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Government Expenditure in Enforcing the Law, Financial Intermediation and Development

Fernando Perera Tallo

CAERP

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Fernando Perera Tallo
C.A.E.R.P. and University of La Laguna

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Abstract

This paper presents a neoclassical growth model with financial intermediation in which government expenditure is used to enforce the law. Government expenditure increases the probability that the financial contract are enforced and reduces financial intermediation costs. There is a feed back process: low per capita capital involves low government expenditure and low probability of enforcing financial contracts, which reduces the incentives to accumulate capital. As a consequence of this feed back process there are three steady states: one without financial intermediation and low per capita capital, another with low probability of enforcing the financial contract and medium per capita capital and other with high probability of enforcing the contract and high per capita capital. The dynamic around the steady state with low enforcing probability is characterized by multiple equilibria and cyclical behavior, the dynamics around the two others steady states presents the typical saddle point dynamics.

"The second duty of the sovereign, that of protecting, as far as possible, every member of the society from the injustice or oppression of every other member of it, or the duty of establishing and exact administration of justice requires two very different degrees of expense in the different periods of society." Adam Smith, *The Wealth of Nations*, book 5, chapter I, part I.

1 Introduction

It is clear since Adam Smith that one of the function of the State is to enforce the law and that part of the government expenditure should be used to this end. This paper presents a model in which the state enforces the law and thus the financial contracts. More precisely, government expenditure increases the probability that financial contract are enforced. Poor societies have high financial intermediation cost because government expenditure is low and thus the probability that financial contract are enforced are also low. This fact implies that the return of savings is not decreasing in the per capita capital, in spite of the fact that the technology of the model is neoclassical and presents decreasing returns with respect to the capital. As a matter of fact the relationship between the return on savings and the per capita capital is governed by two forces that goes in opposite directions: i) financial intermediation costs are decreasing with the per capita capital because to enforce financial contract is easier in rich societies that in poor ones; ii) on the other hand the return on investment is higher in the poor societies for the usual neoclassical reasons: decreasing returns in the capital. This two opposite forces imply that the return on savings does not decreases monotonically with the per capita capital and this has important consequences for the dynamics and long run behavior of the model. Poor countries do not necessary grow faster than rich ones as in the neoclassical growth model and more than one steady state equilibrium may exist

Agents may chose between two alternative technologies in the model: a traditional technology and a modern one. Both of these technologies are neoclassical and essentially the same except for two differences: i) the productivity of modern technology is higher than the productivity of traditional one, ii) modern technology requires financial intermediation whether traditional technology does not. When the per capita capital is low, the probability of enforcing financial contract is so low that financial intermediation becomes prohibitive. In this case, agents need to use traditional technology. There is a threshold per capita level such that if the per capita capital is higher than this threshold level the probability of enforcing financial contract is large enough to make financial intermediation possible. When this happens, financial intermediation and modern technology are used in equilibrium.

The long run behavior of the model is quite different form the neoclassical growth model. There are three steady states: i) The first one has the lowest per capita capital, and neither financial intermediation nor modern technology are used; ii) In the second steady state financial intermediation and modern technology are used, however the probability of enforcing contract are low and thus financial intermediation costs are high; iii) The third steady state is the one with the highest per capita capital, and it is characterized by high probability of enforcing the financial contract, this makes the financial intermediation costs low.

The dynamic behavior of the model is also very different to the neoclassical growth model. The dynamics around the steady state without financial intermediation and the steady state with high probability of enforcing the financial contracts are as in the neoclassical growth model: the typical saddle point dynamics. However the dynamic around the steady state with low probability of enforcing the financial contract is very different: it exhibits cyclical behavior and multiple equilibria.

There are many papers that relate growth and financial intermediation (See among others, Acemoglu and Zilibotti, 1996; Bencivenga and Smith, 1991; Cooley and Smith, 1992; Greenwood-Jovanovic, 1990; Saint-Paul, 1992). Those earlier models do not deal with the role of the government as an enforcer of the law and the financial contracts. The dynamic behavior of those models is very different to the present model

The structure of the paper is as follows: the second section presents the model. The third section characterizes the agents' behavior. The fourth section dills with the existence of steady state and the number of them. The fifth section analyzes the dynamic behavior of the economy. Finally, a conclusion is presented. All the proofs and some technical details are included in the Appendix.

2 The Model

Time is discrete with infinite horizon. There is a single good in the economy that can be used for consumption and investment:

$$y_t = c_t + k_{t+1} - (1-\delta)k_t \tag{1}$$

where y_t denotes per capita production, c_t denotes per capita consumption, k_t denotes per capita capital and $\delta \in (0,1)$ denotes the depreciation rate.

2.1 Consumers

There is a continuum of consumers indexed in $S \equiv \{\aleph\, x\ [0,1]\}$ where $\aleph$ is the set of natural Number: $\aleph \equiv \{1,2,3,4,...\}$. Over this set is defined the measure μ as follow: $\mu(A) = \sum_{i=1}^{\infty} \ell(A \cap \{i \text{ x } [0,1]\})$ where ℓ is the Lebesgue measure. There is a partition of the consumer set S, each of the sets of this partition has measure zero and infinite elements (consumers). Each of the sets of the partition is interpreted as a clan.

Consumers' life is infinite and population is constant. The utility of consumers depends upon the life-time per capita consumption of the clan:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t u(c_t) \tag{2}$$

where $\rho \in \Re_+$, c_t denotes the clan's per-capita consumption at period t and $u(.)$ is the Isoelastic felicity function:

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} & if \quad \sigma \in (0,1) \cup (1,+\infty) \\ ln\,(c) & if \quad \sigma = 1 \end{cases} \tag{3}$$

It is assumed that each consumer has one unit of labor and clans have k_t units of per capita capital, where k_t denotes the amount of per capita capital in the economy.

2.2 Technology

There is two types of technologies: modern and traditional one. Modern technology is more productive than traditional one but it requires financial intermediation due to the fact that the optimal size of the firm is too large to be used directly by clans.

2.2.1 Modern Technology

It is assumed that consumers can neither invest nor produce directly with modern technology. Firms maximize profits and behave competitively. The average production of the firm f y^f is defined as the amount of production that an agent would get if the production of the firm were equally divided between all the agent in the interval $[0,1]$. In similar way is defined the average amount of capital used by firm f "k^f". Modern technology uses two types of labor: workers and managers. The average production of the firm "y^f" is given by the following production function:

$$y^f = \begin{cases} \left(\theta^f\right)^{1-\alpha} A\left((1+\phi)L\right)^\alpha & \text{if } k^f \geq 1 \text{ and } M \geq \frac{\phi}{\theta^f} L \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where $A \in \Re_{++}$, $\alpha \in (0,1)$, and $\phi \in (0, \frac{\alpha}{1-\alpha})$ are constants, θ^f denotes the firm specific stochastic shock, L and M denote respectively the measure or workers and managers hired by the firm:

$$L = \mu\left(\Omega^{L,f}\right) \qquad M = \mu\left(\Omega^{M,f}\right)$$

where $\Omega^{L,f} \subset S$ denotes the set of workers hired by the firm f, $\Omega^{M,f} \subset S$ denotes the set of managers hired by the firm f. .

Firms are very large in comparison with clans: even if all the members of the clans would try to put together all their capital and labor resources to create a firm they would not be able to do it (clans have zero measure).

It follows from the firm production function (4) that when modern technology is used the aggregate production function is a Cobb-Douglas:

$$y = Ak^\alpha \tag{5}$$

where y is the per capita income.

It is assumed that firms specific stochastic shock θ^f is independently distributed across firms. θ^f is distributed according with the Constant Hazard Rate Distribution Function:

$$G(\theta^f) = 1 - e^{-\theta^f} \quad \theta^f \in \Re_+ \tag{6}$$

It is assumed that the stochastic shock θ_t^f is realized after the investment takes place and after the managers are hired (see figure 1). Note that $E(\theta) = 1$.

2.2.2 Traditional Technology

There is also a traditional technology that may be used directly by the consumers without financial intermediaries. The production function of this technology is as follows:

$$\varepsilon K^{\alpha} L^{1-\alpha}$$

where $\varepsilon < \frac{\beta}{\alpha} A$, $\beta \equiv \alpha - (1-\alpha)\phi$. Note that this technology presents constant returns to scale an thus financial intermediaries do not play any roll: consumers may invest their resources and use their labor directly with this technology. The assumption that $\varepsilon < \frac{\beta}{\alpha} A$ implies that modern technology would be always used in equilibrium if the financial intermediation cost were zero.

2.3 Cost of State Verification

There is asymmetric information between firms and lenders. In the first period neither the lenders nor the firms know the value of the stochastic shock. In the second period, the firm observes the stochastic shock free of charge. The lender can observe the firm's cash flow only at some cost, which is refereed to as cost of state verification. The costs of state verification are proportional to the expected cash flow of the firm. The cost of state verification per unit of expected cash flow is denoted by Ψ and satisfy that $\Psi < 1 - \frac{\alpha\varepsilon}{\beta A}$. Financial intermediation and modern technology are used in equilibrium for some level of per capita capital if the assumption that $\Psi < 1 - \frac{\alpha\varepsilon}{\beta A}$ is satisfied. This assumption also involve that if the financial contract were always enforced, modern technology would be used always. However it will shown in next subsection that the probability that financial contract are enforced depends upon the government expenditure on enforcing the law.

2.4 Government

In case that firms default their contracts the government should enforce it. The probability that the government enforces a contract increases with the government expenditure. The probability that the government enforces a contract is denoted by $\chi(g)$, where $\chi(.)$ is an continuous differentiable strictly increasing function and g is the per capita government expenditure in enforcing the law. It is assumed that $\chi(0) = 0$, $\chi(g) < 1$, $\lim_{g\to\infty} \chi(g) = 1$ and that the elasticity of χ with respect to g is a decreasing function, that is, $\frac{\chi'(g)g}{\chi(g)}$ decreases with g. Two examples of the function $\chi(g)$ would be $e^{-\frac{1}{g}}$ and $\frac{g}{1+g}$.

It is assumed that there is not public debt and that the government expenditure is financed with a lump sum tax proportional to the per capita income:

$$g = \tau y \tag{7}$$

where $\tau \in (0,\ 1\text{-}\beta)$ is the tax rate. Since the taxes are proportional to the per capita income (not proportional to personal income), this type of tax is not distortionary, this is the reason why it has been chosen (taxes over labor income would be another example of non distortionary tax).

It is assumed that lenders always can get without any cost.the non depreciated part of the capital $(1 - \delta)$.

2.5 Financial Intermediation

As in Diamond (1984) and Williamson (1986), the existence of financial intermediaries may be justified as agents that reduce the cost of state verification by avoiding multiple monitoring.

Financial intermediaries borrow capital from lenders and then lend this capital to firms. They pay a risk-free interest rate r to lenders. Financial intermediaries are assumed competitive.

3 Agents' Decisions

3.1 Firm's Decisions:

The sequence of decisions that a firm makes is as follows (see figure 1): the firm invests one period before that the production takes place, then at the beginning of the period in which the production takes place the firm hires the managers, after that the stochastic shock is realized and it is observed by the firm, after that firm hires the workers, production takes place and the firm pays to the workers. Then the firm announces the realization of the stochastic shock, the financial intermediary chooses whether to observe (at some cost) or not the realization of the stochastic shock, after that he firm chooses wether to fulfill the contract or not. If the firm fulfill the contract, the firm pays to the financial intermediary and after that it pays the remaining cash flow to the managers. If the firm defaults the contract, the government enforce the contract with probability $\chi(g)$ in which case all the remaining cash flow of the firm goes to the financial intermediaries and managers do not receive anything. If the firm defaults but government does not enforce the contract, then the remaining cash flow goes to the managers and financial intermediaries receives nothing.

3.2 Workers and Managers

When modern technology is used two types of labor are used (see 4): the workers and the managers. It is assumed without loss of generality that the consumers' labor unit is indivisible: consumers can works either as a manager or as a worker and they only can work in a single firm.

There are three differences between mangers and workers: the timing in which they are hired, the information that they have and their wage. First of all manager are hired before the realization of the stochastic shock. Thus the quantity of mangers hired cannot depend upon the realization of the stochastic shock, wether the quantity of workers depends on it. Second managers know the realization of the stochastic shock after the payment to the financial intermediation takes place (see figure 1), wether workers do not know it. This implies that the payment to the workers cannot depend upon the realization of the stochastic shock, wether payment to the managers depends upon the realization of the stochastic shock. As a matter of fact, since there is free entry the zero profit condition of the firm implies that the payment to the manager is equal to the remaining of the production of the firm after the payment to the workers and to the financial intermediaries.

Summarizing the quantity of workers depends upon the realization of the stochastic shock whether the quantity of managers do not. The payment to the managers depends upon the realization of the stochastic shock whether the payment to the workers do not

If consumers of a clan decide to become manager they can perfectly diversify the risk (remember that there are an infinite number of consumers in each clan). Thus, the expected payment to managers should be equal to the payment to workers:

$$\int_0^{\infty} w^M(\theta)dG(\theta) = w \tag{8}$$

where $w^M(\theta)$ is the payment to the managers depending on the realization of the stochastic shock, and w is the wage (the payment to the workers).

3.3 Hiring Decision:

After the stochastic shock is realized the firm (the managers) decides the amount of workers to be hired to maximize his cash-flow:

$$\underset{L}{Max}\ \left(\theta^f\right)^{\alpha} A\left((1+\phi)L^f\right)^{1-\alpha} - wL^f \tag{9}$$

$$s.t:\ L \leq \theta^f \frac{M^f}{\phi}$$

The solution of the above problem is as follows:

$$L^f = \begin{cases} \theta^f \left(\frac{(1-\alpha)A(1+\phi)^{1-\alpha}}{w}\right)^{\frac{1}{\alpha}} & if \quad \left(\frac{(1-\alpha)A(1+\phi)^{1-\alpha}}{w}\right)^{\frac{1}{\alpha}} \leq \frac{M^f}{\phi} \\ \theta^f \frac{M^f}{\phi} & if \quad \left(\frac{(1-\alpha)A(1+\phi)^{1-\alpha}}{w}\right)^{\frac{1}{\alpha}} > \frac{M^f}{\phi} \end{cases} \tag{10}$$

It follows from (10) that the demand of managers by the firm is as follows:

$$M^f(w) = \phi\left(\frac{(1-\alpha)A(1+\phi)^{1-\alpha}}{w}\right)^{\frac{1}{\alpha}} \tag{11}$$

Note that the demand of managers do not depend on the stochastic shock. The reason of this is that managers are hired before the realization of the stochastic shock. It follows from (10) and (11) that the demand of labor by firms is as follows:

$$L^f(w) = \theta^f\left(\frac{(1-\alpha)A(1+\phi)^{1-\alpha}}{w}\right)^{\frac{1}{\alpha}} \tag{12}$$

It is easy to check that the firm optimal cash flow function is proportional to the realization of the stochastic shock:

$$\underset{L^f}{Max}\ \left(\theta^f\right)^{\alpha} A\left((1+\phi)L^f\right)^{1-\alpha} - wL^f\ =\ \theta\pi(w) \tag{13}$$

$$\pi(w)\ =\ \alpha A^{\frac{1}{\alpha}}\left(\frac{(1-\alpha)(1+\phi)}{w}\right)^{\frac{1-\alpha}{\alpha}} \tag{14}$$

where $\pi(w)$ denotes the expected cash-flow of the firm.

3.4 Optimal Incentive Compatible Contract

Following Gale and Hellwig (1985) the optimal incentive compatible contract between a firm and a financial intermediary is the debt contract:

$$
\begin{aligned}
&\underset{\theta_0, r^B}{Max} \int_{\theta_0}^{\infty} [\theta\pi(w) - (\delta + r^B)] dG(\theta) + (1-\chi(g)) \int_{\theta_0}^{\infty} \theta\pi(w) dG(\theta) \\
&s.t. \;\; \theta_0\pi(w) - (\delta + r^B) \; = \; (1-\chi(g))\theta_0\pi(w) \qquad (15) \\
&\int_{\theta_0}^{\infty} (1{+}r^B) dG(\theta) + \int_{0}^{\theta_0} [(1\text{-}\delta){+}\chi(g)\theta\pi(w)\text{-}\Psi\pi(w)] dG(\theta) \; = \; (1{+}r)
\end{aligned}
$$

where r^B denotes the borrower interest rate, r denotes the lenders interest rate and θ_0 denotes bankruptcy point.

The optimal incentive contract is such that the payment to the financial intermediary is fixed as long as the realization of the stochastic shock is large enough to incentive the firm to fulfill the contract. More precisely, the firm fulfills its contract and pays the borrowers interest rate r^B if the realization of its stochastic shock is larger than the bankruptcy point θ_0. In this case the earnings of the firm are equal to $[\theta\pi(w) - (\delta + r^B)]$.

If the realization of its stochastic shock is smaller than the bankruptcy point θ_0, the firm defaults the financial contract. The government enforces the financial contract with probability $\chi(g)$, in which case the financial intermediary receives the cash flow of the firm $\theta\pi(w)$ plus the non depreciated part of the capital $(1-\delta)$ and the firm does not receive anything. If the government does not enforces the contract the firm receives the cash flow $\theta\pi(w)$ and the financial intermediary receives just the non depreciated part of the capital $(1-\delta)$. Thus the expected earnings of the firm in case that the realization of its stochastic shock is smaller than the bankruptcy point θ_0 is equal to $(1-\chi(g))\theta\pi(w)$.

The first restriction in the optimal incentive compatible contract (15) is the definition of bankruptcy point. The bankruptcy point is the realization of the stochastic shock such that the firm is indifferent between fulfill the contract or not. In the first case the firm receives $\theta_0\pi(w) - (\delta + r^B))$, in the second one the expected earnings of the firm are equal to $(1-\chi(g))\theta_0\pi(w)$.

The second restriction in the optimal incentive compatible contract (15) is the financial intermediary zero profit condition. If the realization of the stochastic shock of the firm is larger than the bankruptcy point θ_0 the payment to the financial intermediary is equal to $(1 + r^B)$. If the realization of the stochastic shock of the firm is smaller than the bankruptcy point θ_0 the expected payment to the financial intermediary is equal to (1-δ)+$\chi(g)\theta\pi(w)$. In the case that the firm goes to bankrupt the financial intermediary should incur in the cost of state verification which are equal to $\Psi\pi(w)$. Thus the expected earning of the financial intermediary from a firm are equal to $\int_{\theta_0}^{\infty}(1+r^B)dG(\theta)$+$\int_0^{\theta_0}[(1-\delta)$+$\chi(g)\theta\pi(w)$-$\Psi\pi(w)]dG(\theta)$. Since financial intermediaries are competitive, their expected payment should be equal to the payment to the lenders, which is equal to $(1+r)$.

3.5 Clans

Clans face the following optimization problem:

$$\begin{array}{ll} \underset{\{c_t\}_{t=0}^{\infty}}{Max} & \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t u(c_t) \\ s.t. & (1+r_t)a_t + w_t - \tau y_t = a_{t+1} + c_t \\ & \underset{t\to\infty}{lim} \frac{a_t}{[\prod_{j=0}^{t}(1+r_j)]} \geq 0 \end{array} \tag{16}$$

where a_t denotes the per capita assets of the clan. Clans maximize their utility subject to their budget constraint together with the non-Ponzi Game condition. Since the per capita assets of clans are identical, it follows that in equilibrium $a_t = k_t$.

The Euler Equation the consumers optimization problem (16) is as follows:

$$c_{t+1} = \left(\frac{1+r_{t+1}}{1+\rho}\right)^{\frac{1}{\sigma}} c_t \tag{17}$$

Using the Euler Equation, the Non-Ponzi Game condition, the budget constraint and the transversality condition it follows that

$$\underset{t\to\infty}{lim} \frac{a_t}{[\prod_{j=0}^{t}(1+r_j)]} = 0 \tag{18}$$

4 Long Run Behavior

4.1 Interest rates

It is shown in the Appendix that when modern technology is used, the equilibrium interest rate is as follows:

$$r(k) \equiv \frac{\chi(\tau A k^{\alpha}) - \Psi}{\chi(\tau A k^{\alpha})} \beta A k^{\alpha-1} - \delta = r \tag{19}$$

The equilibrium interest rate has two parts: the marginal product of the capital $\beta A k^{\alpha-1}$ and the fraction of the marginal product of capital that goes to the savers $\frac{\chi(\tau A k^{\alpha})-\Psi}{\chi(\tau A k^{\alpha})}$. The marginal product of capital behaves as in any other neoclassical growth model: it decreases with the per capita capital. Due to imperfections in the financial markets, not all the marginal product of capital goes to the savers, the fraction of the marginal product that goes to savers depend upon two things: the cost of state verification Ψ and the probability that government enforces the contracts χ. The cost of state verification Ψ increases the cost of the financial intermediation and thus it reduces the portion of the marginal product of capital that goes to savers. The higher the probability that government enforces the contracts χ, the higher the incentives of firm to default the financial contract and the lower the financial costs.

The following lemma says that $r(k)$ has inverted U-shape.

Lemma 1 *There exists k^* such that $r(.)$ is strictly increasing in $(0, k^*)$ and strictly decreasing in (k^*, ∞).*

The reason why the interest rate has inverted U-shape may be found in equation (19). There are two forces that goes in opposite directions: the marginal product of capital "$\beta A k^{\alpha-1}$" decreases with per capita capital but the portion of the marginal product of capital that goes to consumers "$\frac{\chi(\tau A k^{\alpha})-\Psi}{\chi(\tau A k^{\alpha})}$" increases with per capita capital. The marginal product of capital decreases with per capita capital for the usual neoclassical reason: decreasing returns. The portion of the marginal product of capital that goes to consumers "$\frac{\chi(\tau A k^{\alpha})-\Psi}{\chi(\tau A k^{\alpha})}$" increases with the expenditure of government, and this expenditure increases with the per capita capital. Thus the portion of the marginal product that goes to saver increases with per capita capital.

Figure 2.a shows the portion of the marginal product that goes to saver when modern technology is used. As it was explained above such portion increases with per capita capital. Figure 2.b shows the marginal product of capital, which decrease with per capita capital. Finally Figure 2.c shows the interest rate: this interest rate has inverted U-shape and is the result of the two opposite forces in figure 2.a and 2.c.

4.2 Financial Development

The broken line in figure 2.b represents the marginal product of capital of traditional technology. Figure 2.b shows the marginal product of capital of traditional technology is always lower than the marginal product of capital of modern technology. The broken line in figure 2.b also represents the marginal product of capital of traditional technology. The marginal product of capital of traditional technology is not always below the interest rate that savers would receive if modern technology is used. As a matter of fact there is threshold level $\overline{k}$ such that if the per capita capital is smaller than $\overline{k}$ the marginal product of capital of traditional technology is smaller than the lenders interest rate that savers would get if modern technology is used, if the per capita capital is larger than $\overline{k}$ the opposite happens. Using equation (19) it is possible to define this threshold per capita capital level $\overline{k}$:

Definition 1 $\overline{k} \Leftrightarrow \frac{\chi(\tau A\overline{k}^{\alpha})-\Psi}{\chi(\tau A\overline{k}^{\alpha})}\beta A = \alpha\varepsilon$.

If the amount of per capita capital is smaller than the threshold level $\overline{k}$ then neither modern technology nor financial intermediation are used in equilibrium, if the per capita capital is larger than $\overline{k}$ then financial intermediation appears in equilibrium.

4.3 Steady State Equilibria

It follows from the Euler Equation (17) that the interest rate in steady state should be equal to the discounting rate of the utility function:

$$r = \rho \tag{20}$$

The following proposition determines the number of steady state and wether there is financial intermediation or not in those steady states. The number of steady state will be importat to analyze the dynamic behavior of the model.

Proposition 2 *There exist* $\underline{\Psi}$, $\overline{\Psi}$ *such that: i) if* $\Psi > \overline{\Psi}$ *then there is a unique steady state in which there is not financial intermediation, ii) if* $\Psi \in \left(\underline{\Psi}, \overline{\Psi}\right)$ *then there is a steady state without financial intermediation and two steady states with financial intermediation, finally if iii)* $\Psi < \underline{\Psi}$ *then there is a unique steady state and there is financial intermediation in it.*

Figure 3 represents the tree cases that the above proposition presents. It follows from equation (19) that the interest rate when the financial intermediation is used decreases with the cost of state verification Ψ. If the cost of state verification Ψ is very high, the interest rate when financial intermediation is used is very low and never arrive to the discount rate of the utility function ρ, this case is represented in figure 3.a. In this case there is only an steady state in which there is not financial intermediation (the level of per capita capital in this steady state is bellow the threshold level $\overline{k}$). If the cost of state verification Ψ is not so very high, the interest rate when financial intermediation is used is higher than the discount rate of the utility function ρ for some values of the per capita capital, this case is represented in figure 3.b. In this case there are three steady states: two in which there is financial intermediation and another in which there is not. Finally if the cost of state verification Ψ is very low, there is an unique steady state which has financial intermediation (the level of per capita capital in this steady state is above the threshold level $\overline{k}$). This case is represented in figure 3.c.

5 Dynamic Behavior

5.1 Dynamic System

It follows from the capital accumulation equation (1), the government budget constraint (7), the per capita production (5), the equilibrium interest rate (19), the Euler Equation (17) and the transversality condition (18) that the dynamic system that describes the equilibrium behavior when $k > \overline{k}$ is as follows

$$c_{t+1} = \left(\frac{1 + r\left(k_{t+1}\right)}{1+\rho}\right)^{\frac{1}{\sigma}} c_t \tag{21}$$

$$k_{t+1} = Ak_t^{\alpha}\left(1 - \tau - \frac{\beta\Psi}{\chi\left(\tau A k_t^{\alpha}\right)}\right) + (1\text{-}\delta)\, k_t - c_t \tag{22}$$

$$\lim_{t\to\infty} \frac{k_t}{\left[\prod_{j=0}^{t}(1+r(k_j))\right]} = 0 \tag{23}$$

Equation (24) is the typical Euler equation. Equation (24) is the capital accumulation equation which is very similar to the one of the neoclassical growth model. The difference with the Neoclassical model is that in the present model a fraction of the per capita income is used for government expenditure (the fraction τ) and another fraction is used for the costs of state verification (the fraction $\frac{\beta\Psi}{\chi}$). The above dynamic system may be rewritten as follows.

$$c_{t+1} = \left(\frac{1+r\left(\Gamma(k_t,c_t)\right)}{1+\rho}\right)^{\frac{1}{\sigma}} c_t \tag{24}$$

$$k_{t+1} = \Gamma(k_t,c_t) \tag{25}$$

$$\lim_{t\to\infty} \frac{k_t}{\left[\prod_{j=0}^{t}(1+r(k_j))\right]} = 0 \tag{26}$$

where $\Gamma(k_t,c_t) = Ak_t^{\alpha}\left(1-\tau-\frac{\beta\Psi}{\chi\left(\tau Ak_t^{\alpha}\right)}\right) + (1\text{-}\delta)\,k_t - c_t$.

The dynamic behavior of the model depends on the number of the steady states (see proposition 2). Obviously the case in which there is not steady state with financial intermediation is not very interesting, thus the case in which $\Psi > \overline{\Psi}$ will not be analyzed. This section will analyze first the case in which there are two steady states with financial intermediation, that is the case in which $\Psi \in \left(\underline{\Psi},\overline{\Psi}\right)$. After that the case in which $\Psi \leq \underline{\Psi}$ will be analyzed.

5.2 Three steady states (case in which $\Psi \in \left(\underline{\Psi},\overline{\Psi}\right)$)

Figure 4. a represents the interest rate as a function of the per capita capital. The point in which the interest rate is equal to the utility function discount rate ρ are the three steady states. The steady state with the lowest per capita level is to the left of the threshold capital $\overline{k}$. This means that neither financial intermediation nor modern technology are used in this steady state, this steady states will be called steady state without financial intermediation (NFI from now on). The other two steady state are to the right of the

threshold level $\overline{k}$, thus financial intermediation are used in both of these steady states. However among these two steady states there are one in which the per capita capital and the probability of enforcing the financial contract are lower than the other one. The first steady state will be called steady state with low enforcing probability (LEP) and the other one will be called steady state with high enforcing probability (HEP).

Figure 4.a shows that when the per capita is to the left to the NFI steady state, the interest rate is above the discounted rate of the utility function, then the consumption growth is positive (see Euler Equation 17). This positive growth rate of consumption is represented in figure 4.b by the vertical arrows in down-up direction. When the per capita capital in between the NFI and the LEP steady state, the interest rate is bellow the discounted rate of the utility function, then the consumption growth is negative. This negative growth rate of consumption is represented in figure 4.b by the vertical arrows in up-down direction. When the per capita capital is between the LEP and HEP steady states, the consumption growth rate is positive, and this is represented in figure 4.b by the vertical arrows in down-up direction. Finally if the per capita capital is above the HEP steady state, the consumption growth is negative, and this is represented in figure 4.b by the vertical arrows in up-down direction.

Figure 4 b also shows that when the consumption is bellow the zero growth rate of capital curve, the capital grows at positive rate (represented by a horizontal arrow in left-right direction). When the consumption is above the zero growth rate of capital curve, the capital grows at negative rate (represented by a horizontal arrow in right-left direction).

Figure 4.b shows that dynamic around both the NFI and the HEP are the typical saddle point dynamic. However the dynamic around the LEP steady state is an spiral, this involve a cyclical dynamic around the LEP steady state and the existence of multiple equilibria (there is more than a consumption level for each per capita capital consistent with the definition of equilibrium). Figure 5 shows that depending on the parameters values, the spiral around the LEP steady state may be stable or unstable.

5.3 One steady state (case in which $\Psi \leq \underline{\Psi}$)

Figure 6 shows the case in which the cost of state verification are low (the case in which $\Psi \leq \underline{\Psi}$)). When this happens there is a unique steady state in which both financial intermediation and modern technology are used (the steady state is who the right of the threshold level $\overline{k}$). Figure 6 shows that the dynamic around this unique steady state is the typical saddle point dynamic. However the consumption growth rate is not monatomically decreasing as in the neoclassical growth models. It follows from figure ... that the consumption growth rate decreases first, increases later and finally decreases again.

6 Conclusion

This paper has presented a model in which the main function of the state is to enforce the law, and more precisely to enforce financial contract. The probability that financial contract are enforce depends upon the government expenditure in enforcing the law. There are two types of technologies: a modern and a traditional one. Modern technology is always more productive than traditional one but it needs financial intermediaries to invest on it. If financial contract were always enforced, modern technology would be always used in equilibrium. However the probability that financial contract are enforced increases with per capita capital, due to the fact that government expenditure in enforcing the law increases with per capita capital. This involve that traditional technology is used when per capita capital is smaller than certain threshold level. Since traditional technology does not need to use financial technology, when the per capita capital is smaller than this threshold capital level, financial intermediation does not appear in equilibrium.

The dynamic behavior of the model is quite different to the Neoclassical growth model, in spite of the fact that the production technology is a typical neoclassical technology. The reason behind this different dynamic is that the interest rate is not a decreasing function of the per capita capital. The reason for that is that there are two opposite forces that drives the behavior of the interest rate: ii) the decreasing returns on the capital which makes the interest rate to decrease with the capital accumulation. ii) The government expenditure in enforcing the law increase with per capita capital and this makes the interest rate to increase with per capita capital. This two forces makes the interest rate to decrease first, until arriving to the point in which financial intermediation and modern technology start to be used, to increase later and finally to decrease again. This behavior of the interest rate involve a dynamic quite different to the typical neoclassical growth model. When the financial intermediation cost are not too low, there are three steady states: one without financial intermediation, another with financial intermediation but with a low probability that financial contract are enforced, and finally another with financial intermediation and with high probability that financial contract are enforced and high per capita capital. The dynamics around the steady state without financial intermediation and steady state with high enforcing probability are the typical monotonic paths of the saddle points. However the dynamics around the steady state with low enforcing probability presents cyclical behavior and multiple equilibria.

7 References

Acemoglu, D. and F. Zilibotti (1996) "Was Prometheus Unbound by Chance? Risk Diversification and Growth" CEPR Discussion Paper, No 1426.

Azariadis, C. (1993) *Intertemporal macroeconomics* Cambridge, Mass. and Oxford: Blackwell.

Bencivenga, V. R. and B. Smith (1991) "Financial Intermediation and Endogenous Growth" Review of Economic Studies 58, April.

Cooley, T.F. and B. Smith (1992) " Financial Markets, Specialization and Learning by Doing", mimeo.

Demirguc Kunt, A. and R. Levine (1995) "Stock Market Development and Financial Intermediaries", Policy Research Working Paper World Bank 1462.

Goldsmith, R. W. (1969), Financial Structure and Development, New Haven, C.T: Yale University Press.

Greenwood, G. and Jovanovic B. (1990) "Financial Development, Growth, and the Distribution of Wealth" Journal of Political Economy 98, Oct.

King, G and R. Levine (1993) " Finance and Growth: Shumpeter Might Be Right", Quarterly Journal of Economics, 717-737.

Levine, R. and S. Zervos (1996) "Stock Market Banks and Economic Growth", Policy Research Working Paper World Bank 1690.

McKinnon, R. I.(1973) Money and Capital in Economic Development, (Washington DC: Brooking Institution.

Quah, D. (1997) "Empirics for Growth and Distribution: Stratification, Polarization and Convergence Clubs" Journal of Economic Growth 2(1), March, 27-59.

Rajan, R. G. and L. Zingales (1996) "Financial Dependence and Growth", NBER Working Paper 5758.

Roubini, N and X. Sala-i-Martin (1992) "Financial Repression and Economic Growth", Journal of Development Economics 39, 5-30.

Saint-Paul, G. (1992) "Technological Choice, Financial Markets and Economic Development", European Economic Review 36, 763-781.

Williamson, S. D. (1986) "Costly Monitoring, Financial Intermediation, and Equilibrium Credit Rationing", Journal of Monetary Economic 18, September, 159-79.

8 Appendix

8.1 Optimal Incentive Compatible Contract

Following Gale and Hellwig (1985) the optimal incentive compatible contract between a firm and a financial intermediary is the debt contract:

$$\begin{aligned} &\underset{\theta_0, r^B}{Max} \int_{\theta_0}^{\infty} [\theta\pi(w) - (\delta + r^B)]dG(\theta) + (1 - \chi(g)) \int_{\theta_0}^{\infty} \theta\pi(w)dG(\theta) \\ &s.t.\;\; \theta_0\pi(w) - (\delta + r^B) \;=\; (1 - \chi(g))\theta_0\pi(w) \\ &\int_{\theta_0}^{\infty} (\delta + r^B)dG(\theta) + \int_0^{\theta_0} [\chi(g)\theta\pi(w)\text{-}\Psi\pi(w)]dG(\theta) \;=\; (\delta + r) \end{aligned} \tag{27}$$

The second restriction of the problem (27) may be written as follows:

$$(\delta + r^B)[1 - G(\theta_0)] = -\int_0^{\theta_0} \chi(g)\theta\pi(w)dG(\theta) + \Psi\pi(w)G(\theta_0) + (\delta + r) \tag{28}$$

Using expression (28), the optimization problem (27) may be rewritten as follows:

$$\underset{\theta_0}{Max}\; [1 - \Psi G(\theta_0)]\pi(w) - (\delta + r) \tag{29}$$

$$s.t.\; \left[\chi(g)\left(\theta_0[1\text{-}G(\theta_0)] + \int_0^{\theta_0} \theta dG(\theta)\right) - \Psi G(\theta_0)\right]\pi(w) \;=\; (\delta + r)$$

Integrating by parts:

$$\int_0^{\theta_0} \theta dG(\theta) = -\theta_0[1 - G(\theta_0)] + \int_0^{\theta_0} [1 - G(\theta)]d(\theta) \tag{30}$$

Since $G(\theta) = 1 - e^{-\theta}$, it follows that $[1 - G(\theta)] = G'(\theta)$. Substituting this in (27) it follows that:

$$\int_0^{\theta_0} \theta dG(\theta) = -\theta_0[1 - G(\theta_0)] + G(\theta_0) \tag{31}$$

Substituting (31) in (29) it follows that:

$$\underset{\theta_0}{Max}\ [1-\Psi G(\theta_0)]\pi(w)-(\delta+r) \tag{32}$$
$$s.t.(\chi(g)-\Psi)G(\theta_0)\pi(w)\ =\ (\delta+r)$$

It follows from the restriction of (32) and the fact that $G(\theta_0)$ is smaller than one that the optimal incentive compatible contract exist if and only if

$$(\chi(g)-\Psi)\pi(w)\ >\ (\delta+r)$$

Substituting the restriction in the objective function it follows that:

$$\pi(w)-(\delta+r)-\frac{\Psi}{\chi(g)-\Psi}(\delta+r)$$

Firms pay managers after paying the financial intermediary. The expected payment to managers should be equal to the wage of workers (see 8):

$$\frac{\pi(w)-\frac{\chi(g)}{\chi(g)-\Psi}(\delta+r)}{M^f(w)}=w \tag{33}$$

8.2 Labor Market Clearing condition:

It follows from (11) and (12) that the labor clearing condition is as follows:

$$k\left[\left(\frac{(1\text{-}\alpha)A(1\text{+}\phi)^{1-\alpha}}{w}\right)^{\frac{1}{\alpha}}\left(\int_0^\infty \theta dG(\theta)+\phi\right)\right]\ =\ 1 \tag{34}$$
$$\Leftrightarrow\ w\ =\ (1-\alpha)(1+\phi)Ak^\alpha \tag{35}$$

Using (33) in (35), it follows that :

$$\delta+r=\frac{\chi(g)-\Psi}{\chi(g)}\beta Ak^{1-\alpha} \tag{36}$$

It follows from the government budget constraint (7), the per capita production (5), the equilibrium interest rate (36) that the equilibrium interest rate is as follows:

$$r(k)\equiv\frac{\chi(\tau Ak^\alpha)-\Psi}{\chi(\tau Ak^\alpha)}\beta Ak^{\alpha-1}-\delta=r \tag{37}$$

8.2.1 Proof Lemma 1

Lemma 3 *There exists k^* such that $r(.)$ is strictly increasing in $(0,k^*)$ and strictly decreasing in (k^*,∞).*

Proof.

$$r'(k) = \frac{\beta A}{\chi(\tau Ak^{\alpha})k^{2-\alpha}}\left[\alpha\Psi\frac{\chi'(\tau Ak^{\alpha})\tau Ak^{\alpha}}{\chi(\tau Ak^{\alpha})} - (1-\alpha)\left(\chi(\tau Ak^{\alpha})-\Psi\right)\right] \Rightarrow$$
$$sign\, r'(k) = sign\left[\alpha\Psi\frac{\chi'(\tau Ak^{\alpha})\tau Ak^{\alpha}}{\chi(\tau Ak^{\alpha})} - (1-\alpha)\left(\chi(\tau Ak^{\alpha})-\Psi\right)\right]$$

Since $\frac{\chi'(\tau Ak^{\alpha})\tau Ak^{\alpha}}{\chi(\tau Ak^{\alpha})}$ is decreasing (by assumption) and $\chi(\tau Ak^{\alpha})$ is strictly increasing , it is follows that $\left[\alpha\frac{\chi'(\tau Ak^{\alpha})\tau Ak^{\alpha}}{\chi(\tau Ak^{\alpha})} - (1-\alpha)\left(\chi(\tau Ak^{\alpha})-\Psi\right)\right]$ is strictly decreasing. Since $\lim\limits_{k\to 0}\left[\alpha\frac{\chi'(\tau Ak^{\alpha})\tau Ak^{\alpha}}{\chi(\tau Ak^{\alpha})} - (1-\alpha)\left(\chi(\tau Ak^{\alpha})-\Psi\right)\right] > (1-\alpha)\Psi$ and $\lim\limits_{k\to\infty}\left[\alpha\frac{\chi'(\tau Ak^{\alpha})\tau Ak^{\alpha}}{\chi(\tau Ak^{\alpha})} - (1\text{-}\alpha)\left(\chi(\tau Ak^{\alpha})\text{-}\Psi\right)\right] = \lim\limits_{k\to\infty}\left[\alpha\frac{\ln\chi(\tau Ak^{\alpha})}{\ln(\tau Ak^{\alpha})} - (1\text{-}\alpha)\left(1\text{-}\Psi\right)\right] = -(1-\alpha)(1-\Psi) < 0$, it follows that there exists k^* such that $r'(.)$ is strictly positive in $(0,k^*)$ and strictly negative in (k^*,∞). ■

8.3 Proof Proposition 2

Proposition 4 *There exist $\underline{\Psi}$, $\overline{\Psi}$ such that if $\Psi > \underline{\Psi}$ then there is a unique steady state in which there is not financial intermediation, if $\Psi \in \left(\underline{\Psi},\overline{\Psi}\right)$ then there is a steady state without financial intermediation and two steady state with financial intermediation, finally if $\Psi < \overline{\Psi}$ then there is a unique steady state and there is financial intermediation in it.*

Proof.

Define

$$\varphi(\Psi) = \underset{k}{Max}\left[\frac{\chi(\tau Ak^{\alpha})-\Psi}{\chi(\tau Ak^{\alpha})}\beta A\frac{1}{k^{1-\alpha}} - \delta\right]$$

The function $\varphi(.)$ is continuous in $(0,1)$, strictly decreasing and $\underset{\Psi\to 0}{Lim}\,\varphi(\Psi) = \infty$, and $\underset{\Psi\to 1}{Lim}\,\varphi(\Psi) = 0$. Thus it is possible to define "$\overline{\Psi}$" as follows:

$$\overline{\Psi} \overset{Definition}{\Leftrightarrow} \varphi(\overline{\Psi}) = \rho$$

Using the implicit function Theorem it is possible to define the function $\overline{k} : \left(0, 1 - \frac{\alpha\varepsilon}{\beta A}\right)$following function:

$$\overline{k}(\Psi) \overset{Definition}{\Leftrightarrow} \frac{\chi(\tau A\overline{k}^{\alpha}) - \Psi}{\chi(\tau A\overline{k}^{\alpha})}\beta A - \alpha\varepsilon = 0$$

The function $\overline{k}(\Psi)$ is continuous in $\left(0, 1 - \frac{\alpha\varepsilon}{\beta A}\right)$, strictly increasing and $\underset{\Psi\to 0}{Lim}$ $\overline{k}(\Psi) = 0$, and $\underset{\Psi\to 1-\frac{\alpha\varepsilon}{\beta A}}{Lim}$ $\overline{k}(\Psi) = \infty$. Thus it is possible to define "$\underline{\Psi}$" as follows:

$$\underline{\Psi} \overset{Definition}{\Leftrightarrow} \alpha\varepsilon\frac{1}{\overline{k}(\underline{\Psi})^{1-\alpha}} - \delta = \rho$$

- Case in which $\Psi > \overline{\Psi}$: In this case $\forall k$ $r(k) \leq \varphi(\Psi) < \varphi(\overline{\Psi}) = \rho \Rightarrow$ There is not steady state with financial intermediation (with modern technology)

- Case in which $\Psi \in \left(\underline{\Psi}, \overline{\Psi}\right)$: In this case: i) $\exists k^*$ s.th. $r(k^*) = \varphi(\Psi) > \varphi(\overline{\Psi}) = \rho$, ii) $r(\overline{k}(\Psi)) = \alpha\varepsilon\frac{1}{\overline{k}(\Psi)^{1-\alpha}} - \delta < \alpha\varepsilon\frac{1}{\overline{k}(\underline{\Psi})^{1-\alpha}} - \delta = \rho$ and iii) $\lim_{k\to\infty} r(k) = -\delta < \rho$. Facts i), ii) and iii) together with lemma 1 imply that there is two steady state with financial intermediation. Fact ii) imply that there is a steady state without financial intermediation.

- Case in which $\Psi < \underline{\Psi}$: In this case: i) $r(\overline{k}(\Psi)) = \alpha\varepsilon\frac{1}{\overline{k}(\Psi)^{1-\alpha}} - \delta$ $> \alpha\varepsilon\frac{1}{\overline{k}(\underline{\Psi})^{1-\alpha}} - \delta = \rho$ and ii) $\lim_{k\to\infty} r(k) = -\delta < \rho$. Facts i) and ii) together with lemma 1 imply that there is one steady state with financial intermediation. Fact i) imply that there is not a steady state without financial intermediation. ■

Figure 1

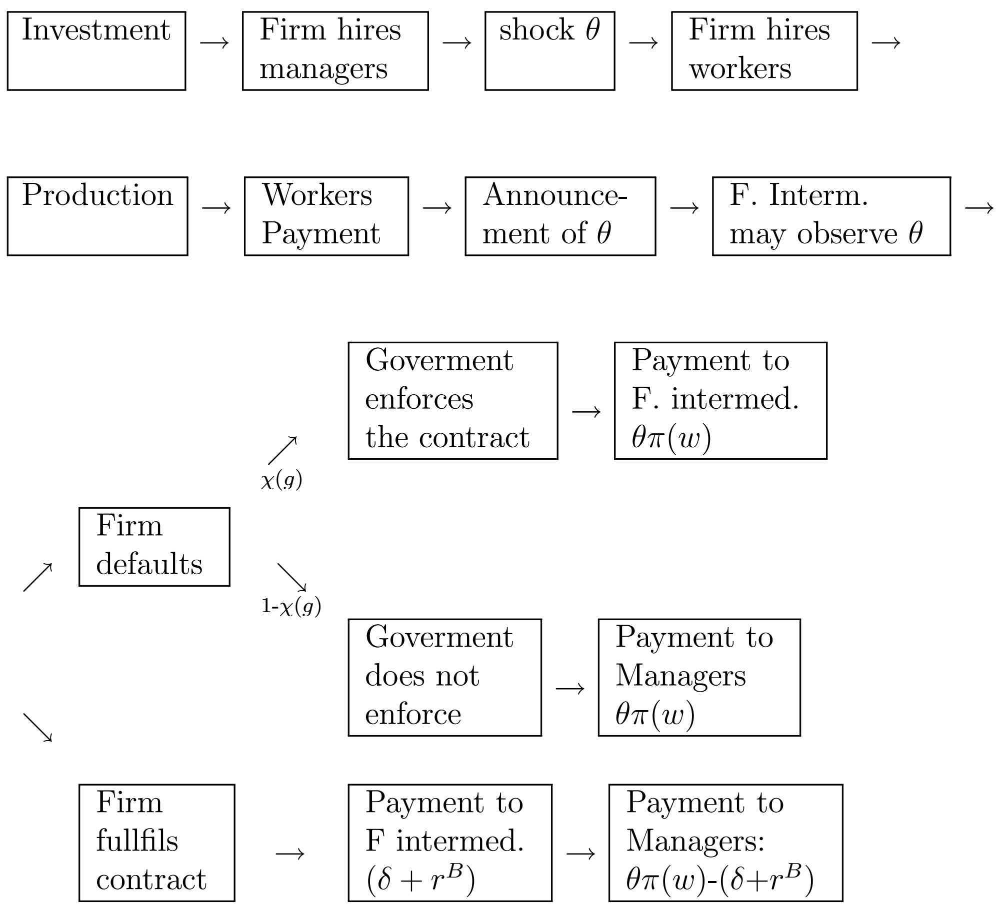

Investment
Firm hires managers
shock θ
Firm hires workers
Production
Workers Payment
Announcement of θ
F. Interm. may observe θ
Goverment enforces the contract
Payment to F. intermed. $\theta\pi(w)$
$\chi(g)$
Firm defaults
1-$\chi(g)$
Goverment does not enforce
Payment to Managers $\theta\pi(w)$
Firm fullfils contract
Payment to F intermed. $(\delta + r^B)$
Payment to Managers: $\theta\pi(w)$-$(\delta$+$r^B)$

Figure 2

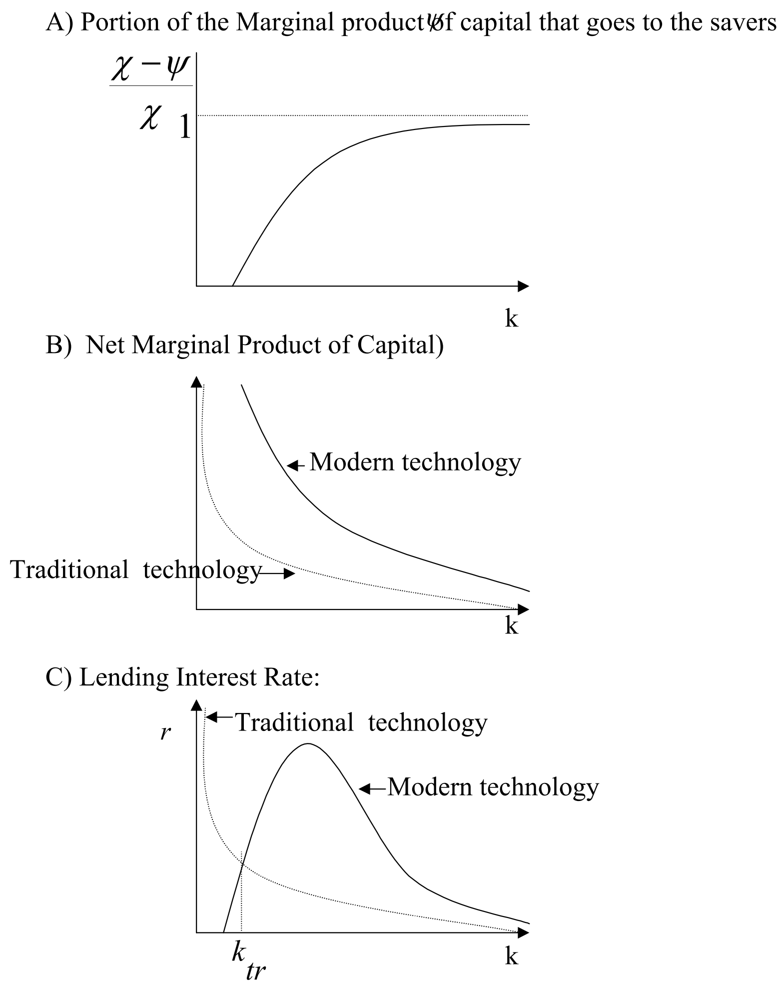
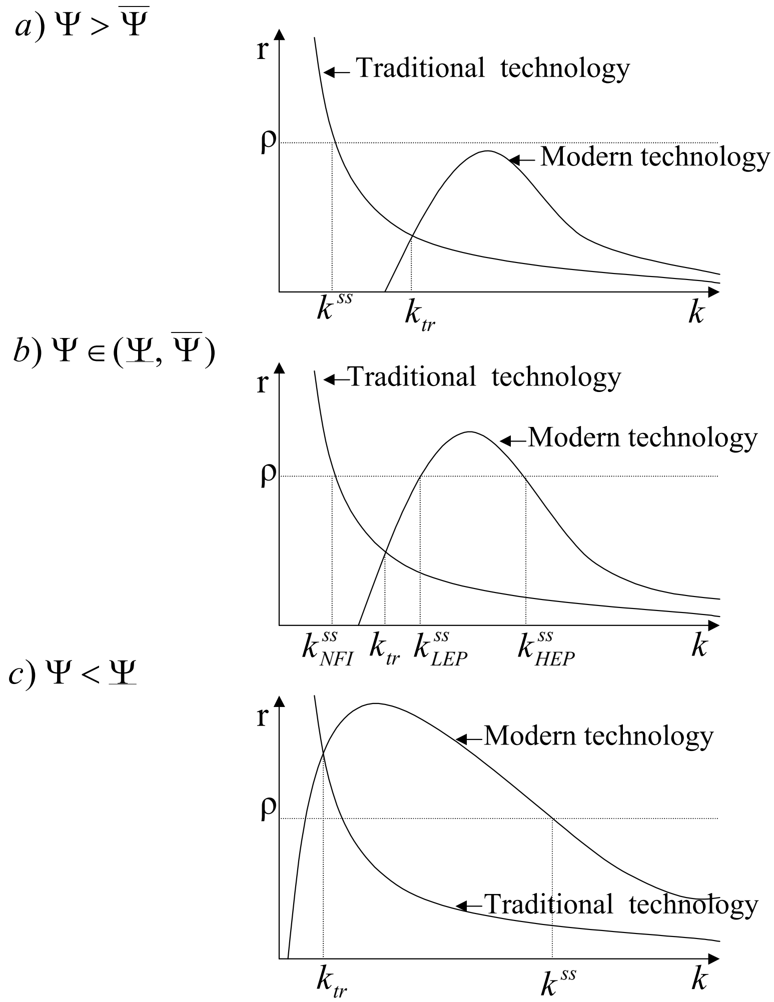

A) Portion of the Marginal product of capital that goes to the savers
$\frac{\chi-\psi}{\chi}$
1
k
B) Net Marginal Product of Capital)
Modern technology
Traditional technology
k
C) Lending Interest Rate:
r
Traditional technology
Modern technology
k_{tr}
k

Figure 3



a) Ψ > Ψ̄
r
Traditional technology
ρ
Modern technology
k^ss
k_tr
k
b) Ψ ∈ (Ψ̲, Ψ̄)
r
Traditional technology
Modern technology
ρ
k^ss_NFI
k_tr
k^ss_LEP
k^ss_HEP
k
c) Ψ < Ψ̲
r
Modern technology
ρ
Traditional technology
k_tr
k^ss
k

Figure 4

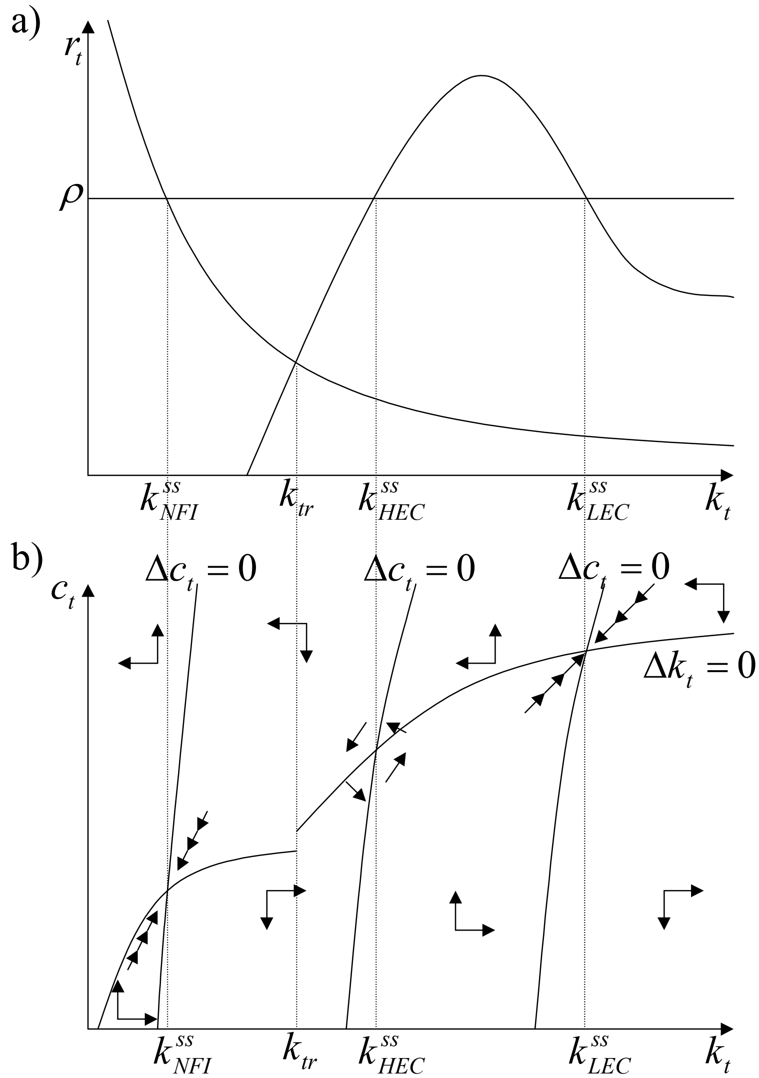
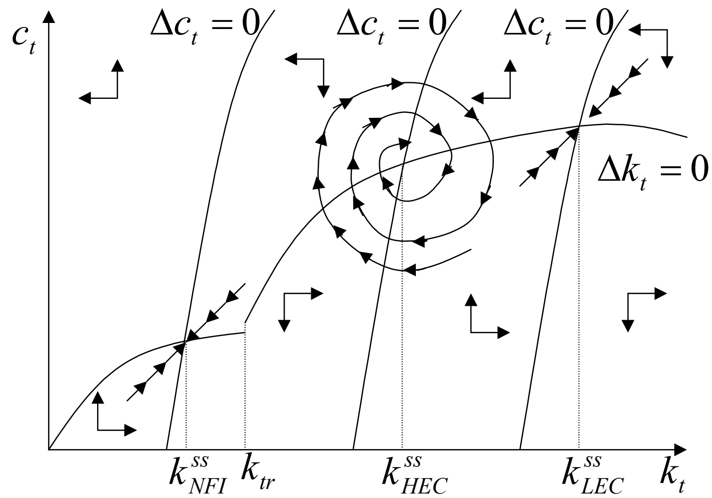

a)
r_t
ρ
k_{NFI}^{ss}
k_{tr}
k_{HEC}^{ss}
k_{LEC}^{ss}
k_t
b)
c_t
$\Delta c_t = 0$
$\Delta c_t = 0$
$\Delta c_t = 0$
$\Delta k_t = 0$
k_{NFI}^{ss}
k_{tr}
k_{HEC}^{ss}
k_{LEC}^{ss}
k_t

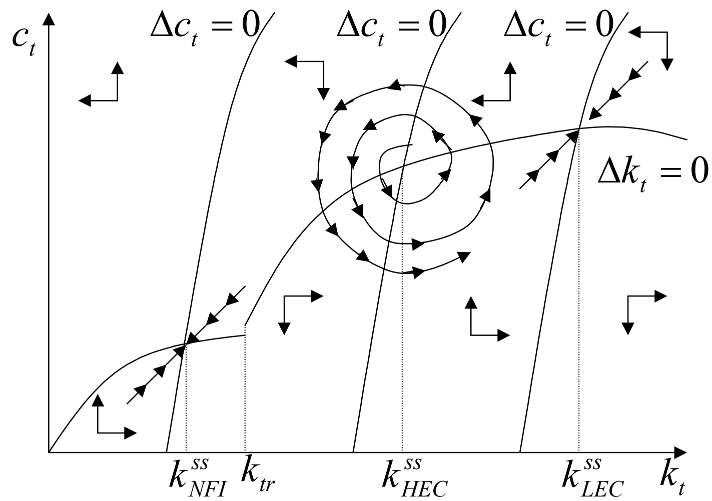
Figure 5

a)



c_t
$\Delta c_t = 0$
$\Delta c_t = 0$
$\Delta c_t = 0$
$\Delta k_t = 0$
k_{NFI}^{ss}
k_{tr}
k_{HEC}^{ss}
k_{LEC}^{ss}
k_t

b)



c_t
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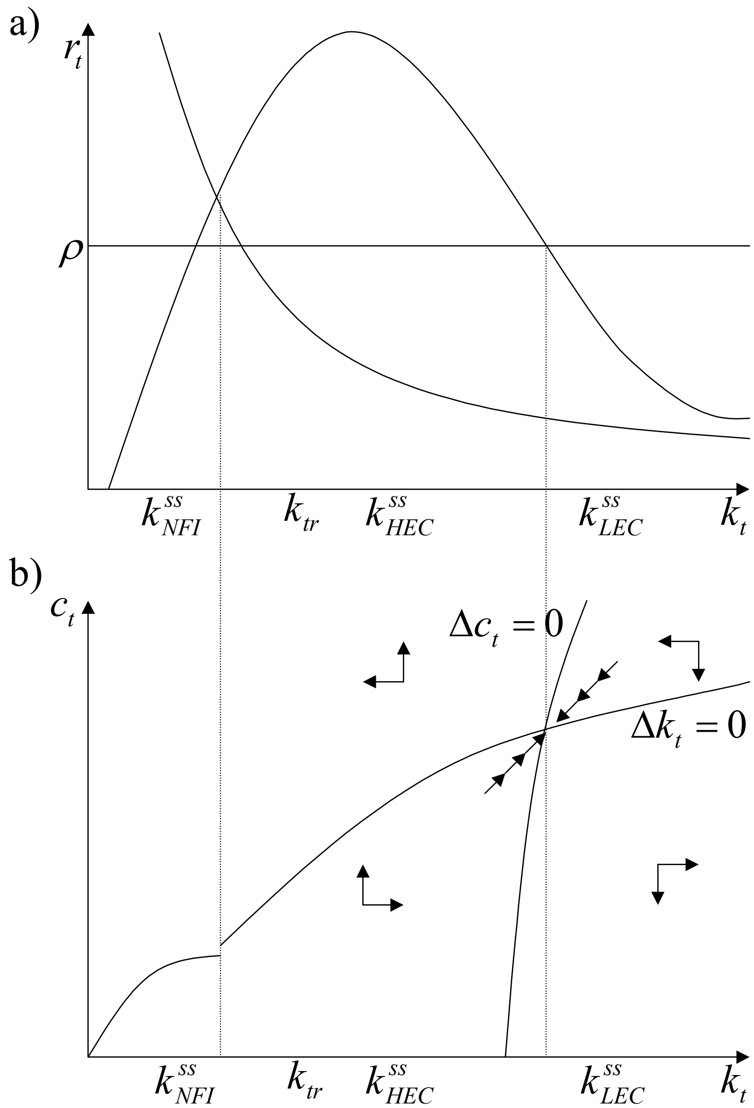
k_t

Figure 6


a)
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k_{tr}
k_{HEC}^{ss}
k_{LEC}^{ss}
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b)
c_t
$\Delta c_t = 0$
$\Delta k_t = 0$
k_{NFI}^{ss}
k_{tr}
k_{HEC}^{ss}
k_{LEC}^{ss}
k_t