factors per capita is normalized at one, therefore $\int_0^1 s^L(i)d\mu = 1$. It is assumed that both $s_0^H(i)$ and $s^L(i)$ are increasing. This means that agents are ordered from poorer to richer: the closer the agent's index "i" is to zero the poorer the agent.

It is assumed that there is $\underline{i} \in [0, 1)$ such that agents in the set $[0, \underline{i}]$ do not have any initial assets (reproducible factors):

$$\forall i \in [0, \underline{i}] \,, \; s_0^H(i) = 0 \tag{4}$$

It is also assumed that agents in the set $[\underline{i}, 1]$ have the same amount of nonreproducible factors, which is denoted by $\overline{s}^L$:

$$\forall i \in [\underline{i}, 1] \,, \; s^L(i) = \overline{s}^L > 0 \tag{5}$$

A property of this wealth distribution is that the richer an agent is the higher the proportion of their income that comes from reproducible factors. The agent that has the same portion of reproducible and nonreproducible factors is denoted by $\overline{i}$:

$$\overline{i} = Inf \left\{ i \in \left[\underline{i}, \overline{i}\right] \quad s.th. \quad s_0^H(i) > s^L(i) = \overline{s}^L \right\}$$

Agents are classified into three "classes": i) Poor agents are those that do not own any assets. They are indexed in the set $[0, \underline{i}]$. ii) Middle class agents are those that own some assets but a higher portion of nonreproducible than reproducible factors: $s_0^H(i) < s^L(i) = \overline{s}^L$. They are indexed in the set $\left(\underline{i}, \overline{i}\right]$. iii) Rich agents are those that own a higher portion of the reproducible factors than the nonreproducible ones: $s_0^H(i) > s^L(i) = \overline{s}^L$. They are indexed in the set $\left(\overline{i}, 1\right]$.

Figure 1 shows the distribution of property rights. It will be shown later that this initial distribution does not change over time. Figure 1.a shows that poor agents ($i < \underline{i}$) do not own reproducible factors, and the reproducible factors owned by agents increase with the index "i". Figure 1.b shows that property rights over nonreproducible factors also increases with "i" and that middle class and rich agents ($i > \underline{i}$) own the same amount of nonreproducible factors (the distribution is flat for those agents).

The assumptions about property rights distribution allows many different cases. For example, every agent may have the same amount of nonreproducible factors; $\underline{i}$ may be zero, in such a case every agent would have some reproducible factors. The important property of this distribution is that the poorer an agent is, the higher the fraction of his income that comes from nonreproducible factors. In other words, the richer an agent is, the higher the portion of his income that comes from reproducible factors. Thus, since biased technological change increases the share of reproducible factors at the expense of the nonreproducible ones, it increases inequality in income distribution.

C. Preferences

The utility of consumers depends upon their consumption "$c_t(i)$" and upon the benchmark consumption level of the society, "v_t"[1]:

$$\begin{cases} \prod_{t=0}^{\infty} \left(c_t(i) - v_t\right)^{\beta^t} & if \quad \forall t \quad c_t(i) \geq v_t \\ \lim\limits_{T\to\infty} \min \left\{ \frac{c_t(i)}{v_t} \right\}_{t=0}^{T} - 1 & if \quad \exists t \quad c_t(i) < v_t \end{cases} \tag{6}$$

where $\beta \in (0,1)$, $c_t(i)$ denotes the consumption of the consumer type i in period t, and v_t is the benchmark level of consumption of the society in period t. The benchmark level can be viewed as the standard of living, which changes over time. When the consumption is always over the benchmark level, the utility function may be rewritten as the time separable logarithmic utility function:

$$\sum_{t=0}^{\infty} \beta^t \ln \left(c_t(i) - v_t\right) \tag{7}$$

When consumption is not always over the benchmark level, the preferences are represented by a Leontieff utility function.

It is assumed that the benchmark consumption level of a society is the minimum consumption level among middle class agents of this society:[2]

$$v_t = \underset{i \in [\underline{i},1]}{ess\,inf}\; c_t(i) \tag{8}$$

where $\underline{i} \in [0,1)$. Since $[\underline{i},1]$ may be a strict subset of $[0,1]$, v_t may be interpreted as a minimum consumption level for a middle class life.

This utility function implies that poor agents have a higher propensity to consume and less elasticity of substitution than rich ones. These facts have a long tradition in growth theory (see among others Alonso-Carrera, Caballe and Raurich, 2001; Carroll, Overland and Weil, 1997, 2000; Fisher and Holf, 2000; Kaldor 1956, 1961; Rebelo, 1992; Sieh, Lai and Chang, 2000) and have empirical support (see Atkenson and Ogaki, 1996; Kuznets, 1961).

III. Agents' Behavior

A. Behavior of Firms

Firms are competitive and maximize profit. It follows from the specification of technology that the maximization problem of the firm is as follows:

[1]It would be more realistic to use a utility function in which the standard of living would have a positive effect on the consumers' utility. For example, a utility as follows: $\prod_{t=0}^{\infty} \left(c_t(i) - v_t\right)^{\beta^t} + \phi \prod_{t=0}^{\infty} \left(v_t\right)^{\beta^t}$. However, such a modification would not change the results of the paper.

[2]$essinf\ c_t^i = Sup\{m\ s.th.\ \mu \left\{i\ s.th.\ c_t^i < m\right\} = 0\}$.

$$\underset{H,L,z}{Max} \; Ae^{\Psi\left(1-Max\left\{\frac{z}{X},1\right\}\right)}H^{1-\frac{1}{z}}L^{\frac{1}{z}} - wL - (r+\delta)H \tag{9}$$

where w denotes the price of nonreproducible factors, and r denotes the net payment to reproducible factors.

When $z > X$, the first order conditions (FOCs) of the above problem are as follows:

$$(r+\delta) = \left(1-\frac{1}{z}\right) Ae^{\Psi\left(1-Max\left\{\frac{z}{X},1\right\}\right)}h^{\frac{-1}{z}} \tag{10}$$

$$w = \frac{1}{z}Ae^{\Psi\left(1-Max\left\{\frac{z}{X},1\right\}\right)}h^{1-\frac{1}{z}} \tag{11}$$

$$\frac{\Psi}{X} = \frac{1}{z^2}\ln(h) \tag{12}$$

where h denotes the ratio of reproducible to nonreproducible factors: $h \equiv H/L$. The first two FOCs (10) and (11) are familiar ones: the price of each factor should be equal to its marginal product. The third FOC (12) states that the marginal cost and the marginal product of adopting a more sophisticated technology should be equal. Such a marginal cost is proportional to the resultant reduction in the state of know-how and increases with the parameter Ψ and decreases with the technological experience X. The marginal product of adopting a more sophisticated technology increases with the ratio of reproducible to nonreproducible factors h.

It follows from the FOCs that the technology which maximizes the profits is as follows:

$$z(X,h) = \begin{cases} \left(\frac{1}{\Psi}X\ln h\right)^{\frac{1}{2}} & if \; h \geq e^{\Psi X} \\ X & if \; h \in \left[1, e^{\Psi X}\right] \\ \frac{1}{(1-\underline{\alpha})} & if \; h \leq 1 \end{cases} \tag{13}$$

The sophistication of the technology chosen by firms in equilibrium increases with the ratio of reproducible to nonreproducible factors h, and with the technological experience X, and decreases with the parameter Ψ. The incentives to use sophisticated technology increase with the ratio of reproducible to nonreproducible factors: when reproducible factors are relatively abundant their relative price is low and firms have more incentives to use technologies more intensive on it. The state of know-how increases with the technological experience and decreases with the parameter Ψ, therefore the incentives to use more sophisticated technologies increase with the technological experience and are reduced with Ψ.

Using the accumulation equation of the technological experience X (3) together with the equation of the technology chosen by firms (13), it follows that the technological experience X grows as follows:

$$\frac{X_{t+1}}{X_t} = \begin{cases} \left(\frac{1}{\Psi}\frac{\ln h_t}{X_t}\right)^{\frac{\eta}{2}} & if\ h_t \geq e^{\Psi X_t} \\ 1 & if\ h_t \in \left[1, e^{\Psi X_t}\right] \\ \left(\frac{1}{(1-\underline{\alpha})X_t}\right)^{\eta} & if\ h_t \leq 1 \end{cases} \tag{14}$$

B. Consumers' Behavior

An agent's net wealth in period t is defined as the assets of this agent in period t plus the present value of his lifetime income of nonreproducible factors, minus the present value of his lifetime benchmark level of consumption:

$$NW_t(i) = A_t(i) + \sum_{j=t}^{\infty} \frac{\left(w_j s^L(i) - v_j\right)}{\prod_{i=t}^{j}(1+r_i)} \tag{15}$$

The consumers with positive net wealth can consume above the benchmark level of consumption in every period, therefore consumers with positive net wealth face the following optimization problem:

$$\begin{array}{ll} \underset{\{c_t(i)\}_{t=0}^{\infty}}{Max} & \sum_{t=0}^{\infty} \beta^t \ln\left(c_t(i) - v_t\right) \\ s.t. & (1+r_t)\, A_t(i) + s^L(i) w_t = A_{t+1}(i) + c_t(i) \\ & A_0(i) = s_0^H(i) h_t \end{array} \tag{16}$$

where $A_t(i)$ denotes the assets owned by consumers of type i in period t.

The following assumption imposes a restriction on the equilibrium path of the benchmark level of consumption: the ratio of assets to net wealth owned by the poorest middle class agents should be bounded.

Assumption 1: $\forall t \geq 0 \ \underset{i \to \underline{i}}{lim} \frac{\left[\int_{\underline{i}}^{i} \left|\frac{A_{t+1}(i)}{NW_t(i)}\right| d\mu\right]}{\mu[\underline{i}, i]} < \infty.$

This assumption precludes the existence of two unreasonable types of equilibria. The first type are equilibria in which the benchmark level of consumption is so high that middle class agents need to get huge amounts of loans in order to finance this benchmark level of consumption. The second unreasonable type of equilibrium is those in which the future benchmark level of consumption is so high that middle class agents are forced to accumulate huge amounts of assets in order to be able to finance this high benchmark level of consumption in the future. In order to avoid these two types of equilibria in which either middle class agents' debts or savings are huge relative to their net wealth, it is assumed that ratio of assets to net wealth in absolute value should be bounded for middle class agents.

Proposition 1 *In equilibrium* $\forall t > 0 \ v_t = \overline{s}^L w_t$.

Proposition 1 says that the benchmark level of consumption is equal to the minimum income among middle class agents. Proposition 1 also implies that poor agents ($i \in [0, \underline{i})$) have negative net wealth, that is, their consumption is under the benchmark level of consumption. Middle class and rich agents ($i \in [\underline{i}, 1]$) has positive net wealth and therefore their consumption is above the benchmark level.

Proposition 1 and the assumptions about wealth distribution imply that middle class and rich consumers face the following optimization problem:

$$\begin{array}{l} \underset{\{c_t(i)\}_{t=0}^{\infty}}{Max} \ \sum_{t=0}^{\infty} \beta^t \ln\left(c_t(i) - \overline{s}^L w_t\right) \\ s.t. \quad (1+r_t)A_t(i) = A_{t+1}(i) + \left(c_t(i) - \overline{s}^L w_t\right) \\ \qquad\quad A_0(i) = s_0^H(i) h_t \end{array} \tag{17}$$

The solution of the above optimization problem implies that middle class and rich agents consume their income from nonreproducible factors plus a constant fraction of their income from reproducible factors:

$$\forall i \in [\underline{i}, 1] \quad c_t(i) = \overline{s}^L w_t + (1-\beta)(1+r_t)A_t(i) \tag{18}$$

Substituting the optimal consumption in the budget constraint, it follows that the asset accumulation is:

$$\forall i \in [\underline{i}, 1] \quad A_{t+1}(i) = \beta(1+r_t)A_t(i) \tag{19}$$

Proposition 1, the assumptions about wealth distribution and the definition of the preferences imply that the following "Euler Equation" should hold for poor agents:

$$\forall i \in [0, \underline{i}) \quad \frac{c_{t+1}(i)}{v_{t+1}} = \frac{c_t(i)}{v_t} \tag{20}$$

Using condition (20), together with the budget constraint and proposition 1, it follows that poor agents consume all their income in every period:

$$\forall i \in [0, \underline{i}) \quad c_t(i) = s^L(i) w_t \tag{21}$$

The aggregation of the individual agents' assets accumulation equation (19) implies the following reproducible factors accumulation equation:

$$h_{t+1} = \beta(1+r_t)h_t \tag{22}$$

Equations (1), (10), (13) and (31) imply that the growth rate of reproducible factors is as follows:

$$\frac{h_{t+1}}{h_t} = \beta\left[(1\text{-}\delta) + \left(1\text{-}\frac{1}{z(X_t,h_t)}\right) A e^{\Psi\left(1-Max\left\{\frac{z(X_t,h_t)}{X_t},1\right\}\right)} h_t^{\frac{-1}{z(X_t,h_t)}}\right] \tag{23}$$

Stationary property rights distribution: Equations (19), (20) together with the assumptions about the initial property rights distribution imply that the initial property right distribution does not change over time. Thus, the stationary property right distribution is such that reproducible factors are more equally distributed than reproducible factors. The preferences are crucial to achieve this result. If the preferences were conventional (with a zero benchmark level of consumption), then the long-run distribution of reproducible factors would be equal to the distribution of nonreproducible factors (see Appendix).

IV. Long-run Behavior

This section characterizes the long-run behavior of the economy, which may be of three types: i) permanent growth, ii) a steady state in which the most unsophisticated technology is used and iii) a continuum of steady states with different technologies, each of which is more sophisticated than the most unsophisticated one.

The discount rate of the utility function will be denoted by ρ from now on:

$$\rho \equiv \frac{1}{\beta} - 1 \Leftrightarrow \beta \equiv \frac{1}{1+\rho}$$

A. Permanent Growth

In order to characterize the long-run behavior of the economy the first thing to know is whether permanent growth is possible. The next proposition gives a set of conditions under which permanent growth is possible.

Proposition 2 *If* $\left[Ae^{-\Psi} - \delta\right] > \rho$*, there exists a non-empty set* Γ *such that if* $(X_0, h_0) \in \Gamma$*, then* $\forall t \geq 0$ $\frac{X_{t+1}}{X_t} > 1$, $\frac{h_{t+1}}{h_t} > 1$, $\lim_{t\to\infty} h_t = +\infty$ *and* $\lim_{t\to\infty} X_t = +\infty$.

Proposition 2 states that permanent growth is possible if A is large enough, and the consumers' discount rate ρ and Ψ are small enough. A affects positively the total factor productivity, therefore growth increases with A. The cost of adopting sophisticated technology increases with Ψ, therefore technological change and growth decreases with Ψ. Finally the larger the discount rate ρ is, the more patient the consumers and the higher the growth.

The behavior of permanent growth paths will be characterized in Section V. The following assumption guarantees the existence of a permanent growth path.

Assumption 2: $\beta\left[(1-\delta) + Ae^{-\Psi}\right] > 1$

B. Low Steady State

The steady state in which the least sophisticated technology $\frac{1}{1-\underline{\alpha}}$ is used will be called low steady state from now on. It follows from the accumulation equation of reproducible factors (23) and the accumulation equation of the technological experience (14) that in such a steady state the following conditions are satisfied:

$$\rho = \underline{\alpha} A \left(h^{low}\right)^{1-\underline{\alpha}} - \delta \tag{24}$$

$$h^{low} < 1 \tag{25}$$

Condition (24) is very familiar: the net marginal product of reproducible factors should be equal to the discount rate of the utility function. Condition (25) should hold in the low steady state because if the ratio of reproducible to nonreproducible factors is larger than one, firms have incentives to choose a technology more sophisticated than $\underline{\alpha}$.

It follows from (24) that the stock of reproducible factors in the low steady state is as follows:

$$h^{low} = \left[\frac{\underline{\alpha} A}{\rho + \delta}\right]^{\frac{1}{1-\underline{\alpha}}} \tag{26}$$

Since in the low steady state the stock of reproducible factors should be smaller than one (25), it follows from (26) that the low steady state exists if and only if the following condition holds:

$$h^{low} = \left[\frac{\underline{\alpha} A}{\rho + \delta}\right]^{\frac{1}{1-\underline{\alpha}}} < 1 \Leftrightarrow \underline{\alpha} < \frac{\rho + \delta}{A} \tag{27}$$

The assumption below guarantees the existence of the low steady state.

Assumption 3[3]: $\underline{\alpha} < \frac{\rho+\delta}{A}$

C. The Other Steady States

It follows from the accumulation equation of reproducible factors (23) and the accumulation equation of the technological experience (14) that in any steady state other than the low one the following condition is satisfied:

$$\rho = \alpha^{ss} A \left(h^{ss}\right)^{-(1-\alpha^{ss})} - \delta \tag{28}$$

$$h^{ss} \in \left[1, e^{\frac{\Psi}{1-\alpha^{ss}}}\right] \tag{29}$$

Condition (28) means that the discount rate of the utility function should be equal to the net marginal product of reproducible factors in steady state. Condition (29)

[3]It is easy to extend the analysis to the case in which assumption 3 is not satisfied.

should hold in steady state because if the per capita reproducible factor h is smaller than one, firms have incentives to choose the least sophisticated technology $\underline{\alpha}$. If h is larger than $e^{\Psi X^{ss}}$, firms have incentives to choose a technology more sophisticated that the one in the steady state (see 13).

It follows from (28) that the per capita reproducible factors in steady state are as follows:

$$h^{ss} = \left[\frac{\alpha^{ss} A}{\rho + \delta}\right]^{\frac{1}{1-\alpha^{ss}}} \tag{30}$$

It follows from (29) and (30) that there is a steady state for each α in the interval $\left[\frac{\rho+\delta}{A}, \frac{\rho+\delta}{Ae^{-\Psi}}\right]$. Another interesting fact is that the fraction of income that goes toward investment (the saving rate) in each steady state is different:

$$\frac{I^{ss}}{Y^{ss}} = \frac{\delta h^{ss}}{A\left(h^{ss}\right)^{\alpha}} = \alpha^{ss}\frac{\delta}{\rho + \delta} \tag{31}$$

where I denotes gross investment.

Summarizing this section, in the long-run differences among countries may persist: countries may either grow or converge to a steady state characterized by stagnation. There is a continuum of steady states in each of which the share of reproducible factors, the technology used and the saving rate are different. Firms do not use more sophisticated technologies because reproducible factors are not abundant and thus to adopt technologies more intensive in reproducible factors is not profitable. Consumers do not have incentives to increase their reproducible factors because firms use technologies that are not intensive in them. In other words, firms do not adopt technologies intensive in reproducible factors because there are insufficient supply of them and consumers do not invest in reproducible factors because there is not enough demand for them either.

V. Dynamic Behavior

The dynamic behavior of the model is determined by the accumulation equations for reproducible factors (23) and the technological experience (14):

$$\frac{h_{t+1}}{h_t} = \beta\left[(1\text{-}\delta) + \left(1\text{-}\frac{1}{z(X_t, h_t)}\right) Ae^{\Psi\left(1-Max\left\{\frac{z(X_t,h_t)}{X_t},1\right\}\right)} h_t^{\frac{-1}{z(X_t,h_t)}}\right] \tag{32}$$

$$\frac{X_{t+1}}{X_t} = \begin{cases} \left(\frac{1}{\Psi}\frac{\ln h_t}{X_t}\right)^{\frac{\eta}{2}} & if\ h_t \geq e^{\Psi X_t} \\ 1 & if\ h_t \in \left[1, e^{\Psi X_t}\right] \\ \left(\frac{1}{(1-\underline{\alpha})X_t}\right)^{\frac{\eta}{2}} & if\ h_t \leq 1 \end{cases} \tag{33}$$

It follows from equation (32) that the growth rate of reproducible factors is zero if the following condition holds:

$$\triangle h_t = 0 \Leftrightarrow h_t = \left[\frac{\left(1 - \frac{1}{z(X_t,h_t)}\right) A e^{\Psi\left(1-Max\left\{\frac{z(X_t,h_t)}{X_t},1\right\}\right)}}{\rho + \delta} \right]^{z(X_t,h_t)} \tag{34}$$

The set of points that satisfy equation (34) is called the *h-zero growth curve*. It has a positive slope for a very intuitive reason: both the share of reproducible factors and the marginal product of the reproducible factors increase with the technological experience. Thus the level of reproducible factors that makes its marginal product equal to the discount rate of the utility function also increases with the technological experience.

It follows from equation (33), that there is no technological progress if the following condition holds:

$$h_t \in [1, e^{\Psi X_t}] \Rightarrow \triangle X_t = 0 \tag{35}$$

The set of points that satisfy equation (35) is the *X-zero growth set*.

Figure 2 shows the phase diagram that describes the dynamic behavior of the model. The h-zero growth curve has a positive slope. The lined area in figure 2 represents the X-zero growth set. The points of intersection of the h-zero growth curve and the X-zero growth set are steady states, those steady states are represented by the thicker $SS1$-$SS2$ curve.

The low steady state is on the vertical axis under the line $h_t = 1$. It follows from equation (13) that under the line $h_t = 1$, the technology used is the most unsophisticated one. Thus, the model behaves like the neoclassical model: the amount of reproducible factors converges monotonically to the low steady state level and the share of reproducible factors stays constant over time.

The points above the X-zero growth set and below the h-zero growth curve are combination of state variables where both the technological sophistication level and the amount of reproducible factors grow to a positive rate. When the initial values of the state variables are in this area, permanent growth is possible.

There is a convergent path toward the steady state $SS2$. If the initial values of the state variables are above this $SS2$-convergent path, the economy converges toward a permanent growth path. There is also a convergent path toward the steady state $SS1$. If the initial values of the state variables are below this path, the economy converges toward the low steady state. If the initial values of the state variables are in between the $SS1$ and the $SS2$-convergent paths, the economy converges toward one of the steady states on the $SS1$-$SS2$ curve.

Conditional Convergence: The sophistication of the technology does not change when the initial values of the state variables are in the X-zero growth set, therefore economies in which their initial values are in the X-zero growth set behave like the neoclassical growth model. When the initial amount of reproducible factors is smaller than one, the same happens (see equation 13): the technology used is the most unsophisticated, does not change over time and the model behaves as the neoclassical one does.

Each steady state has a convergent path along which the share of reproducible factors stays constant and the model behaves as the neoclassical one does. However, there is a difference between the convergent path toward each steady state: the share of investment is different along each path. More precisely, it follows from equation (31) that the saving rate and the fraction of income invested increase with the sophistication of the technology and the per capita income in the steady state. Therefore if there is a set of countries in which the initial values of the state variable are below the curve $e^{\Psi X_t}$, those countries exhibit conditional convergence.

VI. The Permanent Growth Path and Inter-personal Distribution

In this subsection the permanent growth path is characterized. It follows from proposition 2 and the phase diagram in figure 2 that if the initial values of the state variables are above the X-zero growth set and below the h-zero growth curve then the equilibrium exhibits permanent growth of both state variables. Such set will be denoted from now on as Γ. That is, Γ denotes the set of values of the state variables where both state variables grow permanently without bounds.

Proposition 3 *If* $(X_0, h_0) \in \Gamma$ *then* $\lim_{t \to \infty} \frac{h_{t+1}}{h_t} = \beta \left[(1\text{-}\delta) + Ae^{-\Psi} \right]$.

To interpret this proposition, consider the FOC (12): the marginal cost of adopting a marginally more sophisticated technology increases with the parameter Ψ. Thus, the parameter Ψ reduces the incentives to adopt sophisticated technologies and consequently the speed of technological change and the long-run growth. The other factors that affect growth are very familiar from the endogenous growth literature: the higher the total factor productivity A the higher the growth, the more patient that agent are (the higher β) the higher the growth. If the parameter Ψ were zero, the technology used along the permanent growth path would be that in which the share of nonreproducible factors is zero. In that case the model would be a typical AK model (Rebelo 1994) with growth rate $\beta\left[(1\text{-}\delta) + A\right]$.

Corollary 4 *If* $(X_0, h_0) \in \Gamma$ *then* $\frac{w_{t+1}}{w_t} < \frac{y_{t+1}}{y_t}$, $\lim_{t \to \infty} \frac{w_{t+1}}{w_t} = \lim_{t \to \infty} \frac{y_{t+1}}{y_t} = \beta \left[(1\text{-}\delta) + Ae^{-\Psi} \right]$, *where* y *denotes the per capita income.*

Corollary 4 says that per capita income grows faster than payment of nonreproducible factors along the permanent growth path. Poor agents only own nonreproducible factors, thus the model predicts that poor agents' income grows more slowly than the per capita income.

Gini's coefficient over income distribution I^y_{Gini} and over consumption I^c_{Gini} are defined as follows:

$$I^y_{Gini} \equiv 1 - \int_0^1 \left(\frac{\int_0^x y(i)d\mu}{y\mu[0,x]} \right) dx \quad I^c_{Gini} \equiv 1 - \int_0^1 \left(\frac{\int_0^x c(i)d\mu}{c\mu[0,x]} \right) dx \tag{36}$$

where $y(i)$ denotes the (gross) income of type i. If Gini's coefficient is equal to zero then all agents in the economy enjoy the same level of income. The higher Gini's coefficient is, the higher the inequality.

Proposition 5 *If* $(X_0, h_0) \in \Gamma$, *then* $\forall t \geq 0$ $I^y_{Gini_{t+1}} > I^y_{Gini_t}$, $I^c_{Gini_{t+1}} > I^c_{Gini_t}$.

Proposition 5 says that along the permanent growth path income inequality increases. Nonreproducible factors are more equally distributed than reproducible ones (see figure 1), thus since the share of nonreproducible factors is declining along the permanent growth path, inequality rises over time.

Proposition 5 also says that inequality not only increases in terms of income distribution but in terms of consumption distribution as well. It follows from the Euler Equation that if the benchmark level of consumption were zero, the consumption growth of every agent would be the same, and the consumption distribution would not change over time whatever the initial income distribution was (see Chatterjee 1992).

Figure 3 shows the effect of biased technological change on income distribution. The lined area in figure 3 represents the income from nonreproducible factors (divided by the per capita income). The right hand part of figure 3 shows that biased technological change reduces the portion of the agents' income that comes from nonreproducible factors and increases the portion from reproducible factors. The portion of the agents' income from nonreproducible factors is higher for poor and middle class agents than for rich ones (see figure 1). Thus, biased technological change reduces the portion of per capita income that poor and middle class agents own and increases the rich agent's portion of per capita income (see figure 3).

The result of figure 3 is stated formally in the next proposition.

Proposition 6 *Let* $(X_0, h_0) \in \Gamma$, *a) If* $i < \bar{i}$ $\frac{s^y_{t+1}(i)}{s^y_t(i)} < 1$, *if* $i > \bar{i}$ $\frac{s^y_{t+1}(i)}{s^y_t(i)} > 1$, *where* $s^y_t(i)$ *is the share of per capita income of agent i:* $s^y_t(i) \equiv y_t(i)/y_t$. *b)* $\frac{y_{t+1}(i)}{y_t(i)}$ *increases with* i.

Proposition 6.a formalizes the results of figure 3: along the permanent growth path the share of the per-capita income of poor and middle class agents decreases, the share of the per-capita income of rich agents increases. Proposition 6.b means that along the permanent growth path, the richer the agent the faster the growth of his income.

Summarizing along the permanent growth path, the share of the more equally distributed factor (nonreproducible factors) decreases in favor of the less equally distributed factor (reproducible factors). As a consequence inequality increases along the permanent growth path.

VII. Some Extensions

A. The productivity slow down

Most of the models that try to explain the increase of wage inequality in U.S.A. predict that the technological change is faster and thus that TFP grows at a faster rate. This prediction is completely contra-factual. In this section the basic model presented until now will be modified introducing an externality of the same type that the one used by Romer (1986) in order to explain the productivity slow down.

The technology considered in this section is as follows:

$$F^z(H, L; h) = A\,(B\,h)^{\frac{\gamma}{z}}\, e^{\Psi\left(1-Max\left\{\frac{z}{X},1\right\}\right)}(H)^{1-\frac{1}{z}} L^{\frac{1}{z}} \tag{37}$$

where $B \in \Re_{++}$, $\gamma \in [0, 1]$ and h is the per capita capital. This means that the per capita reproducible factors have positive external effects over the total factor productivity. The dynamic behavior of the model is as described in figure 2 when $\gamma < 1$ and as described in figure 4 when $\gamma = 1$. The asymptotic growth rate along a permanent growth path is as follows:

$$\lim_{t\to\infty} \frac{h_{t+1}}{h_t} = \beta\left[(1-\delta) + Ae^{-\Psi(1-\gamma)}\right] \tag{38}$$

The externality has a positive long-run growth effect (γ positively affects the long-run growth), even though it is not essential in order to generate long-run growth. When γ is equal to one, there is an externality of the Romer's type (Romer 1986) and therefore even in the case in which there is no biased technological change, long-run growth is possible. If the initial values of the state variables are in the X-zero growth set (lined area in figure 4), then the growth rate is as follows:

$$\frac{h_{t+1}}{h_t} = \beta\left[(1-\delta) + \left(1 - \frac{1}{X_t}\right) AB^{\frac{1}{X_t}}\right] \tag{39}$$

It is quite straightforward that even if the long-run growth rate (38) were larger than along the transition (39) the TFP growth rate would decrease after biased technological change starts.

B. Increasing dispersion in wages

An empirical puzzle observed in the American labor market is the increasing dispersion in wages inside the same educational groups (see Violante 2002). The model may capture this by introducing a stochastic shock in the productivity of the individual reproducible factors. In this case individual income would be as follows:

$$y_t(i) = w_t s^L(i) + r_t \theta_t(i) s^H(i) h_t$$

where $\theta_t(i)$ is a stochastic shock independently distributed with an expected value equal to one. In this case biased technology change would reduce the share of the non-risky factor in favor of the risky one and would thus increase the dispersion among workers with the same education level.

VIII. Conclusion

This paper has presented a model in which technological change reduces the share of nonreproducible factors in favor of reproducible ones. The incentives to adopt more sophisticated technologies depend upon the technological experience of the country and the relative abundance of reproducible factors. The technological experience increases the productivity of sophisticated technologies, the abundance of reproducible factors provides incentives to adopt sophisticated technologies which are more intensive in them. Technological change increases the demand for reproducible factors and thus the incentive to accumulate them. This feed-back process may generate growth but may also generate stagnation: There is a continuum of steady states with different technologies, shares of reproducible factors and saving rates. When reproducible factors and technological experience are not abundant enough, economies converge to one of these steady states. Otherwise converge to a permanent growth path in which the share of reproducible factors increases permanently.

The preferences presented have the property of "keeping up with the Joneses", which implies that nonreproducible factors are more equally distributed than reproducible factors. This fact implies that biased technological change increases income distribution inequality.

IX. References

Acemoglu, D. (1998) "Why Do New Technologies Complement Skills? Directed Technical change and Wage Structures" Quarterly Journal of Economics 113, 1055 1090.

Acemoglu, D. (2000) "Labor and Capital Augmenting Technological Change" NBER Working Paper No 7544.

Acemoglu, D (2002) "Technical Change, Inequality, and the Labor Market" Journal of Economic Literature 40(1): 7-72

Aghion, P. and P. Howitt (1992) "A Model of Growth through Creative Destruction", Econometrica, LX, 323-51.

Alonso-Carrera, J., J. Caballe and X. Raurich (2001) "Growth, Habit Formation and Catching-up with the Joneses" mimeo.

Arrow, K. (1962) "The Economic Implications of Learning by Doing", Review of Economic Studies 29, 155-73.

Atkenson, A. and Ogaki, M., (1996) "Does the Intertemporal Elasticity of Substitution Vary with Wealth and Does that Matter for Aggregate Savings?, Journal of Monetary Economics 38-3, 507-534.

Azariadis, C. (1996) "The Economics of Poverty Traps: Part One: Complete Markets" Journal of Economic Growth 1(4): 449-96.

Azariadis, C. and A. Drazen (1990) "Threshold Externalities in Economic Development" Quarterly Journal of Economics 105(2): 501-26

Carroll C. , J. Overland and D. Weil (1997) "Comparison Utility in a Growth Model" Journal of Economic Growth 2, 339-367.

Carroll C. , J. Overland and D. Weil (2000) "Saving and Growth with Habit Formation" American Economic Review 90, 1-15.

Caselli, F. (1999) "Technological Revolutions" American Economic Review 87, 78-102

Chatterjee, S. (1994) "Transitional Dynamics and the Distribution of Wealth in a Neoclassical Growth Model" Journal of Public Economics 54(1): 97-119

Deardorff, A. (2001) "Rich and Poor Countries in Neoclassical Trade and Growth" Economic Journal 111(470), 277-94.

Easterly, W. (1994) "Economic Stagnation, Fixed Factors, and Policy Thresholds" Journal of Monetary Economics 33(3): 525-57

Easterly, W. (2001) "The Lost Decades: Developing Countries' Stagnation in Spite of Policy Reform 1980-1998" Journal of Economic Growth 6(2), 135-57.

Fisher, W. and F. Holf (2000) "Relative Consumption, Economic Growth and Taxation" Journal of Economics 72, 241-262."

Galor, O. (1996) "Convergence: Inferences from Theoretical Models" Economic Journal 106(437), 1056-69

Galor, O. and O. Moav (2000) "Ability Biased Technological Transition, Wage Inequality, and Economic Growth" Quarterly Journal of Economics 115(2), 469-97.

Galor, O. and D. Tsiddon (1997) "Technological Progress, Mobility, and Economic Growth" American Economic Review 87(3), 363-82.

Greenwood J., and M. Yorukoglu, (1997) "1974" Carnegie Rochester Conference Series on Public Policy.

Grossman, G. and Helpman H: (1991) "Quality Ladders in the Theory of Growth", Review of Economic Studies, LVIII, 43-61.

Kaldor, N. (1956) "Alternative Theories of Distribution", Review of Economic Studies 23, 83-100.

Kaldor, N. (1961) "Capital Accumulation and Economic Growth" in Lutz and Hague (eds.) The Theory of Capital , St. Martin's Press, New York.

Krusell, P., L. Ohanian, V. Rios-Rull and G. Violante (2000) "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis" Econometrica 68(5): 1029-53.

Kuznets, S. (1966) Modern Economic Growth: Rate Structure and spread, Yale university Press, New Haven.

Levi, F. and R.J. Murnane (1992) "U.S. Earnings Levels and Earnings Inequality: A Review of Recent Trends and Proposed Explanations" Journal of Economic Literature, 30(3): 1333-81.

Lucas, R. (1988) "On the Mechanics of Economic Development", Journal of Monetary Economics 22, 3-42.

Matsuyama, K. (1992) "Agricultural Productivity, Comparative Advantage, and Economic Growth", Journal of Economic Theory, 58, 317-334.

Quah, Danny (1996) "Twin Peaks: Growth and Convergence in Models of Distribution Dynamics" Economic Journal 106(437), 1045-55.

Quah, Danny (1997) "Empirics for Growth and Distribution: Stratification, Polarization, and Convergence Clubs" Journal of Economic Growth 2(1), 27-59.

Rebelo (1992) "Growth in Open Economies", Carnegie-Rochester Conferences Series in Public Policy 36, 5-46.

Rebelo, S. (1994) "Long Run Policy Analysis and Long Run Growth", Journal of Political Economy, October, 1002-37.

Romer, P. (1986) "Increasing Returns and long run Growth", Journal of Political Economy, 94, p 1002-37.

Romer, P. (1990) "Endogenous Technical Change", Journal of Political Economy, Oct, S71-S102.

Sieh, J. C. Lai and W. Chang (2000) "Addictive Behavior and Endogenous Growth" Journal of Economics 72, 263-273.

Stokey, Nancy L. (1988) "Learning by Doing and the Introduction of New Goods" Journal of Political Economy, XCVI, 701-17.

Violante, G. (2002)" Technological Acceleration, Skill Transferability, and the Rise in Residual Inequality" Quarterly Journal of Economics 117(1), 297 338.

X. Appendix

A. Proof Proposition 1

It follows from the budget constraint that:

$$A_t(i) + \sum_{j=t}^{\infty} \frac{w_j s^L(i)}{\prod_{i=t}^{j}(1+r_i)} = \sum_{j=t}^{\infty} \frac{c_{j(i)}}{\prod_{i=t}^{j}(1+r_i)} \tag{40}$$

It follows from (40) and the definitions of Net Wealth NW_t and benchmark level of consumption v_t that:

$$\begin{aligned}
\underset{i\in[\underline{i},1]}{ess\inf}\; NW_t(i) &= \underset{i\in[\underline{i},1]}{ess\inf} \left[A_t(i) + \sum_{j=t}^{\infty} \frac{w_j s^L(i)}{\prod_{i=t}^{j}(1+r_i)}\right] - \sum_{j=t}^{\infty} \frac{v_j}{\prod_{i=t}^{j}(1+r_i)} \\
&= \underset{i\in[\underline{i},1]}{ess\inf} \left[A_t(i) + \sum_{j=t}^{\infty} \frac{w_j s^L(i)}{\prod_{i=t}^{j}(1+r_i)}\right] - \sum_{j=t}^{\infty} \frac{\underset{i\in[\underline{i},1]}{ess\inf}\; c_{j(i)}}{\prod_{i=t}^{j}(1+r_i)} \\
&= \underset{i\in[\underline{i},1]}{ess\inf} \left[A_t(i) + \sum_{j=t}^{\infty} \frac{w_j s^L(i)}{\prod_{i=t}^{j}(1+r_i)}\right] - \underset{i\in[\underline{i},1]}{ess\inf} \left[\sum_{j=t}^{\infty} \frac{c_{j(i)}}{\prod_{i=t}^{j}(1+r_i)}\right] \\
&= \underset{i\in[\underline{i},1]}{ess\inf} \left[A_t(i) + \sum_{j=t}^{\infty} \frac{w_j s^L(i)}{\prod_{i=t}^{j}(1+r_i)}\right] - \underset{i\in[\underline{i},1]}{ess\inf} \left[A_t(i) + \sum_{j=t}^{\infty} \frac{w_j s^L(i)}{\prod_{i=t}^{j}(1+r_i)}\right] \\
&= 0
\end{aligned}$$

Thus:

$$\underset{i\in[\underline{i},1]}{ess\inf}\; NW_t(i) = 0 \tag{41}$$

It follows from (41) and assumption 1 that:

$$\underset{i\in[\underline{i},1]}{ess\inf}\; A_t(i) = 0 \tag{42}$$

Using (42) in the budget constraint:

$$\begin{aligned}
\underset{i\in[\underline{i},1]}{ess\inf}\; A_{t+1}(i) &= (1+r_t)\underset{i\in[\underline{i},1]}{ess\inf}\; A_t(i) + \underset{i\in[\underline{i},1]}{ess\inf}\; s^L(i)w_t - \underset{i\in[\underline{i},1]}{ess\inf}\; c_t(i) \Rightarrow \\
\underset{i\in[\underline{i},1]}{ess\inf}\; c_t(i) &= \underset{i\in[\underline{i},1]}{ess\inf}\; s^L(i)w_t = \overline{s}^L(i)w_t
\end{aligned} \tag{43}$$

■

B. Proof Proposition 2:

Lemma 7 *If* $\beta\left[(1-\delta)+Ae^{-\Psi}\right]>1$ *then there exists a non-empty set* Γ *such that if* $(X_0,h_0)\in\Gamma$, *then* $\forall t\geq 0$ $\frac{X_{t+1}}{X_t}>1$, $\frac{h_{t+1}}{h_t}>1$.

Proof. Lemma 7

Define:

$$\underline{X} \equiv Max\left\{\frac{\beta Ae^{-\Psi}}{\beta\left[Ae^{-\Psi}+(1-\delta)\right]-1},\frac{1}{1-\overline{\alpha}},1+\frac{1}{\Psi}\right\} \tag{44}$$

$$\Omega \equiv \left\{(h,X) \quad s.th \quad h>e^{\Psi\underline{X}}, \quad X>\underline{X} \quad and \quad h>e^{\Psi X}\right\} \tag{45}$$

Since it is assumed that $\beta\left[(1-\delta)+Ae^{-\Psi}\right]>1$, $\underline{X}$ is well defined. Define $g^h(h,X)$ and $g^X(h,X)$ as the function that relates the growth rate of reproducible factors and technological experience with the state variables:

$$g^h(h,X) = \beta\left[(1\text{-}\delta)+\left(1\text{-}\frac{1}{z(X,h)}\right)Ae^{\Psi\left(1-\frac{z(X,h)}{X}\right)}h^{-\frac{1}{z(X,h)}}\right]-1 \tag{46}$$

$$g^X(h,X) = \left(\frac{z(X,h)}{X}\right)^{\eta}-1 \tag{47}$$

It follows from (13), (44) and (46) that if $X>\underline{X}$:

$$g^h(e^{\Psi X},X) = \beta\left[(1\text{-}\delta)+\left(1\text{-}\frac{1}{X}\right)Ae^{-\Psi}\right]\text{-}1 > \beta\left[(1\text{-}\delta)+\left(1\text{-}\frac{1}{\left(\frac{\beta Ae^{-\Psi}}{\beta[Ae^{-\Psi}+(1-\delta)]-1}\right)}\right)Ae^{-\Psi}\right]\text{-}1 = 0 \tag{48}$$

$$\lim_{h\to\infty} g^h(h,X) = \beta(1\text{-}\delta)-1 < 0 \tag{49}$$

It follows from (13), (44), (45) and (46) that if $(h,X)\in\Omega$

$$\frac{\partial g^h(h,X)}{\partial h} = \theta(h,X)\left[-1+\frac{1}{\left(1\text{-}\frac{1}{z(X,h)}\right)\ln h}\right] < \theta(h,X)\left[\text{-}1+\frac{1}{\left(1\text{-}\frac{1}{\underline{X}}\right)\ln e^{\Psi\underline{X}}}\right] \leq \theta(h,X)\left[\text{-}1+\frac{1}{\left(1\text{-}\frac{1}{1+\frac{1}{\Psi}}\right)\left(1+\frac{1}{\Psi}\right)\Psi}\right] = 0 \tag{50}$$

where $\theta(h,X)=\beta\frac{1}{z(X,h)}\left(1\text{-}\frac{1}{z(X,h)}\right)Ae^{\Psi\left(1-\frac{z(X,h)}{X}\right)}h^{-\frac{1}{z(X,h)}-1}$. If follows from (48), (49), (50) and the Implicit Function Theorem that there exist a continuous differentiable function $h_{\triangle h=0}$: $(\underline{X},+\infty)\to\left(e^{\Psi\underline{X}},+\infty\right)$ such that: $g^h(h_{\triangle h=0}(X),X)=0$, where

$h_{\triangle h=0}(X) > e^{\Psi X}$ and $\frac{\partial h_{\triangle h=0}(h,X)}{\partial h} = -\frac{\frac{\partial g^h(h,X)}{\partial X}}{\frac{\partial g^h(h,X)}{\partial h}} > 0$. The function $h_{\triangle h=0}(X)$ gives the value of reproducible factors which make the growth rate of reproducible factors zero for a given value of the technological Experience. In the same way, define $h_{\triangle X=0}(X)$ as the function that gives the value of reproducible factors which make the growth rate of technological experience zero for a given value of the technological Experience

$$h_{\triangle X=0}(X) = e^{\Psi X}$$

Remember that $\forall X > \underline{X} \Rightarrow h_{\triangle h=0}(X) > e^{\Psi X}$. Thus, it is possible to define the set Γ as follows:

$$\Gamma = \{(h, X) \in \Omega \ \ s.th. \ \ h_{\triangle h=0}(X) > \ h > h_{\triangle X=0}(X)\}$$

It follows from (33), (50) and the definition of $h_{\triangle h=0}(X)$ and $h_{\triangle X=0}(X)$ that:

$$\text{If } (h_t, X_t) \in \Gamma \Rightarrow h_{t+1} > h_t \text{ and } X_{t+1} > X_t \tag{51}$$

It follows from the Technological Experience accumulation equation (3) and (51) that if $(h_t, X_t) \in \Gamma$:

$$\begin{aligned} X_{t+2} &= X_{t+1}^{1-\eta} z(h_{t+1}, X_{t+1})^{\eta} > X_t^{1-\eta} z(h_t, X_t)^{\eta} = X_{t+1} \Rightarrow \\ h_{t+1} &> h_{\triangle X=0}(X_{t+1}) \end{aligned} \tag{52}$$

Define $h_{+1}(h, X)$ as the function that relates the reproducible factors in period t+1 with the state variables in period t:

$$h_{+1}(h, X) = \beta \left[(1-\delta)h + \left(1 - \frac{1}{z(X,h)}\right) A e^{\Psi\left(1 - \frac{z(X,h)}{X}\right)} h^{1 - \frac{1}{z(X,h)}} \right] \tag{53}$$

Since $h_{+1}(h, X)$ increases with both h and X it follows from (51) and the fact that $h_{\triangle h=0}(X_{t+1})$ is an increasing function that:

$$\begin{aligned} h_{\triangle h=0}(X_{t+1}) &= h_{+1}(h_{\triangle h=0}(X_{t+1}), X_{t+1}) > h_{+1}(h_{\triangle h=0}(X_t), X_t) \\ &> h_{+1}(h_t, X_t) = h_{t+1} \end{aligned} \tag{54}$$

It follows from equations (51), (52) and (54) that

$$(h_t, X_t) \in \Gamma \Rightarrow (h_{t+1}, X_{t+1}) \in \Gamma \tag{55}$$

■Lemma 7

Lemma 8 *If* $(X_0, h_0) \in \Gamma$, $\lim_{t\to\infty} h_t = +\infty$ *and* $\lim_{t\to\infty} X_t = +\infty$.

Proof. Lemma 8 (by contradiction)

Assume that $(X_0, h_0) \in \Gamma$ and $\lim_{t\to\infty} h_t = \overline{h} \neq +\infty$ or $\lim_{t\to\infty} X_t = \overline{X} \neq +\infty$.

- If $\overline{h} \neq h_{\triangle h=0}(\overline{X}) \Rightarrow \lim_{t\to\infty} g^h(h,X) = g^h(\overline{h},\overline{X}) > 0 \Rightarrow \lim_{t\to\infty} h_t = +\infty \Rightarrow\Leftarrow$
- If $\overline{h} = h_{\triangle h=0}(\overline{X})$ and $\overline{X} \neq +\infty \Rightarrow \lim_{t\to\infty} g^X(h,X) = g^X(\overline{h},\overline{X}) > 0 \Rightarrow \lim_{t\to\infty} X_t = +\infty \Rightarrow \overline{h} = \lim_{X\to\infty} h_{\triangle h=0}(X) = +\infty \Rightarrow\Leftarrow$
- If $\overline{h} = h_{\triangle h=0}(\overline{X})$ and $\overline{X} = +\infty \Rightarrow \overline{h} = \lim_{X\to\infty} h_{\triangle h=0}(X) = +\infty \Rightarrow\Leftarrow$
- If $\lim_{t\to\infty} h_t = +\infty \Rightarrow \lim_{t\to\infty} X_t = \lim_{h_t\to\infty} \overline{X}^{1-\eta} z(h_t,\overline{X})^\eta = +\infty \Rightarrow\Leftarrow \blacksquare$

C. Proof Proposition 3

Lemma 9 *If* $(h_0, X_0) \in \Gamma$ *then* $\lim_{t\to\infty} \frac{X_{t+1}}{X_t} = 1$

Proof. Lemma 9

Let $(h_0, X_0) \in \Gamma$. It follows from the definition of Γ and proposition 2 that $h_t > 1$. Thus, it follows from (46) that:

$$\begin{aligned}\frac{h_{t+1}}{h_t} &< \beta\,[(1\text{-}\delta)+A] \Rightarrow \limsup_{t\to\infty} \frac{\ln h_{t+1}}{\ln h_t} \leq \limsup_{t\to\infty} \frac{\ln h_t + \ln\left(\beta\,[(1\text{-}\delta)+A]\right)}{\ln h_t} = 1 \\ &\Rightarrow \lim_{t\to\infty} \frac{\ln h_{t+1}}{\ln h_t} = 1 \end{aligned} \tag{56}$$

It follows from equations (3) and (13):

$$\begin{aligned}\frac{X_{t+1}}{X_t} &= \left(\frac{\ln h_t}{\ln h_{t-1}}\right)^{\frac{\eta}{2}} \left(\frac{X_t}{X_{t-1}}\right)^{1-\frac{\eta}{2}} = \\ &\left(\frac{\ln h_t}{\ln h_{t-1}}\right)^{\frac{\eta}{2}} \left(\frac{\ln h_{t-1}}{\ln h_{t-2}}\right)^{\frac{\eta}{2}\left(1-\frac{\eta}{2}\right)} \cdots \left(\frac{\ln h_{\tau+1}}{\ln h_\tau}\right)^{\frac{\eta}{2}\left(1-\frac{\eta}{2}\right)^{t-\tau-1}} \left(\frac{X_{\tau+1}}{X_\tau}\right)^{\left(1-\frac{\eta}{2}\right)^{t-\tau}}\end{aligned} \tag{57}$$

It follows from (56) that $\exists \varepsilon > 0, \tau$ s. th. $\forall t > \tau$ $\frac{\ln h_{t+1}}{\ln h_t} < (1+\varepsilon)$, therefore:

$$\begin{aligned}\limsup_{t\to\infty} \frac{X_{t+1}}{X_t} &= \limsup_{t\to\infty} \left(\frac{\ln h_t}{\ln h_{t-1}}\right)^{\frac{\eta}{2}} \cdots \left(\frac{\ln h_{\tau+1}}{\ln h_\tau}\right)^{\frac{\eta}{2}\left(1-\frac{\eta}{2}\right)^{t-\tau-1}} \left(\frac{X_{\tau+1}}{X_\tau}\right)^{\left(1-\frac{\eta}{2}\right)^{t-\tau}} \\ &< \limsup_{t\to\infty} (1+\varepsilon)^{1-\left(1-\frac{\eta}{2}\right)^{t-\tau}} \left(\frac{X_{\tau+1}}{X_\tau}\right)^{\left(1-\frac{\eta}{2}\right)^{t-\tau}} = (1+\varepsilon)\end{aligned} \tag{58}$$

Since the above is true for every $\varepsilon > 0$ it follows that $\lim_{t\to\infty} \frac{X_{t+1}}{X_t} = 1$. $\blacksquare$

Proof. Proposition 3

It follows from (13) that when $h_t \geq e^{\Psi X_t}$:

$$\ln(h_t^{-\frac{1}{z_t}}) = -\left(\frac{\Psi \ln h_t}{X_t}\right)^{\frac{1}{2}} = -\frac{\Psi\left(\frac{1}{\Psi}\ln h_t X_t\right)^{\frac{1}{2}}}{X_t} = -\Psi\frac{z_t}{X_t} \Rightarrow \tag{59}$$

$$h_t^{-\frac{1}{z_t}} = e^{-\Psi\frac{z_t}{X_t}} \tag{60}$$

It follows from (3), (46), (60), Proposition 2 and lemma 9 that:

$$\lim_{t\to\infty}\frac{h_{t+1}}{h_t} = \lim_{t\to\infty}\beta\left[(1-\delta)+\left(1\text{-}\frac{1}{z_t}\right)Ae^{\Psi\left(1-2\frac{z_t}{X_t}\right)}\right] =$$

$$\lim_{t\to\infty}\beta\left[(1-\delta)+\left(1\text{-}\frac{1}{z_t}\right)Ae^{\Psi\left(1-2\left(\frac{X_{t+1}}{X_t}\right)^{\frac{1}{\eta}}\right)}\right] = \beta\left[(1-\delta)+Ae^{-\Psi}\right] \quad (61)$$

■

D. Proof Corollary 4

$$\frac{w_{t+1}}{w_t} = \frac{z_t}{z_{t+1}}\frac{y_{t+1}}{y_t} < \frac{y_{t+1}}{y_t} \quad (62)$$

It follows from (3) and (60) that along the permanent growth path:

$$y_t = Ae^{\Psi\left(1-\frac{2z(X_t,h_t)}{X_t}\right)}h_t \quad (63)$$

It follows from (3) and lemma 9 that

$$\lim_{t\to\infty}\frac{z_{t+1}}{z_t} = \lim_{t\to\infty}\left(\frac{X_{t+2}}{X_{t+1}}\right)^{\frac{1}{\eta}}\left(\frac{X_{t+1}}{X_t}\right)^{1-\frac{1}{\eta}} = 1 \quad (64)$$

It follows from (3), (62), (63) and lemma 9 that:

$$\lim_{t\to\infty}\frac{w_{t+1}}{w_t} = \lim_{t\to\infty}\frac{z_t}{z_{t+1}}\frac{h_{t+1}}{h_t}e^{\Psi 2\left(\frac{z_t}{X_t}-\frac{z_{t+1}}{X_{t+1}}\right)} = \quad (65)$$

$$\lim_{t\to\infty}\frac{z_t}{z_{t+1}}\frac{h_{t+1}}{h_t}e^{\Psi 2\left(\left(\frac{X_{t+1}}{X_t}\right)^{\frac{1}{\eta}}-\left(\frac{X_{t+2}}{X_{t+1}}\right)^{\frac{1}{\eta}}\right)} = \lim_{t\to\infty}\frac{h_{t+1}}{h_t} = \beta\left[(1\text{-}\delta)+Ae^{-\Psi}\right]$$

■

E. Proof Proposition 5

$$I_{Gini} \equiv 1-\int_0^1\left(\frac{\int_0^x y(i)d\mu}{y\mu\,[0,x]}\right)dx =$$

$$1-\int_0^1\left(\frac{\frac{1}{z}\int_0^x s^L(i)d\mu+\left(1-\frac{1}{z}\right)\int_0^x s^H(i)d\mu}{\mu\,[0,x]}\right)dx \quad (66)$$

It follows from the assumptions about $s^L(i)$ and $s^H(i)$ that $\forall x<1$ $\int_0^x s^H(i)d\mu < \int_0^x s^L(i)\mu$. Therefore, it follows that the Gini's coefficient is an increasing function of z. ■

XI. Appendix II

A. Stationary Property Right Distribution with CES utility function

Consider the following consumer's optimization problem

$$\begin{aligned} &\underset{\{c_t(i)\}_{t=0}^{\infty}}{Max} \ \textstyle\sum_{t=0}^{\infty} \beta^t u(c_t(i)) \\ &s.t. \quad (1+r_t)H_t(i) + s^L(i)w_t = H_{t+1}(i) + c_t(i) \\ &\qquad\ A_0^i = s_0^H(i)H_t \end{aligned} \tag{67}$$

where the notation used is as in the main text and where the instantaneous utility function $u(.)$ is the CES utility function:

$$u(c_t(i)) = \begin{cases} \frac{(c_t(i))^{1-\sigma}}{1-\sigma} & if \quad \sigma \in (0,1) \cup (1,+\infty) \\ ln(c_t(i)) & if \quad \sigma = 1 \end{cases} \tag{68}$$

Using conventional methods, it is follows that:

$$c_t(i) \ = \xi_t \left(\left(1+r_t\right) H_t s_t^H(i) + \lambda_t w_t s^L(i) \right) \tag{69}$$

where ξ_t and λ_t are defined as follows:

$$\xi_t \equiv \left[\textstyle\sum_{\tau=t}^{\infty} \prod_{j=t+1}^{\tau} \frac{1+g_j^c}{1+r_j} \right]^{-1} \quad \lambda_t \equiv \left[\textstyle\sum_{\tau=t}^{\infty} \prod_{j=t+1}^{\tau} \frac{1+g_j^w}{1+r_j} \right] \tag{70}$$

where g_t^c and g_t^w denotes the consumption and the wages growth rates:

$$(1+g_t^c) \equiv \frac{c_t}{c_{t-1}} = \left(\beta \left(1+r_t\right)\right)^{\frac{1}{\sigma}} \quad (1+g_t^w) \equiv \frac{w_t}{w_{t-1}} z \tag{71}$$

It follows from (67) and the budget constraint in (69) that:

$$s_{t+1}^H(i) = \frac{1}{(1{+}g_{t+1}^H)} \left[(1\text{-}\xi_t) \left(1{+}r_t\right) s_t^H(i) + (1\text{-}\xi_t\lambda_t) \frac{w_t}{H_t} s^L(i) \right] \tag{72}$$

Integrating (72) with respect to i:

$$1 = \frac{1}{(1+g_{t+1}^H)} \left[(1-\xi_t) \left(1+r_t\right) + (1-\xi_t\lambda_t) \frac{w_t}{H_t} \right] \tag{73}$$

Using (72) and (73):

$$\frac{s_{t+1}^H(i)}{s_t^H(i)} = \left[1 + \frac{1}{(1+g_{t+1}^H)} (1-\xi_t\lambda_t) \frac{w_t}{H_t} \left(\frac{s^L(i)}{s_t^H(i)} - 1 \right) \right] \tag{74}$$

It follows from the above expression that the property rights stay constant if either $i)$ $\forall t \ \xi_t\lambda_t = 1$ or $ii)$ for almost every i $s_t^H(i) = s^L(i)$. The condition $i)$ is only satisfied if the growth rate of wages is always equal to the growth rate of consumption, this only happen in a balanced growth path (or a zero growth steady state).

Since in the model presented in the main text there is not a balanced growth path with positive growth rate, the only stationary property rights distribution under the assumption of CES instantaneous utility function would be the property right distribution in which the distribution of reproducible factors would be equal to the distribution of nonreproducible factors (condition *ii*).

B. Stable Property Right Distribution with Biased technological change and logarithmic utility function

Consider that the initial conditions, the parameter values and the price path are such that the following condition is satisfied:

$$\forall t \quad (1+g_t^w) < \left(1+g_t^H\right) \tag{75}$$

The above condition would be satisfied along any permanent growth path with biased technological change. Assume that $\sigma = 1$, that is, the instantaneous utility function is logarithmic. It follows from the definition of ξ_t (70) that in the case of the logarithmic utility function:

$$\xi_t = (1-\beta) \tag{76}$$

It is going to be proved first that $\xi_t\lambda_t < 1$. Suppose that $\xi_t\lambda_t \geq 1$, then it follows from (73), (76) that the following condition should be satisfied

$$\left(1+g_{t+1}^H\right) = \left[\beta\left(1+r_t\right) + (1\text{-}\xi_t\lambda_t)\frac{w_t}{H_t}\right] \leq \beta\left(1+r_t\right) = (1+g_t^c) \tag{77}$$

It follows from (75) and (77) that

$$\left(1+g_{t+1}^w\right) < \left(1+g_{t+1}^H\right) \leq \left(1+g_{t+1}^c\right) \tag{78}$$

It follows from definitions of ξ_t, λ_t (70) and (78) that:

$$\xi_{t+1}\lambda_{t+1} \geq \xi_t\lambda_t \geq 1 \tag{79}$$

Using the same argument as before it follows that:

$$\forall i > 1 \ \left(1+g_{t+i}^c\right) > \left(1+g_{t+i}^w\right) \tag{80}$$

But it follows from (70) and (80) that $\xi_t\lambda_t < 1$, which is contradiction with the initial assumption. Therefore in equilibrium $\xi_t\lambda_t < 1$.

It follows from the fact that $\xi_t\lambda_t < 1$ together with equation (74) that along the equilibrium path the distribution of property rights over reproducible factors becomes closer each period to the distribution of property rights over nonreproducible factors.

Figure 1: Initial Property Rights Distribution

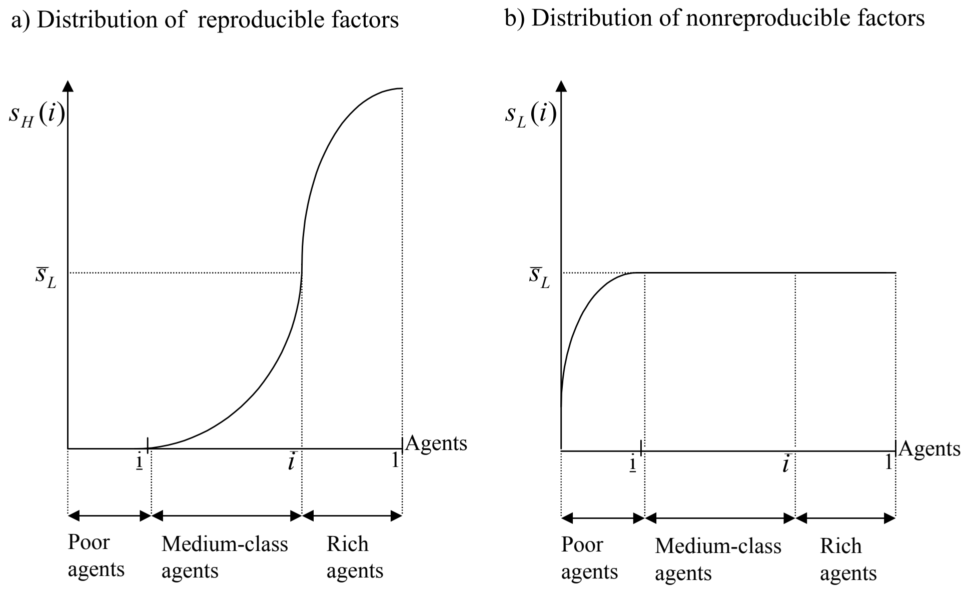
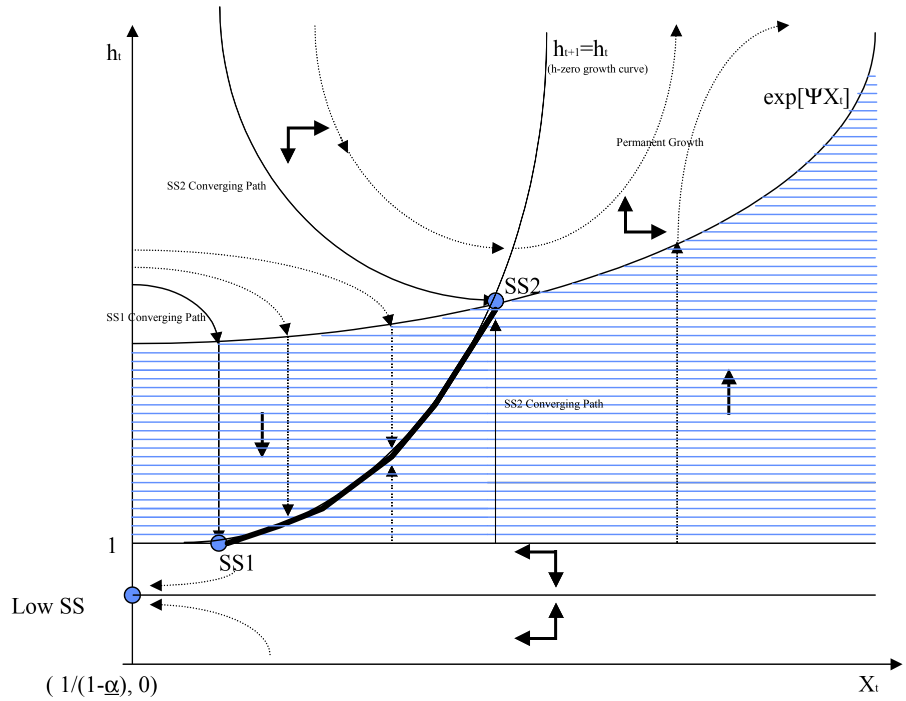


Figure 2



h_t
$h_{t+1}=h_t$
(h-zero growth curve)
exp[ΨX$_t$]
Permanent Growth
SS2 Converging Path
SS1 Converging Path
SS2
SS2 Converging Path
1
SS1
Low SS
(1/(1-α), 0)
X_t

Figure 3:
Effect of Technological Progress on Income Distribution

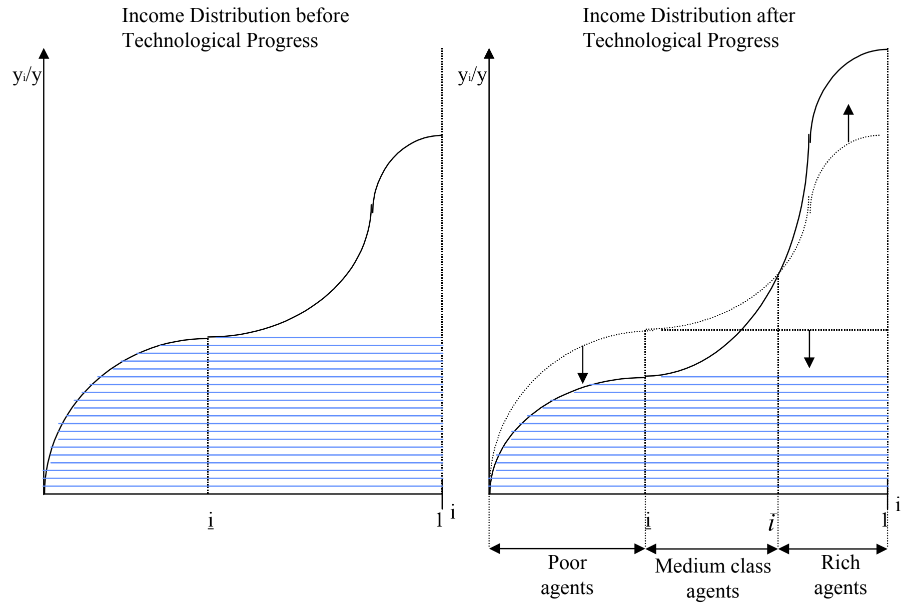


Figure 4:
Externality a la Romer (γ =1)