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Technological Progress, Slow Growing Economies and Polarization

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Abstract

Slow technological progress and financial sectors with low productivity are endemic among developing countries. This paper presents a model in which technological progress affects the productivity of the financial sector. When the technological progress is fast, the financial intermediation costs are low and this increases the incentives to invest in new technology. This feed back process involves the existence of two types of balanced growth path equilibria: one in which the productivity of the financial sector is high and the technological progress fast, and other in which the productivity of the financial sector is low and the technological progress slow. It also may appear an steady state in which there is neither financial sector nor technological progress. Multiple equilibria and indeterminacy of equilibria may arise: for given initial conditions, there are several equilibrium paths converging to different balanced growth paths with different growth rates.

Key Words: Growth Theory, Technological Progress, Development Traps, Financial Intermediation, Multiple Equilibria.

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1 Introduction

It has been widely recognized that technological progress has played a very important role in growth, however not too much attention has been paid to the effect that technological progress has on the financial system. Nevertheless it is quite evident that the financial sector has widely benefited from the technological progress. It is hard to imagine the actual financial system without cash machines, credit cards, telephones, faxes, computers and electronic communications. During the last few decades a large number of new financial instruments and institutions have appeared. Technological progress has played a very important role in the implementation of these innovations in the financial markets. New technologies can lower the cost of providing new financial services and instruments and thus make them profitable. For example, improvements in computer technology allowed the extension of the use of bank credit cards and the introduction of securitization. New technologies that allow instantaneous transmission of information have completely changed the way in which Stock markets work. Internet has revolutionized the financial system. It is also evident that the financial system in different countries uses different technologies. The financial system in the Sub-Saharan Africa does not use the same technology than in U.S.A. To offer bank services by Internet in countries in which most of the population does not have access to it would not be profitable.

This paper explores the behavior of growth, in an environment in which technological progress affects the financial system. The introduction of this new element will be useful to better understand the observed huge empirical difference in growth performance across countries.

This paper presents a model that formalizes two ideas: *i*) technological progress affects the financial system. The financial system is not impervious to technological progress; e.g. the widespread use of computers and electronic communications has revolutionized the methods used by the financial system. *ii*) The financial system affects technological progress. Investment is needed to innovate and the financial system affects the cost of raising funds for this investment[1]. This interaction between technological

[1]Many empirical studies illustrate the close relationship between the financial system

progress and the financial system involve the existence of two types of balanced growth path equilibria: with low financial intermediation costs and fast technological progress (LIC), with high financial intermediation costs and slow technological progress (HIC). When the technological progress is fast the intermediation costs are low and this increases the incentives to invest in new technology. It may also appear another type of long run equilibria in which there is neither financial intermediation nor technological progress. This result is consistent with the polarization in the cross-country per capita income distribution, which has been empirically observed by D. Quah (1997).

Another interesting aspect of the model is the dynamic behavior outside of the balanced growth path. The LIC balanced growth path is the typical saddle point equilibria with a unique equilibrium path converging to it. However, the HIC balanced growth path is either a node or a focus. When the HIC balanced growth path is unstable, there is a unique equilibrium trajectory that converges to the LIC balanced growth path. When the HIC balanced growth path is stable, multiple equilibria arise for levels of wealth small enough. When the wealth level is large enough this multiple equilibria may disappears and the unique equilibrium trajectory converges to the LIC balanced growth path.

There is an important literature about poverty traps (See among others Azariadis and Drazen (1990), Gali (1995) and Zilibotti (1995)). The mechanism that generates poverty traps in those papers is not the effect of technological change in the financial sector. Besides that, these papers generate poverty traps in which there is stagnation, that is the growth rate is zero. In the present paper economies that are in poverty traps grow at a positive but low growth rate (the HIC balanced growth path).

There are also many papers that relate growth and financial intermediation (See among others, Acemoglu and Zilibotti, 1996; Bencivenga and Smith, 1991; Cooley and Smith, 1992; Greenwood-Jovanovic, 1990; Khan, 2001; Saint-Paul, 1992). Those earlier models do not deal with the effect that technological change has on the financial system. The present paper is not focused on financial development and is more macroeconomically oriented.

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and economical development. See among others Demirguc and Levine (1995), Goldsmith (1969), King and Levine (1993), Levine and Zervos (1996), McKinnon (1973) and Rajan and Zingales (1996).

The structure of the paper is as follows: the second section presents the model. The third section characterizes the agents' behavior. The fourth section analyzes the behavior of the economy along the balanced growth path. The fifth section analyzes the dynamic behavior of the economy outside the balanced growth path. Finally, a conclusion is presented. All the proofs and some technical details are included in the Appendix I. Appendix II shows that the financial technology used in the paper may be rationalized as an optimal contract with asymmetric information and cost state verification.

2 The Model

Time is discrete with infinite horizon. There is a single good in the economy that can be used for consumption and investment:

$$y_t = c_t + k_{t+1} - (1-\delta)k_t \tag{1}$$

where y_t denotes per capita production, c_t denotes per capita consumption and k_{t+1} denotes per capita capital, and $\delta \in (0,1)$ denotes the depreciation rate (the population growth rate is zero).

2.1 Preferences

The Agents' life is infinite and population is constant. Consumers' preferences are given by a time separable utility function:

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t u(c_t) \tag{2}$$

where $\rho > 0$ is the discount rate of the utility function, c_t denotes the consumption at period t, and $u(.)$ is the isoelastic felicity function:

$$u(c_t) = \begin{cases} \frac{{c_t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} & if \quad \sigma \in (0,1) \cup (1,+\infty) \\ ln(c_t) & if \quad \sigma = 1 \end{cases} \tag{3}$$

2.2 Technology in the Real Sector:

The sector that produces physical goods will be called "real sector". There is a continuum of types of technologies that are differentiated by their level of sophistication. The level of sophistication of a technology is indexed by $z \in \Re$, where a higher index z means a higher level of sophistication. Each technology uses capital and labor, denoted by L. The technology z is represented by the following production function at the *firm level*:

$$F(k_f, L_f; z, z\text{-}h) = \begin{cases} e^z e^{-(1-\alpha)(z-h)}(L_f\text{-}1)^\lambda & If\ k_f \geq e^z\phi \quad L_f \geq 1 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where α, $\lambda \in (0,1)$, $\lambda+\alpha < 1$, $\phi \in \Re_+$, k_f and L_f are respectively the capital and the labor used by the firm; h is the *technological experience*, defined as a weighted average of the level of sophistication of the technologies used in the past:

$$h_t \equiv \sum_{i=1}^{\infty} \eta_i z_{t-i} \tag{5}$$

where $\eta_i \in \Re_+$, $\sum_{i=1}^{\infty} \eta_i = 1$. The productivity of each technology increases with its level of sophistication and it decreases with the difference between its level of sophistication and that used in the past (the technological experience).

The function $e^{-(1-\alpha)(z-h)}$ may be interpreted as the state of know-how. If a technology is more sophisticated than h then the state of know-how is smaller than one. Agents in the society are not able to use this technology at its maximum potential productivity.

The interpretation of equation (5) is that the use of highly sophisticated technologies at the present time increases the agents' capability of using even more sophisticated technology in the future. Permanent growth is possible due to this learning-by-doing process (for other models with learning by doing see among others Arrow, 1962; Krugman, 1988; Lucas, 1986,1992; Matsuyana, 1992; Stokey, 1988; and Young, 1991).

It follows from (4) that in order to use the technology "z", a fixed amount of $e^z\phi$ units of capital are required. That is, the more sophisticated the technology, the greater the fixed cost required for using this technology.

2.3 Firms

It is assumed that consumers can neither invest nor produce directly and there is a continuum of potential firms that are free to enter the market. Firms maximize profits and behave competitively.

2.4 Financial Sector

Financial intermediaries borrow capital from lenders and lend to borrowers. They pay a risk-free interest rate r to lenders. Financial intermediaries are assumed to be competitive.

To transfer the payment of the lender to the borrowers, $\frac{\varphi}{e^z}$ units of labor per (gross) borrowing interest rate are required. Note that and z is the technology used in production, this mean that the technology used by the real sector has a positive externality over the financial sector. Thus financial intermediaries must incur a financial intermediation cost, a constant fraction "ψ" of the (gross) borrowing interest rate is required in order to transfer such interest rate from borrowers to lenders:

$$\psi \equiv \varphi w / e^z \tag{6}$$

where w denotes the wage. The financial intermediation costs may be interpreted in different ways: as monitoring costs (as Diamond, 1984), as informational costs (as Williamson, 1986) or simply as transaction costs. The important point is that this intermediation technology is positively affected by the technology used in the real sector "z".

Appendix II presents an optimal contract with asymmetric information that gives as a result the same type of financial technology that the one presented here.

Since financial intermediaries are competitive, the following zero profit condition should be satisfied:

$$(\delta + r_B) = (\delta + r) + \psi(\delta + r_B) \tag{7}$$

where r_B denotes the borrowing net interest rate and r the lending net interest rate. Equation ((7) says that the gross borrowing interest rate $(\delta + r_B)$ should be equal to the gross lending interest rate $(\delta + r)$ plus the financial intermediation costs $\psi(\delta + r_B)$.

3 Agent Decisions

3.1 Firms

The profit maximization problem of the firm may be interpreted in two different ways: as a technological or as a capital choice.

Technological Choice: Firms maximize profits choosing the amount of labor and the technology:

$$\underset{L_f,z}{Max}\; e^z e^{-(1-\alpha)(z-h)}(L_f-1)^{\lambda} - wL - (\delta + r_B)\,\phi e^z \tag{8}$$

The first order conditions of the maximization problem of the firm are as follows:

$$\begin{aligned} w &= \lambda e^z e^{-(1-\alpha)(z-h)}\left(\frac{1}{L_f-1}\right)^{1-\lambda} &(9)\\ r_B &= \alpha e^{-(1-\alpha)(z-h)}(L_f-1)^{\lambda} - \delta &(10)\end{aligned}$$

Equation (10) may be rewritten as follows

$$z = \ln\left(\alpha\frac{e^{(1-\alpha)h}}{\phi^{\alpha}}\frac{(L_f-1)^{\lambda}}{(\delta+r_B)}\right)^{\frac{1}{1-\alpha}} \tag{11}$$

The sophistication level of the technology is a decreasing function of the borrowing interest rate. High borrowing interest rate reduces the incentives to make the large investment that highly sophisticated technology require. Thus, the technological progress is a decreasing function of the borrowing interest rate.

Capital Choice: It follows from (4) that the amount of capital used by the firm is equal to ϕe^z, thus the firm maximization problem (8) may be rewritten as follows:

$$\underset{L_f,k_f}{Max}\; e^{(1-\alpha)h}\left(\frac{k_f}{\phi}\right)^{\alpha}(L_f-1)^{\lambda} - wL - (\delta + r_B)\,k_f \tag{12}$$

where k_f denotes the amount of capital used by the firm. Since $\lambda + \alpha < 1$, the average cost function of the firm has U-shape.

The first order conditions of the maximization problem of the firm are well known: the marginal product of a factor should be equal to his renting price:

$$w = \lambda \frac{e^{(1-\alpha)h}}{\phi^{\alpha}} k_f^{\alpha} \left(\frac{1}{L_f - 1} \right)^{1-\lambda} \tag{13}$$

$$r_B = \alpha \frac{e^{(1-\alpha)h}}{\phi^{\alpha}} \frac{1}{k_f^{1-\alpha}} (L_f - 1)^{\lambda} - \delta \tag{14}$$

Equation (14) means that the borrowing interest rate is equal to the net marginal product of capital, the lending interest rate will be smaller than the net marginal product of capital due to the financial intermediation costs (see 7).

3.2 Consumers

Consumers face the following optimization problem:

$$\underset{\{c_t\}_{t=0}^{\infty}}{Max} \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^{t} u(c_t) \tag{15}$$
$$s.t: \; (1+r_t)a_t + w_t = a_{t+1} + c_t$$

where a denotes the assets that consumers hold, that is, the consumers' wealth. Since consumers do not borrow at equilibrium, the results of the paper would not change if the borrowing interest rate for consumers were different from the lending interest rate, or if consumers were not allowed to borrow.

The Euler Equation and the trasverslity condition of the consumers' optimization problem (15) is as follows:

$$\frac{c_{t+1}}{c_t} = \left(\frac{1+r_{t+1}}{1+\rho} \right)^{\sigma} \tag{16}$$

$$\lim_{t \to \infty} \left(\frac{1}{1+\rho} \right)^{t} c_t^{-\frac{1}{\sigma}} a_{t+1} = 0 \tag{17}$$

4 Balanced Growth Path

This section analyzes the balanced growth path and shows that there are two types of balanced growth path: one with high financial intermediation costs (HIC from now) and the other with low financial intermediation costs (LIC from now).

Equilibrium is when agents optimize and markets clear. A balanced path equilibrium is an equilibrium path such that the variables c_t, w_t, e^{z_t}, e^{h_t}, x_t and a_t grow at a constant rate and the interest rate r_t, the intermediation costs ψ_t, and the per capita number of firms P_t stay constant.

As in any other endogenous growth model[2], it is necessary to assume the discount rate of the utility small enough and the productivity of the technology large enough to generate positive growth rates:

Assumption 1: $\rho < \overline{\rho}$, $\Gamma > \underline{\Gamma}$, where $\overline{\rho}$ and $\underline{\Gamma}$ are two constants defined in the Appendix.

4.1 Production Growth Rate:

It is shown in the Appendix that along the balanced growth path the production growth rate is as follows:

$$(1+g) = \left[\frac{\Gamma}{(\delta+r_B)}\right]^{\gamma} \tag{18}$$

where g denotes the production growth rate, γ and Γ are two constants defined in the Appendix. Equation (18) means that production growth rate is negatively related with borrowing interest rates. The cost of financing technology increases with the borrowing interest rate. Thus, the higher the borrowing interest rate, the lower are the incentives to invest in sophisticated technology and the slower the production growth (see 11).

It is shown in the Appendix that along the balanced growth path the relationship between the borrowing and the lending interest rate is as follows:

$$(\delta+r) = (\delta+r_B)\text{-}\xi\,(\delta+r_B)^2 \Leftrightarrow (\delta+r_B) = \frac{1 \pm \sqrt{1\text{-}4\xi\,(\delta+r)}}{2\xi} \tag{19}$$

[2] In a "AK" model this condition would be: $\rho < A - \delta$.

where ξ is a constant defined in the Appendix. Equation (19) means that for a given lending interest rate, there are two borrowing interest rates. This is due to complementarities between the real and the financial sector.

Figure 1.a represents equation (18): the production growth rate decreases with the borrowing interest rate. Figure 1.b represents equation (19): given a lending interest rate r^* there are two borrowing interest rates: one with high financial intermediation costs r_B^{HIC} and the other with low financial intermediation costs r_B^{HIC}. Obviously the borrowing interest rate is higher with HIC ($r_B^{HIC} > r_B^{LIC}$). Figure 1.a shows that these two borrowing interest rates imply two growth rates: g^{HIC} and g^{LIC}. Since the growth rate decreases with the borrowing interest rate and the borrowing interest is higher with HIC, the growth rate is slower with HIC ($g^{HIC} < g^{LIC}$)). Finally, figure 1.c relates the lending interest rate with the production growth rate. Given a lending interest rate r^*, there are two growth rates: g^{HIC} and g^{LIC}.

4.2 Consumption Growth Rate

It follows from Euler Equation (16) that along a balanced path equilibrium the following equation should hold:

$$(1 + g_c) = \left(\frac{1 + r}{1 + \rho}\right)^{\sigma} \tag{20}$$

where g_c denotes the consumption growth rate. Substituting (19) in (20), it follows that:

$$(1 + g_c) = \left[\frac{(1 + r_B) - \xi\,(\delta + r_B)^2}{1 + \rho}\right]^{\sigma} \tag{21}$$

Figure 2.c represents Euler Equation (20) as an up-sloping curve. This curve is called "Consumption Growth" and relates the consumption growth rate with the lending interest rate. Figure 2.a represents equation (21), which is also called "Consumption Growth" and relates the consumption growth rate with the borrowing interest rate.

4.3 Balanced Growth Path Equilibria

As it was previously explained, the production curves in figures 2.a and 2.c relate the production growth rate with the borrowing and the lending interest rate respectively. The consumption curves in figures 2.a and 2.c represent Euler Equations (20) and (21), and relate the consumption growth rate with the borrowing and the lending interest rate respectively. The points at which the consumption and production curves intercept[3] in figure 2.a and 2.b are the interest rate-growth rate pair along the balanced growth path, where consumption and production grow at the same rate. There are two balanced growth paths: one with high intermediation costs (HIC) and other with low intermediation costs (LIC). The lending interest rate is lower along the HIC path, the opposite happens with the borrowing interest rate. The reason is that the financial intermediation costs are higher along the HIC path.

Figure 2.a shows that the borrowing interest rate is lower along the LIC path. Thus, firms have more incentives to invest in sophisticated technology. As a result, both technological progress and production growth is faster along that path. Figure 2.b shows that the lending interest rate is higher along the LIC path. As a result, consumers have more incentives to save, thus, consumption grows faster. Roubini and Sala-i-Martin, X. (1995) found that countries with low interest rates were countries with slow growth. The results presented here are consistent with this empirical finding.

The above results are summarized in the following proposition.

Proposition 1 *Two balanced growth path equilibria with positive growth rate exist. In one of them (along the HIC path) the intermediation costs are higher than the other (the LIC path). The lending interest rate and the growth rate are higher along the LIC path, and the borrowing interest rate is lower.*

There is feedback between the real and the financial sector. Since technological progress in the real sector positively affects the financial intermediation technology, when technological progress in the real sector is fast, the financial intermediation costs are low. As a consequence, the borrowing interest rate is low and this increases the incentives for firms to adopt more sophisticated technologies, which generate faster technological progress. When the technological progress is slow the opposite happens: slow technological progress implies high financial intermediation costs, which implies high borrowing interest rate that reduces incentives to invest in sophisticated technology, which make the technological progress to be slow.

[3]Assumption 1 guaranty that the consumption and the production curves intercept.

5 Dynamic Behavior

This section analyzes the dynamic behavior of the model outside the balanced growth path, see the Appendix for technical details.

In order to simplify, it is assumed that the technological experience accumulation equation (5) takes the following form:

$$h_{t+1} = Max\left\{\eta z_t + (1-\eta)h_t, h_t\right\} \tag{22}$$

Note that the above technological experience accumulation equation is a particular case of equation (5) in which it is imposed the restriction that technical progress cannot be negative.

5.1 Traditional Technology[4]

It is assumed that there is a "traditional technology" with the following production function:

$$\varepsilon\; e^{(1-\alpha)h} K^{\alpha} L^{1-\alpha}$$

where $\varepsilon < \overline{\varepsilon}$, where $\overline{\varepsilon}$ is a constant defined in the Appendix. Since the traditional technology presents constant returns to scale, financial intermediation is not needed, it is assumed that consumers may invest directly in the traditional technology. When the traditional technology is used the technological experience does not increase the next period. In order to distinguish the other technologies from the traditional one, the technologies that are not the traditional one will be called "sophisticated technologies" from now on.

The traditional technology is inefficient from the static and dynamic point of view. There are always sophisticated technologies that are more productive than the traditional one, thus the traditional technology is inefficient from the static point of view. Further more, the traditional technology does not have positive external effects on the technological experience and thus is also inefficient from the dynamic point of view. However, when the financial intermediation costs are prohibitive, the traditional technology is used in equilibrium.

[4]When the per labor capital is very low, the intermediate cost are prohibitive, and there is not equilibrium. It is necessary to introduce a traditional technology in order to guaranty the existence of equilibrium for low levels of capital.

5.2 Financial Development

Per labor capital and the per labor consumption are defined as follows: $\underline{k}_t \equiv \frac{k_t}{e^{h_t}}$, $\underline{c}_t \equiv \frac{c_t}{e^{h_{t+1}}}$. It is shown in the Appendix that the equilibrium intermediation costs are a decreasing function of the per labor capital as is shown in figure 3.a. This result is quite intuitive: the productivity of the financial sector increases with wealth.

Figure 3 also shows the borrowing and lending interest rate as a function of the per labor capital. The borrowing interest rate is equal to the net marginal product of capital (see equation 14), which decreases with the per labor capital as in the neoclassical growth model. However figure 3.c shows that the lending interest rate when sophisticated technologies are used is very different to the neoclassical model: have a inverted U-shape. The reason for this shape may be found in the financial intermediaries zero profit condition (7), which implies that the lending interest rate is equal to the borrowing interest rate minus the financial intermediation cost:

$$(\delta + r) = (\delta + r_B) - \psi(\delta + r_B) \tag{23}$$

Both the borrowing interest rate (marginal product of capital) and the intermediation costs decrease with the per labor capital (see figure 3.a and 3.b). Since the lending interest rate is equal to the borrowing interest rate minus the financial intermediation cost, these two forces go in opposite directions and as a result the relationship between lending interest rate and per labor capital has an inverted U-shape. This inverted U-shape relationship between interest rate and per labor capital plays a key role in the dynamic behavior of the model.

The broken line in figure 3.b and 3.c represents the marginal product of capital if traditional technology is used. Figure 3.b shows that the marginal product of capital of the traditional technology is always bellow the marginal product of capital of the sophisticated technologies. However figure 3.c shows that when the per labor capital is lower than the threshold level $\underline{k}^{tr}$, the marginal product of capital of the traditional technology is above the lending interest rate that would appear in equilibrium if the sophisticated technologies were used. As a consequence, when the per labor capital is lower of the threshold level $\underline{k}^{tr}$ the traditional technology is used in the economy and consequently the financial sector does not appear in equilibrium. This last result is similar to other models of "financial development" (see Bencivenga and Smith, 1991; Cooley and Smith, 1992; Greenwood-Jovanovic, 1990): in order that the financial sector appears, a minimum amount of wealth is required. The present model also predicts that the financial sector improve its productivity with the increase of wealth (see figure 3.a). Further more, there are three possible types of financial sector in the long-run (see figure 6.a): i) Not existence of financial intermediation (NFI path from now on), this would be the case in the steady state in which the traditional technology is used; ii) A financial sector with low productivity (HIC path); iii) A financial sector with high productivity (LIC path). The following proposition establish the conditions under which NFI steady state exists.

Proposition 2 *There exists $\underline{\varepsilon} \in (0, \overline{\varepsilon})$ such that if $\varepsilon \in (\underline{\varepsilon}, \overline{\varepsilon})$ there is not NFI steady state, if $\varepsilon \in (o, \underline{\varepsilon})$ there is NFI steady state.*

Figure 4.a and 6.a represents respectively the case in which there is not and there is NFI steady state.

5.2.1 Dynamic Behavior when there is not NFI steady state

The curve called technological progress in figure 4.a represents the technological progress growth rate as a function of the per labor capital, where technological progress growth rate is defined as the growth rate of the exponential of the technological experience (e^{h_t}). It follows from the production function in (12) that this concept of technological progress is very similar to the Sollow's total factor productivity growth rate. When the per labor capital is bellow the threshold level $\underline{k}^{tr}$, the technology used is the traditional one, which means that the there is no technological progress, this is reflected in figure 4.a, in which the technological progress curve is flat at zero level when the per labor capital is smaller than $\underline{k}^{tr}$. When the per labor capital is larger than $\underline{k}^{tr}$, the technological progress rises with the per labor capital. When the per labor capital is large, borrowing interest rate is low, and thus agents have more incentives to invest in more sophisticated technology (see 11)

The consumption curve in figure 4.a represents the consumption growth rate as a function of the per labor capital. It follows from the Euler Equation (16) that the consumption growth rate is an increasing function of the lending interest rate, figure 4.a shows the consumption growth rate behaves as the lending interest rate (see figure 3.c). When the per labor capital is smaller than the threshold level $\underline{k}^{tr}$ the technology used is the traditional one and there is not financial intermediation, thus the lending interest rate is equal to the marginal product of capital. Consequently, the lending interest rate and the consumption growth rate behaves as in the neoclassical model: they decrease with the per labor capital. As it was previously explained, when the per labor capital is larger than the threshold level $\underline{k}^{tr}$ there are two opposite forces that determine the evolution of the lending interest: both the financial intermediation cost and the marginal product of capital decrease with the per labor capital, the reduction of the financial intermediation costs makes the lending interest rate to increase, the reduction of the marginal product of capital has the opposite effect. When the per labor capital is between $\underline{k}^{tr}$ and $\underline{k}^{*}$ (see figure 4.a) the predominant force is the reduction of the intermediation costs that makes both lending interest rate and consumption growth rate to rise with the per labor capital. When the per labor capital is larger than $\underline{k}^{*}$ the predominant force is the reduction of the marginal product of capital that makes the lending interest rate and the consumption growth rate to fall with the per labor capital.

Figure 4.b displays the phase diagram the describes the dynamic behavior of the economy. Figure 4.a shows that the consumption growth rate equalizes the technological progress growth rate along the HIC and LIC balanced growth paths (g^{HIC} and g^{LIC} respectively). When the per labor capital is smaller than the threshold level $\underline{k}^{tr}$, the consumption grows at faster rate than the technological progress (with zero growth rate), this is reflected in the vertical arrows to the left of the $k_{t+1} = \underline{k}^{tr}$ schedule in phase diagram 4.b. When the per labor capital is between $\underline{k}^{tr}$ and $\underline{k}^{HIC}$, figure 4.a shows that the consumption grows more slowly than the technological progress and thus the vertical arrows in the region in between the $k_{t+1} = \underline{k}^{tr}$ and $k_{t+1} = \underline{k}^{HIC}$ schedules indicates that the per labor consumption grows at a negative rate. Per labor consumption grows at a positive rate in the region in between the $k_{t+1} = \underline{k}^{HIC}$ and $k_{t+1} = \underline{k}^{LIC}$ schedules and a negative rate in the region to the left of the $k_{t+1} = \underline{k}^{LIC}$ schedules.

The $\triangle c = 0$ schedules are not vertical lines as in models with continuous time. The reason for this may be found in the Euler Equation, which depends upon the future interest rate when time is discrete (see Azariadis 1993 for details).

Figure 4.b shows the behavior of the per labor capital reflected in the

horizontal arrows, such a behavior is conventional: when the per labor consumption is low the per labor capital grows at a positive growth rate, when the per labor consumption is high the per labor capital grows at a negative rate. Trajectories bellow (above) the $\triangle k = 0$ schedule correspond to and increasing (decreasing) per labor capital, as represents horizontal arrows.

It is shown in the Appendix that the LIC balanced growth path is a saddle, wether the HIC is either a node (as in figure 5.b) or a focus (as in figure 6.a). There are three possible types of dynamic behavior: i) a unique equilibrium trajectory (figure 5.a), ii) multiple equilibria for low levels of per labor capital and unique equilibrium path for high enought per labor capital (figure 5.b and 6.a), iii) multiple equilibria for any initial conditions (figure 6.b).

When HIC balanced growth path is unstable the behavior of the economy is as described in figure 5.a. In this case there is not any trajectory converging to HIC balanced growth path, and therefore the trajectory converging to the LIC balanced growth path is the unique one that satisfiy the transversality condition and consequently the unique equilibrium path.

When HIC balanced growth path is stable the behavior of the economy may be as described in figure 5.b or 6.a. There are multiple equilibria when the per labor capital is smaller than certain threshold level: the set of equilibrium consumption for a given per labor capital would be a single point and an interval $(\{\underline{c}_1(\underline{k})\} \cup (\underline{c}_2(\underline{k}), \underline{c}_3(\underline{k}))$ where $\underline{c}_1(\underline{k}) < \underline{c}_2(\underline{k}) < \underline{c}_3(\underline{k}))$. The single point correspond to the consumption of the trajectory, which converges to the LIC balanced growth path, the interval to trajectories converging to the HIC one. The shadow area in figure 5.b and 6.a represents the set of tragectories, which converges toward the HIC balanced growth path.

The third posibility is as figure 6.b shows: there are multiple equilibria for any ininitial conditions. The set of equilibrium consumption for a given per labor capital would be an interval $[\underline{c}_1(\underline{k}), \underline{c}_2(\underline{k}))$ where the lower bound of the interval $\underline{c}_1(\underline{k})$ is the consumption of the trajectory, which converges to the LIC balanced growth path and the rest of the interval to trajectories converging to the HIC one.

5.3 Dynamic Behavior when there is NFI steady state

When the productivity of the traditional technology is low enough there is NFI steady state (see proposition 1). Figure 6 displays the phase diagram corresponding to the behavior of the economy, which is similar to the case in which there is not NFI steady state. The main differnce is that now besides the paths explained above there is a path that converges to the NFI steady state.

6 Conclusion

This paper have presented a model in which the technological progress not only affects the real sector but also affect the financial sector: the productivity of the financial sector improves with technological change. The technological change also is affected by the financial sector since to adopt new technologies requires fund to invest on it. As a result of this interaction there are two types of long run equilibria: one in which the financial sector has low productivity and in which the technological progress is slow; and other in which the financial sector has a high productivity and the technological change is fast. It may also appear another in which there are neither financial sector nor technological progress and in which the economy is stagnated.

The productivity of the financial sector increases with the wealth level. When the wealth level is low the productivity of the financial technology is also low and the financial sector does not appear in equilibrium, instead it is used a traditional technology that does not generate technological progress. In this situation to invest in new technologies is not profitable since to get funds for this propose is too expensive due to the low productivity of the financial technology. When the per-labor capital is larger that certain threshold level, financial intermediation appears in equilibrium and the productivity of the financial system is high enough to incentivates firm to invest in new technologies and to generate technological progress.

It may exist multiple equilibria for given initial conditions, converging to different balanced growth paths with different growth rates. More precisely, there are three possible types of dynamic behavior: i) a unique equilibrium trajectory , ii) multiple equilibria for low levels of per labor capital and unique equilibrium path for high enought per labor capital and, iii) multiple equilibria for any initial conditions.

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8 Appendix I

Definition 1 $\gamma \equiv \left[(1-\alpha)\sum_{i=1}^{\infty}\eta_i i\right]^{-1}, \qquad \Gamma \equiv \frac{\alpha}{\phi}\left(\frac{\lambda}{1-\alpha-\lambda}\right)^{\lambda}, \qquad \xi \equiv \frac{\varphi(1-\alpha-\lambda)\phi}{\alpha}$

8.1 Balanced Growth Path

It follows from the maximization problem of the firm (8) that the profit denoted by "π", the technological choice z, the demand of labor by the firm "L_f", the demand of capital by the firm "k_f" and the per firm production denoted by "x" are as follows:

$$\pi = (1-\alpha-\lambda)x - w \tag{24}$$

$$e^z = \frac{\alpha x}{\phi(\delta + r_B)} \tag{25}$$

$$L_f = \left(\frac{\lambda x}{w}\right) + 1 \tag{26}$$

$$k_f = \phi e^z = \frac{\alpha x}{(\delta + r_B)} \tag{27}$$

$$x = \left[e^{(1-\alpha)h}\left(\frac{\lambda}{w}\right)^{\lambda}\left(\frac{\alpha}{\phi(\delta + r_B)}\right)^{\alpha}\right]^{1/(1-\alpha-\lambda)} \tag{28}$$

8.1.1 Zero Profit condition

Since firms are competitive and entry is free the following zero profit condition should be satisfied:

$$(1-\alpha-\lambda)x = w \tag{29}$$

8.1.2 Capital Market Equilibrium

The per capita demand of capital is equal to demand of capital by firms k_f (see 27) multiplied by the per capita number of firms (P). The supply of capital should be equal to the assets that consumers hold, which obviously should be equal to the per capita capital. The capital market clears when demand of capital is equal to supply of capital. :.

$$k_f P = \frac{\alpha x}{(\delta + r_B)} P = k \tag{30}$$

where $P \in [0,1]$ denotes the per capita number of firms.

8.1.3 Labor Market

The per capita demand of labor is equal to the per capita demand of labor by the real sector plus the per capita demand of labor by financial sector. The per capita labor supply is equal to one. Thus it follows from the definition of the financial intermediation technology (section 2), (26) and (**??**) that the labor market clearing condition is as follows:

$$L_f P + \frac{\varphi}{e^z}(\delta + r_B)k_f P = \underbrace{\left(\frac{\lambda x}{w} + 1\right) P}_{\text{Real Sector}} + \underbrace{\frac{\varphi}{e^z}(\delta + r_B)\frac{\alpha x}{(\delta + r_B)} P}_{\text{Financial Sector}} = 1 \tag{31}$$

8.1.4 Production growth rate along the balanced growth path:

Lemma 1: Along a Balanced growth path equilibrium the technological experience accumulation equation is as follows:

$$\underline{Z} = (1+g)^{1/(1-\alpha)\gamma} \tag{32}$$

where $\underline{Z} \equiv \frac{e^{z_t}}{e^{h_t}}$ and $\gamma \equiv \left[(1-\alpha)\sum_{i=1}^{\infty} \eta_i i\right]^{-1}$.

Proof. Lemma 1:

It follows from the technological experience accumulation equation (5) that:

$$\frac{e^{h_{t+1}}}{e^{h_t}} = \frac{e^{\sum_{i=1}^{\infty} \eta_i z_{t+1-i}}}{e^{h_t}} = \frac{1}{e^{h_t}} \prod_{i=1}^{\infty} \left(e^{z_t} \frac{e^{z_{t-i+1}}}{e^{z_t}}\right)^{\eta_i} = \frac{e^{z_t}}{e^{h_t}} \prod_{i=2}^{\infty} \left(\frac{e^{z_{t-i+1}}}{e^{z_t}}\right)^{\eta_i} \tag{33}$$

It follows from the definition of balanced growth path equilibrium that:

$$\frac{e^{h_{t+1}}}{e^{h_t}} = (1+g) \tag{34}$$

$$\frac{e^{z_{t+1}}}{e^{z_t}} = (1+g) \Rightarrow \left(\frac{e^{z_{t-i+1}}}{e^{z_t}}\right) = (1+g)^{-(i-1)} \tag{35}$$

$$\frac{e^{z_t}}{e^{h_t}} = \underline{Z} \tag{36}$$

Using equations 33 to 36:

$$(1+g) = \underline{Z}(1+g)^{-\sum_{i=2}^{\infty} \eta_i (i-1)} \Leftrightarrow$$

$$(1+g)^{\left(1-\sum_{i=2}^{\infty} \eta_i\right) + \sum_{i=2}^{\infty} \eta_i i} = {}^{5}(1+g)^{\sum_{i=1}^{\infty} \eta_i i} = (1+g)^{1/(1-\alpha)\gamma} = \underline{Z}$$

■

Define $\underline{w}_t \equiv \frac{w_t}{e^{h_t}}$, $\underline{x}_t \equiv \frac{x_t}{e^{h_t}}$, $\underline{Z}_t \equiv \frac{e^{z_t}}{e^{h_t}}$, $(1+g_{t+1}) \equiv \frac{e^{h_{t+1}}}{e^{h_t}}$. It follows from the definition of balanced path equilibrium that $\underline{w}_t$, $\underline{x}_t$, $\underline{Z}_t$, $(1+g_{t+1})$ should be constant along the balanced path equilibrium. Thus,the following equation should hold along the balanced path equilibrium:

$$(1-\alpha-\lambda)\underline{x} = \underline{w} \tag{37}$$

$$\underline{Z} = \underline{x}\left[\frac{\alpha}{(\delta+r_B)\phi}\right] \tag{38}$$

$$\underline{x} = \left[\left(\frac{\lambda}{\underline{w}}\right)^{\lambda}\left(\frac{\alpha}{(\delta+r_B)\phi}\right)^{\alpha}\right]^{1/(1-\alpha-\lambda)} \tag{39}$$

$$\psi = \frac{\varphi\underline{w}}{\underline{Z}} \tag{40}$$

$$(\delta+r_B) = (\delta+r)+\psi(\delta+r_B) \tag{41}$$

where equation 37 come from the Zero Profit Condition 29, equation 38 and 39 from the optimal choice of the technology and production of the firm (equations 25 and 28), 40 come from the definition of financial intermediation costs (6) and 41 come from the financial intermediation zero profit condition (equation 7).

Equations 32, 37, 38 and 39 imply equation 18 in the main text:

$$(1+g) = \left[\frac{\Gamma}{(\delta+r_B)}\right]^{\gamma} \tag{42}$$

where $\Gamma \equiv \frac{\alpha}{\phi}\left(\frac{\lambda}{1-\alpha-\lambda}\right)^{\lambda}$.

Equations 37, 38, 40 and 41 imply equation 19 in the main text

$$(\delta+r) = (\delta+r_B)\text{-}\xi\,(\delta+r_B)^2 \Leftrightarrow (\delta+r_B) = \frac{1 \pm \sqrt{1\text{-}4\xi\,(\delta+r)}}{2\xi} \tag{43}$$

where $\xi \equiv \frac{\varphi(1-\alpha-\lambda)\phi}{\alpha}$.

8.1.5 Existence of Balanced Growth Path Equilibrium (Proposition 1)

$F(r_B)$ is defined as follows:

$$F(r_B) \equiv g_c(r_B)^{\frac{1}{\sigma}} - g_y(r_B)^{\frac{1}{\sigma}} = \left(\frac{(1+r_B) - \xi\,(\delta + r_B)^2}{1+\rho}\right) - \left[\frac{\Gamma}{(\delta{+}r_B)}\right]^{\frac{\gamma}{\sigma}} \quad (44)$$

Define $\widehat{r}_B$ as the borrowing interest rate that make the consumption growth factor zero $\widehat{r}_B \equiv \left(\frac{1+\sqrt{1+4\xi(1-\delta)}}{2\xi} - \delta\right)$. Note that:

$$\underset{r_B \to -\delta}{Lim} F(r_B) = -\infty \quad (45)$$

$$F\left(\widehat{r}_B\right) = -\left[\frac{\Gamma}{(\delta{+}\widehat{r}_B)}\right]^{\frac{\gamma}{\sigma}} \quad (46)$$

Note also that the consumption growth and lending interest rates arrive to their maximum at $r_B = \frac{1}{2\xi} - \delta$. Thus, to have a HIC and a LIC balanced growth path the following condition is needed:

Assumption 1.a: $\rho < \overline{\rho} \equiv \min\left\{\frac{1}{4\xi} - \delta,\ \frac{(1-\delta)+\frac{1}{4\xi}-[2\Gamma\xi]^{\frac{\gamma}{\sigma}}}{[2\Gamma\xi]^{\frac{\gamma}{\sigma}}}\right\}$

Assumption 1.a implies that $F\left(\frac{1}{2\xi} - \delta\right) > 0$. It follows from the above assumption together with (45) and (46) that there is a balanced growth path in the interval $\left(-\delta, \frac{1}{2\xi} - \delta\right)$ and other in the interval $\left(\frac{1}{2\xi} - \delta, \widehat{r}_B\right)$. The balanced growth path with the borrowing interest rate in the interval $\left(-\delta, \frac{1}{2\xi} - \delta\right)$ is the LIC balanced growth the other is the HIC balanced growth path.

The following assumption guaranty that there is positive growth rate along the HIC and the LIC balanced growth path:

Assumption 1.b: $\Gamma > \underline{\Gamma} \equiv \frac{1+\sqrt{1\text{-}4\xi\delta}}{2\xi}$

Assumption 1.b implies that $F\left(\frac{1+\sqrt{1\text{-}4\xi(\delta+\rho)}}{2\xi} - \delta\right) < 1$, then $r_B^{HIC} \in \left(\frac{1}{2\xi} - \delta,\ \frac{1+\sqrt{1\text{-}4\xi(\delta+\rho)}}{2\xi} - \delta\right) \Rightarrow r^{HIC} \in \left(\rho,\ \frac{1}{4\xi} - \delta\right) \Rightarrow g_c^{HIC} = g^{HIC} > 0$.

The uniqueness of the LIC balanced growth path is obvious since $g_c'(r_B^{HIC}) > 0$. To prove the uniqueness of the HIC balanced growth path consider

$F'(r_B^{HIC})$:

$$F'(r_B^{HIC}) \; = \; \left(\delta{+}r_B^{HIC}\right) \left[\frac{\left(\delta{+}r_B^{HIC}\right) - 2\xi \left(\delta{+}r_B^{HIC}\right)^2}{1+\rho} - \frac{\gamma}{\sigma} \left[\frac{\Gamma}{(\delta{+}r_B^{HIC})} \right]^{\frac{\gamma}{\sigma}} \right] \; =$$
$$\frac{\delta{+}r_B^{HIC}}{1{+}\rho} \left[\left(1\text{-}\frac{\gamma}{\sigma}\right) \left[\left(\delta{+}r_B^{HIC}\right) - \xi \left(\delta{+}r_B^{HIC}\right)^2 \right] - \xi \left(\delta{+}r_B^{HIC}\right)^2 - \frac{\gamma}{\sigma}(1\text{-}\delta) \right] \; <$$
$$\frac{\left(\delta + r_B^{HIC}\right)}{1+\rho} \left[\left(1 - \frac{\gamma}{\sigma}\right) \left[\frac{1}{2\xi} - \xi \left(\frac{1}{2\xi}\right)^2 \right] - \xi \left(\frac{1}{2\xi}\right)^2 - \frac{\gamma}{\sigma}(1-\delta) \right] \; =$$
$$-\frac{\gamma}{\sigma} \frac{\left(\delta + r_B^{HIC}\right)}{1+\rho} \left[\frac{1}{4\xi} + (1-\delta) \right] \; < \; 0$$

where in the second equality has been used the fact that $F(r_B^{HIC}) = 0$. Since $F'(r_B^{HIC}) < 0$, it follows that HIC balanced growth path is unique. ■

8.2 Dynamics (Outside the Balanced Growth Path)

The following condition should hold in equilibrium when sophisticated technology is used (equations 23, 31, 30, 29, 25, 28, 6, 5, 1 and 16):

$$\text{Labor Market} \quad [\lambda + \alpha\psi_t] \left(\tfrac{x_t}{w_t}\right) P_t = 1 - P_t \tag{47}$$
$$\text{Capital Market} \quad e^{z_t}\phi P_t = k_t \tag{48}$$
$$\text{Zero Profit Condition} \quad (1-\alpha-\lambda)x_t = w_t \tag{49}$$
$$\text{Technology} \quad e^{z_t} = \alpha x_t \left[\tfrac{(1-\psi_t)}{(r_t+\delta)\phi}\right] \tag{50}$$
$$\text{Ouput per firm} \quad x_t = \left[e^{(1-\alpha)h_t} \left(\tfrac{\lambda}{w_t}\right)^{\lambda} \left(\tfrac{(1-\psi_t)\alpha}{(r_t+\delta)\phi}\right)^{\alpha} \right]^{1/(1-\alpha-\lambda)} \tag{51}$$
$$\text{Intermediation Cost} \quad \psi_t \equiv \tfrac{\varphi w_t}{e^{z_t}} \tag{52}$$
$$\text{Technological Experience} \quad h_{t+1} = Max\left\{\eta z_t + (1-\eta)h_t, h_t\right\} \tag{53}$$
$$\text{Capital Accumulation} \quad y_t = c_t + k_{t+1} - (1-\delta)k_t \tag{54}$$
$$\text{Euler Equation} \quad c_{t+1} = \left(\tfrac{1+r_{t+1}}{1+\rho}\right)^{\sigma} c_t \tag{55}$$
$$\text{Transversality Condition} \quad \lim_{t\to\infty} \tfrac{k_t}{\left[\prod_{i=0}^{t}(1+r_i)\right]} = 0 \tag{56}$$

It follows from Equations 49 and 51 that:

$$x_t = e^{h_t}\left[\Gamma\left(\frac{(1-\psi_t)}{(r_t+\delta)}\right)^{\alpha}\left(\frac{\phi}{\alpha}\right)^{1-\alpha}\right]^{1/(1-\alpha)} \tag{57}$$

It follows from Equations 50 and 57 that:

$$e^{z_t} = e^{h_t}\left[\Gamma\frac{(1-\psi_t)}{(r_t+\delta)}\right]^{1/(1-\alpha)} \tag{58}$$

It follows from equations 49, 50 and 52 that:

$$\frac{(r_t+\delta)}{(1-\psi_t)} = \frac{\psi_t}{\xi} \tag{59}$$

It follows from equations 47 and 49 that:

$$P_t = \frac{1-\alpha-\lambda}{1-\alpha(1-\psi_t)} \tag{60}$$

Equations 48, 58 and 60 imply:

$$\frac{(r_t{+}\delta)}{(1\text{-}\psi_t)} = \Gamma\left(\frac{\phi\,(1\text{-}\alpha\text{-}\lambda)}{1\text{-}\alpha(1\text{-}\psi_t)}\right)^{1-\alpha}\left(\frac{e^{h_t}}{k_t}\right)^{1-\alpha} = \alpha A(\psi_t)\left(\frac{1}{\underline{k}_t}\right)^{1-\alpha} \tag{61}$$

where $A(\psi) \equiv \frac{\Gamma}{\alpha}\left(\frac{\phi(1\text{-}\alpha\text{-}\lambda)}{1\text{-}\alpha(1\text{-}\psi_t)}\right)^{1-\alpha}$. Using equation 59 and 61, it is possible to define a function $\psi(.)$ that relates the financial intermediation costs with the per labor capital:

$$\begin{aligned} \psi_t &= \psi(\underline{k}_t) \\ \psi(\underline{k}) \;\Leftrightarrow\; \psi - \xi\left[\frac{\Gamma}{\alpha}\left(\frac{\phi\,(1\text{-}\alpha\text{-}\lambda)}{1\text{-}\alpha(1\text{-}\psi_t)}\right)^{1-\alpha}\right]\alpha\left(\frac{1}{\underline{k}}\right)^{1-\alpha} &= 0 \end{aligned} \tag{62}$$

It follows from the Implicit Function Theorem that $\psi(.)$ is an decreasing function and $\lim_{\underline{k}\to\infty}\psi(\underline{k}) = 0$. It follows from 48, 50, 61 and 59 that:

$$y_t = P_t x_t = \frac{(r_t+\delta)}{\alpha(1-\psi(\underline{k}_t))}k_t = \frac{\psi(\underline{k}_t)}{\alpha\xi}k_t \tag{63}$$

8.2.1 Existence of steady state without financial intermediation

Define $\underline{k}^{tr}$ as the capital level such that the lending interest rate would be the same if firms uses sophisticated technology as if firm would use traditional technology:

$$\underline{k}^{tr} \Leftrightarrow A(\psi(\underline{k}^{tr}))(1-\psi(\underline{k}^{tr})) = \varepsilon$$

The following assumption imply that the HIC balanced growth path exist ($\underline{k}^{HIC} > \underline{k}^{tr}$).

Assumption 2: $\varepsilon < \overline{\varepsilon} \equiv A(\psi^{HIC})(1-\psi^{HIC})$

Proposition: There exists $\underline{\varepsilon} \in (0, \overline{\varepsilon})$ such that if $\varepsilon \in (\underline{\varepsilon}, \overline{\varepsilon})$ there is not NFI steady state, if $\varepsilon \in (o, \underline{\varepsilon})$ there is NFI steady state.

Proof.

Define $\underline{k}^{tr}(\varepsilon)$ such that $A(\psi(\underline{k}^{tr}(\varepsilon)))(1-\psi(\underline{k}^{tr}(\varepsilon))) = \varepsilon$. Since $A(\psi(\underline{k}))(1-\psi(\underline{k}^{tr}))$ is a strictly increasing function it follows from the Implicit Function Theorem that $\underline{k}^{tr}(\varepsilon)$ is well defined and is an increasing continuous function. Define $\underline{\varepsilon}$ such that $\frac{\alpha\underline{\varepsilon}}{\underline{k}^{tr}(\underline{\varepsilon})^{1-\alpha}} - \delta = \rho$. Note that $\frac{\alpha\overline{\varepsilon}}{\underline{k}^{tr}(\overline{\varepsilon})^{1-\alpha}} - \delta = r^{HIC} > \rho$, note also that follows from the definition of $\underline{k}^{tr}(\varepsilon)$ that:

$$\frac{\alpha\underline{\varepsilon}}{\underline{k}^{tr}(\underline{\varepsilon})^{1-\alpha}} - \delta = \frac{\alpha A(\psi(\underline{k}^{tr}(\underline{\varepsilon})))(1-\psi(\underline{k}^{tr}(\underline{\varepsilon})))}{\underline{k}^{tr}(\underline{\varepsilon})^{1-\alpha}} - \delta = \frac{\left(1-\psi(\underline{k}^{tr}(\underline{\varepsilon}))\right)\psi(\underline{k}^{tr}(\underline{\varepsilon}))}{\xi} - \delta$$

thus

$$\frac{\partial\left(\frac{\alpha\underline{\varepsilon}}{\underline{k}^{tr}(\underline{\varepsilon})^{1-\alpha}} - \delta\right)}{\partial\varepsilon} = \frac{\partial\left(\frac{\left(1-\psi(\underline{k}^{tr}(\underline{\varepsilon}))\right)\psi(\underline{k}^{tr}(\underline{\varepsilon}))}{\xi} - \delta\right)}{\partial\Psi} \frac{\partial\Psi(\underline{k}^{tr}(\underline{\varepsilon}))}{\partial\underline{k}} \frac{\partial\underline{k}^{tr}(\underline{\varepsilon})}{\partial\underline{\varepsilon}} =$$

$$\underbrace{\frac{1-2\psi(\underline{k}^{tr}(\underline{\varepsilon}))}{\xi}}_{(-)} \underbrace{\Psi'(\underline{k}^{tr}(\underline{\varepsilon}))}_{(-)} \underbrace{\underline{k}^{tr}(\underline{\varepsilon})}_{(+)} > 0 \tag{64}$$

where it has been used the fact that

$$\psi(\underline{k}^{tr}(\underline{\varepsilon})) > \psi^{HIC} = = \frac{1+\sqrt{1\text{-}4\xi\,(\delta+r^{HIC})}}{2} > \frac{1}{2}$$

It follows from the definition of $\underline{\varepsilon}$ and (64) that:

- if $\varepsilon > \underline{\varepsilon} \Rightarrow \frac{\alpha\varepsilon}{\underline{k}^{tr}(\varepsilon)^{1-\alpha}} - \delta > \frac{\alpha\underline{\varepsilon}}{\underline{k}^{tr}(\underline{\varepsilon})^{1-\alpha}} - \delta = \rho \Rightarrow$ There is not NFI steady state.

- if $\varepsilon < \underline{\varepsilon} \Rightarrow \frac{\alpha\varepsilon}{\underline{k}^{tr}(\varepsilon)^{1-\alpha}} - \delta < \frac{\alpha\underline{\varepsilon}}{\underline{k}^{tr}(\underline{\varepsilon})^{1-\alpha}} - \delta = \rho \Rightarrow$ There is NFI steady state.

■

8.2.2 Dynamic Behavior

The dynamic behavior of the economy depends on the initial per labor capital. If the per labor capital is smaller than "$\underline{k}^{tr}$" firms in the economy use traditional technology, otherwise firms use sophisticated technology.

Case in which $\underline{k} < \underline{k}^{tr}$: In this case the dynamic equations of the system is very similar to the conventional Ramsey model:

$$\begin{aligned} e^{h_{t+1}} &= e^{h_t} \\ k_{t+1} &= \varepsilon\, e^{(1-\alpha)h_t} k_t^{\alpha} + (1-\delta)k_t - c_t \\ \frac{c_{t+1}}{c_t} &= \left[\frac{1}{1+\rho}\left((1-\delta) + \alpha\varepsilon\left(\frac{e^{h_t}}{k_{t+1}}\right)^{1-\alpha}\right)\right]^{\sigma} \end{aligned}$$

The above dynamic system may be rewritten as follows:

$$\underline{k}_{t+1} = \varepsilon\, \underline{k}_t^{\alpha} + (1-\delta)\underline{k}_t - c_t \tag{65}$$

$$\underline{c}_{t+1} = \left[\frac{1}{1+\rho}\left((1-\delta) + \alpha\varepsilon\left(\frac{1}{\underline{k}_{t+1}}\right)^{1-\alpha}\right)\right]^{\sigma} \underline{c}_t \tag{66}$$

where $\underline{k}_t \equiv \frac{k_t}{e^{h_t}}$ and $\underline{c}_t \equiv \frac{c_t}{e^{Max\{\eta z_t + (1-\eta)h_t, 1\}}} = \frac{c_t}{e^{h_{t+1}}} = \frac{c_t}{e^{h_t}}$.

Case in which $\underline{k} > \underline{k}^{tr}$: It follows from 58 and 53 and 59 that[6]:

$$e^{h_{t+1}} = e^{h_t} Max\left\{\left(\frac{\Gamma\xi}{\psi(\underline{k}_t)}\right)^{\gamma}, 1\right\} \tag{67}$$

It follows from 54 and 63 that:

$$k_{t+1} = \left((1\text{-}\delta) + \frac{\psi(\underline{k}_t)}{\alpha\xi}\right) k_t - c_t \tag{68}$$

It follows from 55 and 59 that:

$$\frac{c_{t+1}}{c_t} = \left[\frac{1}{1+\rho}\left((1-\delta) + \frac{(1-\psi(\underline{k}_{t+1}))\,\psi(\underline{k}_{t+1})}{\xi}\right)\right]^{\sigma} \tag{69}$$

[6] For this technological experience accumulation equation $\gamma \equiv \frac{\eta}{1-\alpha}$.

67, 68and 69 imply the following dynamic system:

$$\underline{k}_{t+1} = \left[(1\text{-}\delta) + \frac{\psi(\underline{k}_t)}{\alpha\xi}\right] \min\left\{\left(\frac{\psi(\underline{k}_t)}{\Gamma\xi}\right)^{\gamma}, 1\right\} \underline{k}_t - \underline{c}_t \tag{70}$$

$$\underline{c}_{t+1} = \left[\frac{(1-\delta) + \frac{(1\text{-}\psi(\underline{k}_{t+1}))\psi(\underline{k}_{t+1})}{\xi}}{1+\rho}\right]^{\sigma} \min\left\{\left[\frac{\psi(\underline{k}_{t+1})}{\Gamma\xi}\right]^{\gamma}, 1\right\} \underline{c}_t \tag{71}$$

where $\underline{c}_t \equiv \frac{c_t}{e^{Max\{\eta z_t+(1-\eta)h_t,1\}}} = \frac{c_t}{e^{h_t} Max\left\{\left[\frac{\Gamma\xi}{\psi(\underline{k}_t)}\right]^{\gamma},1\right\}} = \frac{c_t}{e^{h_{t+1}}}$.

8.2.3 Dynamic System

It follows from (65), (66), (70) and (70) that the dynamic system that describes the behavior of the economy is as follows:

$$\underline{k}_{t+1} = F_{\underline{k}}(\underline{k}_t) - \underline{c}_t \tag{72}$$

$$\underline{c}_{t+1} = F_c\left(\underline{k}_{t+1}\right)\ \underline{c}_t = F_c\left(F_{\underline{k}}(\underline{k}_t) - \underline{c}_t\right)\underline{c}_t \tag{73}$$

where:

$$F_{\underline{k}}(\underline{k}) \equiv \begin{cases} \varepsilon\underline{k}^{\alpha} + (1-\delta)\underline{k} & if \quad \underline{k} < \underline{k}^{tr} \\ \left[\frac{\psi(\underline{k})\underline{k}}{\alpha\xi} + (1\text{-}\delta)\underline{k}\right] \min\left\{\left(\frac{\psi(\underline{k})}{\Gamma\xi}\right)^{\gamma}, 1\right\} & if \quad \underline{k} \geq \underline{k}^{tr} \end{cases} \tag{74}$$

$$F_c(\underline{k}) \equiv \begin{cases} \left[\frac{(1-\delta)+\alpha\varepsilon\left(\frac{1}{\underline{k}}\right)^{1-\alpha}}{1+\rho}\right]^{\sigma} & if \quad \underline{k} < \underline{k}^{tr} \\ \left[\frac{(1-\delta)+\frac{(1\text{-}\psi(\underline{k}_{t+1}))\psi(\underline{k}_{t+1})}{\xi}}{1+\rho}\right]^{\sigma} \min\left\{\left(\frac{\psi(\underline{k}_{t+1})}{\Gamma\xi}\right)^{\gamma}, 1\right\} & if \quad \underline{k} \geq \underline{k}^{tr} \end{cases} \tag{75}$$

8.2.4 Dynamic Behavior: Local Analysis

The dynamic system 72-73 may be linearized around the steady state:

$$\begin{bmatrix} \underline{k}_{t+1}\text{-}\underline{k}^{ss} \\ \underline{c}_{t+1}\text{-}\underline{c}^{ss} \end{bmatrix} = \begin{bmatrix} F'_{\underline{k}}(\underline{k}^{ss}) & \text{-}1 \\ F'_c\left(\underline{k}^{ss}\right) F'_{\underline{k}}(\underline{k}^{ss})\ \underline{c}^{ss} & \text{-}F'_c\left(\underline{k}^{ss}\right)\underline{c}^{ss}\text{+}1 \end{bmatrix} \begin{bmatrix} \underline{k}_t\text{-}\underline{k}^{ss} \\ \underline{c}_t\text{-}\underline{c}^{ss} \end{bmatrix} \tag{76}$$

The corresponding eigenvalues are:

$$\lambda = \frac{F'_{\underline{k}}(\underline{k}^{ss})\text{-}F'_c(\underline{k}^{ss})\,\underline{c}^{ss}{+}1 \pm \sqrt{\left(F'_{\underline{k}}(\underline{k}^{ss})\text{-}F'_c(\underline{k}^{ss})\,\underline{c}^{ss}{+}1\right)^2 - 4F'_{\underline{k}}(\underline{k}^{ss})}}{2} =$$

$$1{+}\frac{F'_{\underline{k}}(\underline{k}^{ss})\text{-}F'_c(\underline{k}^{ss})\,\underline{c}^{ss}\text{-}1 \pm \sqrt{\left(F'_{\underline{k}}(\underline{k}^{ss})\text{-}F'_c(\underline{k}^{ss})\,\underline{c}^{ss}\text{-}1\right)^2 - 4F'_c(\underline{k}^{ss})\,\underline{c}^{ss}}}{2}$$

LIC balanced growth path: in this case $F'_c(\underline{k}^{ss}) < 0$. Therefore both eigenvalues are real and one of them is larger than one and the other smaller, so this balanced growth path is a saddle.

HIC balanced growth path: in this case $F'_c(\underline{k}^{ss}) > 0$. There are four possibilities:

- $F'_{\underline{k}}(\underline{k}^{HIC})\text{-}F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC}\text{-}1 > 0$ and $\left(F'_{\underline{k}}(\underline{k}^{HIC})\text{-}F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC}\text{-}1\right)^2 - 4F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC} > 0$. Both eigenvalues are real and larger than one. The *HIC* balanced growth path is and unstable node.

- $F'_{\underline{k}}(\underline{k}^{HIC})\text{-}F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC}\text{-}1 < 0$ and $\left(F'_{\underline{k}}(\underline{k}^{HIC})\text{-}F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC}\text{-}1\right)^2 - 4F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC} > 0$. Both eigenvalues are real and smaller than one. The *LIC* balanced growth path is an stable node.

- $F'_{\underline{k}}(\underline{k}^{HIC})\text{-}F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC}\text{-}1 > 0$ and $\left(F'_{\underline{k}}(\underline{k}^{HIC})\text{-}F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC}\text{-}1\right)^2 - 4F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC} < 0$. The eigenvalues are a complex conjugate pair, with real part larger than one. The *HIC* balanced growth path is an unstable focus.

- $F'_{\underline{k}}(\underline{k}^{HIC})\text{-}F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC}\text{-}1 < 0$ and $\left(F'_{\underline{k}}(\underline{k}^{HIC})\text{-}F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC}\text{-}1\right)^2 - 4F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC} < 0$. The eigenvalues are a complex conjugate pair, with real part smaller than one. The *HIC* balanced growth path is an stable focus.

The important thing is that if $F'_{\underline{k}}(\underline{k}^{HIC}) < F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC} + 1$ then there is a ball arround the *HIC* "steady state" such that any of these points has a path that satisfied the dynamic system 72-73 and tend to the *HIC* balanced growth path. This means that all of those path satisfy transversality condition and thus they are equilibrium. Therefore the equilibrium is localy indeterminate: for a capital level close enought to the HIC level, there are a interval of consumptions levels such that the paths that satisfy 72-73 and start at such consumption level tend to the HIC "steady state". Thus, all the consumptions in such interval are consistent with the definition of equilibrium.

Proposition: If $\alpha \leq \frac{1}{2}$ there is a $\underline{\eta} < 2\alpha$ such that if $\eta > \underline{\eta}$ then there is a interval around k^{HIC} such that the equilibrium is indeterminated.

Proof.

It follows from the definition of "$F_{\underline{k}}(\underline{k})$" that if $\underline{k} > \underline{k}^{tr}$

$$F'_{\underline{k}}(\underline{k}) = \frac{F_{\underline{k}}(\underline{k})}{\underline{k}} + \left[\frac{F_{\underline{k}}(\underline{k})}{\underline{k}} - \frac{1-\delta}{1+g(\underline{k})}\right]\frac{\psi'(\underline{k})\underline{k}}{\psi(\underline{k})} + \gamma\frac{F_{\underline{k}}(\underline{k})}{\underline{k}}\frac{\psi'(\underline{k})\underline{k}}{\psi'(\underline{k})} =$$
$$\frac{1}{1+g(\underline{k})}\left\{\frac{\psi(\underline{k})}{\alpha\xi}\left[1+(1+\gamma)\frac{\psi'(\underline{k})\underline{k}}{\psi(\underline{k})}\right] + (1\text{-}\delta)\,(1+\gamma)\,\frac{\psi'(\underline{k})\underline{k}}{\psi(\underline{k})}\right\}$$

$\left[\frac{\psi(\underline{k})\underline{k}}{\alpha\xi} + (1\text{-}\delta)\underline{k}\right]$It follows from the definition of "$\psi(\underline{k})$" and Implicit Function Theorem that the elasticity of the finanicar intermediation costs with respect to the per labor capital is as follows:

$$\frac{\psi'(\underline{k})\underline{k}}{\psi(\underline{k})} = -\frac{(1-\alpha)}{1+\frac{(1-\alpha)\alpha\psi(\underline{k})}{(1-\alpha(1\text{-}\psi(\underline{k})))}} < -\frac{1-\alpha}{1+\alpha}$$

$$F'_{\underline{k}}(\underline{k}^{HIC}) < \frac{F_{\underline{k}}(\underline{k}^{HIC})}{\underline{k}^{HIC}}\left[1-\frac{(1-\alpha)(1+\gamma)}{1+\alpha}\right] + \frac{1-\delta}{1+g^{HIC}}\frac{1-\alpha}{1+\alpha} =$$
$$\frac{F_{\underline{k}}(\underline{k}^{HIC})}{\underline{k}^{HIC}}\left[\frac{2\alpha-\eta}{1+\alpha}\right] + \frac{1-\delta}{1+g^{HIC}}\frac{1-\alpha}{1+\alpha} <$$
$$\frac{1}{1+g^{HIC}}\left[\frac{1}{\varphi(1-\alpha-\lambda)\phi}\left[\frac{2\alpha-\eta}{1+\alpha}\right] + (1-\delta)\left[\frac{1+\alpha-\eta}{1+\alpha}\right]\right]$$

$\left[(1\text{-}\delta) + \frac{\psi(\underline{k}_t)}{\alpha\xi}\right]$

If $\eta \geq 2\alpha$ $F'_{\underline{k}}(\underline{k}) < 1$, then there is $\underline{\eta} < 2\alpha$ such that if $\eta < \underline{\eta}$ then $F'_{\underline{k}}(\underline{k}^{HIC}) < F'_c\left(\underline{k}^{HIC}\right)\underline{c}^{HIC} + 1.$ ■

Proposition: If $\alpha \leq \frac{1}{2}$ and $\eta > \underline{\eta}$ then there is $\widetilde{\varepsilon} \in (0, \overline{\varepsilon})$ such that if $\varepsilon > \widetilde{\varepsilon}$ there is a interval around k^{HIC} such that the equilibrium is indeterminated.

$$\psi(\underline{k}) \Leftrightarrow \psi - \xi \left[\frac{\Gamma}{\alpha} \left(\frac{\phi(1\text{-}\alpha\text{-}\lambda)}{1\text{-}\alpha(1\text{-}\psi_t)} \right)^{1-\alpha} \right] \alpha \left(\frac{1}{\underline{k}} \right)^{1-\alpha} = 0$$

9 Appendix II: Optimal Incentive Compatible Contract

As it was mentioned in the main text, the financial intermediation costs presented in this paper may be interpreted in different ways. It may be interpreted as a monitoring costs (as Diamond, 1984), as an informational costs (as Williamson, 1986) or simply as a transaction costs. In this section the model is slightly modified in order to show that the financial technology presented in the main text is the same as the result of an optimal incentive compatible contract with asymmetric information.

The production function of firms is very similar to the main text but now the firm production depends upon a firm specific stochastic shock denoted by θ^i:

$$F(k_f, L_p, L_m; z, z\text{-}h, \theta^i) = \begin{cases} \left(\theta^i\right)^{1-\lambda} e^z e^{-(1-\alpha)(z-h)} L_p^\lambda & If\ k_f \geq e^z \phi\ L_m \geq 1 \\ 0 & \text{otherwise} \end{cases} \tag{77}$$

where L_p denotes the number of workers that work directly in production and L_m the number of managers. This mean that besides the workers that work directly in production, the firm needs a manager in order to organize production. It is easy to check by substituting the equality $L_f = L_p + L_m = L_p + 1$ in (77) that the above production function (77) is the same that in the one in the main text (4) but with a stochastic shock.

The firm specific stochastic shock θ^i is independently distributed across firms and is distributed according with the Constant Hazard Rate Distribution Function:

$$G(\theta^i) = 1 - e^{-\theta^i} \qquad \theta^i \in \Re_+ \tag{78}$$

There is asymmetric information between firms and lenders. In the first period neither the lender nor the firm know the value of the stochastic shock. In the second period, the firm observes the stochastic shock free of charge. The lender can observe the firm's cash flow only at some cost that is refereed to as an enforcement costs. The enforcement costs are proportional to the expected cash flow of the firm. The enforcement costs per unit of expected cash flow are denoted by ψ.

The sequence of decisions that a firm makes is as follows (see figure bellow): the firm invests one period before that the production takes place. At the beginning of the period in which the production takes place the firm hires a manager, after that the stochastic shock is realized and it is observed by the manager who hires the labor. Thus, the production takes and the manager pays the workers, before paying the financial intermediary. After paying the workers the managers announces the realization of the stochastic shock, the financial intermediator chooses whether to observe (at some cost) or not the realization of the stochastic shock, after that the payment of the financial intermediator takes place. Finally the firm pays his manager (the free entry condition implies that the firm's profit is zero).

$$\boxed{\textit{Investment}} \rightarrow \boxed{\begin{array}{l}\textit{Hiring}\\ \textit{Manager}\end{array}} \rightarrow \boxed{\theta} \rightarrow \boxed{\begin{array}{l}\textit{Hiring}\\ \textit{Labor}\end{array}} \rightarrow \boxed{\begin{array}{l}\textit{Payment}\\ \textit{wor\underline{k}ers}\end{array}} \rightarrow$$

$$\boxed{\begin{array}{l}\textit{Announc.}\\ \textit{Shoc\underline{k}}\ \theta\end{array}} \rightarrow \boxed{\begin{array}{l}\textit{Financial Int.}\\ \textit{may observ}\ \theta\end{array}} \rightarrow \boxed{\begin{array}{l}\textit{Payment}\\ \textit{Financ. I.}\end{array}} \rightarrow \boxed{\begin{array}{l}\textit{Payment}\\ \textit{Manager}\end{array}}$$

In order to analyze the firm's decision backward induction is applied, hiring decision is analyzed first, investment decision is analyzed later.

9.1 Hiring Decision:

After the stochastic shock is realized the firm decides the amount of workers to be hired to maximize his cash-flow:

$$\underset{L_p}{Max}\left(\theta^i\right)^{1-\lambda} e^{(1-\alpha)h}\left(\frac{k_f}{\phi}\right)^{\alpha} L_p^{\lambda} - wL_p \tag{79}$$

It is easy to check that the firm optimal cash flow function is proportional to the realization of the stochastic shock:

$$\underset{L_p}{Max}\ \theta^{\alpha} e^{(1-\alpha)h}\left(\frac{k_f}{\phi}\right)^{\alpha} L_p^{\lambda} - wL_p = \theta\pi(w, k_f) \tag{80}$$

where $\pi(w, k_f)$ denotes the expected cash-flow of the firm:

$$\pi(w, k_f) = (1\text{-}\lambda)\left(\frac{e^{(1-\alpha)h}\left(\frac{k_f}{\phi}\right)^{\alpha}\lambda^{\lambda}}{w^{\lambda}}\right)^{\frac{1}{1-\lambda}} \tag{81}$$

Note that the expected cash flow of the firm when there is stochastic shock is the same that the cash flow when there is not stochastic shocks:

$$\pi(w,k_f) = \underset{L_p}{Max}\; e^{(1-\alpha)h}\left(\frac{k_f}{\phi}\right)^{\alpha} L_p^{\lambda} - wL_p \tag{82}$$

9.2 Optimal Incentive Compatible Contract

Following Gale and Hellwig (1985) the optimal incentive compatible contract between a firm and a financial intermediator is the debt contract:

$$\underset{k_f,\theta_0,r^B}{Max} \int_{\theta_0}^{\infty} \left[\theta\pi(w,k_f) - (r^B+\delta)k_f\right] dG - w \tag{83}$$

$$s.t.\;\; \theta_0\pi(w,k_f) = (\delta+r^B)k_f \tag{84}$$

$$\int_{\theta_0}^{\infty}(\delta+r^B)k_f dG(\theta) + \int_0^{\theta_0}[\theta\pi(w,k_f)\text{-}\psi\pi(w,k_f)]dG(\theta) = (\delta+r)k_f \tag{85}$$

where r^B denotes the borrowing interest rate, r denotes the lending interest rate, θ_0 denotes bankruptcy point.

The bankruptcy point is the value such that if a firm receives a realization of the stochastic shock smaller than such value the firm will not be able to pay that borrowing interest rate. When the realization of the stochastic shock is larger than the bankruptcy point, the financial intermediator will not observe the realization of the stochastic shock and will receive the borrowing interest rate. When the realization of the stochastic shock is smaller than the bankruptcy point the financial intermediator will observe the realization of the stochastic shock and will receive as a payment the firms cash flow minus the enforcement costs.

The opportunity cost of the manager is to work directly in production and to get the salary "w". This opportunity cost has been subtracted from the profits of the firm.

The first restriction (84) is the definition of bankruptcy point and the second one (85) is the financial intermediator zero profit condition.

Substituting (84) in (83) and (85) and integrating by parts[7], the debt contract 83-85 may be rewritten as follows:

[7] $\int_0^{\theta_0}\theta dG(\theta) = -\theta_0[1\text{-}G(\theta_0)] + \int_0^{\theta_0}[1\text{-}G(\theta)]d\theta$

$$\underset{k_f,\theta_0}{Max}\ \pi(w,k_f)-w-(r+\delta)k_f-\psi G(\theta_0)\pi(w,k_f) \tag{86}$$
$$s.t.\left[\int_0^{\theta_0}[1-G(\theta)]d\theta-\psi G(\theta_0)\right]\pi(w,k_f)-(r+\delta)k_f\ =\ 0$$

Since $G(\theta)=1-e^{-\theta}$, it follows that

$$[1-G(\theta)]=G'(\theta)\Rightarrow\int_0^{\theta_0}[1-G(\theta)]d\theta=\int_0^{\theta_0}G'(\theta)d\theta=G(\theta_0)$$

Substituting the above expression in (86) it follows that:

$$\underset{k_f,\theta_0}{Max}\ \pi(w,k_f)-w-(r+\delta)k_f-\psi G(\theta_0)\pi(w,k_f)c \tag{87}$$
$$s.t.(1-\psi)G(\theta)\pi(w,k_f)\ =\ (r+\delta)k_f$$

Substituting the restriction in the objective function it follows that:

$$\underset{k_f}{Max}\ \pi(w,k_f)-w-(r+\delta)k_f-\frac{\psi}{1-\psi}(r+\delta)k_f \tag{88}$$

Substituting (82) in (88) it follows that:

$$\underset{k_f}{Max}\left[\underset{L_p}{Max}\ e^{(1-\alpha)h}\left(\frac{k_f}{\phi}\right)^{\alpha}L_p^{\lambda}-wL_P\right]-w-\frac{(r+\delta)}{1-\psi}k_f\ \Leftrightarrow$$
$$\underset{k_f,L_p}{Max}\ e^{(1-\alpha)h}\left(\frac{k_f}{\phi}\right)^{\alpha}L_p^{\lambda}-w\left(L_p+1\right)-\frac{(r+\delta)}{1-\psi}k_f\ \Leftrightarrow$$
$$\underset{k_f,L}{Max}\ e^{(1-\alpha)h}\left(\frac{k_f}{\phi}\right)^{\alpha}\left(L_f-1\right)^{\lambda}-wL-\frac{(r+\delta)}{1-\psi}k_f \tag{89}$$

where $L_f\equiv L_p+L_m\equiv L_p+1$. That is "$L_f$" is the expected number of workers that hire the firm, which is equal to the expected number of workers that work directly in production "L_p" plus the manager that organize production. It follows from equation (7) in the main text that the firm maximization problem (89) is exactly the same than the one presented in the main text (12).

Figure 1: Production Growth Rate

a) Growth rate and borrowing interest rate **b)** Lending and borrowing interest rates

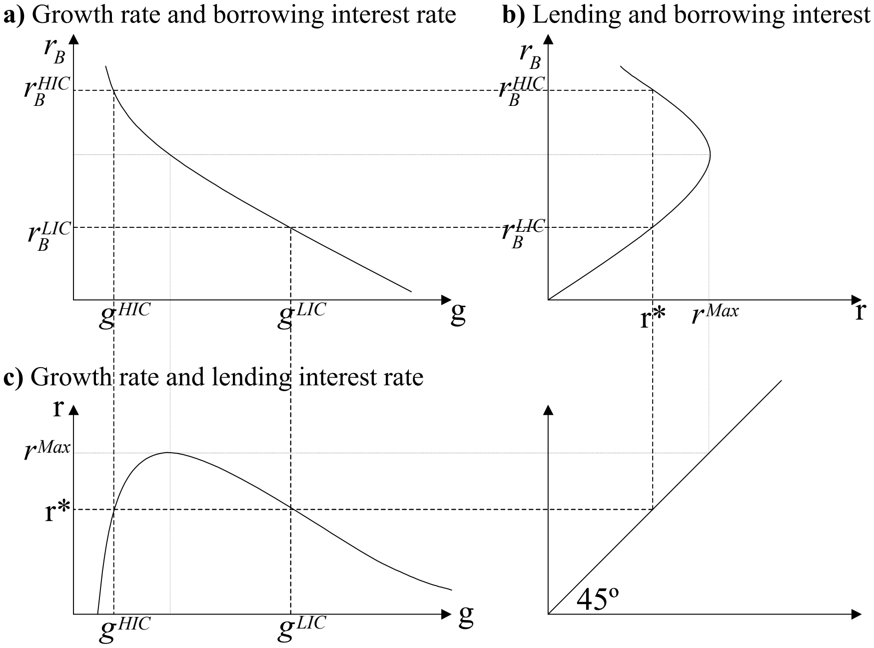


Figure 2

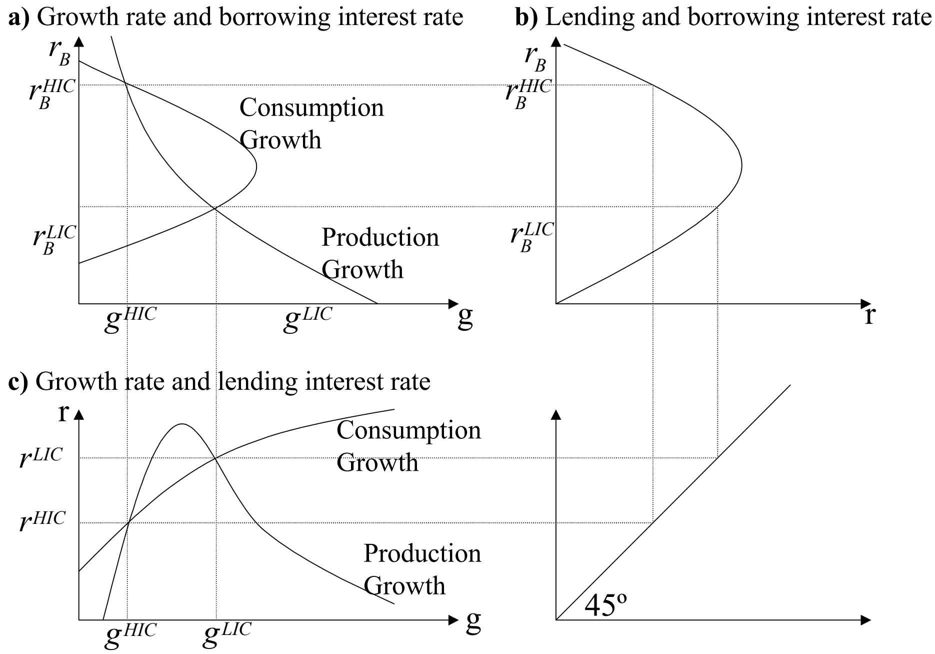
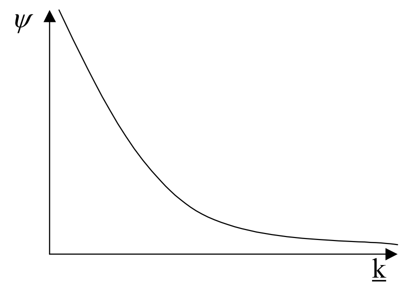
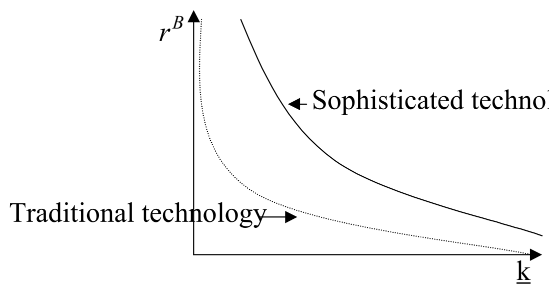

a) Growth rate and borrowing interest rate
b) Lending and borrowing interest rate
r_B
r_B^{HIC}
r_B^{LIC}
Consumption Growth
Production Growth
g^{HIC}
g^{LIC}
g
r
c) Growth rate and lending interest rate
r
r^{LIC}
r^{HIC}
Consumption Growth
Production Growth
g^{HIC}
g^{LIC}
g
45º

Figure 3

A) Financial Intermediation Costs ψ



B) Borrowing Interest Rate (Net Marginal Product of Capital)



C) Lending Interest Rate: $r = \left(r^B + \delta\right)(1-\psi) - \delta$

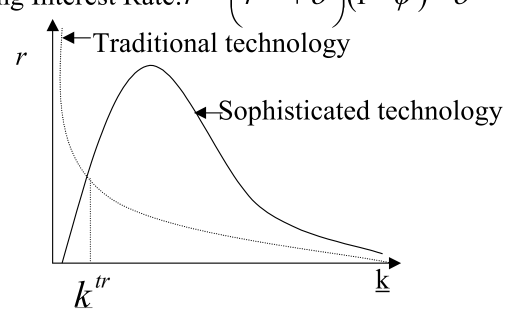


Figure 4

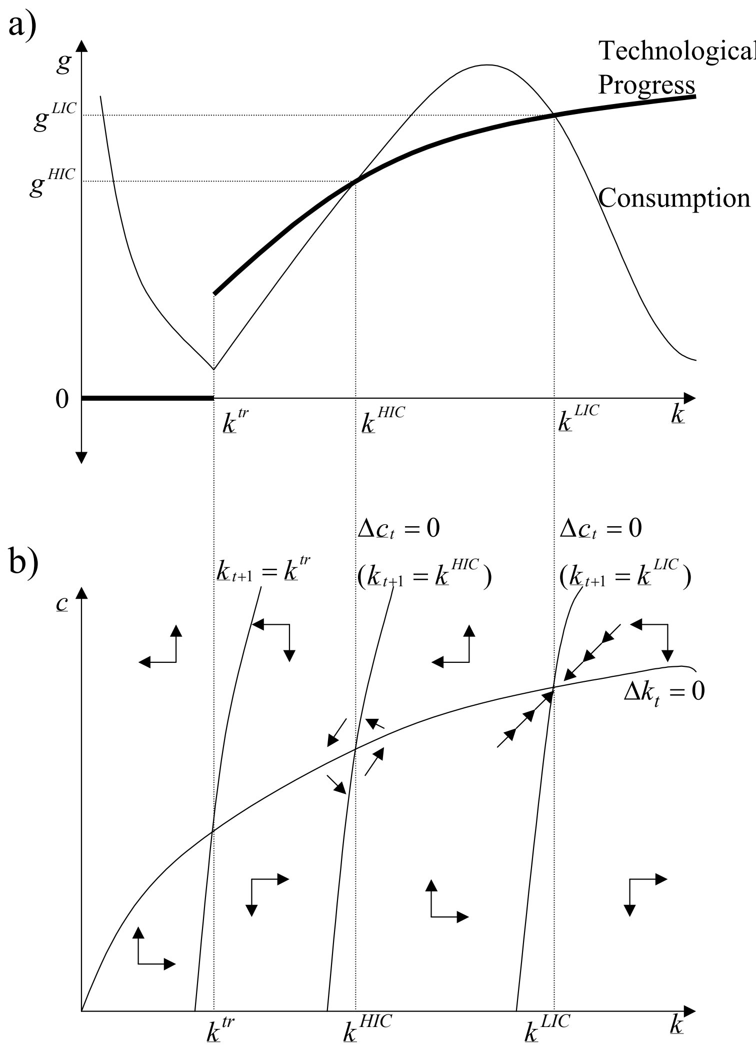
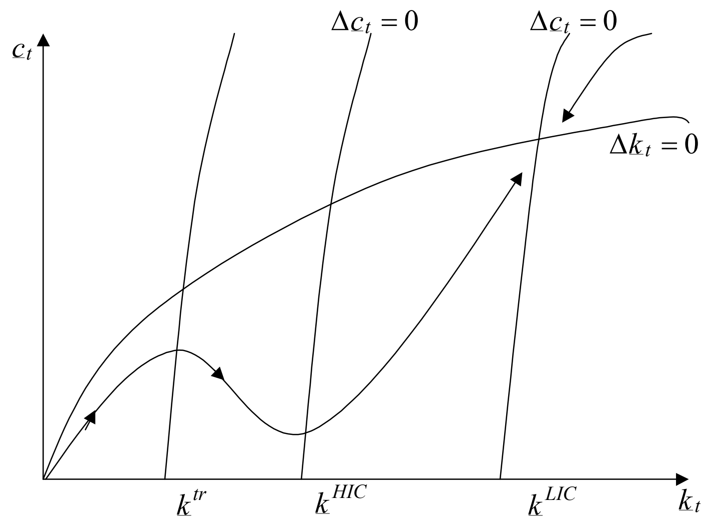

a)
g
g^{LIC}
g^{HIC}
Technological Progress
Consumption
0
k^{tr}
k^{HIC}
k^{LIC}
k
b)
c
$k_{t+1} = k^{tr}$
$\Delta c_t = 0$
$(k_{t+1} = k^{HIC})$
$\Delta c_t = 0$
$(k_{t+1} = k^{LIC})$
$\Delta k_t = 0$
k^{tr}
k^{HIC}
k^{LIC}
k

Figure 5

a) HIC Balanced growth path is an unstable node or focus.



b) HIC Balanced growth path is a stable focus.

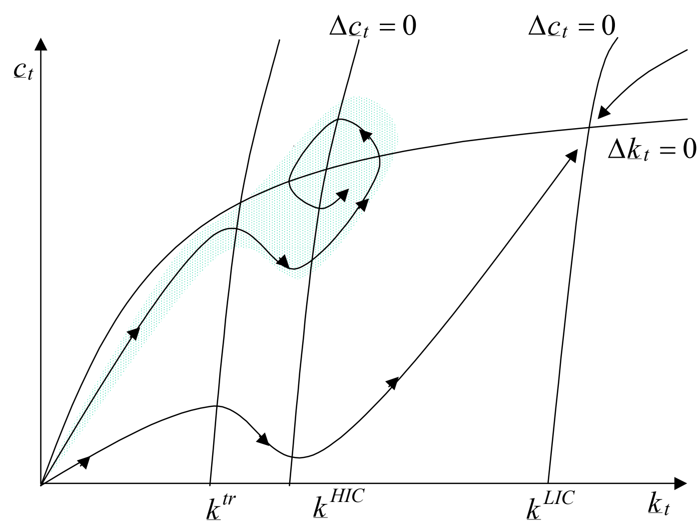
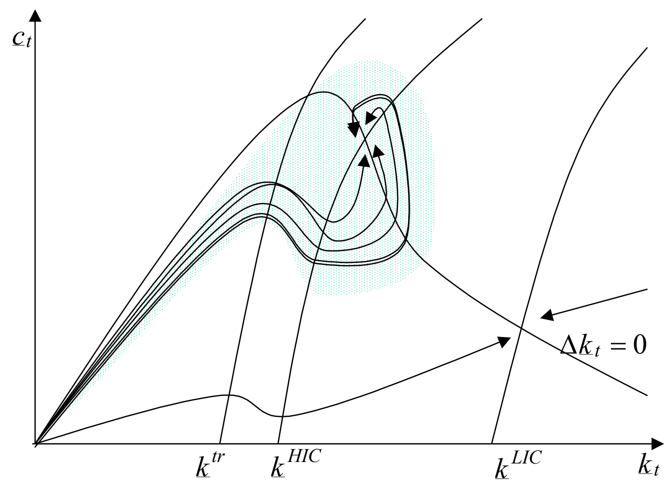


Figure 6

c) HIC Balanced growth path is an stable node



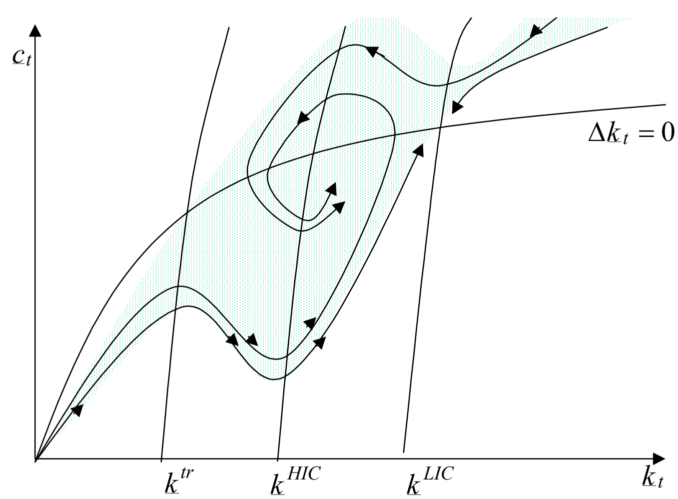
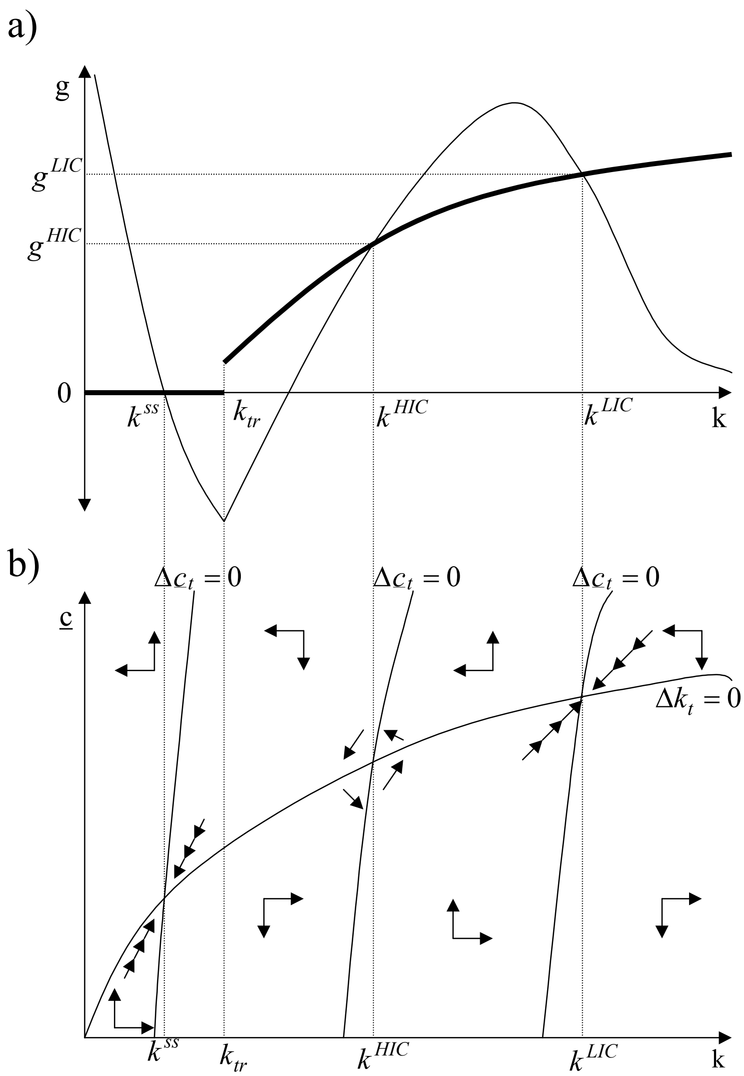


Figure 7



a)
g
g^{LIC}
g^{HIC}
0
k^{ss}
k_{tr}
k^{HIC}
k^{LIC}
k
b)
$\Delta c_t = 0$
$\Delta c_t = 0$
$\Delta c_t = 0$
c
$\Delta k_t = 0$
k^{ss}
k_{tr}
k^{HIC}
k^{LIC}
k