Precautionary Savings or Working Longer Hours?

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September 2003

Abstract

This paper quantifies the size of precautionary savings implied by a dynamic general equilibrium model with heterogeneous agents when explicitly considering the labor supply decision of households. Key parameters as the intertemporal elasticities of substitution of consumption and leisure are obtained from observed household behavior by calibrating the model to reproduce in equilibrium relevant statistics from micro data. I find that precautionary savings are smaller than if they were measured by use of a model economy without labor decision and that they can even be negative. In addition, the incomplete markets economy is smaller in size than its complete markets counterparts. That is to say, aggregate output is smaller, between 82% and 94% of aggregate output in the complete markets economies. These result are in stark contrast to the ones implied by models without labor choice as Aiyagari (1994) and are due to the importance of hours as a mechanism to confront wage fluctuations.

Keywords: Incomplete Markets; Precautionary Savings; Labor Supply

JEL Classification: E21; D31; J22; C68

Contents

1   Introduction

There is a vast economic literature trying to understand the motives for savings.[1] One specific reason for saving is the precautionary motive, the accumulation of resources in order to smooth out consumption variations across different states of the world. There are several attempts to measure the size of the savings held for precautionary reasons. Dynamic general equilibrium macroeconomic models quantify precautionary savings from 3% to more than 100% of total wealth.[2] In spite of this disparity of results, there is a widespread agreement in that (1) precautionary savings are positive and therefore (2) incomplete markets economies are bigger in size.[3] A key ingredient of the models leading to these conclusions is that they treat earnings as exogenous, stochastic and uninsurable, abstracting from the labor/leisure choice of the individuals.

The purpose of this paper is to quantify the size of precautionary savings implied by a dynamic general equilibrium model with heterogeneous agents when explicitly considering the labor decision of households. Key parameters as the intertemporal elasticities of substitution of consumption and leisure are obtained from observed household behavior by calibrating the model to reproduce in equilibrium relevant statistics from micro data.

There are at least two good reasons to think that the explicit consideration of labor supply is an essential ingredient to understand the savings behavior. First, the labor decision may act as an additional channel to smooth out fluctuations in labor productivity. The choice of hours worked lets households increase their earnings once a bad productivity shock has arrived. Second, the allocation of work effort across different states has important implications for the aggregate labor of the economy as measured in efficiency units. Variations in aggregate labor have important implications for prices and therefore allocations.

I find that precautionary savings are smaller than if they were measured by use of a model economy without labor decision and that they can even be negative. Precisely, precautionary savings are about 11.2% of aggregate capital when compared to a complete markets economy where everybody enjoys the same level of consumption and they can fall to -2.1% of aggregate capital in a complete markets economy with the same consumption distribution as in the incomplete markets world. In addition, I find that the incomplete markets economy is smaller in size than its complete markets counterparts. That is to say, aggregate output

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[1]See Browning and Lusardi (1996) for a complete survey.

[2]The wide range of estimates is explained by the size of the non-permanent component of the idiosyncratic uncertainty in the earnings process that different authors consider. See Díaz, Pijoan-Mas, and Ríos-Rull (2003) for details.

[3]See for example Huggett (1997) for a theoretical proof.

is smaller, between 82% and 94% of aggregate output in the complete markets economies. These results are in stark contrast to the ones implied by models without labor choice as the early work of Huggett (1993) or Aiyagari (1994). Together with these results I also find that compared to complete markets economies, aggregate hours worked in an incomplete markets world are higher, between 20% and 84% higher depending on the complete markets economy we look at. In contrast, aggregate labor, measured in efficiency units, is nevertheless much smaller.

The reason for these results is as follows. In an incomplete markets economy, the marginal product of capital is lower than in the complete markets economies. This is by reason of the variation of future consumption due to the uncertainty in labor earnings as described by Aiyagari (1994). Lower marginal product of capital implies higher capital labor ratio. In models without labor choice higher capital labor ratio is equivalent to higher aggregate capital. However, in an economy where households choose their labor supply a higher capital labor ratio can be achieved by means of higher aggregate capital, lower aggregate labor or a combination of both. It turns out that in the incomplete markets economy calibrated to observed household behavior the main channel is a fall in aggregate labor, with capital slightly increasing or even falling (negative precautionary savings) and aggregate output being thus smaller. Under complete markets, households substitute leisure across different states, working long hours when their market productivity is high and working few or none hours when their market productivity is low. State contingent assets allow consumers to transfer resources between states and keep the marginal utility of consumption equal across states. In contrast, in the incomplete markets world households are not so willing to substitute labor across states precisely because the ability to transfer resources between states is limited. In equilibrium, a large fraction of low productivity workers are also asset-poor. This type of households will supply many hours in spite of not being very productive because the marginal utility of consumption is very high. The productivity of the average hour worked in the incomplete markets economy can be halved with respect to a world with complete markets.

Notice therefore that the reason for the quantitative results of the paper is that households choose to work long hours when bad shocks come instead of building in advance stocks of assets large enough to avoid having to work too much when they are unproductive. Hence, the answer to the title question of this paper is that households rely more on working long hours when bad times come than on building big stocks of assets. Another way of seeing this result is that whereas the measure of precautionary savings range from -2% to 11%, the share of hours worked in the incomplete markets economy in excess of the amount of

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hours worked in the complete markets economies range from 17% to 46%.

An important idea emphasized throughout the paper is that the complete markets version of the model economy used in this paper is not a representative agent economy. I define precautionary savings as the difference in aggregate capital between the incomplete and the complete markets economies. There are many (infinite) possible complete markets economies. I will use two of them as comparisons to the baseline model economy. I show that if one used the representative agent counterpart instead the results would be completely different and in line with those obtained in models without labor leisure choice.

There are very few papers trying to address the issue of precautionary savings with labor supply. Marcet, Obiols-Homs, and Weil (2002) also argue that precautionary savings may be negative although, contrary to the results presented here, in their model this goes necessarily through a decrease in hours worked. However, Marcet, Obiols-Homs, and Weil (2002) do not directly compare savings between incomplete and complete markets economies nor consider substitution of work effort across states. Low (2003) examines the life-cycle consumption and labor allocation in a partial equilibrium context and finds that for a given set of parameters, a model with endogenous labor supply generates higher savings. However, he does not measure precautionary savings.

The reminder of the paper is organized as follows. In section 2 I describe the model economies. In sections 3 and 4 I give the mathematical details of the incomplete and complete market economies respectively. Then, in section 5 I explain how the model economies are calibrated to data. Before seeing the results I discuss the exact concept of precautionary savings used throughout the paper in section 6 and the experiments to be performed in section 7. Then, the model economies are simulated and their results presented and discussed in section 8. Section 9 performs some robustness checks of the results. Finally, section 10 concludes.

2   The model economies

The economies analyzed in this paper are growth economies with production, populated by a measure one of households that live forever. We will only look at steady states.

2.1 Preferences

Households derive utility from consumption and leisure. Current consumption is denoted by c and leisure by l. Future utilities are discounted at the rate $\beta \in (0,1)$. We write the per period utility as $u(c,l)$, and total expected utility at time τ as $E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} u(c_t, l_t)$.

2.2 Production technology

Each period households receive a shock to their efficiency units of labor $\varepsilon \in \Upsilon \equiv \{\varepsilon_1, ..., \varepsilon_{n_\varepsilon}\}$. This shock is Markov with transition matrix Γ, with $\Gamma_{\varepsilon\varepsilon'}$ stating $Pr\left(\varepsilon_{t+1} = \varepsilon' | \varepsilon_t = \varepsilon\right)$.

Aggregate output Y is produced according to an aggregate neoclassical production function $F(K,L)$ that takes as inputs capital K and efficient units of labor L. The aggregate labor input comes from aggregating over all agents' efficiency units of labor worked. Aggregate capital results from aggregation of all assets. Capital depreciates at an exogenous rate $\delta \in [0,1]$.

2.3 Market arrangements

We distinguish between two types of market arrangements. Our benchmark economy is an incomplete markets economy. By incomplete markets I mean that there are no state contingent markets for the household specific shock ε. Households hold assets $a \in A \equiv [\underline{a}, \infty)$ that pay interest at rate r. I assume that households are restricted by a lower bound on their assets holdings $\underline{a}$. This lower bound may arise endogenously as the quantity that ensures that the household is capable of repaying its debt in all states of the world or we can just set it exogenously as a borrowing constraint.[4]

In the economy with complete markets we assume there is a costless insurance technology against the realization of the idiosyncratic shocks. Households can trade one-period state contingent bonds $b'(\varepsilon')$ which deliver one unit of consumption good at the beginning of next period if the household productivity shock is ε'.

The state contingent bonds are traded between households and insurance companies. The realizations of the efficiency units endowments are perfectly observable. Since probabilities of next period endowments depend on the current period realization, insurance companies trade state-contingent bonds $b'(\varepsilon')$ with households at a price $q(\varepsilon, \varepsilon')$ which depends on households current efficiency unit endowment ε. Notice that $q(\varepsilon, \varepsilon')$ is the amount of consumption units

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[4]See Huggett (1993) and Aiyagari (1994) for details.

that the household of type ε has to give today to the insurance company in order to get one consumption unit tomorrow in case the realization of the idiosyncratic shock is ε'.

Between two consecutive periods, insurance companies hold all the capital of the economy (the proceeds from selling state contingent bonds). They rent this capital to production firms taking the rental rate r as given.

This asset structure completes the market structure in this economy. Therefore, in equilibrium the consumption of any household will be the same regardless of the realization of her efficiency units shock.[5]

3 Incomplete markets economies

In this section I detail the mathematical structure that describes the equilibrium allocations and prices of the incomplete markets economy. Sections 3.1 and 3.2 describe the optimization problems faced by firms and households whereas section 3.3 states the conditions that define the steady state equilibrium.

3.1 The firm problem

The representative firm behaves as a price taker, renting the production factors in competitive markets. The objective of the firm is to maximize profits by choice of capital K and labor L:

$$\max_{K,L} \left\{ F\left(K, L\right) - (r + \delta)K - wL \right\}$$

where w is the rental prices of a unit of labor and r is the rental price, net of depreciation, of a unit of capital.

3.2 The household problem

The individual household's state variables are its shock and its assets $\{\varepsilon, a\}$.[6] The problem that the household solves is:

$$v\left(\varepsilon, a\right) = \max_{c,l,a'} \left\{ u(c, l) + \beta \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} v\left(\varepsilon', a'\right) \right\} \tag{1}$$

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[5]This statement is true if preferences are separable between leisure and consumption. Otherwise, we would have that households equalize marginal utilities of consumption across states but consumption would not be equalized.

[6]Since there is no aggregate uncertainty and since we only look at steady states, there are no aggregate state variables.

$$\text{s.t.:} \qquad c + a' = w\varepsilon\,(1 - l) + (1 + r)\,a \tag{2}$$

$$c > 0, \quad 1 \geq l \geq 0 \quad \text{and} \quad a' \geq \underline{a}$$

where r and w are the return on assets and the rental rate per efficiency units of labor.

Under certain conditions problems of this type have a solution that we denote $a' = g^a(\varepsilon, a)$, $c = g^c(\varepsilon, a)$ and $l = g^l(\varepsilon, a)$ with an upper bound on asset holdings, $\bar{a}$ such that $\bar{a} \geq g^a(\varepsilon, a) \geq \underline{a}$ for all $\varepsilon \in \Upsilon$ and all $a \in [\underline{a}, \bar{a}]$. Hereafter I will also use the more compact notation $s \equiv \{\varepsilon, a\}$ and $S \equiv \Upsilon \times [\underline{a}, \bar{a}]$.

It is possible to construct a Markov process for the individual state variables, from the Markov process on the shocks and from the decision rules of the agents (see Huggett (1993) or Hopenhayn and Prescott (1992) for details). Let $\mathcal{B}$ be the σ-algebra generated in S by, say, its open intervals. A probability measure μ over $\mathcal{B}$ exhaustively describes the economy by stating how many households are of each type.

Let $Q(s, B)$ denote the probability that a type $s = \{\varepsilon, a\}$ has of becoming of a type in $B \subset \mathcal{B}$. Given the objects defined so far, we can express Q as:

$$Q(s, B) = \sum_{\varepsilon' \in B_\varepsilon} \Gamma_{\varepsilon\varepsilon'} \mathbf{I}_{g(\varepsilon, a) \in B_a}$$

where $\mathbf{I}$ is an indicator function that takes value 1 if its argument is true and 0 otherwise, B_ε is the projection of B in Υ and B_a is the projection of B in $[\underline{a}, \bar{a}]$. The function Q naturally describes how the economy moves over time by generating a probability measure for tomorrow μ' given a probability measure μ today. The exact way in which this occurs is

$$\mu'(B) = \int_S Q(s, B)\,d\mu \tag{3}$$

As shown by Huggett (1993), if the process for the earnings shock is nice in the sense that it has a unique stationary distribution, then so has the economy.[7] Furthermore, this unique stationary distribution is the limit to which the economy converges under any initial distribution.[8]

3.3 Equilibrium

A steady state equilibrium for this economy is a set of functions $\{v, g^a, g^c, g^l\}$, a measure of households μ, and a pair of prices $\{w, r\}$ such that:

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[7]For example if it satisfies the American–dream American–nightmare condition stated in Ríos-Rull (1998), then there is a unique stationary distribution of households over earning shocks and asset holdings.

[8]This does not mean that this will happen in equilibrium outside the steady state. The transition Q has been constructed under the assumption that the households think that prices are constant.

1. Given a pair of prices $\{w, r\}$, the functions $\{v, g^a, g^c, g^l\}$ solve the households' decision problem

2. Prices are given by marginal productivities:

$$
\begin{aligned}
w &= F_L(K, L) \\
r &= F_K(K, L) - \delta
\end{aligned}
$$

3. Factor inputs are obtained aggregating over households:

$$
\begin{aligned}
L &= \int \varepsilon \left(1 - g^l\right) d\mu \\
K &= \int g^a d\mu
\end{aligned}
$$

4. The measure of households is stationary:

$$
\mu(B) = \int_S Q(s, B) d\mu
$$

5. By virtue of the Walras law, the aggregate resource constraint of the economy is automatically satisfied:

$$
C + K' = F(K, L) + (1 - \delta) K
$$

4 Complete markets economies

The representative firm problem is the same as in the incomplete markets case (see section 3.1). In section 4.1 we look at the problem faced by the insurance firms. In section 4.2 we analyze the problem of the household. The conditions that define the steady state equilibrium are stated and explained in section 4.3.

4.1 The insurance company problem

The representative insurance company trades the state contingent bonds with the households. At a given period it sells for each $\varepsilon' \in \Upsilon$ an amount $b(\varepsilon')$ of bonds at the market price $q(\varepsilon, \varepsilon')$. The insurance company takes the proceeds from selling the state contingent bonds and transforms them into capital, which is rented to firms. Next period the insurance company gets paid the rental rate r in return for the capital services. At the same time it

honors its liabilities paying the bonds to households. Then, the insurance firm chooses $b\left(\varepsilon'\right)$ such that:

$$\max_{b(\varepsilon')} \left\{(1+r)\ q\left(\varepsilon,\varepsilon'\right) b\left(\varepsilon'\right) - \Gamma_{\varepsilon\varepsilon'} b\left(\varepsilon'\right)\right\} \quad \forall \varepsilon \in \Upsilon \text{ and } \forall \varepsilon' \in \Upsilon$$

which delivers the optimality condition:

$$q\left(\varepsilon,\varepsilon'\right) = (1+r)^{-1}\,\Gamma_{\varepsilon\varepsilon'} \quad \forall \varepsilon \in \Upsilon \text{ and } \forall \varepsilon' \in \Upsilon \tag{4}$$

This is an equilibrium or no-arbitrage condition for the insurance sector, which states that the price of the bond reflects the (discounted) probability of an event taking place.

4.2 The household problem

Under complete markets, the individual household's state variables are its shock and its asset holdings $\{\varepsilon, b\left(\varepsilon\right)\}$. The problem that the household solves is:

$$v\left(\varepsilon, b\left(\varepsilon\right)\right) = \max_{c,l,b'(\varepsilon')} \left\{u(c,l) + \beta \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} v\left(\varepsilon', b'\left(\varepsilon'\right)\right)\right\} \tag{5}$$

$$\text{s.t.:} \qquad c + \sum_{\varepsilon'} q\left(\varepsilon,\varepsilon'\right) b'\left(\varepsilon'\right) = w\varepsilon\left(1-l\right) + b\left(\varepsilon\right) \tag{6}$$

$$c > 0 \quad \text{and} \quad 1 \geq l \geq 0$$

The solution to this problem is given by the policy functions $b'\left(\varepsilon'\right) = g^b(\varepsilon, b, \varepsilon')$, $c = g^c(\varepsilon, b)$ and $l = g^l(\varepsilon, b)$.

For a household of type $\{\varepsilon, b\left(\varepsilon\right)\}$ we have an euler equation for each state contingent bond $b'\left(\varepsilon'\right)$:

$$u_c\left(c,l\right) = \beta \frac{\Gamma_{\varepsilon\varepsilon'}}{q\left(\varepsilon,\varepsilon'\right)}\ u_c\left(c',l'\right) \quad \forall \varepsilon' \in \Upsilon$$

where l is given by the intratemporal first order condition:

$$u_c\left(c,l\right) w\varepsilon = u_l\left(c,l\right) \tag{7}$$

l' by the same condition forwarded one period, c is given by the household budget constraint (equation 6) and c' by the same budget constraint forwarded one period.

Imposing equilibrium in the insurance market (equation 4) the previous equation can be rewritten as:

$$u_c\left(c,l\right) = \beta\left(1+r\right)\ u_c\left(c',l'\right) \quad \forall \varepsilon' \in \Upsilon$$

Notice that the marginal value of one unit of consumption today does not depend on ε'. Therefore, the marginal value of consumption tomorrow will not depend on ε' either. This tells us that households will choose their purchases of state contingent bonds such that the next period marginal utilities of consumption are equalized across states.[9]

For a steady state equilibrium with a balanced growth path to exist we will require $\beta(1+r) = 1$.[10] This condition further simplifies the Euler equations for the state contingent bonds:

$$u_c(c, l) = u_c(c', l') \quad \forall \varepsilon' \in \Upsilon$$

This is the standard complete markets relationship. The households chooses $b'(\varepsilon')$ such that the marginal utility of consumption is equalized across different states of the world and different points in time.

4.3 Equilibrium

A steady state equilibrium for the complete markets economy is a set of functions $\{v, g^b, g^c, g^l\}$, a measure of households μ, a pair of prices $\{w, r\}$ and a pricing function $q(\varepsilon, \varepsilon')$ such that:

1. Given a pair of prices $\{w, r\}$ and the pricing function $q(\varepsilon, \varepsilon')$, the functions $\{v, g^b, g^c, g^l\}$ solve the households' decision problem.

2. The pair of prices $\{w, r\}$ are given by marginal productivities:

$$w = F_L(K, L) \tag{8}$$
$$r = F_K(K, L) - \delta \tag{9}$$

3. Factor inputs are obtained aggregating over households:

$$L = \int \varepsilon(1 - g^l) \, d\mu \tag{10}$$
$$K = \int \sum_{\varepsilon'} q(\varepsilon, \varepsilon') g^b(\varepsilon, b(\varepsilon), \varepsilon') \, d\mu \tag{11}$$

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[9]Or the consumption level itself if consumption and leisure are separable in the utility function.
[10]Notice that if $\beta(1+r) > 1$ we would need

$$u_c(c, l) > u_c(c', l') \quad \forall \varepsilon' \in \Upsilon$$

which asks for consumption (and therefore assets) to grow forever. If $\beta(1+r) < 1$ the opposite would be true.

4. The pricing function $q\left(\varepsilon, \varepsilon'\right)$ satisfies the no-arbitrage condition in the insurance industry:

$$q\left(\varepsilon, \varepsilon'\right) = (1+r)^{-1} \Gamma_{\varepsilon \varepsilon'}$$

5. The steady state condition $\beta\left(1+r\right) = 1$ holds.

6. By virtue of the Walras law, the aggregate resource constraint of the economy is automatically satisfied:

$$C + K' = F\left(K, L\right) + (1 - \delta) K$$

This equilibrium is not unique. There is a unique set of prices r, w and $q\left(\varepsilon, \varepsilon'\right)$ and capital labor ratio K/L pinned down by the equilibrium conditions 5, 2 and 4. Given a unique set of prices, condition 1 implies a unique set of functions $\left\{v, g^b, g^c, g^l\right\}$. However, there are several ways to achieve the unique capital labor ratio. Condition 3, the only one left, does not imply a unique distribution of households μ. There are potentially infinite different distributions μ compatible with the unique capital labor ratio determined by conditions 5 and 2. These different distributions will generate different levels of aggregate capital and labor and therefore of hours worked, output and aggregate consumption.

5   Calibration

The calibration strategy I follow is the following. First, I choose a process for the efficiency units of labor that matches the volatility and persistence of the non-permanent component of the wages observed in micro data. Then, I choose the remaining model parameters such that in the steady state equilibrium the incomplete markets economy matches some characteristics, both macro and micro, of actual data. This implies solving for the equilibrium as many times as needed until the statistics from data are matched.[11] In a sense, this calibration strategy can be seen as an exactly identified generalized method of moments estimation. Finally, given these parameters the complete markets economies are solved. In the remaining of this section I give more details about this process.

The model is set such that there is no fixed heterogeneity. In this world any household will be at some point at the top and at some other point at the bottom of the earnings distribution. When mapping this process to actual wage data this would imply that a computer

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[11]Other recent articles that follow a similar calibration strategy by use of a similar model are Castañeda, Díaz-Giménez, and Ríos-Rull (2003) and Heathcote, Storesletten, and Violante (2003).

engineer may end up being as productive as an assembly line worker (and viceversa). This huge variance of productivities does not seem to reflect the real volatility of wages faced by people. Therefore, instead of calibrating the wage process to the overall dispersion of the observed wage distribution I will take as reference the wage distribution net of fixed heterogeneity. [12]

The idiosyncratic process of efficiency units, then, is taken from Conesa and Krueger (2002). They set a seven-states Markov chain ($n_\varepsilon = 7$), which is a discretization of the non-permanent component of the stochastic process estimated by Storesletten, Telmer, and Yaron (2002) by means of PSID data. This process generates a sizeable dispersion of wages giving rise to a gini coefficient of 0.44. Likewise, the wage process is quite persistent, with the probability of being in the same state next year being larger than 92% in all cases.

The model period is set equal to one year. The production function is the standard Cobb-Douglas:

$$F(K, L) = K^{1-\theta} L^{\theta}$$

The chosen utility function is:

$$u(c, l) = \frac{c^{1-\sigma} - 1}{1 - \sigma} + \lambda \frac{l^{1-\nu} - 1}{1 - \nu}$$

The Cobb-Douglas production function is a standard choice that is consistent with the non-trended factor shares observed in US data for the post-war years. The utility function is also a standard choice. It gives enough parameters to have distinct intertemporal elasticities for consumption and leisure which let us match observed individual behavior.[13]

There are 6 parameters we want to pin down: the preference parameters β, σ, ν, λ and the technology parameters θ and δ.[14] I will calibrate these values so that in equilibrium the incomplete markets economy matches some statistics of data.

The time discount factor β is set such that the capital output ratio in the steady state equilibrium equals 3.0. The depreciation rate of capital δ is set to deliver an investment over capital ratio of 0.25 and the labor share θ is set equal to 0.640. These are standard target macroeconomic ratios. The parameter λ is chosen such that in the steady state equilibrium the average fraction of the time devoted to market activities over all households equals 0.33.

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[12]The crudest way of doing so would be to get rid of the variance due to education and age.

[13]Notice however that $\sigma \neq 1$ is inconsistent with balanced growth path in a representative agent economy with positive growth. See Heathcote, Storesletten, and Violante (2003) for details.

[14]The lower level on asset holdings $\underline{a}$ is set equal to the *natural borrowing limit*. Since in this model there are no transfers and labor income depends on hours supplied the *natural borrowing limit* turns out to be zero. Less stringent borrowing limits are tried in section 9.2.

Table 1: **Calibration targets and model parameters.**

parameter	target	value
σ	$corr(h, \varepsilon) = 0.01$	1.045
ν	$cv(c)/cv(h) = 3.00$	1.116
λ	$H = 0.33$	1.658
β	$K/Y = 3.00$	0.942
θ	$wL/Y = 0.64$	0.640
δ	$I/Y = 0.25$	0.083

The curvature parameter for consumption σ is chosen such that the correlation between hourly wages and hours of work equals 0.01.[15,16] The curvature parameter for leisure ν is chosen such that in equilibrium the coefficient of variation of consumption relative to the coefficient of variation of hours equals 3.[17,18] In table 1 there is a summary of the parameter values and the calibration targets.[19]

The chosen wage process together with the calibrated parameters generate the distributional moments for hours, earnings and wealth reported in table 2. The first thing to notice is that this very simple model accounts for the large concentration of the earnings and wealth distribution observed in the Survey of Consumer Finances. The gini coefficient for the distribution of the model earnings is 0.48 whereas in data it is 0.61. The gini coefficient for wealth is matched almost exactly at 0.80 and so are the share of wealth held by each quintile of the distribution.

Therefore, contrary to most infinitely-lived models with exogenous earnings, this model properly calibrated to micro data can generate a high degree of concentration of wealth. The infinitely-lived models have problems fitting the upper tail of the wealth distribution. The top 5% of the wealth distribution holds 43% of overall wealth in the model whereas it holds up to 58% in data.[20] However, the fact that we are missing something in the upper

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[15]This value is obtained from the 2002 NBER Merged Outgoing Rotation Groups sub-sample of the CPS data set. If one takes away from wages its permanent component (i.e., age, education and sex) the correlation between hours and wages is 0.01 for the whole sample and -0.01 for men only. Heathcote, Storesletten, and Violante (2003) report a value of 0.02 for the period 1967-1996 using PSID data.

[16]In section 9.1 I provide a summary of results obtained with σ equal to 1.5 and 2.0.

[17]This value is taken from Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

[18]Notice that this implies a value for ν equal to 1.116 and therefore an intertemporal elasticity of substitution of leisure slightly below one. Empirical estimates using micro data on male household heads tend to be lower than this number. However, according to Domeij and Floden (2003) estimates that do not take into account borrowing constraints may be seriously downward biased.

[19]Notice that since the parameters are calibrated to equilibrium statistics, any of them affects all calibration targets. However, in the text I highlight the statistic that is most influenced by each parameter.

[20]These values are not reported in the tables.

Table 2: **Distributional statistics.**

variable	cv	gini	q_1	q_2	q_3	q_4	q_5
earnings							
model	1.05	0.48	3.5%	8.3%	13.6%	22.0%	52.6%
data	2.65	0.61	−0.2%	4.0%	13.0%	22.9%	60.2%
wealth							
model	2.07	0.80	0.0%	0.0%	1.6%	13.9%	84.5%
data	6.53	0.80	−0.3%	1.3%	5.0%	12.2%	81.7%
hours							
model	0.28	0.12	0.17	0.34	0.37	0.38	0.39
data	0.27	0.12	0.19	0.33	0.34	0.34	0.43

Note: cv refers to coefficient of variation. $q_1, ..., q_5$ refer, for earnings and wealth, to the share held by all people in the corresponding quintile with respect to the total. However, for hours it is the average number of hours worked by people in the corresponding quintile. Wealth and earnings data refers to the SCF98 and is quoted from Budría, Díaz-Gimenez, Quadrini, and Ríos-Rull (2002). Data on hours are from CPS2002.

tail is understandable since data sets as the PSID that serve as source for data on wages miss the upper tails of the wage distribution and only provide wage measures that are top-coded.[21] Among the infinitely-lived models that try to reproduce the wealth distribution in US data, the only one that has a better fit in the upper tail is the one by Krusell and Smith (1998), although in a less parsimonious approach since these authors give different (stochastic) discount factors to households to ensure that rich households have a reason strong enough to increase their wealth.[22]

6 The measurement of precautionary savings

The objective of the paper is to give a measurement of the precautionary savings implied by a dynamic general equilibrium model with labor/leisure choice. The term precautionary savings refers to the amount of assets held for the sole purpose of equating consumption across different states of nature. Therefore, the measure of precautionary savings used throughout the article will be the difference between the aggregate capital in the steady state equilibrium of the incomplete markets economy and the aggregate capital in the steady state equilibria of its complete markets counterparts divided by the aggregate capital in the incomplete markets economy. This statistics gives us a measure of which proportion of the observed

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[21]In contrast, our source of data for earnings and wealth is the non top-coded SCF which actually over-samples rich households to ensure that we do not miss them.

[22]For a survey on heterogeneous agents models trying to fit distributional statistics of US data see Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

capital is due to the absence of state contingent markets.

In model economies without endogenous labor supply the allocations of the complete markets economy coincide with those of the representative agents abstraction. Completing the markets is equivalent to giving to everybody the unconditional mean of the shock with probability one. However, when households have a choice of hours the equivalence between the complete markets economy and the representative agent formulation vanishes. In the complete markets economies households equate marginal utilities of consumption across states but work more when productivity is high and less when productivity is low, using the state contingent bonds to transfer resources between states. This yields higher aggregate labor than in a representative agent economy.

In economies without labor/leisure choice computing precautionary savings is quite a straightforward calculation. One typically needs to calibrate the incomplete markets economy to the observed aggregate wealth. This implies setting a value for the time preference parameter β. Then, given β the representative agent economy implies a unique interest rate $\frac{1}{\beta} - 1$ which in turn implies a unique aggregate level of capital. The (relative) difference between the capital level observed in data (and matched by the incomplete markets model) and the capital level implied by the complete markets or representative agent economy is the measure of precautionary savings.

In economies with labor/leisure choice, nevertheless, precautionary savings are not uniquely determined. Precisely, whereas the interest rate and hence the capital labor ratio are still uniquely determined under complete markets, the total level of capital is not. The same capital to labor ratio can be achieved by multiple combinations of aggregate labor and aggregate capital. In other words, the difference between the capital labor ratio under complete and incomplete markets cannot be wholly attributed to savings. Part (if not all) may come from differences in aggregate labor. As we have seen, there are many (infinite) distributions of households over their individual state space that sustain a steady state equilibrium in complete markets. All these distributions will therefore give the same capital labor ratio. However, different distributions will generate different levels of aggregate capital and aggregate labor.

7 Experiments

The economic experiment performed in this paper is the comparison of allocations of the steady state equilibrium of an incomplete markets economy calibrated to reproduce certain

statistics of data with the allocations of the steady state equilibria of different complete markets counterparts.

As we have seen, there are multiple complete markets economies to use. The issue, then, is which are the relevant ones to use as benchmark to provide measurements of precautionary savings and the value of labor as insurance mechanism. I will provide two different cases: (1) a complete markets economy where all households enjoy the same consumption and (2) a complete markets economy where the distribution of consumption is an scaled version of that under incomplete markets. In addition, I also solve for the representative agent economy to show that it may differ much from the complete markets economies. In the next subsections I explain the nature of and how to solve for the two chosen complete markets counterparts.

7.1 Characterization 1

The first chosen complete markets characterization is one in which all households enjoy the same level of consumption. This is equivalent to an economy where all households have the same permanent income and corresponds to a social planner problem where all households are given the same weight.

The steady state equilibrium for this economy is fully characterized by the following equations:

1. The steady state condition

$$\beta \left(1 + F_K\left(K, L\right) - \delta\right) = 1 \tag{12}$$

2. The n_ε optimal decisions of hours

$$l^\nu = \frac{\lambda}{F_L\left(K, L\right)\varepsilon} \, c^\sigma \tag{13}$$

   which can be written as a function $l\left(c, \varepsilon\right)$.

3. The n_ε budget constraints

$$c + \beta \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} b\left(\varepsilon'\right) = b\left(\varepsilon\right) + F_L\left(K, L\right)\varepsilon\left(1 - l\right) \tag{14}$$

   where the optimal decision of consumption is included by imposing $b\left(\varepsilon\right) = b'\left(\varepsilon\right)$.

4. The market clearing conditions for the production inputs

$$L = \int \varepsilon (1 - l) \, d\mu \tag{15}$$

$$K = \int \beta \sum_{\varepsilon'} \Gamma_{\varepsilon \varepsilon'} b (\varepsilon') \, d\mu \tag{16}$$

Notice that we have already substituted the pair of prices $\{w, r\}$ and the pricing function $q (\varepsilon, \varepsilon')$ by their equilibrium conditions. This set of equations exhaust the definition of equilibrium stated in section 4.3. We have $2n_\varepsilon + 3$ equations and the same number of unknowns: K, L, c, $b (\varepsilon)$ and $l (c, \varepsilon)$.

7.2 Characterization 2

The second complete markets characterization is one in which the distribution of consumption is exactly the same as in the incomplete markets economy. In terms of the social planner problem, it corresponds to the set of weights that give the same distribution of consumption as in the incomplete markets economy.

To solve for this characterization I take the consumption level enjoyed by each household in the incomplete markets economy and solve the equations 13 and 14 for each of them. Then I aggregate (equations 15 and 16) and check if the steady state is verified (equation 12). If it is not, the initial distribution of consumption is rescaled by a constant and the process done again until the steady state is found.

8 Results

In this section I present selected statistics from the model economies. I want to focus on four different results. Firstly, precautionary savings are not big and may even be negative. They range from 11.2% to -2.1% of aggregate capital. Secondly, aggregate hours worked are higher in the incomplete markets economies, between 20% to 84%. In contrast, aggregate labor is lower. This makes the average hour worked in the incomplete markets economies less productive. Thirdly, incomplete markets economies are not necessarily bigger than complete market economies. Indeed, aggregate output in the incomplete markets economy is between 6% and 18% smaller than in the complete markets economies whereas aggregate consumption is between 11% and 23% smaller. Finally, the measurements of precautionary savings and relative size of the economy are dramatically different if instead of solving for the complete markets economies we worked with the representative agent model.

8.1 The size of precautionary savings

In the first row of table 3 I report precautionary savings. We observe that compared to the complete markets economy with the same consumption for everybody (economy CM_1) precautionary savings are 11.2% whereas they fall to -2.1% when compared to our second characterization (economy CM_2). In other words, aggregate capital is lower in the incomplete markets economy than in the complete markets economy CM_2.

Table 3: **Precautionary savings.**

	CM_1	CM_2	RA
aggregate capital	11.2%	-2.1%	26.5%
capital-labor ratio	24.5%	24.5%	24.5%

Note: the first row is the relative difference of aggregate capital in the incomplete markets economy and the corresponding economy in the table. The second row is the relative difference of the ratio between aggregate capital and aggregate labor in the incomplete markets economy and the corresponding economy in the table.

Are these big or small numbers? In the second row of the same table 3 I present a similar statistic but instead of measuring it over aggregate capital I do it over the capital labor ratio. This would be the the type of precautionary savings captured by models without labor choice since it directly arises from the lower marginal product of capital in the incomplete markets economy. In a model without labor choice, the whole difference between capital labor ratios would be imputed to aggregate capital. We can see that *precautionary savings* measured over capital labor ratios are higher, up to 24%. That is to say, one fourth of the aggregate capital per efficiency units of labor in the incomplete markets economy is due to the absence of state contingent markets. This is the type of effect highlighted by Aiyagari (1994) and it is solely due to the uninsurable variation of labor earnings when the markets are incomplete.[23] There are two possible ways whereby the capital labor ratio can be higher in the incomplete markets economy: (a) a higher aggregate capital and/or (b) a lower aggregate labor. The numbers in table 3 suggest that aggregate labor must be lower in the incomplete markets economy. We look into this issue in the next section.

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[23]It is actually the interaction of uncertainty with the convexity of the marginal utility, a property of the utility function known as prudence. This assumption implies that the average of the marginal utilities is larger than the marginal utility of the average consumption. With borrowing constraints more stringent than the natural borrowing limit we may obtain precautionary savings regardless of the sign of the third derivative of the utility function. See Aiyagari (1994) or Huggett and Ospina (2001) for details.

8.2 Aggregate hours and labor

The complete and incomplete markets economies imply distinctly different household's behavior in the labor market. A first evidence of this are the disparate measures of aggregate hours worked and aggregate labor in the different model economies. In the first row of table 4 we see that aggregate hours worked are much higher in the incomplete markets economy than in the complete markets counterparts. In the incomplete markets economy aggregate hours are calibrated to reproduce the 0.33 value found in data. This number is a 84% higher than the equivalent in the first complete markets characterization and 20% higher than in the second one. In contrast to this result, aggregate labor is lower in the incomplete markets economy. The second row of table 4 shows that aggregate labor in the incomplete markets economy is 0.328 whereas it is 0.386 and 0.443 in the complete markets economies. The fact that hours worked are higher and aggregate labor lower in the incomplete markets economy suggests that the average efficiency per hour worked must be lower in the incomplete markets economy. In the third row of table 4 we see that the average amount of efficiency units per hour worked are between 50% and 100% higher in the complete markets economies. We also see in table 4 that the total productivity of hour worked, measured as output per hour, is clearly higher in the complete markets economies.

Table 4: **Statistics of simulated economies.**

	IM	CM_1	CM_2	RA
H	0.330	0.179	0.274	0.320
L	0.328	0.386	0.443	0.320
L/H	0.994	2.156	1.617	1.000
Y/H	3.485	6.816	5.109	3.166

The reasons for these results are quite straightforward. Under complete markets, households base the variation of hours worked across states entirely on the variation of efficiency units. The first order condition for the labor decision is given by equation 7. In the complete markets economy the consumption level is the same regardless of the realization of ε. Therefore, l adjusts to movements on ε alone. In other words: the realization of the idiosyncratic shock does not carry any wealth effect and the variations of hours worked respond only to the substitution effect. However, under incomplete markets the realization of the shock does change consumption levels. Low ε imply low consumption and therefore high marginal utility of consumption. Therefore, a household with low ε chooses to work more in the incomplete markets economy due to the high value of the wage obtained. This effects is strengthened when the idiosyncratic shocks are persistent. Overall, the average productivity

per hour worked must be lower in the incomplete markets economy because labor is used to smooth consumption fluctuations across states and therefore does not fully respond to variations of productivity.
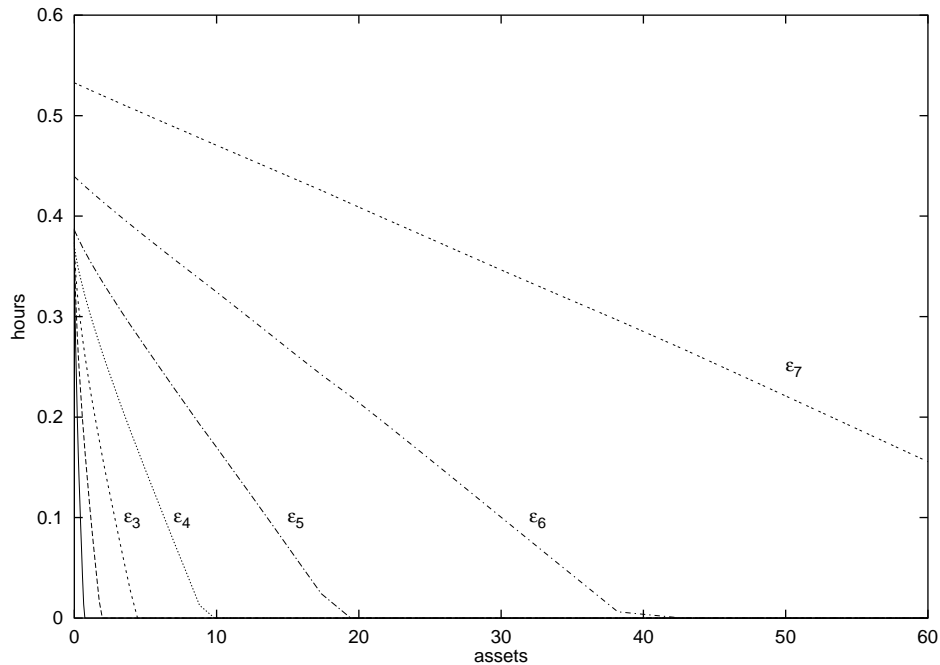

Figure 1: **Policy functions for hours. Economy IM.**

Note: Productivity shocks are labelled such that $\varepsilon_7 > \varepsilon_6 > ... > \varepsilon_1$. Hours are reported as fraction of total available time. Assets are reported in levels, where 1.15 corresponds to the aggregate output of the economy.

To quantify the differences in labor market behavior, let us first look at the graphical counterpart of equation 7. In figure 1 I plot the policy function for hours worked in the incomplete markets economy.[24] We observe how, other things equal, the work effort increases with the efficiency endowment and decreases with wealth. However, as shown in table 5, the correlation between efficiency units and assets is a very high 0.70. We can hardly analyze the labor decision under the *other things equal* abstraction because in equilibrium good shocks are associated to high wealth and bad shocks to low wealth. The reason behind this is that shocks are very persistent. Households receive long series of good or bad shocks, which make them deplete or accumulate wealth. Then, in equilibrium, we observe that many of the very productive households are also wealth rich and therefore they do not work much because the marginal value of each unit of consumption is low. On the other hand, we observe that

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[24]In terms of the policy functions defined in section 3.2 it corresponds to 1-$g^l(\varepsilon, a)$

many of the low productive households are also wealth poor, supplying a lot of hours in spite of their low return because the marginal value of an extra unit of consumption is very high for them. The exact amount in which this happens is determined at the calibration point by the correlation between efficiency units and hours worked, which is set equal to 0.01 as in data.

Table 5: **Statistics of simulated economies.**

	IM	CM_1	CM_2
$corr\left(a, \varepsilon\right)$	0.70	-0.99	-0.51
$corr\left(h, \varepsilon\right)$	0.01	0.93	0.59

Marcet, Obiols-Homs, and Weil (2002), in a related work, claim that hours worked are lower under incomplete markets. In a similar model, they set up household uncertainty as a work opportunity. Households have a choice of hours only if given a (stochastic) work opportunity. These authors find that the number of hours worked will be smaller in an incomplete markets economy because employed workers are richer in an incomplete markets world and therefore consume more leisure. The argument is based solely on the wealth effect of only one type of household. Their setting, therefore, does not allow for substitution of labor across different states of the world. This is the main channel operating here: complete markets allow households to work many hours when the market return is high and work few or none hours when the market return is low. The substitution of labor across states in the complete markets economies makes the average return per hour worked much higher and therefore lifetime income higher for everybody. Households are therefore richer in the complete markets economies and this wealth effect makes them demand more leisure.

8.3   The size of the economy

The third result to analyze is that the incomplete markets economy is not necessarily bigger in size than its complete markets counterparts. In table 6 we can see that aggregate capital under incomplete markets is 12% larger than in our complete markets characterization 1 but about 2% smaller than in characterization 2. Aggregate labor is between 15% and 20% lower. As a result, aggregate output in the incomplete markets world is between 6% and 18% smaller and aggregate consumption between 11% and 23% smaller.

These results are in contrast to the standard result in Huggett (1993) or Aiyagari (1994). They are a direct consequence of the different allocation of work effort in the different economies. The use of hours as a smoothing mechanism implies a lower average efficiency per

Table 6: **Statistics of simulated economies.**

	IM	CM_1	CM_2	RA
K	3.44	3.06	3.51	2.53
L	0.33	0.39	0.44	0.32
Y	1.15	1.22	1.40	1.00
C	0.86	0.97	1.11	0.80

hour worked and therefore a lower aggregate labor than in the complete markets economies. Therefore, aggregate capital does not need to increase so much in order to achieve the higher capital labor ratio of the incomplete markets economy equilibrium. If the fall in aggregate labor is large enough, the larger capital labor ratio of the incomplete markets economy can be achieved with even a fall in aggregate capital. The allocation of work effort is the missing ingredient in the seminal analysis of Huggett (1993) and Aiyagari (1994).

8.4 The comparison between the complete markets and the representative agent economies.

In the previous section I have been referring to the comparison between the incomplete markets economy and two complete markets counterparts. Yet, in the tables I also presented results for the representative agent economy. Looking back to the tables it is clear that the answer to this paper question would have been very different had I compared the incomplete markets economy to the representative agent formulation. Precisely, the incomplete markets economy is bigger than the representative agent counterpart. Compared to the representative agent economy, precautionary savings would have been 26.5% and the incomplete markets economy 15% bigger in terms of output.

The reason for this is that the complete markets economies and the representative agent economy turn out to be very different. In both cases consumption is equalized across states and therefore marginal propensities to consume are linear in wealth.[25] However, the representative agent receives always the same efficiency endowment whereas the households in the complete markets economies receive different endowments in different states of the world. Consequently, the representative agent works always the same amount of hours whereas the households in the complete markets world substitute leisure across states to work when they are more productive and to enjoy leisure when they are less productive. The average efficiency unit per hour worked is much larger in the complete markets economies, as seen in

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[25]As opposed to the convex consumption policies that arise with uninsured idiosyncratic risk.

table 4. Accordingly, aggregate labor is lower in the representative agent economy and, for the equilibrium to hold, so is aggregate capital.

9 Robustness

In this section I want to show how the qualitative results prevail once we change certain parameters of the model. In section 9.1 I look at changes in σ, the parameter driving the intertemporal elasticity of substitution. I do so because this is a parameter typically taken off the shelves in macroeconomic models. The qualitative results of the paper do not change but quantitative results do. It is important to highlight that the model fit to data diminishes substantially, which suggest that the standard practice in macroeconomics of taking a *reasonable* value for σ from some empirical paper may be very misleading. In section 9.2 I show how relaxing the borrowing constraints of households in the incomplete markets economy does not affect the results.

9.1 The intertemporal elasticity of substitution of consumption

The quantitative results of the paper depend on the households willingness to substitute leisure and consumption across time and states of the world. This behavior is mainly determined by ν and σ. The choice of these parameters, therefore, should be inferred from households observed behavior. The parameter ν is obtained from the observed relative volatility between consumption and hours worked. The parameter σ drives the correlation between hours and wages. As we have seen, $\sigma = 1.044$ produces a correlation between hours and wages equal to 0.01, in line with empirical measurements.

Table 7: **Calibration targets and model parameters.**

parameter	target	value	value	value
σ	$-$	1.045	1.500	2.000
ν	$cv(c)/cv(h) = 3.00$	1.116	2.972	5.429
λ	$H = 0.33$	1.658	0.987	0.474
β	$K/Y = 3.00$	0.942	0.939	0.933
θ	$wL/Y = 0.64$	0.640	0.640	0.640
δ	$I/Y = 0.25$	0.083	0.083	0.083

In this section, I solve the model economies with σ equal to 1.5 and equal to 2.0. Changing a parameter value implies that the model will not be consistent with certain dimensions of

data that we regard as important. However, there is value in doing so because it allows us to understand the economic mechanisms at work. For these experiments, I choose the same calibration targets as for the benchmark case except for the correlation between hours and wages, which is kept free. Table 7 presents the parameter values obtained for the two new economies together with the benchmark economy.

Table 8: **Distributional statistics.**

variable	cv	gini	q_1	q_2	q_3	q_4	q_5
earnings							
model $\sigma = 1.04$	1.05	0.48	3.5%	8.3%	13.6%	22.0%	52.6%
model $\sigma = 1.50$	0.84	0.41	5.2%	10.0%	15.3%	23.0%	46.5%
model $\sigma = 2.00$	0.75	0.38	5.9%	10.6%	16.0%	23.5%	44.0%
data	2.65	0.61	−0.2%	4.0%	13.0%	22.9%	60.2%
wealth							
model $\sigma = 1.04$	2.07	0.80	0.0%	0.0%	1.6%	13.9%	84.5%
model $\sigma = 1.50$	1.80	0.75	0.0%	0.5%	4.3%	18.0%	77.2%
model $\sigma = 2.00$	1.56	0.69	0.0%	1.4%	7.0%	21.0%	70.6%
data	6.53	0.80	−0.3%	1.3%	5.0%	12.2%	81.7%
hours							
model $\sigma = 1.04$	0.28	0.12	0.17	0.34	0.37	0.38	0.39
model $\sigma = 1.50$	0.24	0.13	0.21	0.30	0.34	0.38	0.42
model $\sigma = 2.00$	0.22	0.12	0.23	0.29	0.33	0.37	0.43
data	0.27	0.12	0.19	0.33	0.34	0.34	0.43

Note: *cv* refers to coefficient of variation. $q_1, ..., q_5$ refer, for earnings and wealth, to the share held by all people in the corresponding quintile with respect to the total. However, for hours it is the average number of hours worked by people in the corresponding quintile. Wealth and earnings data refers to the SCF98 and is quoted from Budría, Díaz-Gimenez, Quadrini, and Ríos-Rull (2002). Data on hours are from CPS2002.

The first result to highlight is that the correlations between hours and wages for σ equal to 1.5 and 2.0 are −0.44 and −0.63 respectively. These values are clearly inconsistent with the almost zero equivalent measure in both the CPS and PSID data sets. What they tell us is that by increasing the curvature of both consumption and leisure, households that want very smooth consumption and leisure patterns will respond more to wealth effects (marginal utility of consumption) than to substitution effects (wage) when taking their labor decisions.

In table 8 I report the distributional statistics generated by these new calibrations together with the benchmark one. We observe that as σ is increased the fit of the model to data, in terms of the earnings and wealth distribution, diminishes. The inequality statistics fall and the concentration of each variables diminishes. The curvature of consumption in the utility function turns out to be a very important parameter for a model that wants to be consistent with earnings and wealth inequality facts. As we have just seen, σ drives

the correlation between wages and hours, a key determinant of the earnings and wealth distributions.[26]

Table 9: **Precautionary savings.**

variable	model	CM_1	CM_2
aggregate capital	$\sigma = 1.04$	11.2%	-2.1%
	$\sigma = 1.50$	17.0%	8.7%
	$\sigma = 2.00$	23.3%	18.0%
capital-labor ratio	$\sigma = 1.04$	24.5%	24.5%
	$\sigma = 1.50$	27.3%	27.3%
	$\sigma = 2.00$	32.8%	32.8%

Note: the first three rows are the relative differences of aggregate capital in the incomplete markets economy and the corresponding economy in the table. The next three rows are the relative differences of the ratio between aggregate capital and aggregate labor in the incomplete markets economy and the corresponding economy in the table.

Table 9 reports precautionary savings. With the new parameterizations, precautionary savings are positive in all cases. However, they are still smaller than when measured by capital labor ratios. As before, this implies that aggregate labor in the incomplete markets economy must be smaller than in the complete markets counterparts. In table 10 we can confirm that, in fact, aggregate labor is smaller in the incomplete markets economies.

Table 10: **Statistics of simulated economies.**

variable	model	IM	CM_1	CM_2
K	$\sigma = 1.04$	3.44	3.06	3.52
	$\sigma = 1.50$	3.11	2.58	2.84
	$\sigma = 2.00$	2.99	2.30	2.45
H	$\sigma = 1.04$	0.33	0.18	0.27
	$\sigma = 1.50$	0.33	0.20	0.26
	$\sigma = 2.00$	0.33	0.24	0.27
L	$\sigma = 1.04$	0.33	0.39	0.44
	$\sigma = 1.50$	0.30	0.34	0.37
	$\sigma = 2.00$	0.29	0.33	0.35
Y	$\sigma = 1.04$	1.15	1.22	1.40
	$\sigma = 1.50$	1.03	1.05	1.16
	$\sigma = 2.00$	1.00	0.99	1.05

The other two results also hold. First, the amount of aggregate hours worked is larger in the incomplete markets economy. Second, the incomplete markets economy is not necessarily

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[26]See Heathcote, Storesletten, and Violante (2003).

bigger. In table 10 we observe that aggregate output in the incomplete markets economy is smaller than in the complete markets economies for the case $\sigma = 1.50$ and it is also smaller than the CM_2 economy for the case $\sigma = 2.00$. However, in this latter case output in the CM_1 economy is slightly smaller than in the incomplete markets economy.

A final comment to makes is that precautionary savings are increasing in σ. Higher risk aversion in consumption implies a higher need to smooth consumption across states. As we have seen this can be achieved by means of building higher stocks of savings and /or by working longer hours when bad shocks come. We observe that both things happen together, with precautionary savings increasing with σ and with the correlation between hours and wages falling with σ.

9.2   The borrowing constraints

A possible objection to the results shown so far is that allowing households to borrow may diminish the use of hours as a smoothing mechanism. To probe this issue, I calibrate three new economies to the same targets as before. In these new economies, households are allowed to borrow one fourth, one half and the whole of the average annual income of the economy.[27] Table 11 presents the parameter values for these three economies.

Table 11: **Calibration targets and model parameters.**

parameter	target	$\underline{a} = 0.00$ value	$\underline{a} = -0.25Y$ value	$\underline{a} = -0.50Y$ value	$\underline{a} = -1.00Y$ value
σ	$corr(h, \varepsilon) = 0.01$	1.045	1.035	1.026	1.010
ν	$cv(c)/cv(h) = 3.00$	1.116	1.257	1.389	1.641
λ	$H = 0.33$	1.658	1.572	1.495	1.358
β	$K/Y = 3.00$	0.942	0.944	0.945	0.946
θ	$wL/Y = 0.64$	0.640	0.640	0.640	0.640
δ	$I/Y = 0.25$	0.083	0.082	0.082	0.082

The distributional statistics can be found in table 12. We observe that working behavior is almost unchanged. The distribution of hours worked does not seem to depend much on the borrowing limits once we force the models to be consistent with the same observed household behavior, namely average of hours worked, correlation of hours with wages and relative volatility of consumption and hours. The distribution of labor earnings also remains

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[27]Notice that for this parametric experiment we are not taking into consideration that households may prefer not to work and repudiate the debt and the problems for the existence of a credit market that this would pose.

almost the same. There is some more action in terms of wealth, where we find sizeable amounts of households holding negative asset holdings, which makes the inequality measures much higher.

Table 12: **Distributional statistics.**

variable	*cv*	*gini*	q_1	q_2	q_3	q_4	q_5
earnings							
model $\underline{a} = 0.00$	1.05	0.48	3.5%	8.3%	13.6%	22.0%	52.6%
model $\underline{a} = -0.25Y$	1.04	0.48	3.5%	8.3%	13.6%	22.0%	52.6%
model $\underline{a} = -0.50Y$	1.04	0.48	3.6%	8.3%	13.5%	22.1%	52.5%
model $\underline{a} = -1.00Y$	1.03	0.48	3.7%	8.3%	13.5%	22.1%	52.4%
data	2.65	0.61	-0.2%	4.0%	13.0%	22.9%	60.2%
wealth							
model $\underline{a} = 0.00$	2.07	0.80	0.0%	0.0%	1.6%	13.9%	84.5%
model $\underline{a} = -0.25Y$	2.16	0.85	-1.7%	-1.6%	0.7%	14.9%	87.8%
model $\underline{a} = -0.50Y$	2.23	0.90	-3.4%	-3.2%	0.1%	15.8%	90.8%
model $\underline{a} = -1.00Y$	2.39	0.99	-6.7%	-6.1%	-1.0%	17.5%	96.3%
data	6.53	0.80	-0.3%	1.3%	5.0%	12.2%	81.7%
hours							
model $\underline{a} = 0.00$	0.29	0.13	0.17	0.34	0.37	0.38	0.39
model $\underline{a} = -0.25Y$	0.29	0.13	0.17	0.34	0.37	0.38	0.39
model $\underline{a} = -0.50Y$	0.29	0.13	0.17	0.33	0.37	0.38	0.40
model $\underline{a} = -1.00Y$	0.29	0.14	0.17	0.32	0.36	0.38	0.41
data	0.27	0.12	0.19	0.33	0.34	0.34	0.43

Note: *cv* refers to coefficient of variation. $q_1, ..., q_5$ refer, for earnings and wealth, to the share held by all people in the corresponding quintile with respect to the total. However, for hours it is the average number of hours worked by people in the corresponding quintile. Wealth and earnings data refers to the SCF98 and is quoted from Budría, Díaz-Gimenez, Quadrini, and Ríos-Rull (2002). Data on hours are from CPS2002.

As for the statistics of interest in this paper, they are almost unchanged. In table 13 we observe that precautionary savings measured over aggregate capital are always lower than measured over capital labor ratios and that precautionary savings can reach negative values. Likewise, aggregate labor is higher in the incomplete markets economy, aggregate hours lower and aggregate output lower, with quantitative values very close to those in the non-borrowing economy (and not reported here).

Table 13: **Precautionary savings.**

variable	model	CM_1	CM_2
aggregate capital	$\underline{a} = 0.00$	11.2%	−2.1%
	$\underline{a} = -0.25Y$	11.5%	−2.4%
	$\underline{a} = -0.50Y$	11.4%	−2.5%
	$\underline{a} = -1.00Y$	10.5%	−2.4%
capital-labor ratio	$\underline{a} = 0.00$	24.5%	24.5%
	$\underline{a} = -0.25Y$	23.4%	23.4%
	$\underline{a} = -0.50Y$	22.4%	22.4%
	$\underline{a} = -1.00Y$	20.8%	20.8%

Note: the first four rows are the relative differences of aggregate capital in the incomplete markets economy and the corresponding economy in the table. The next four rows are the relative differences of the ratio between aggregate capital and aggregate labor in the incomplete markets economy and the corresponding economy in the table.

10 Conclusions

The seminal works of Huggett (1993) and Aiyagari (1994) have taught us important properties of model economies where households face uninsurable idiosyncratic shocks to labor income. This type of models have been largely used to understand the determinants of the distributions of earnings and wealth and they have also been used to evaluate the impact of policy reform over these same distributions.[28] One of the most commonly agreed upon properties of this type of economies is that they are bigger in size, both in terms of output and aggregate capital, than economies where there are markets to insure against earnings risk. The absence of these markets is seen as one reason for savings in actual economies, giving rise to what has been termed as *precautionary savings*.

In this paper I show that this result is an artifact of ignoring the labor supply decision of households. Households can use hours of work as well as savings to confront fluctuations. The use of hours generate smaller, even negative, precautionary savings. In addition, aggregate output can also be smaller in the incomplete markets economy than in the complete markets economies.

What we learn from the quantitative exercise performed in this paper is that the use of hours as an insurance mechanism is very important. I write a dynamic general equilibrium model with incomplete markets and use it to infer the preference parameters from observed household behavior. It turns out that once we solve the complete markets economy with these

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[28]A review on the papers that use this type of models to understand the wealth and earnings distributions can be found in Castañeda, Díaz-Giménez, and Ríos-Rull (2003). Examples of policy experiments with this type of economies are Ventura (1999) or Conesa and Krueger (2002).

preference parameters, the amounts of aggregate capital and aggregate labor in equilibrium are very different from the ones observed in data and matched by the incomplete markets model economy. The aggregate labor in the incomplete markets economy is much lower despite the fact that aggregate hours are higher. Compared to complete markets economies, workers spend many hours working when they are not very productive just because the marginal product of consumption is very high. Household could avoid this by raising their precautionary savings and avoiding to ever reach that situation. They choose not to do so, making an extensive use of labor supply as a self-insurance mechanism against wage fluctuations.

References

AIYAGARI, S. R. (1994): "Uninsured Idiosyncratic Risk, and Aggregate Saving," *Quarterly Journal of Economics*, 109(3), 659–684.

BROWNING, M., AND A. LUSARDI (1996): "Household Saving: Micro Theories and Micro Facts," *Journal of Economic Literature*, 34, 1797–1855.

BUDRÍA, S., J. DÍAZ-GIMENEZ, V. QUADRINI, AND J.-V. RÍOS-RULL (2002): "New Facts on the U.S. Distribution of Earnings, Income and Wealth," *Federal Reserve Bank of Minneapolis Quarterly Review*, 26(3), 2–35.

CASTAÑEDA, A., J. DÍAZ-GIMÉNEZ, AND J.-V. RÍOS-RULL (2003): "Accounting for U.S. Earnigns and Wealth Inequaltiy," Forthcoming, *Journal of Political Economy*.

CONESA, J. C., AND D. KRUEGER (2002): "On the Optimal Progressivity of the Income Tax Code," Mimeo, CRES, Universidad de Barcelona.

DÍAZ, A., J. PIJOAN-MAS, AND V. RÍOS-RULL (2003): "Precautionary Savings and Wealth Distribution Under Habit Formation Preferences," *Journal of Monetary Economics*, 50(6).

DOMEIJ, D., AND M. FLODEN (2003): "The Labor-Supply Elasticity and Borrowing Constraints: Why Estimates are Biased.," Mimeo.

HEATHCOTE, J., K. STORESLETTEN, AND G. VIOLANTE (2003): "The Macroeconomic Implications of Rising Wage Inequality in the US," Mimeo.

HOPENHAYN, H., AND E. C. PRESCOTT (1992): "Stochastic Monotonicity and Stationary Distributions for Dynamic Economies," *Econometrica*, 60, 1387–1406.

HUGGETT, M. (1993): "The Risk Free Rate in Heterogeneous-Agents, Incomplete Insurance Economies," *Journal of Economic Dynamics and Control*, 17(5/6), 953–970.

——— (1997): "The One-Sector Growth Model with Idiosyncratic Shocks: Steady States and Dynamics," *Journal of Monetary Economics*, 39(3), 385–403.

HUGGETT, M., AND S. OSPINA (2001): "Aggregate Precuationary Savings: When is the Third Derivative Irrelevant?," *Journal of Monetary Economics*, 48, 373–396.

KRUSELL, P., AND A. SMITH (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), 867–896.

LOW, H. (2003): "Self-Insurance in a Life-cycle Model of Labor Supply and Savings," Unpublished Manuscript.

MARCET, A., F. OBIOLS-HOMS, AND P. WEIL (2002): "Incomplete markets, Labor Supply and Capital Accumulation," Unpublished manuscript, Universitat Pompeu Fabra.

RÍOS-RULL, J.-V. (1998): "Computing Equilibria in Models with Heterogenous Agents," in *Computational Methods for the Study of Dynamic Economics*, ed. by R. Marimon, and A. Scott, chap. 9. Oxford University Press.

STORESLETTEN, K., C. TELMER, AND A. YARON (2002): "Consumption and Risk Sharing over the Life Cycle," Forthcoming, *Journal of Monetary Economics.*

VENTURA, G. (1999): "Flat Tax Reform: a Quantitative Exploration," *Journal of Economic Dynamics and Control*, 23, 1425–1458.