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Missing Markets for Human Capital and Differences in Growth*

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Abstract

There are some empirical facts that growth models usually cannot explain: *i*) the differences in consumption growth rates across countries when international capital markets are considered, *ii*) the low growth and low levels of education in developing countries where the return on education is very high. This paper introduces a generational structure that implies that the return on human capital is higher than the return on physical capital and that consumption growth rates vary across countries when international capital markets are included. The human capital technology of the paper implies that poor countries grow more slowly and invest a smaller share of income on education, in spite of an extraordinarily high return on education and the existence of international capital markets.

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I. Introduction

Empirical evidence suggests that stagnation and low growth plague many developing countries (see Easterly 1994, 2001). Easterly (1994) shows that the rate of growth of per capita GDP is significantly positive in only 41 out of the 86 developing countries included in the Summer and Heston (1988) data set. However, growth of per capita GDP is significantly positive in all the OECD countries. Quah (1996, 1997) has found that income distribution among countries is becoming increasingly more unequal.

Some growth models have been proposed to explain these facts, but the results of most of these models do not hold if international capital markets are introduced. To be more precise, under standard preferences, growth models that include international capital markets imply that consumption should grow at the same speed in every country.

Psacharopoulos (1981, 1994) found that the return on human capital in developing countries is very high. Thus it is even more difficult to understand why many developing countries are economically stagnated when these countries have such a high return on education.

This paper is focused on solving the two empirical puzzles mentioned above: i) the existence of differences in consumption growth rate when international capital markets are taken into account, ii) stagnation and low levels of education in developing countries where the return on human capital is very high.

Under standard preferences[1], the Euler Equation implies that consumption growth rates are an increasing function of interest rates. When international capital markets are introduced, agents face the same interest rate in every country, the Euler Equation thus implies that the consumption growth rates should be equal across countries. Rebelo (1991) has pointed out that these models imply an "implausible solution" to the development problem: instead of concentrating on removing imperfections in internal markets that impede the growth of developing countries, it may be more effective to remove barriers to the international mobility of capital. Thus, the citizens of growth stagnated countries would be able to invest in developed countries where the return on capital is high. This policy would make developing countries' consumption increase at the same rate as that of developed countries.

This paper presents a conventional endogenous growth model with human capital[2] (see Uzawa (1965) and Lucas (1988)). The main innovation is the generational structure introduced. Agents in this model have a "life cycle": when they are younger they save in order to finance their children's education in the future, later in life, they spend these savings on their children's education[3]. Thus, the generational structure of the model implies that dynasties cannot invest in human capital in every period. If dynasties were able to invest in human capital in every period, the return on human capital would never be higher than the return on physical capital in equilibrium, because in this case no one would invest in physical capital. However, the generational structure presented in the paper implies a different result: parents that do not have children being educated, save (in physical capital) in order to be able to finance their education in the future, even when the return on human capital is higher than on physical capital. Thus, the return

[1]Most papers in growth theory have infinite life agents with time separable preferences with constant elasticity of substitution felicity function.

[2]The benchmark model may be found in Barro and Sala-i-Martin (1995), page 172.

[3]An intuitive way to see the model is that agents save in the periods in which their children are kids in order to finance their university education when their children are older.

on human capital may be higher than the return on physical capital in equilibrium, since even in this case there are agents that invest in physical capital because they do not have children being educated. Summarizing, the generational structure introduced in the model implies that the return on human capital is higher than on physical capital. The empirical evidence (Psacharopoulos, 1981, 1994) supports this prediction of the model.

The Euler Equation in this model implies that the consumption growth rate is an increasing function of the return on human capital. Since the return rate on human capital is different from the interest rate in equilibrium, the Euler Equation implies that consumption growth rates are different across countries as long as returns on human capital are different across countries. Thus, even when international capital markets are considered and agents of different countries face the same interest rate, the model generates differences in consumption growth rates across countries

The stagnation of developing countries is even more striking when the returns on human capital are observed. Psacharopoulos (1981, 1994) found that developing countries have a very high return on education. Thus it is more difficult to understand why countries with such a high return on education grow so slowly and invest so little in education. In order to solve this empirical puzzle, the paper presents a human capital technology in which a minimum amount of investment is required in order to invest in education and in which the return on human capital decreases with the amount of investment. Since the return on human capital decreases with the amount of investment and poor countries invest less in human capital than rich countries it follows that the return on education in poor countries is higher than in rich countries. Thus, this type of technology is consistent with the empirical evidence found by Psacharopoulos (1981, 1994). The model also predicts that the share of investment in education in the income of poor countries is small and thus these countries grow slowly, in spite of the fact that the return on education in poor countries is very high. Thus the model explains why poor countries in which the return on human capital is so high grow so slowly and invest so little in education in comparison with rich countries.

The paper also analyzes the case in which past human capital has a positive external effect in the production of human capital. In this case, poverty traps appear and consumption in poor countries may be stagnated forever, in spite of the free capital mobility in international markets and the return on human capital in poor countries being so high.

There are some papers in which the return on human capital may be larger than the return on physical capital. In some of these papers this result is due to the preferences, which are not represented by the conventional time separable utility with altruism (infinite-life). More precisely, there is not perfect altruism (Caballe, 1995; Drazen, 1978; and Rangazas, 1996) or altruism is absent (Buiter and Kletzer, 1995). By contrast, the preferences are absolutely standard in the model presented here: agents care about the discount value of he whole utility path of the dynasty. Besides that the first three mentioned papers (Caballe, 1995; Drazen, 1978; and Rangazas, 1996) do not deal with international capital markets and differences in growth rates in Buiter and Kletzer (1995) are due to differences in taste parameters.

There are other papers (Barro, Mankiw and Sala-i-Martin, 1995; and Galor and Zeira, 1993) in which the source of the result is not the preferences[4], but the introduction of imperfections in financial markets. Those models have in common that long run growth is not allowed due to the decreasing returns on human capital. If the assumption of

[4]Preferences are not standard in Galor and Zeida (1993).

constant returns on human capital is introduced in order to generate growth, the results of the former papers disappear. Further more, those models predict that the unique countries in which growth rate is positive are those in which the per capita capital is under the steady state level, that is, "poor countries". Thus, those models are no not suitable to analyze the topics in which this paper is focus on: the difference in growth rates (no levels) among countries and why many poor countries grow so slowly in comparison with developed countries.

The paper is organized as follows. Section II presents the basic model. Section III analyzes agents' behavior. Section IV studies the aggregate behavior of the economy (under the assumption of a closed economy), this section shows that the generational structure of the model implies that in equilibrium the return on human capital is higher than on physical capital. Section V modifies the basic model to generate differences in growth rates among countries when international capital markets are taken into account. Section VI explains the possible existence of poverty traps in an open economy environment. Section VII concludes. The proofs and some technical details may be founded in the Appendix.

II. The Model

This section presents the basic model used in this paper. It will be slightly modified in section V to generate differences in consumption growth rates among countries benefiting from international capital markets

A. Technology

Time is discrete with an infinite horizon. There is a single good in the economy, which can be used for consumption and investment in human and physical capital. The production technology of this good is given by the following production function:

$$A\ L_t^{1-\alpha} K_t^{\alpha}$$

where K denotes physical capital, L denotes the labor efficiency units, $A \in \Re_{++}$ is the total factor productivity and $\alpha \in \left(0, \frac{1}{2}\right)$. It will be shown later that the number of labor efficiency units depends on the amount of human capital.

The production is used for consumption and investment in human and physical capital:

$$A\ L_t^{1-\alpha} K_t^{\alpha} = C_t + I_t^K + I_t^H \tag{1}$$

C (aggregate) consumption, I^K (aggregate) investment in physical capital, and I^H(aggregate) investment in human capital.

The physical and human capital accumulation equation is the typical neoclassical one:

$$\begin{aligned} K_{t+1} &= (1-\delta)K_t + I_t^K & (2) \\ H_{t+1} &= (1-\delta)H_t + I_t^H & (3) \end{aligned}$$

where H denotes human capital and $\delta \in [0, 1]$ is the depreciation rate.

The amount of labor efficiency units of a worker is proportional to his human capital:

$$L(H_t) = H_t \tag{4}$$

where $L(H_t)$ denotes the amount of labor efficiency units that a worker with H_t units of human capital has.

B. Preferences

The preferences of a dynasty are given by a time separable, constant elasticity of substitution utility function:

$$V_t^i = \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau^i) \tag{5}$$

where c_τ^i denotes the consumption of dynasty i in period τ, $\beta \in (0,1)$, and $u(.)$ is an isoelastic felicity function:

$$u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma} & if\ \gamma \in (0,1) \cup (1,+\infty) \\ \\ \ln(c) & if\ \gamma = 1 \end{cases} \tag{6}$$

Agents do not differentiate between the consumption of different members of the dynasty, they only care about their aggregate consumption.

C. Generational Structure

The model is up to now an absolutely conventional endogenous growth model with human capital. Its innovative aspect is the generational structure, which is essential to the results of the paper.

In each country there are two types of dynasties, odd and even. Agents belonging to odd dynasties are born in odd periods, agents belonging to even dynasties are born in even periods. Agents live four periods, but they only take economic decisions and work in the third and fourth periods of their lives. They have an offspring in the third period of their lives. This means that in each dynasty there are always two agents alive: an agent who works, referred to as "middle-aged", and an agent who does not, referred to as "young"[5]. Agents receive education in the second period of their youth, this mean that dynasties cannot invest in human capital in every period. Finally bequest cannot be negative.

This generational structure implies that the dynasty whose middle-aged agent starts working at period t faces the following optimization problem:

$$Max \textstyle\sum_{n=0}^{\infty} \beta^n u(c_{t+n}^i) \tag{7}$$

$s.t.\ :$

$$w_{t+n} L(H_{t+n}^i) + r_{t+n} K_{t+n}^i = I_{t+n}^{H,i} + I_{t+n}^{K,i} + c_{t+n}^i \tag{8}$$

$$K_{t+n+1}^i = I_{t+n}^{K,i} + (1\text{-}\delta)\, K_{t+n}^i \tag{9}$$

$$H_{t+n+1}^i = I_{t+n}^{H,i} + (1\text{-}\delta)\, H_{t+n}^i \tag{10}$$

$$K_{t+n}^i \geq 0 \qquad \text{if } n \in \aleph^{even} \tag{11}$$

$$I_{t+n}^{H,i} = 0 \qquad \text{if } n \in \aleph^{even} \tag{12}$$

[5]The model may be modified in a straightforward way to introduce retirement simply by assuming that agents live for six periods: in the first two periods agents are educated, in the third and fourth period agents works and in the fifth and sixth period agents are retired. Since the model does not differentiate between the consumption of members of different ages within the dynasty, this modification does not change the behavior of the model at all.

where w_{t+n} denotes wage at period $t+n$, r_{t+n} denotes gross interest rate at period $t+n$, denotes the natural number, included zero: $\aleph \equiv \{0,1,2,3,4,5...\}$, and $\aleph^{even}$ are the even numbers: $\aleph^{even} \equiv \{0,2,4,6,8,...\}$. Dynasties maximize its utility which depend upon the consumption path of the dynasty. The model does not differentiate between consumption of the middle-aged and the young member of the dynasty: the dynasty's consumption is the aggregate consumption of the middle-aged and young members of the dynasty.

Equation (8) is a typical budget constraint: the dynasty's income is given by the return on the physical capital owned by the dynasty "$r_{t+n}K^i_{t+n}$" plus the labor income of the middle-aged dynasty member"$w_{t+n}L(H^i_{t+n})$". This member's labor income depends on his human capital. This income is spent on the consumption of the dynasty "c^i_{t+n}", the education of the young member of the dynasty "$I^{H,i}_{t+n}$", and investment in physical capital"$I^{H,i}_{t+n}$". Equations (9) and (10) are also conventional equations of capital accumulation. The part of the model that is not conventional are equations (11) and (12).

Equation (11) means bequest cannot be negative. When middle-aged agents die, they cannot bequeath liabilities to their children. middle-aged agents (parents) can use their future labor income to finance their consumption, but they cannot use young agents' future labor income in order to finance their consumption or to finance the young agents' education. This implies that dynasties can only borrow if their middle-aged member should still be alive in the next period. Bequest cannot be negative for legal reasons. If a finance intermediary lends to someone that does not have any wealth, and this person dies, the finance intermediary cannot take his heir as a slave. The borrower's heir does not inherit any legal responsibility for his parent's debts. This point has been used before in the finance intermediary literature (Hart and Moore, 1994).

Equation (12) means that the education of the young member takes place in the second period of his life. A possible interpretation is that the education in the first period in the child's life is free. This assumption simply models the fact that education at the beginning of a child's life is cheaper in at less in three aspects: i) Living costs: university education often requires moving from the parents' home. ii) Tuition fees are more expensive at university than at school. iii) Finally the forgone income is much smaller when children are at school age than at university since children cannot work or at less they have smaller salaries

The feature the model tries to capture is that, during the life cycle, in some periods, dynasties incur more expenditure or earn less income. Agents cannot invest in human capital in order to smooth consumption. Dynasties that wish to do this must invest in physical capital, even in the case where the return on human capital is higher than on physical capital.

An intuitive way to interpret this model is that parents save (physical capital) when their children are kids in order to be able to finance their education later in life (when their children go to the college).

The generational structure of the model is crucial to get the results of the paper. More precisely, section IV shows that this structure means the return on human capital is higher than on physical capital in equilibrium. This result is the basis of all the rest of the paper.

III. Agents' Decisions

A. Firms

Firms are perfectly competitive and maximize profit at each point in time. Thus, the optimization problem of firms at time t is the following:

$$\underset{L_t,\, K_t}{Max}\ A\ L_t^{1-\alpha} K_t^{\alpha} - w_t L_t - r_t K_t \tag{13}$$

where r_t denotes gross interest rate, w_t denotes the wage per labor efficiency unit. The first order conditions of this problem are the following:

$$\begin{aligned} r_t &= \alpha A \left(\tfrac{K_t}{L_t}\right)^{-(1-\alpha)} \\ w_t &= (1-\alpha) A \left(\tfrac{K_t}{L_t}\right)^{\alpha} \end{aligned} \tag{14}$$

The above first order conditions are familiar ones: the payment of a productive factor should be equal to its marginal productivity.

B. The Dynasties' Decisions

To simplify notation, the super-index that denotes the type of dynasty (odd or even) is dropped in this section.

Maximization problem (7) of dynasty whose middle-aged agent starts working at period t may be rewritten as follows:

$$\begin{aligned} & V_t(K_t, H_t, \{w_\tau, r_\tau\}_{\tau=t}^{\infty}) = \\ & \underset{K_{t+1}, H_{t+1}, K_{t+2}, H_{t+2}}{Max} \quad \{u\left((1\text{-}\delta{+}w_t)\, H_t + (1\text{-}\delta{+}r_t)\, K_t - H_{t+1} - K_{t+1}\right) + \\ & \beta\ u\left((1\text{-}\delta{+}w_{t+1})\, H_{t+1} + (1\text{-}\delta{+}r_{t+1})\, K_{t+1} - H_{t+2} - K_{t+2}\right) + \\ & \beta^2 V_{t+2}\left(K_{t+2}, H_{t+2}, \{w_\tau, r_\tau\}_{\tau=t+2}^{\infty}\right)\Big\} \\ & s.t.\ H_{t+1} \ = \ (1-\delta)H_t,\ K_{t+2} \ \geq \ 0 \end{aligned} \tag{15}$$

The income of each dynasty is equal to the physical capital income $r_t K_t$ plus the labor income $w_t H_t$ that is proportional to its middle-aged member's human capital. The consumption of a dynasty is equal to its income minus the investment in physical and human capital. However given the generational structure of the model, dynasties do not invest in human capital in every period. More precisely, if the middle-aged agent of the dynasty starts working in period t, the dynasty does not invest in the education of its young member until period $t+1$, thus the investment in human capital in period t is zero. The human capital of the dynasty in the periods t and $t+1$ is the human capital of its middle-aged agent (born in period $t-2$) and thus does not change since middle-aged agents do not invest in their education after they start working. This is the reason why the first restriction ($H_{t+1}{=}(1-\delta)H_t$) appears in the dynasty optimization problem. Since bequest cannot be negative the physical capital in period $t+2$ cannot be negative ($K_{t+2} \geq 0$). Dynasties may borrow against their middle-aged member's future labor income but not against their young member's future labor income. That is, K_{t+1} may be negative but K_{t+2} should be positive.

The first order conditions of the consumer optimization problem are the following Euler equations:

$$
\begin{aligned}
c_{t+1}/c_t &= (\beta R_{t+1})^{\sigma} && (16)\\
c_{t+2}/c_{t+1} &= (\beta R_{H,t+2})^{\sigma} && (17)
\end{aligned}
$$

where σ denotes the elasticity of substitution: $\sigma \equiv \frac{1}{\gamma}$; R and R_H denote the return factor on physical and human capital $R \equiv (1-\delta+r)$, $R_H \equiv (1-\delta+w)$. Thus the Euler Equations are very typical: consumption growth is increasing with the return rate on the investment of dynasties. Combining these two Euler Equations it is possible to get the average consumption growth of the dynasty, which along the balance growth path is the same as the average consumption growth of the country:

$$
\sqrt{c_{t+2}/c_t} = \left(\beta\sqrt{R_{t+1}R_{H,t+2}}\right)^{\sigma} \tag{18}
$$

Thus the consumption growth rate is an increasing function of the geometric average of the return on physical capital and the return on human capital.

In the usual growth models the return on human and physical capital are the same and consequently Euler Equation (18) simply implies that the consumption growth rate is an increasing function on the return on physical capital. Thus if there is perfect capital mobility in the international markets, every country faces the same interest rate, this Euler Equation implies that consumption in every country grows at the same rate. However, next section shows that the generational structure of the model implies that the return on human capital is higher than on physical capital in equilibrium. Thus, even if there is perfect international capital mobility and consumers in every country face the same interest rate, Euler Equation (18) implies that differences in consumption growth rates among countries appear if there are also differences in the return on human capital.

IV. Balanced growth path (Closed Economy)

This section shows that the generational structure of the model implies that under certain conditions the return on human capital is higher than on physical capital in a closed economy. This finding will be crucial for the remaining results of the paper.

Definition 1 *Given the initial wealth allocation* $\left\{K_0^{odd}, H_0^{odd}, K_0^{even}, H_0^{even}\right\}$, *a competitive equilibrium is a sequence of wealth and consumption* $\left\{K_t^{odd}, H_t^{odd}, c_t^{odd}, K_t^{even}, H_t^{even}, c_t^{even}\right\}_{t=0}^{\infty}$, *aggregate factors* $\{K_t, H_t\}_{t=0}^{\infty}$ *and prices* $\{w_t, r_t\}_{t=0}^{\infty}$ *such that for all* $t \in \aleph$*:*

1. *Both odd and even dynasties maximize their utility according with the following maximization problems:*

$$
\begin{aligned}
& Max \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_{\tau}^{odd}) \\
& s.t: \\
(1\text{-}\delta+w_{\tau})\, H_{\tau}^{odd} + (1\text{-}\delta+r_{\tau})\, K_{\tau}^{odd} &= K_{\tau+1}^{odd} + H_{\tau+1}^{odd} + c_{\tau}^{odd} \\
K_{\tau}^{odd} \geq 0 \qquad H_{\tau+1}^{odd} &= (1\text{-}\delta)\, H_{\tau}^{odd} \qquad \text{if } \tau \in \aleph^{odd}
\end{aligned}
$$

$$Max \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_{\tau}^{even})$$

$s.t$:

$$(1\text{-}\delta\text{+}w_{\tau})\, H_{\tau}^{even} + (1\text{-}\delta\text{+}r_{\tau})\, K_{\tau}^{even} = K_{\tau+1}^{even} + H_{\tau+1}^{even} + c_{\tau}^{even}$$
$$K_{\tau}^{even} \geq 0 \qquad H_{\tau+1}^{even} = (1\text{-}\delta)\, H_{\tau}^{even} \qquad \text{if } \tau \in \aleph^{even}$$

2 *Factors get paid their marginal product:*

$$r_t = \alpha A \left(\frac{K_t}{L_t}\right)^{-(1-\alpha)}$$
$$w_t = (1-\alpha) A \left(\frac{K_t}{L_t}\right)^{\alpha}$$

3 *Markets clear:*

$$K_t = K_t^{odd} + K_t^{even}$$
$$H_t = H_t^{odd} + H_t^{even}$$

Definition 2 *Balanced growth path is a competitive equilibrium such that:*

1. *The (Aggregate) Human and physical capital grows at constant rate:*

   $\frac{K_{t+1}}{K_t} = \frac{H_{t+1}}{H_t} = (1+g)$

2 *Dynasties' wealth and consumption grow at constant rate each two periods:*

$\frac{K_{t+2}^{odd}}{K_t^{odd}} = \frac{H_{t+2}^{odd}}{H_t^{odd}} = \frac{c_{t+2}^{odd}}{c_t^{odd}} = \frac{K_{t+2}^{even}}{K_t^{even}} = \frac{H_{t+2}^{even}}{H_t^{even}} = \frac{c_{t+2}^{even}}{c_t^{even}} = (1+g)^2$

3 *Prices are constant:* $w_t = w$, $r_t = r$.

Conventional growth models predict that the return on human capital should be equal to the return on physical capital. The reason for this conventional result is simple: if the return on human capital were larger, no one would invest in physical capital, thus the return on human and physical capital should be equal in equilibrium. This model has a different result. The reason is that even in the case of the return on human capital being higher, dynasties that do not have young members being educated will invest in physical capital. Thus, even when the return on physical capital is lower than on human capital, there is positive investment in physical capital. This implies that the return on human capital may be higher than the return on physical capital in equilibrium.

Assumption 1: $A\,(1-\alpha)^{1-\alpha}\,\alpha^{\alpha} \geq \left(\frac{1-\delta}{1-2\alpha}\right)^{\frac{1}{\sigma}} \frac{1}{\beta} - (1-\delta)$

The above assumption is similar to the standard assumptions that guaranty positive growth in endogenous growth models[6]: the productivity of the economy "A" should be large enough and the discount rate of the utility function should be small enough. This assumption guaranties that along the balanced growth path the return on human capital is higher than on physical capital.

[6]It is easy to check that the condition that guaranties positive growth if the model would not have the present generational structure would be as follows (See Barro and Sala-i-Martin, 1995): $A\,(1-\alpha)^{1-\alpha}\,\alpha^{\alpha} \geq \frac{1}{\beta} - (1-\delta)$

Proposition 1 *The return on human capital is higher than on physical capital along the balanced growth path:* $R_H > R$.

Proof. see Appendix

The generational structure of the model is crucial to this result. If dynasties were able to invest in human capital in every period then the return on it would always be the same as the return on physical capital, otherwise no one would invest in physical capital.

It was already explained in the introduction that the empirical evidence supports this result. Psacharopoulos' estimates of the return on human capital are larger than the usual interest rates (see Psacharopoulos, 1981, 1994).

Corollary 2 *The ratio of physical to human capital along the balanced growth path is greater than the one when markets are complete (when negative bequest are possible).*

Proof. If markets are complete the ratio of physical to human capital is optimal when $R_H = R \Leftrightarrow k = \frac{\alpha}{1-\alpha}$. Along the balanced growth equilibrium $R_H > R \Leftrightarrow k > \frac{\alpha}{1-\alpha}$. ■

As a consequence of the generational structure of the model, dynasties cannot invest in human capital in every period. They invest in physical capital in some periods, even if the return on human capital is higher. As a consequence the ratio of physical to human capital is greater than if markets were complete (when negative bequest are possible). When markets are complete the equilibrium is Pareto Optima, thus Corollary 2 means that the ratio capital to human capital is inefficiently high in de incomplete market economy.

Proposition 3 *There is* $\overline{\sigma} > 1$ *such that if* $\sigma < \overline{\sigma}$ *then the balanced growth path exist.*

Assumption 2: $\sigma < \overline{\sigma}$

Assumption 2 guaranties the existence of balanced growth path. This type of assumption is very standard in the endogenous growth literature. The growth factor along the balanced growth path is $\left(\beta\sqrt{R_H R}\right)^{\sigma}$ (see equation 18 and Appendix). In order that the consumers' problem is well defined when $\sigma > 1$ and growth is positive, the following condition is needed[7]:

$$\sqrt{R_H R} > \left(\beta\sqrt{R_H R}\right)^{\sigma} \Leftrightarrow \sigma < \frac{\frac{1}{2}\ln\left(R_H R\right)}{\frac{1}{2}\ln\left(R_H R\right) + \ln\beta}$$

If $\sigma \to \infty$ (linear utility) the consumers problem is not well defined when growth is positive. Then the balanced growth path exists if the elasticity of substitution is smaller than a certain constant $\overline{\sigma}$.

V. Differences in growth rates in open economy

This paper attempts to explain differences in consumption growth rates and the low growth of poor countries when international capital markets are taken into account. This low growth is even more striking given Euler equation (18) and the empirical evidenced found by Psacharopoulos (1981, 1994). As the return on human capital is higher in poor

[7]In the Rebelo's AK model (Rebelo, 1994) such condition would be as follows: $A > (\beta A)^{\sigma} \Leftrightarrow \sigma < \frac{\ln A}{\ln A + \ln\beta}$

countries than in rich ones, Euler Equation (18) would predict that the consumption growth rate would be higher in poor than in rich countries. This conclusion would be in contradiction with the empirical evidence found by Easterly (1994, 2001) and Quah (1996, 1997). This section presents a model in which the return on human capital is higher in poor countries than in rich countries. However their consumption growth rate is slower and the proportion of saving that goes into investment in human capital is less.

Consider now that the amount of labor efficiency units instead of being as in (4) it is as follows:

$$L(H) = \begin{cases} \varepsilon & if \quad H \in [0, \varepsilon) \\ \underline{L} + \Psi(H - \varepsilon) & if \quad H \in \left[\varepsilon,\ \overline{H}\right) \\ H & if \quad H \geq \overline{H} \end{cases} \tag{19}$$

where $\overline{H} \geq \underline{L} > 2\ \varepsilon$, and $\Psi(H - \varepsilon)$ is an increasing continuous twice differentiable function[8].

Assumption 3: a) $\Psi(0) = 0$, $\Psi\left(\overline{H} - \varepsilon\right) = \left(\overline{H} - \underline{L}\right)$, $\Psi'\left(\overline{H} - \varepsilon\right) = 1$; b) $\forall H \in \left(\varepsilon,\ \overline{H} - \varepsilon\right)$ $\Psi"(H) > 0$.

Assumption 3.a implies that $L(H)$ is continuous and differentiable in (ε, ∞). Assumption 3.b guaranties that the return on education decreases with the amount of human capital.

This human capital technology is represented in figure 1 and it has three important characteristics: i) There is a minimum amount of investment in education ε. ii) The return on education decreases with the amount of human capital. iii) Human capital technology is linear when the amount of human capital is large enough (when it is larger than $\overline{H}$).

The economic meaning of the minimum amount of human capital ε is that agents need to have a minimum amount of knowledge if they want to work as qualified workers. If agents invest less than the minimum level ε, they will not have enough knowledge to work as qualified workers and their human capital will be useless.

Another important characteristic of human capital technology (19) is that the return on human capital decreases with the amount of human capital (see thick line in figure 2.a):

$$R_H(H) = \begin{cases} 1 - \delta + w\left(\frac{\underline{L} + \Psi(H - \varepsilon)}{H}\right) & if \quad H \in \left[\varepsilon,\ \overline{H}\right) \\ 1 - \delta + w = R_H & if \quad H > \overline{H} \end{cases} \tag{20}$$

It follows from assumption 3.b that the return on human capital decreases with the human capital in the interval $\left(\varepsilon, \overline{H}\right)$ (a proof of this statement may be founded in the Appendix). The intuition is that the marginal return on human capital (the thick broken line in figure 2.a) is always bellows the return on human capital in the interval $(\varepsilon,\ \overline{H})$. The marginal labor income that is added by a extra unit of human capital is smaller than the average labor income and thus the (average) return of human capital decreases in the interval $(\varepsilon,\ \overline{H})$. The intuition is exactly the same as the fact that the average productivity decreases when the marginal productivity is bellow of it.

Since rich countries invest more in education than poor countries, the model predicts that the return on education is higher in poor countries than in rich ones. The empirical evidence supports this fact (see Psacharopoulos 1981, 1994).

[8] An example of $\Psi(.)$ would be the following function: $\Psi(x) = (\overline{H} - \underline{L})\left(\frac{x}{\overline{H} - \varepsilon}\right)^{\gamma}$ $\gamma \equiv \left(\frac{\overline{H} - \varepsilon}{\overline{H} - \underline{L}}\right)$.

The human capital technology is linear when the human capital is larger than $\overline{H}$. This assumption makes the existence of balanced growth path possible and implies that when the amount of human capital is large enough, the behavior of the model is exactly as described in the last section. More precisely, countries that have a per-capita human capital above a certain threshold level $\widehat{H}$ behave exactly as in the last section. These countries will be referred to as rich countries. The threshold level $\widehat{H}$ will be defined more precisely later.

Figure 2.a plots the return on human capital (equation 20). Since the minimum investment ε is required in order to invest in human capital, the return on human capital for an amount of human capital smaller than ε is zero. The return on human capital has its highest level at the minimum amount ε and decreases until it arrives at $\overline{H}$. From this level the return on human capital is constant and equal to that in section IV. This means that when the per-capita human capital is larger than $\overline{H}$, countries behave as described in section IV.

Since rich countries have a much higher weight in the international capital markets, it is quite reasonable to assume that they determine the international interest rate.

Assumption 4: The international interest rate is equal to the interest rate along the balanced growth path in a closed economy (denoted by r): $\forall t\ r_t = r$.

The above assumption means that the international interest rate is equal to the rate that rich countries would have along the balanced growth path in a closed economy. Poor countries have little weight in the international capital markets, as a consequence the international interest rate is the same as it would be if all countries in the World were rich. This assumption is standard, see for example Barro, Mankiw and Sala-i-Martin (1995).

It follows from the above assumption together with the firms' first order conditions that the per labor efficiency unit wage is the same in every country (the per worker wage is proportional to the human capital and thus it is different in different countries):

$$\begin{aligned} r_t &= r = \alpha A\left(k\right)^{-(1-\alpha)} \Rightarrow \\ w_t &= (1-\alpha)A\left(k\right)^{\alpha} = w \end{aligned} \tag{21}$$

where k is the ratio of physical to human capital along the balanced growth path in a closed economy, w is the wage per labor efficiency unit along the balanced growth path in a closed economy.

A. Threshold Level $\widehat{H}$:

If countries have a per-capita human capital larger than a certain threshold level $\widehat{H}$ they behave exactly as in the last section. More precisely, when dynasties in the economy have enough human capital, they invest all their savings in education in the period in which the education of their young member takes place. This threshold level $\widehat{H}$ will be the amount of human capital such that dynasties will be indifferent between investing ε units in education and the rest of their savings in physical capital or investing all their savings in education. If $w\Psi'(0) > r$ such threshold level is ε, otherwise $\widehat{H}$ is defined as follows:

$$\widehat{H} \Leftrightarrow wL(\varepsilon) + r\left(\widehat{H} - \varepsilon\right) = wL(\widehat{H}) \tag{22}$$

The left hand side of the above equation is the income that a dynasty would get on investing the minimum amount ε in education and the rest of their savings $\left(\widehat{H}-\varepsilon\right)$ in physical capital. The right hand side of equation (22) is the income if they invest all their savings $\widehat{H}$ in human capital.

$\triangle r_H$ is defined as follows:

$$\triangle r_H(H) = w\frac{\triangle L}{\triangle H} = w\frac{L(H)-L(\varepsilon)}{H-\varepsilon} = \begin{cases} w\frac{\Psi(H-\varepsilon)}{H-\varepsilon} & if \quad H\in\left[\varepsilon,\ \overline{H}\right] \\ w\frac{H-\underline{L}}{H-\varepsilon} & if \quad H\geq\overline{H} \end{cases} \tag{23}$$

$\triangle r_H$ is the average increment in the return on human capital relative to the return on the minimum investment ε. $\triangle r_H$ is the average increment in the labor income when the human capital increases over the minimum level ε. This concept is similar to the return rate on human capital, but taking the minimum level ε as a benchmark point instead of zero. Figure 2.b shows that $\triangle r_H$ increases with the human capital (see Appendix).

It follows from equation (22) that the threshold level $\widehat{H}$ is the amount of human capital at which the interest rate is equal to the average increment in the return on human capital $\triangle r_H$:

$$r = w\frac{L(\widehat{H})-L(\varepsilon)}{\widehat{H}-\varepsilon} = \triangle r_H(\widehat{H}) \tag{24}$$

Figure 2.b shows that when the amount of human capital is below the threshold level $\widehat{H}$ then the average increment in the return on human capital $\triangle r_H$ is below the return rate of physical capital r. Thus, agents will be better off investing the minimum amount ε in human capital and the rest of their savings in physical capital. When the amount of human capital is above the threshold level $\widehat{H}$ then the average increment in the return on human capital $\triangle r_H$ is above the return rate of physical capital r and thus agents will be better off investing all their savings in human capital. The threshold level $\widehat{H}$ may be higher or lower than $\overline{H}$. The case in which $\widehat{H}$ is higher than $\overline{H}$ (figure 2.b) will now analyzed, the other case will be analyzed later.

B. Case in which $\widehat{H}>\overline{H}$:

In this case, it follows from (23) that equation (24) may be rewritten as follows:

$$r = w\frac{\widehat{H}-\underline{L}}{\widehat{H}-\varepsilon} \Leftrightarrow \tag{25}$$

$$\widehat{H} = \frac{w\underline{L}-r\varepsilon}{(w-r)} \tag{26}$$

It follows from the definition of the threshold level that if a dynasty's savings are smaller than $\widehat{H}$ in the period in which they invest in education, they will invest the minimum amount ε in human capital and the rest of their savings in physical capital. If their savings are larger than $\widehat{H}$, they will invest all their savings in human capital:

$$\begin{array}{llll} If\ Z_{t+2}<\varepsilon & \Rightarrow & H_{t+2}=0 & K_{t+2}=Z_{t+2} \\ If\ Z_{t+2}\in\left[\varepsilon,\widehat{H}\right) & \Rightarrow & H_{t+2}=\varepsilon & K_{t+2}=Z_{t+2}-\varepsilon \\ If\ Z_{t+2}\geq\widehat{H} & \Rightarrow & H_{t+2}=Z_{t+2} & K_{t+2}=0 \end{array} \tag{27}$$

where $Z = K + H$ and "$t+1$" is the period in which the education of the young member of the dynasty takes place. Thus, rich countries invest a higher proportion of their savings in education than poor countries, in spite of the fact that the return on education is lower than in poor countries (see equation 20 and figure 2.a).

When the amount of human capital is positive ($H_t > 0$) the consumer optimization problem is very similar to the one presented in section III (see equation 15):

$$\begin{aligned} V(K_t, H_t) &= \underset{K_{t+1},H_{t+1},K_{t+2},H_{t+2}}{Max} \{u\left(R_H(H_t)\ H_t + RK_t - H_{t+1} - K_{t+1}\right) + \\ &\beta\ u\left(R_H(H_{t+1})\ H_{t+1} + RK_{t+1} - H_{t+2} - K_{t+2}\right) + \beta^2 V\left(K_{t+2}, H_{t+2}\right)\Big\} \\ &s.t. \quad H_{t+1} = H_t, \quad K_{t+2} \geq 0 \end{aligned} \tag{28}$$

The first order (necessary) conditions of the above optimization problem are the following Euler equations[9]:

$$c_{t+1}/c_t = (\beta R)^\sigma \tag{29}$$

$$c_{t+2}/c_{t+1} = \begin{cases} (\beta R)^\sigma & if \quad Z_{t+2} \in (0,\varepsilon) \cup \left(\varepsilon, \widehat{H}\right) \\ (\beta R_H)^\sigma & if \quad Z_{t+2} > \widehat{H} \end{cases} \tag{30}$$

When the middle-aged agent of a dynasty starts working in period t, he invests in physical capital in the first period and consequently the growth of its consumption (equation 29) will be a function of the return on physical capital R. In the period $t+1$ this agent invests in the education of the dynasty's young agent. If the amount of investment that this dynasty makes is smaller than $\widehat{H}$ then it also invests in physical capital and the growth of its consumption (equation 30) is a function of the return on physical capital R. If this dynasty's investment is larger than $\widehat{H}$ then it invests all its savings in human capital and consequently the growth of its consumption (equation 30) is a function of the marginal return on human capital. Since $\widehat{H}$ is larger than $\overline{H}$ the marginal return[10] on human capital is R_H when dynasties invest above the threshold level $\widehat{H}$ (see equation 20 and figure 2).

Combining Euler equations (29) and (30):

$$\sqrt{c_{t+2}/c_t} = \begin{cases} (\beta R)^\sigma & if \quad Z_{t+2} \in (0,\varepsilon) \cup \left(\varepsilon, \widehat{H}\right) \\ \left(\beta\sqrt{RR_H}\right)^\sigma & if \quad Z_{t+2} > \widehat{H} \end{cases} \tag{31}$$

The above equation shows that the consumption growth factor in poor countries is proportional to the return on physical capital $[\beta R]^\sigma$. The consumption growth factor in rich countries is proportional to the geometrical average of the return on human and physical capital$\left[\beta\sqrt{RR_H}\right]^\sigma$. It was shown in section IV that the return on human capital is higher than on physical capital in rich countries ($R_H > R$). Thus the consumption growth rate is higher in rich countries than in poor countries.

Poor countries invest in human capital less than rich countries. When a dynasty's savings in the period in which the education of the young agent takes place are smaller

[9]Since $L(H)$ is not differentiable in ε, the marginal return on human capital is not defined when $H = \varepsilon$. Therefore Euler Equation (28) is not satisfied when $H_{t+2} = \varepsilon$.

[10]The marginal return of human capital is defined as the left-hand derivative of the labor income with respect to the human capital: $\lim\limits_{\substack{\varepsilon \to 0 \\ \varepsilon < 0}} \frac{w * L(H+\varepsilon) - w * L(H)}{\varepsilon}$.

than the threshold level $\widehat{H}$ then dynasties either will not invest in education or will invest the minimum amount ε in human capital and the rest in physical capital. If the savings are larger than the threshold level $\widehat{H}$, they will invest all their savings in human capital in the period in which they invest in the education of their young member.

Thus this model is consistent with the empirical facts presented in the introduction: *i*) Poor countries' consumption growth rates are lower than rich countries' consumption growth rate in spite of the free capital mobility in international markets. *ii*) The portion of savings that goes to the investment in human capital is smaller in poor countries in spite of the fact that the return on human capital is higher.

C. Case in which $\widehat{H} < \overline{H}$:

When $\widehat{H} < \overline{H}$ the Euler equations are as follows:

$$\begin{aligned} c_{t+1}/c_t &= (\beta R)^\sigma \\ c_{t+2}/c_{t+1} &= \begin{cases} (\beta R)^\sigma & if \quad Z_{t+2} \in (0,\varepsilon) \cup \left(\varepsilon, \widehat{H}\right) \\ (\beta (1-\delta + w\Psi'(H-\varepsilon)))^\sigma & if \quad Z_{t+2} \in \left(\widehat{H}, \overline{H}\right) \\ (\beta R_H)^\sigma & if \quad Z_{t+2} > \overline{H} \end{cases} \end{aligned} \tag{32}$$

Combining the Euler equations (32) and (32):

$$\sqrt{c_{t+2}/c_t} = \begin{cases} (\beta R)^\sigma & if \quad Z_{t+2} \in (0,\varepsilon) \cup \left(\varepsilon, \widehat{H}\right) \\ \left(\beta \sqrt{R\,(1-\delta + w\Psi'(H-\varepsilon))}\right)^\sigma & if \quad Z_{t+2} \in \left(\widehat{H}, \overline{H}\right) \\ \left(\beta \sqrt{R R_H}\right)^\sigma & if \quad Z_{t+2} > \overline{H} \end{cases} \tag{33}$$

The above Euler Equation shows that there are three types of countries:

i) Poor countries ($Z_{t+2} \in \left(0, \widehat{H}\right)$): dynasties in these countries only invest the minimum amount ε in human capital in the period in which their young members' education takes place (or if they are very poor they do not invest in human capital at all). The rest of their savings are invested in physical capital. These countries have the slowest growth and the lowest income share of investment in human capital, in spite of the fact that their returns on human capital are the highest ones.

ii) Medium wealth countries ($Z_{t+2} \in \left(\widehat{H}, \overline{H}\right)$): dynasties in these countries invest all their savings in human capital in the period in which their young members' education takes place. These countries always grow faster than poor countries but slower than rich countries. In these countries the return on human capital is smaller than in poor countries and larger than in rich countries.

iii) Rich countries ($Z_{t+2} > \overline{H}$): These countries behave exactly as in section IV.

D. Marginal return on human capital

Define the marginal return factor on human capital as follows:

$$MgR_H(H) = \begin{cases} 1-\delta + w\ \frac{L-\varepsilon}{\varepsilon} & if \quad H = \varepsilon \\ 1-\delta + w\ \Psi'(H-\varepsilon) & if \quad H \in \left[\varepsilon,\ \overline{H}\right) \\ R_H & if \quad H > \overline{H} \end{cases} \tag{34}$$

When human capital is larger than the minimum level ε the marginal return on human capital is defined as the derivative of labor income with respect to human capital plus the part of human capital that remains after depreciation. Since labor income is not differentiable at ε, the marginal return on human capital is defined in discrete terms. Note that the assumption that $\underline{L} > 2\varepsilon$ implies that the marginal return when the human capital at ε is larger than R_H. This means that the marginal return on human capital in poor countries is always larger than in rich countries.

Figure 2.a plots the marginal return on human capital as a thick broken line. The marginal return on human capital has its maximum at the minimum human capital level ε, which is represented by a black point in figure 2.a. It follows from the assumption that $\Psi"(.) > 0$ that after that it jumps down and increases until arriving the rich countries' level R_H. In between these two points the marginal return on human capital is lower than in rich countries. However when $\widehat{H} \geq \overline{H}$ no-one invest in the interval $\left(\varepsilon, \overline{H}\right)$ and therefore those point are not observable.

VI. Poverty Traps

As was stated in the introduction, many developing countries suffer stagnation (see Easterly 1994, 2001). As Rebelo (1993) pointed out the poverty trap literature fail to gave an explanation for stagnation when international capital markets are introduced. Another problem is that the poverty trap literature (Azariadis and Drazen, 1990) predicts that the return of human capital in poor countries is lower than in rich countries and the formers do not invest in education. As it was explained in the introduction, both of these results are contra-factual.

In the model so far, poverty traps have not been generated, as poor countries grow more slowly than rich countries but eventually they reach the wealth level that allows dynasties to invest in human capital and to grow at the same rate as rich countries. In this subsection the human capital technology is modified slightly to generate poverty traps and stagnation in an environment with international capital markets. In those poverty traps both the marginal and the average return on human capital is higher than in rich countries.

Consider that the amount of labor efficiency units in countries in which the average human capital in the last period was smaller than ε instead of being as in (19) is as follows[11]:

$$If\ \widetilde{H}_{-1} \leq \varepsilon :$$

[11]This human capital technology may also be written as follows:

$$\begin{cases} \left(\frac{\min\left\{\widetilde{H}_{-1},\ \varepsilon\right\}}{\varepsilon}\right)^{\phi} \varepsilon & if\ H \in \left[0, \min\left\{\widetilde{H}_{-1},\ \varepsilon\right\}\right) \\ \left(\frac{\min\left\{\widetilde{H}_{-1},\ \varepsilon\right\}}{\varepsilon}\right)^{\phi} \left[\underline{L}+\Psi\left(H\text{-}\widetilde{H}_{-1}\right)\right] & if\ H \in \left[\min\left\{\widetilde{H}_{-1},\ \varepsilon\right\},\ \left(\overline{H}\text{-}\varepsilon+\min\left\{\widetilde{H}_{-1},\ \varepsilon\right\}\right)\right) \\ \left(\frac{\widetilde{H}_{-1}}{\varepsilon}\right)^{\phi} H & if\ H \geq \left(\overline{H}\text{-}\varepsilon+\min\left\{\widetilde{H}_{-1},\ \varepsilon\right\}\right) \end{cases}$$

$$L\left(H,\ \widetilde{H}_{-1}\right) = \begin{cases} \left(\frac{\widetilde{H}_{-1}}{\varepsilon}\right)^{\phi}\varepsilon & if\ H\in\left[0,\widetilde{H}_{-1}\right) \\ \left(\frac{\widetilde{H}_{-1}}{\varepsilon}\right)^{\phi}\left[\underline{L}+\Psi\left(H\text{-}\widetilde{H}_{-1}\right)\right] & if\ H\in\left[\widetilde{H}_{-1},\ \left(\overline{H}\text{-}\varepsilon+\widetilde{H}_{-1}\right)\right) \\ \left(\frac{\widetilde{H}_{-1}}{\varepsilon}\right)^{\phi}H & if\ H\geq\left(\overline{H}\text{-}\varepsilon+\widetilde{H}_{-1}\right) \end{cases} \quad (35)$$

where $\phi\in(0,1)$ and $\widetilde{H}_{-1}$ is the average human capital in the economy in the period before (the hat$\tilde{}$means that this human capital is perceived by agents as an externality). This human capital technology is very similar to the one presented in the last section (19), the main difference is that in this section the past human capital has positive external effects in the human capital technology. The assumption is quite reasonable: the average amount of human capital probably has a positive effect on productivity in education, if agents in a country have a high level of education, it is easier to transmit knowledge to new generations. This type of externalities has often been used in the endogenous growth literature (see among others Azariadis and Drazen, 1990; and Lucas, 1988).

It follows from the human capital technology (35) that when the average human capital in the last period was smaller than ε the (average) return on human capital is as follows:

$$R_H = \begin{cases} 1-\delta+w\left(\frac{\widetilde{H}_{-1}}{\varepsilon}\right)^{\phi}\frac{\underline{L}+\Psi\left(H-\widetilde{H}_{-1}\right)}{H} & if\ H\in\left[\widetilde{H}_{-1},\ \left(\overline{H}-\varepsilon+\widetilde{H}_{-1}\right)\right) \\ 1-\delta+w\left(\frac{\widetilde{H}_{-1}}{\varepsilon}\right)^{\phi} & if\ H\geq\left(\overline{H}-\varepsilon+\widetilde{H}_{-1}\right) \end{cases} \quad (36)$$

Figure 3.a compares the return of two different levels of past human capital. When the investment in human capital is small the return on human capital is larger for the low level of past human capital than for the high one. However when the investment in human capital is large the opposite happens: the return on human capital is smaller for the low level of past human capital than for the high one

As in the last section $\triangle r_H$ is the average increment in the return on human capital relative to the return on the minimum investment (which in this section is $\widetilde{H}_{-1}$):

$$\triangle r_H(H,\widetilde{H}_{-1}) = w\frac{\triangle L}{\triangle H} = w\frac{L(H,\widetilde{H}_{-1})-L(H_{-1},\widetilde{H}_{-1})}{H-\widetilde{H}_{-1}} \Leftrightarrow \quad (37)$$

$$\triangle r_H(H,\widetilde{H}_{-1}) = \begin{cases} w\left[\left(\frac{\widetilde{H}_{-1}}{\varepsilon}\right)^{\phi}\frac{\Psi\left(H-\widetilde{H}_{-1}\right)}{H-H_{-1}}\right] & if\ H\in\left[\widetilde{H}_{-1},\ \left(\overline{H}-\varepsilon+\widetilde{H}_{-1}\right)\right) \\ w\left(\frac{\widetilde{H}_{-1}}{\varepsilon}\right)^{\phi}\frac{H-\underline{L}}{H-H_{-1}} & if\ H\geq\left(\overline{H}-\varepsilon+\widetilde{H}_{-1}\right) \end{cases} \quad (38)$$

Figure 3.b plots the average increment in the return on capital $\triangle r_H$ for two different levels of past human capital. It shows that when the past human capital is very small the average increment in the return on capital $\triangle r_H$ is below the return on physical capital r. In this case agents do not have incentives to increase the human capital above the past level and consequently countries are stuck in a poverty trap. When the past human capital is larger than certain threshold level, the average increment in the return on human capital $\triangle r_H$ is above the return on physical capital r. In this case agents have incentives to increase the human capital above the past level and consequently these countries will converge to the balanced growth path of rich countries. This threshold level is denoted by $\underline{H}$ and it is defined as the amount of human capital at which $\triangle r_H$ is equal to the return

on physical capital r when the human capital tends toward infinity:

$$r = \lim_{H\to\infty} \triangle r_H\left(H, \underline{H}\right) = \lim_{H\to\infty} w\left(\frac{\underline{H}}{\varepsilon}\right)^{\phi} \frac{H-\underline{L}}{H-\underline{H}} = w\left(\frac{\underline{H}}{\varepsilon}\right)^{\phi} \tag{39}$$

$$\Leftrightarrow \quad \underline{H} = \left(\frac{r}{w}\right)^{\frac{1}{\phi}} \varepsilon \tag{40}$$

If $H_{-1} \in (0, \underline{H})$ the human capital and the Domestic Income of the economy stay constant forever. If $H_{-1} > \underline{H}$, the economy converges to the balanced growth path with permanent positive growth. Countries with a per capita human capital lower than $\underline{H}$ will be referred to as poor countries in this section.

The aggregate behavior of poor countries depends on the equilibrium interest rate in rich countries (which depends on the parameter values). There are two cases:

1. If the equilibrium interest rate in rich countries is equal to or smaller than the utility discount rate $\left(R \leq \frac{1}{\beta}\right)$, then poor countries will be stagnated.
2. If the equilibrium interest rate in rich countries is larger than the utility discount rate $\left(R > \frac{1}{\beta}\right)$, then poor countries consumption will grow at a positive rate but slower than in rich countries, and will export capital.

All of this would happen in spite of poor countries having a higher rate of return on human capital than rich countries (both average and marginal).

A. Stagnation: $R \leq \frac{1}{\beta}$

In this case, there is a continuum of steady states. There is a steady state for each point in the interval $[0, \underline{H}]$, where dynasties invest the same amount in education that the past generation did. The aggregate consumption also stays constant. However each dynasty's consumption is different in even and odd periods. More precisely, the Euler equations in this case is as follows:

$$c^i_{t+1} = (\beta R)^{\sigma} c^i_t \tag{41}$$

$$c^i_{t+2} = \frac{1}{(\beta R)^{\sigma}} c^i_{t+1} \tag{42}$$

where $i \in \{\text{odd, even}\}$, the dynasty i is the one in which its middle-aged agent starts working in period t, and thus the dynasty invests in education in period $t+1$. This means that the dynasty's consumption grows at a positive rate in the period in which the dynasty does not invest in education and at a negative rate in the period in which the dynasty invest in the education. Given that on average the growth rate is zero, this simply means that the dynasty's consumption is greater in the periods in which it does not invest in education than in the other periods[12]. It follows from (41) and (42) that dynasties' consumption at steady state is as follows:

$$c^E = \frac{(\beta R)^{\sigma}\left[1+R\right] wH}{\left[R+(\beta R)^{\sigma}\right]} \tag{43}$$

$$c^{NE} = \frac{\left[1+R\right] wH}{\left[R+(\beta R)^{\sigma}\right]} \tag{44}$$

[12]This result come from the fact that $L(.)$ is not differentiable when $H = H_{-1}$ (see 19).

where c^E is the dynasties' consumption in the periods in which the education of their young members take place and c^{NE} is the dynasties' consumption in the other periods.

The marginal return on human capital when $\widetilde{H}_{-1} \leq \varepsilon$ is as follows:

$$MgR_H = \begin{cases} 1-\delta+w\left(\frac{\widetilde{H}_{-1}}{\varepsilon}\right)^{\phi}\frac{\underline{L}-\varepsilon}{\widetilde{\widetilde{H}}_{-1}} & if \quad H=\varepsilon \\ 1-\delta+w\left(\frac{\widetilde{H}_{-1}}{\varepsilon}\right)^{\phi}\Psi'(H-\varepsilon) & if \quad H\in\left[\varepsilon,\ \overline{H}\right) \\ R_H & if \quad H>\overline{H} \end{cases}$$

The average and marginal returns on human capital are as follows at steady state ($H < \underline{H} < \varepsilon$):

$$R_H = 1-\delta+w\left(\frac{H}{\varepsilon}\right)^{\phi}\frac{\varepsilon}{H} = 1-\delta+w\left(\frac{\varepsilon}{H}\right)^{1-\phi} > 1-\delta+w = R_H \quad (45)$$

$$MgR_H = 1-\delta+w\left(\frac{H}{\varepsilon}\right)^{\phi}\frac{\underline{L}-\varepsilon}{H} = 1-\delta+w\left(\frac{\varepsilon}{H}\right)^{1-\phi}\frac{\underline{L}-\varepsilon}{\varepsilon} > R_H \quad (46)$$

Thus the return on human capital (both average and marginal) decreases with the human capital: the poorer the country the higher the return on human capital. Thus, the predictions of the model are consistent with the empirical evidence found by Psacharopoulos (1981, 1994).

B. Positive growth rate: $R > \frac{1}{\beta}$

In this case, the consumption growth rate will be $(\beta R)^{\sigma}$ in poor countries and $\left(\beta\sqrt{RR_H}\right)^{\sigma}$ in rich countries. It was proven in section IV that the return on human capital is greater than the return on physical capital ($R_H > R$). Thus, the model predicts that the consumption growth rate is higher in rich countries.

Since the human capital in poor countries (countries with human capital lower than $\underline{H}$) stays constant, the Domestic Income in poor countries is stagnated. However consumption growth rate may be positive because the capital owned by consumers in these countries grows without bounds. Thus, these countries eventually become capital exporters.

Summarizing, the model predicts that the consumption growth rate in poor countries is lower than the consumption growth rate in rich countries despite the return on human capital (both marginal and average) in poor countries being greater.

VII. Conclusion

Empirical evidence suggests that many developing countries suffer stagnation and low growth. Usually growth models cannot explain these facts where international capital mobility is allowed. This paper has presented a growth model with a generational structure that implies differences in growth rates among countries, even if international capital markets are introduced.

Another empirical fact that is difficult to understand with the existing growth models is that countries that are stagnated have an extraordinarily high return on human capital.

The paper has explained why countries with a high return on education may grow slower and invest a smaller fraction of their savings in education than countries with a lower return on human capital.

The generational structure of the model implies that the return on human capital is higher than on physical capital in equilibrium. This result is consistent with the empirical evidence found by Psacharopoulos (1981, 1994) and implies that even when there is free capital mobility in international markets, differences in consumption growth rates among countries appear. The intuition is that even when the interest rate is the same in every country, the return rate on human capital is different to the interest rate and consequently it may be different from one country to another. Since the consumption growth rate increases with the geometrical average of the return factor on human and physical capital, the consumption growth rate varies between countries.

The paper has presented a human capital technology in which a minimum investment is required in order to invest in human capital and in which the return on human capital decreases with the investment. This technology implies that the portion of investment dedicated to human capital is smaller in poor countries than in rich ones, in spite of the higher return on human capital in poor countries.

Finally, the paper also shows that when the past human capital has positive external effects in the human capital technology, there are poverty traps. In this case, the consumption growth rate in poor countries may remain stagnated forever, in spite of the high return on human capital.

VIII. References

Azariadis, C. and A. Drazen (1990) "Threshold external effects in Economic Development", Quarterly Journal of Economics, 105.

Barro, R., N. G. Mankiw, and X. Sala-*i*-Martin (1995) "Capital Mobility in Neoclassical Models of Growth", American Economic Review, 85 (1).

Barro, R. and X. Sala-i-Martin (1995), *Economic Growth*, Mc-Graw Hill.

Barro, R. (1974) "Are Government Bonds Net Wealth?", Journal of Political Economy, 82 (6).

Buiter, W. H. and K. M. Kletzer "Capital Mobility, Fiscal Policy, and Growth under Self-Financing of Human Capital Formation" Canadian Journal of Economics, 28(0), Nov. 1995, p. S163-94.

Caballe, J. (1995) "Endogenous Growth, Human Capital, and Bequest in a Life-Cycle Model", Oxford Economic Papers, 47.

Drazen, A. (1978) "Government Debt, Human capital, and Bequest in a Life-Cycle Model", Journal of Political Economy, 86 (3).

Easterly, W. (1994) "Economic Stagnation, Fixed Factors, and Policy Threshold", Journal of Monetary Economics 28.

Easterly, W. (2001) "The Lost Decades: Developing Countries' Stagnation in Spite of Policy Reform 1980-1998" Journal of Economic Growth 6(2), June 2001, pages 135-57.

Galor, O. and J. Zeira (1993) "Income Distribution and Macroeconomics", Review of Economic Studies, 60 (1).

Hart, O. and J. Moore (1994) "A Theory of Debt Based on the Inalienability of Human Capital", Quarterly Journal of Economics, 109 (4).

Ljungqvist, L. (1993) "Economic Underdevelopment, the Case of a Missing Market for Human Capital", Journal of Development Economics, 40.

Lucas, R. (1988) "On the Mechanics of Economic Development", Journal of Monetary Economics 22.

Lucas, R. (1990) "Why Doesn't Capital flow from Rich to Poor Countries?", American Economic Review, 80 (2).

Psacharopoulos, G. (1981) "Returns to Education: an Updated International Comparison", Comparative Education, 17.

Psacharopoulos, G. (1994) "Returns to Investment in Education: A Global Update" World Development; 22(9), September, p. 1325-43.

Rebelo, S. (1994) "Long Run Policy Analysis and Long Run Growth", Journal of Political Economy, 99 (3).

Rebelo, S. (1992) "Growth in Open Economies", Carnegie-Rochester Conference Series on Public Policy, 36.

Rangazas, P. (1996) "Fiscal Policy and Endogenous Growth in a Bequest-Constrained Economy", Oxford Economic Papers, 48.

Quah, Danny (1996) "Twin Peaks: Growth and Convergence in Models of Distribution Dynamics" Economic-Journal; 106(437), 1045-55.

Quah, Danny (1997) "Empirics for Growth and Distribution: Stratification, Polarization, and Convergence Clubs" Journal-of-Economic-Growth; 2(1), 27-59.

Uzawa, H. (1965) "Optimal Technical Change in an Aggregative Model of Economic Growth", International Economic Review, 6.

IX. Appendix

Proposition 1: The return on human capital is higher than on physical capital along the balanced growth path: $R_H > R$.

Proof. (by contradiction)

The return on human capital may be equal or larger than the return on physical capital. Assume that the return on physical and human capital is the same (R=R_H). Note that:

$$\begin{aligned} R &= R_H \Leftrightarrow H = (1-\alpha)Z \Rightarrow && (47)\\ R_H &= R = (1-\delta) + A\,(1-\alpha)^{1-\alpha}\,\alpha^{\alpha} && (48) \end{aligned}$$

where $Z \equiv H + K$. The consumer's budget constraint in this case may be rewritten as follows:

$$\begin{aligned} Z_{t+1}^{i} &= RZ_t^i - c_t^i \Rightarrow \\ RZ_t^i &= \sum_{n=0}^{\infty} \frac{c_{t+n}^i}{R^n} \end{aligned}$$

where $Z^i \equiv H^i + K^i$. It follows from the Euler Equations (16) and (17) that consumption grows factor is equal to $(\beta R)^{\sigma}$. Using this in the above budget constraint it follows that:

$$\begin{aligned} RZ_t^i &= \sum_{n=0}^{\infty} \frac{\left[(\beta R)^{\sigma}\right]^n c_t^i}{R^n} = \frac{c_t^i}{1-\beta^{\sigma} R^{\sigma-1}} \Rightarrow \\ c_t^i &= \left[R - \beta^{\sigma} R^{\sigma}\right] Z_t^i \end{aligned}$$

It follows from the above equation and the budget constraint that:

$$Z_{t+1}^i = (\beta R)^{\sigma}\, Z_t^i \tag{49}$$

Let's i denotes the poorer dynasty and j the richer one (they may also have the same amount of wealth). Let's denote ϕ the portion of total wealth owned by dynasty i: $\phi \equiv \frac{Z_t^i}{Z_t^i + Z_t^j} \in \left(0, \frac{1}{2}\right]$. It follows from (49) that wealth distribution do not change over time, that is, ϕ is constant. Let's t a period in which the middle age member of dynasty i start working. It follows from the budget constraint, the definition of ϕ and (49) that:

$$\begin{aligned} &H_t \equiv H_t^i + H_t^j \leq Z_t^i + (1-\delta)H_{t-1}^j \leq \\ &\phi Z_t + (1-\delta)(1-\phi)Z_{t-1} \leq \frac{1}{2}\left[Z_t + \frac{(1-\delta)}{(\beta R)^{\sigma}} Z_t\right] \Rightarrow \\ &\frac{H_t}{Z_t} \leq \frac{1}{2}\left[1 + \frac{(1-\delta)}{\left[\beta\left(1-\delta + A\,(1-\alpha)^{1-\alpha}\,\alpha^{\alpha}\right)\right]^{\sigma}}\right] && (50) \end{aligned}$$

It follows from (47) and (50) that if the return of human capital and physical capital are the same the following condition should be satisfied:

$$\frac{1}{2}\left[1 + \frac{(1-\delta)}{\left[\beta\left(1-\delta + A\,(1-\alpha)^{1-\alpha}\,\alpha^{\alpha}\right)\right]^{\sigma}}\right] \geq (1-\alpha) \tag{51}$$

It follows from assumption 1 that (51) does not hold. $\Rightarrow\Leftarrow$ $\blacksquare$

Corollary 2: The ratio of physical to human capital along the balanced growth path is greater than the one when markets are complete (when negative bequest are possible).

Proof. If markets are complete the ratio of physical to human capital is optimal when $R_H = R \Leftrightarrow k = \frac{\alpha}{1-\alpha}$. Along the balanced growth equilibrium $R_H > R \Leftrightarrow k > \frac{\alpha}{1-\alpha}$. ■

Proposition 3: There is $\overline{\sigma} > 1$ such that if $\sigma < \overline{\sigma}$ then the balanced growth path exist.

It follows from the definition of balanced growth path and proposition 1 that along the balanced growth path the following conditions should be satisfied:

$$\begin{aligned}
\frac{K_{t+1}}{K_t} &= \frac{H_{t+1}}{H_t} = G && (52)\\
\frac{H_{t+2}^{i(t)}}{H_t^{i(t)}} &= \frac{H_{t+3}^{j(t)}}{H_{t+1}^{j(t)}} = G^2 && (53)\\
R &= (1\text{-}\delta) + \alpha A\left(\frac{K_t}{H_t}\right)^{-(1-\alpha)} && (54)\\
R_H &= (1\text{-}\delta) + (1-\alpha)A\left(\frac{K_t}{H_t}\right) && (55)\\
H_t &= H_t^{i(t)} + (1-\delta)\, H_{t-1}^{j(t)} && (56)\\
R_H(1\text{-}\delta)H_t^{i(t)}+RK_{t+1}\text{-}H_{t+2}^{i(t)} &= (\beta R)^{\sigma}\left(\left(R_H\text{-}\,(1\text{-}\delta)\right)H_t^{i(t)}\text{-}K_{t+1}\right) && (57)\\
\left(R_H\text{-}\,(1\text{-}\delta)\right)H_t^{j(t)}\text{-}K_{t+1} &= (\beta R_H)^{\sigma}\left(R_H(1\text{-}\delta)H_{t-1}^{j(t)}+RK_t\text{-}H_{t+1}^{j(t)}\right) && (58)
\end{aligned}$$

where G denotes growth factor and $i(t)$, $j(t)$ are two functions $\aleph \rightarrow \{odd, even\}$ defined as follows:

$$i(t) = \begin{cases} \text{odd if } t \in \aleph^{odd} \\ \text{even if } t \in \aleph^{even} \end{cases} ; \qquad j(t) = \begin{cases} \text{odd if } t \in \aleph^{even} \\ \text{even if } t \in \aleph^{odd} \end{cases}$$

Equations (52), (53) come directly from point 2 and 3 of the definition of Balanced growth path. Equations (54), (55) means that factors get paid its marginal product (point 2 in the definition of equilibrium). Equation (56) is the market clearing condition in point 3 of the definition of equilibrium, where it has been introduced the restriction that agents only can invest in the second period of their working life (12). Equations (57) and (58) come from Euler Equations Equation (16) and (17) together with the budget constraints.

It follows from (52), (53) and (56) that:

$$\frac{H_{t+1}^{j(t)}}{H_t^{i(t)}} = G \tag{59}$$

It follows from (56) and (59) that:

$$\frac{H_t^{i(t)}}{H_t} = \frac{H_{t+1}^{j(t)}}{H_{t+1}} = \frac{G}{G+(1-\delta)} \tag{60}$$

It follows from (52), (57), (59) and (60) that:

$$\left(R_H \frac{(1\text{-}\delta)}{G+(1\text{-}\delta)} + Rk - \frac{G^2}{G+(1\text{-}\delta)}\right) = (\beta R)^{\sigma}\left(\frac{R_H\text{-}\,(1\text{-}\delta)}{G+(1\text{-}\delta)} - k\right) \tag{61}$$

where $k_t \equiv \frac{K_t}{H_t}$, it follows from (52) that along the balanced growth path k_t is constant. It follows from (52), (57), (58) and (59) that:

$$G = \beta^{\sigma}\left(R_H R\right)^{\frac{\sigma}{2}} \tag{62}$$

It follows from (54), (55), (61) and (62) that along the balanced growth path:

$$F(k) = \frac{R_H(k)\,(1\text{-}\delta)\,\text{-}G(k)^2}{1-\delta+G(k)} + R(k)k - (\beta R(k))^{\sigma}\left(\frac{R_H(k)\text{-}\,(1\text{-}\delta)}{1\text{-}\delta\text{+}G(k)}\text{-}k\right) = 0$$

where $R_H(k) = (1-\delta) + (1-\alpha)\,Ak^{-(1-\alpha)}$; $R(k) = (1-\delta) + (1-\alpha)\,Ak^{-(1-\alpha)}$, $G(k) = \left(\beta\sqrt{R_H(k)R(k)}\right)^{\sigma}$. It follows from assumption 1 that:

$$\frac{(1-\delta)}{G\left(\frac{\alpha}{1-\alpha}\right)} < 1-2\alpha$$

Taking account the above equation and the fact that $R_H\left(\frac{\alpha}{1-\alpha}\right) = R\left(\frac{\alpha}{1-\alpha}\right)$ it follows that:

$$\begin{aligned}
&F\left(\frac{\alpha}{1-\alpha}\right) = \\
&\frac{R\left(\frac{\alpha}{1-\alpha}\right)\frac{(1\text{-}\delta)}{G\left(\frac{\alpha}{1-\alpha}\right)}\text{-}G\left(\frac{\alpha}{1-\alpha}\right)}{\frac{(1\text{-}\delta)}{G\left(\frac{\alpha}{1-\alpha}\right)}+1} + R\left(\frac{\alpha}{1\text{-}\alpha}\right)\frac{\alpha}{1\text{-}\alpha} - G\left(\frac{\alpha}{1\text{-}\alpha}\right)\left(\frac{\frac{R\left(\frac{\alpha}{1\text{-}\alpha}\right)}{G\left(\frac{\alpha}{1\text{-}\alpha}\right)}\text{-}\frac{(1\text{-}\delta)}{G\left(\frac{\alpha}{1\text{-}\alpha}\right)}}{\frac{(1\text{-}\delta)}{G\left(\frac{\alpha}{1\text{-}\alpha}\right)}+1}\text{-}\left(\frac{\alpha}{1\text{-}\alpha}\right)\right) = \\
&\left(R\left(\frac{\alpha}{1\text{-}\alpha}\right)+G\left(\frac{\alpha}{1\text{-}\alpha}\right)\right)\left[\frac{\frac{(1\text{-}\delta)}{G\left(\frac{\alpha}{1\text{-}\alpha}\right)}}{\frac{(1\text{-}\delta)}{G\left(\frac{\alpha}{1\text{-}\alpha}\right)}+1} - \frac{1}{\frac{(1\text{-}\delta)}{G\left(\frac{\alpha}{1\text{-}\alpha}\right)}+1} + \frac{\alpha}{1\text{-}\alpha}\right] < \\
&\left(R\left(\frac{\alpha}{1\text{-}\alpha}\right)+G\left(\frac{\alpha}{1\text{-}\alpha}\right)\right)\left[\frac{(1-2\alpha)}{2(1-\alpha)} - \frac{1}{2(1-\alpha)} + \frac{\alpha}{1-\alpha}\right] = 0
\end{aligned}$$

If $\sigma < \frac{1}{\alpha}$ then $\lim_{k\to\infty} F(k) = +\infty$. Thus there exist $k^* \in \left(\frac{\alpha}{1\text{-}\alpha}, +\infty\right)$ such that $F(k^*) = 0$. Let $\widetilde{\sigma}$ be defined such that:

$$\widetilde{\sigma} = \sup\left\{\sigma^* \geq \frac{1}{\alpha} \text{ s.th. } \forall\ \sigma < \sigma^*\ \exists k^* \in \left(\frac{\alpha}{1\text{-}\alpha}, +\infty\right) \text{ s.th. } F(k^*) = 0\right\}$$

If $\sigma \leq 1$ the balanced growth path exist. If $\sigma \in (1, \widetilde{\sigma})$ the balanced growth path exist if the following condition is satisfied:

$$\sqrt{R_H(k^*)R(k^*)} > \left(\beta\sqrt{R_H(k^*)R(k^*)}\right)^{\sigma} \Leftrightarrow R_H(k^*)R(k^*) < \left(\frac{1}{\beta}\right)^{\frac{2\sigma}{\sigma-1}}$$

Let $k^{BGP} : (1, \widetilde{\sigma}) \to \left(\frac{\alpha}{1-\alpha}, +\infty\right)$ be a correspondence such that $k^{BGP}(\sigma) = \{k \text{ s.th. } F(k) = 0\}$. Let's $\overline{\sigma}$ be defined as follows:

$$\overline{\sigma} = \sup\left\{\sigma^* \in (1, \widetilde{\sigma}) \text{ s.th. } \forall\ \sigma < \sigma^*\ \exists k^* \in k^{BGP}(\sigma) \text{ s.th. } R_H(k^*)R(k^*) < \left(\frac{1}{\beta}\right)^{\frac{2\sigma}{\sigma-1}}\right\} \blacksquare$$

Lemma 4 *The return factor on human capital R_H (defined in 20) is a strictly decreasing function of H in the interval $\left(\varepsilon, \overline{H}\right)$*

Proof.

If $H \in \left(\varepsilon, \overline{H}\right) \Rightarrow \frac{\partial R_H}{\partial H} = \frac{w}{H^2}\left(-\underline{L} - \Psi(H-\varepsilon) + \Psi'(H-\varepsilon)H\right)$

$\frac{\partial(-\underline{L} - \Psi(H-\varepsilon) + \Psi'(H-\varepsilon)H)}{\partial H} = \Psi"(H-\varepsilon)H > 0$

$\Rightarrow \underset{H \in \left(\varepsilon, \overline{H}\right]}{Max} \; -\underline{L} - \Psi(H-\varepsilon) + \Psi'(H-\varepsilon)H = -\underline{L} - \Psi(\overline{H}-\varepsilon) + \Psi'(\overline{H}-\varepsilon)\overline{H} = 0$

$\Rightarrow \forall H \in \left(\varepsilon, \overline{H}\right) \; \frac{\partial R_H}{\partial H} < 0.$ ■

Lemma 5 $\frac{\Psi(x)}{x}$ *is increasing in* $\left(\varepsilon, \; \overline{H}\right)$

Proof.

$\frac{\partial\left(\frac{\Psi(x)}{x}\right)}{\partial x} = \frac{1}{x^2}\left(\Psi'(x)x - \Psi(x)\right)$

$\frac{\partial(\Psi'(x)x - \Psi(x))}{\partial x} = \Psi"(x)x > 0 \Rightarrow \Psi'(x)x - \Psi(x) > \Psi'(0)0 - \Psi(0) = 0.$ ■

X. Appendix II

In this appendix the model presented in section II is modified in order to introduce investment in education in the two periods that youth endure. Consider that the same model than in the main text but now the human capital accumulation equation is different. The human capital of an agent which born at period t (and start working at period $t+2$) is as follows:

$$H_{t+2}^{i} = (1-\delta)H_{t+1}^{i} + B\min\left\{I_{t+1}^{H,i}, \frac{I_t^{H,i}}{\zeta}\right\}$$

where $i \in \{\text{odd, even}\}$, $\zeta \in [0,1]$, $B \in [1,+\infty)$ and $\zeta \leq \frac{B-1}{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}+(1\text{-}\delta)}$. This equation means that parents invest in the education of their children in the two periods in which their children are young. More precisely, they invest in education in the first period of child's life a proportion ζ of what invest in the second period:

$$I_t^{H,i} = \zeta I_{t+1}^{H,i}$$

The maximization problem of dynasty whose middle-aged agent starts working at period t is as follows:

$$\begin{aligned}
&Max \sum_{n\in\aleph^{even}} \beta^n \left\{u\left(w_{t+n}H_{t+n}+r_{t+n}K_{t+n}\text{-}\left(K_{t+n+1}\text{-}(1\text{-}\delta)K_{t+n}\right)\text{-}\frac{\zeta(H_{t+n+2}-(1-\delta)^2H_{t+n})}{B}\right)\right.\\
&\left.+\beta\ u\left(w_{t+n+1}(1\text{-}\delta)H_{t+n}+r_{t+n+1}K_{t+n+1}\text{-}\left(K_{t+n+2}\text{-}(1\text{-}\delta)K_{t+n+1}\right)\text{-}\frac{(H_{t+n+2}\text{-}(1\text{-}\delta)^2H_{t+n})}{B}\right)\right\}\\
&s.t. \qquad K_{t+n} \geq 0 \qquad n \in \aleph^{even}
\end{aligned} \tag{63}$$

There are two differences between above optimization problem (63) and the presented in section III in the main text (15): i) parents invest the amount $\frac{\zeta}{B}(H_{t+2}-(1-\delta)H_{t+1})$ in the first period of the education of their children, ii) the restriction $H_{t+1} = (1-\delta)H_t$ has been introduced directly in the optimization problem. The first order conditions of the consumer optimization problem are the following Euler equations:

$$\begin{aligned}
u'(c_t)\frac{\zeta}{B} + \frac{\beta}{B}u'(c_{t+1}) &= \beta^2 u'(c_{t+2})\left(\frac{\zeta(1\text{-}\delta)^2}{B}+w_{t+2}\right) + \beta^3 u'(c_{t+3})\left(\frac{(1\text{-}\delta)^2}{B}+w_{t+3}(1\text{-}\delta)\right)\\
u'(c_t) &= \beta u'(c_{t+1})R_{t+1}\\
u'(c_{t+2}) &= \beta u'(c_{t+3})R_{t+3}
\end{aligned}$$

Along the balanced growth path (in which r and w are constant) the above equation may be rewritten as follows:

$$c_{t+1}/c_t = (\beta R)^{\sigma} \tag{64}$$

$$c_{t+2}/c_{t+1} = (\beta R_H)^{\sigma} \tag{65}$$

where R and R_H denote the return factor on physical and human capital along the balanced growth path: $R \equiv (1-\delta+r)$, $R_H \equiv \frac{(1\text{-}\delta)^2\left(\zeta+\frac{1}{R}\right)+Bw\left(1+\frac{1-\delta}{R}\right)}{\zeta R+1}$. Since there is not liquidity restriction in the first period in which middle age agent works, along the balanced growth path the consumption path and the human capital path that generates optimization problem (15) are the same as the following optimization problem:

$$
\begin{aligned}
V(K_t, H_t) &= \underset{K_{t+1},,K_{t+2},H_{t+2}}{Max} \{u\left(R_H\ H_t + RK_t - K_{t+1}\right) + \\
&\beta\ u\left(RK_{t+1} - H_{t+2} - K_{t+2}\right) + \beta^2 V\left(K_{t+2}, H_{t+2}\right)\Big\} \qquad (66)\\
&s.t. \quad K_{t+2} \geq 0
\end{aligned}
$$

Lemma 6 *Let k^* be defined as the ratio physical to human capital such that $R_H = R$, $k^* \leq \frac{\alpha}{1-\alpha}$.*

Proof.

$$
\begin{aligned}
R_H &= R \Leftrightarrow R = \frac{(1\text{-}\delta)^2\left(\zeta + \frac{1}{R}\right) + Bw\left(1 + \frac{1-\delta}{R}\right)}{\zeta R + 1} \Leftrightarrow \\
\left(R^2 - (1-\delta)^2\right)(\zeta R + 1) &= Bw\left(R + 1 - \delta\right) \\
r\left(r + 2(1-\delta)\right)(\zeta R + 1) &= Bw\left(R + 1 - \delta\right) \Leftrightarrow \\
r\left(R + 1 - \delta\right)(\zeta R + 1) &= Bw\left(R + 1 - \delta\right) \Leftrightarrow \\
r\left(\zeta R + 1\right) - Bw &= 0 \Leftrightarrow \chi(k) = 0
\end{aligned}
$$

where

$$
\chi(k) = \alpha A\left(\frac{1}{k}\right)^{1-\alpha}\left(\zeta\left(\alpha A\left(\frac{1}{k}\right)^{1-\alpha} + (1\text{-}\delta)\right) + 1\right) - B(1\text{-}\alpha)Ak^{\alpha}
$$

Using the assumption that $\zeta \leq \frac{B-1}{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}+(1\text{-}\delta)}$:

$$
\begin{aligned}
\chi\left(\frac{\alpha}{1-\alpha}\right) &= A\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\left[\zeta\left(A\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha} + (1\text{-}\delta)\right) + 1 - B\right] = \\
A\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}&\left(A\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha} + (1\text{-}\delta)\right)\left[\zeta - \frac{B-1}{A\alpha^{\alpha}(1-\alpha)^{1-\alpha} + (1\text{-}\delta)}\right] \leq 0
\end{aligned}
$$

Since $\chi(.)$ is decreasing and $\underset{k\to 0}{lim}\ \chi(k) = \infty$ it follows from the definition of k^* that $k^* \leq \frac{\alpha}{1-\alpha}$. ■

Proposition 1': The return on human capital is higher than on physical capital along the balanced growth path: $R_H > R^*$.

Proof. (by contradiction)

Assume that the return on physical and human capital is the same (R=R_H). It follows from Lemma 5 that:

$$
R_H = R \Rightarrow k^* \leq \frac{\alpha}{1-\alpha} \Rightarrow H \geq (1-\alpha)Z \qquad (67)
$$

$$
k^* \leq \frac{\alpha}{1-\alpha} \Rightarrow R \geq (1-\delta) + A\left(1-\alpha\right)^{1-\alpha}\alpha^{\alpha} \qquad (68)
$$

where $Z \equiv H + K$. It follows from the Euler Equations (64) and (65) that consumption grows factor is equal to $(\beta R^*)^{\sigma}$. Using standard techniques (see Appendix I) it follows that Z grows at constant rate:

$$
Z^i_{t+1} = \left(\beta R^*\right)^{\sigma} Z^i_t \qquad (69)
$$

where $Z^i \equiv H^i + K^i$. Let's i denotes the poorer dynasty and j the richer one (they may also have the same amount of wealth). Let's denote ϕ the portion of total wealth owned by dynasty i: $\phi \equiv \frac{Z_t^i}{Z_t^i + Z_t^j} \in \left(0, \frac{1}{2}\right]$. It follows from (69) that wealth distribution do not change over time, that is, ϕ is constant. Let's t a period in which the middle age member of dynasty i starts working. It follows from the budget constraint and Lemma 5:

$$\begin{aligned}
&H_t \equiv H_t^i + H_t^j \leq Z_t^i + (1-\delta) H_{t-1}^j \leq \\
&\phi Z_t + (1-\delta)(1-\phi) Z_{t-1} \leq \frac{1}{2}\left[Z_t + \frac{(1-\delta)}{(\beta R^*)^\sigma} Z_t\right] \Rightarrow \\
&\frac{H_t}{Z_t} \leq \frac{1}{2}\left[1 + \frac{(1-\delta)}{\left[\beta\left(1-\delta+A(1-\alpha)^{1-\alpha}\alpha^\alpha\right)\right]^\sigma}\right]
\end{aligned} \tag{70}$$

It follows from (67) and (70) that if the return of human capital and physical capital are the same the following condition should be satisfied:

$$\frac{1}{2}\left[1 + \frac{(1-\delta)}{\left[\beta\left(1-\delta+A(1-\alpha)^{1-\alpha}\alpha^\alpha\right)\right]^\sigma}\right] \geq (1-\alpha) \tag{71}$$

It follows from assumption 1 (in the main text) that (71) does not hold. $\Rightarrow\Leftarrow$ $\blacksquare$

the results of the model are consistent with the empirical evidence: There are two types of countries poor and rich, poor countries has a higher marginal return on human capital than rich ones. The only case in which the marginal return on human capital behavior in the model is contra-factual is when $\widehat{H} < \overline{H}$ and countries have medium wealth ($Z_{t+2} \in \left(\widehat{H}, \overline{H}\right)$). In such case, medium wealth countries has a higher average return on human capital than rich countries, the marginal return on human capital is larger than the interest rate but smaller than the return on human capital in rich countries.

It follows from the assumption that $\Psi"(.) > 0$ that when human capital is in between ε and $\overline{H}$ the marginal return on human capital is smaller than w. Then, the marginal return on human capital in those points is low in comparison with the balanced growth path level. However those points or part of them are not "observable" since no one invest at these levels.

Figure 1:
Labor efficiency units and human capital

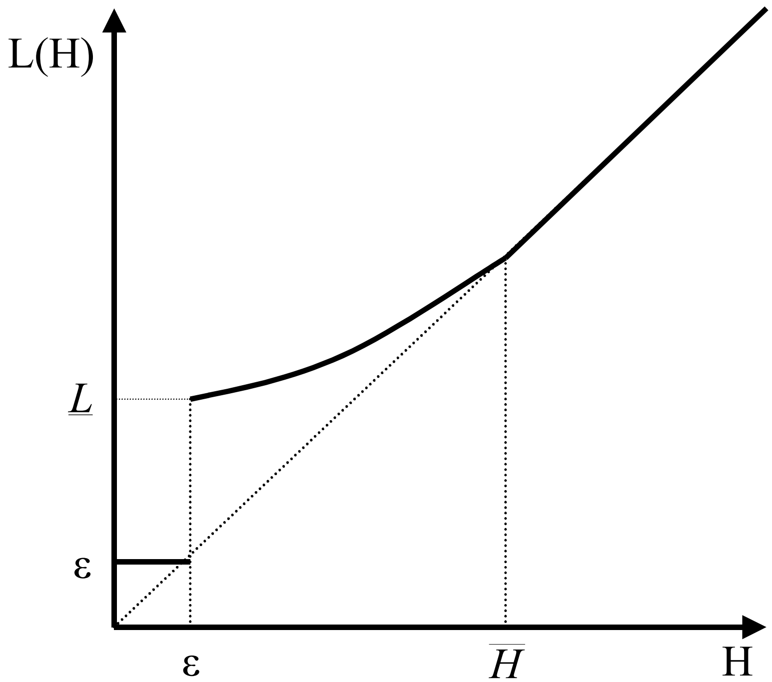
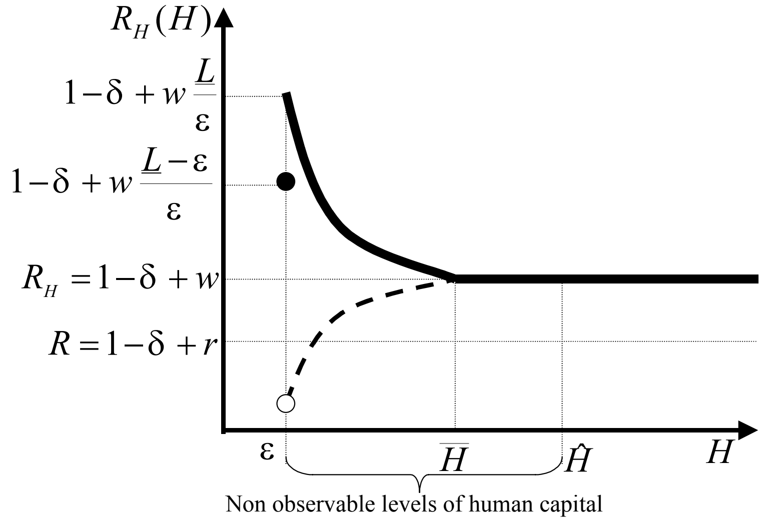


Figure 2: The Return on Human Capital

a) Average and marginal return on human capital



b) Average Increment on the return on human capital

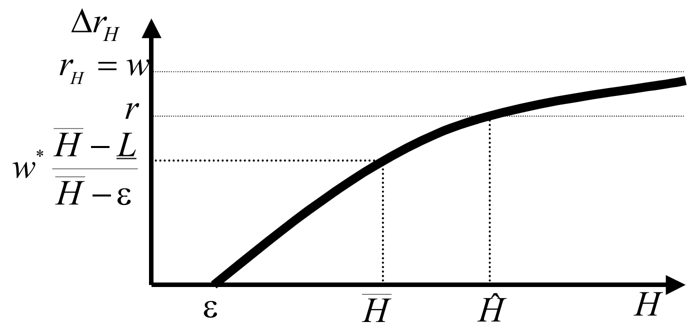
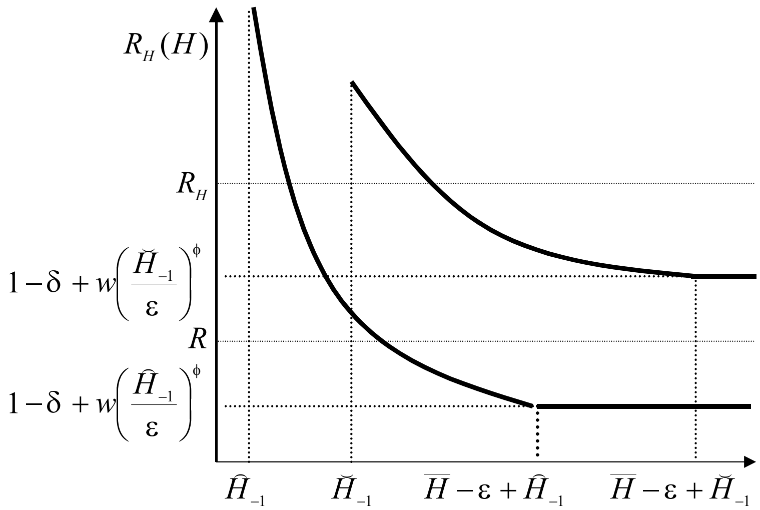


Figure 3

a) Return on human capital for two past human capitals



b) Average Increment on the return on human capital