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R&D, Increasing Technological Complexity and the Productivity Slow Down

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Abstract

This paper presents an endogenous growth model with *R&D* in order to jointly explain the productivity slowdown and the rising wage inequality suffered by developed Wold during the last three decades. An increase in technological complexity makes new technology more intensive in skilled labor, rising the demand for this factor and its wage. This increment in skill wage reduces the demand for new technologies, which are more intensive in skilled labor, inhibiting *R&D* and technological change.

I. Introduction

Total factor productivity has slow down during the last three decades. Table 1 shows that total factor productivity growth rate were significantly higher during the period 1950-73, that during the period 1973-1993 in OECD countries. Total factor productivity growth rate during the period 1973-93 was less than one third the rates in the period 1950-1973 for most countries. Since total factor productivity growth is conventionally explained as the result of technological improvements, these data seems to suggest that technological change has slow down during the last three decades. At the same time in many developed countries unskilled workers have suffered reduced relative wages (specially in United State and United Kingdom) and increased unemployment (especially in Continental Europe)[1].

Table 1: Total factor productivity 1950-1993

	1950-73	1974-93		1950-73	1973-93
Australia	1.7	0.8	Ireland	2.9	2.2
Austria	3.0	0.6	Japan	4.6	1.0
Belgium	3.7	1.2	Netherl.	3.5	1.2
Canada	2.0	0.3	Norway	2.4	0.3
Denmark	2.3	0.9	Spain	2.5	1.1
Finland	2.7	1.5	Sweden	2.5	0.6
France	3.8	1.3	Swithzer.	1.8	0.0
Germany	2.5	1.0	U.K.	2.5	1.1
Grece	1.9	0.0	U.S.	1.5	0.0
Italy	4.1	1.2	OECD	2.7	0.8

Source: Eglander and Gurney (1994)

This paper argues that both of these empirical phenomena may be the result of the same process: increasing technological complexity. Complex technologies are intensive in skilled labor, thus an increase in technological complexity produces a shift in the demand for skilled labor, rising the skill wage. Since new technologies are more intensive in skilled labor, this shift in skill wage make new technologies less profitable, reducing the demand for them and slowing down technological change.

There is an increasing consensus among economist that rising wage inequality has been the result of biased technological change. However most papers in the literature argue that the aceleration of technological change speed has produced this biased

[1]See for example Beman, Bound and Machin (1998) or Machin and Van Reenen (1998) for empirical evidence

technological change, which is the culprit of rising wage inequality. By contrast, this paper argue that biased technological change has reduced technological change speed, and henceforth it is the cause of both productivity slow down and labor demand shifts against unskilled workers. The following diagram displays the difference between previous literature and the present paper:

Standard literature:

Speed up Technological Change $\Rightarrow$ Biased Technological Change $\Rightarrow$ Rise in Skill wage

This paper:

Biased Technological Change $\Rightarrow$ Rise in Skill wage $\Rightarrow$ Slow down Technological Change

This paper presents a endogenous growth model with $R\&D$, in which new technologies are more intensive in skilled labor than standard ones and in which technologies become standard by a process of learning by doing. Skilled labor is relatively scare and expensive with respect to unskilled labor. In this context a biased technological change that makes the share of skilled labor to rise, not only increases wage inequality but also reduces technological change speed. There are three reasons for this: i) The rise in the relative wage of skilled labor resulting from biased technological change makes new technologies less profitable since these are more intensive in skilled labor than standard ones. As a result the demand for new technologies and the profitability of $R\&D$ falls, slowing down technological change. ii) Since skilled labor is relatively scare, a biased technological change that increases the share of the relatively scare factor at the expense of the relatively abundant one (unskilled labor), reduces the production for a given amount of resources. This also reduces the speed of technological change. iii) If biased technological change only affects new technologies, the profitability of new technologies will fall, since skilled labor is the relatively expensive factor. Thus the demand for new technologies shifts downward, reducing $R\&D$ and technological change speed.

The paper identifies four factors that rise wage inequality and slow down productivity simultaneously: i) a biased technological change that affects only new technologies; ii) a biased technological change that affects all technologies (new and standard); iii) a slow down in the standardization process; iv) a rise in the minimum talent required to work as skilled worker. All of these changes may be interpreted as an intensification in technological complexity. More complex technologies require more skilled labor; thus biased technological change may be caused by an increase in technological complexity. Complex technologies are also more difficult to learn how to use it; thus, a reduction in the speed of the standardization process may also be interpreted as an increase in technological complexity. Skilled workers that use complex technologies may need more talent. Summarizing, complex technologies are intensive in skilled labor, and rise wage inequality. It also makes new technologies less profitable, since these are intensive in the factor that has become more expensive (skilled labor) and hence reduces the demand for new technologies and discourages *R&D*.

There is a recent and important literature about technological change and wage inequality (see among others Acemoglu, 1998, 2000, 2002; Caselli, 1999; Krusell, Ohanian, Rios-Rull and Violante, 2000; Galor and Moav, 2000; Greenwood and Yorukoglu, 1997; Violante, 2002). In such literature faster technological change increases the demand for skilled workers causing biased technological change and rising wage inequality. The causality direction is quite different here: biased technological change rises the demand for skilled workers and its wage. This reduces the demand for new technologies, which are more intensive in skilled labor, slowing down technological change. Thus, in contrast with previous papers, this model predicts a decline in technological change speed.

The paper is also related with endogenous growth literature, (Romer, 1986; Lucas, 1988; Rebelo, 1992). Especially relevant is the literature on endogenous technological change (see among others Aghion and Howitt, 1992; Grossman and Helpman, 1991; Romer, 1990).

The paper is organized as follows. The model is set in next section. Section III analyzes agents' behavior. Section IV characterizes balanced growth path. The dynamics outside such balanced growth path is studied in section V. Section VI proposes several extensions to the benchmark model. The conclusions are reached in section VII. Finally all the proofs and some technical details are relegated to the Appendix.

II. The Benchmark Model

A. Technology

Time is indexed by the real numbers $t \in \Re$, that is, it is continuous and infinite. There is a single consumption good that will be called final good, which is produced using unskilled labor L, skilled labor H and a continuum of intermediate goods x_i indexed by $i \in [0,1]$, according with the following production function:

$$\int_0^1 \left[A_i H^{\alpha\left(\frac{A_i}{Z}\right)} L^{1-\alpha\left(\frac{A_i}{Z}\right)}\right]^{1-\beta} x_i^{\beta}\, di \tag{1}$$

where A_i is the quality index of the intermediate good i, Z is the state of know how, $\beta \in (0,1)$ and $\alpha(A_i - Z)$ is defined as follows:

$$\alpha\left(\frac{A_i}{Z}\right) = \underline{\alpha} + \varphi \ln\left(\min\left\{\frac{A_i}{Z}, 1\right\}\right) \tag{2}$$

where $\underline{\alpha} \in (0,1)$ is the minimum share of skilled labor and $\varphi \in \Re_{++}$. Technologies with technological level bellow the state of know how will be called standard ($A_i \leq Z$), the others will be called new ($A_i > Z$). The share of skilled labor $\alpha\left(\frac{A_i}{Z}\right)$ is lower for standard technologies the for new ones. As a matter of fact, the share of skilled labor $\alpha\left(\frac{A_i}{Z}\right)$ increases with the technological level, new technologies requires more skilled labor and therefore such technologies are more intensive in such factor. The share of skilled labor also increases with φ, thus an increase in φ may be interpreted as a rise in technological complexity.

The quality index A_i is an state variable and increases with the investment in research and development I_i, according with the following accumulation equation:

$$\dot{A_i} = \psi\left(\frac{A_i}{Z}\right) I_i \tag{3}$$

where $\psi\left(\frac{A_i}{Z}\right)$ is the marginal productivity of investment, which is defined as follows:

$$\psi\left(\frac{A_i}{Z}\right) = B\left[1 - \gamma \ln\left(\frac{A_i}{Z}\right)\right] \tag{4}$$

where $B \in \Re_{++}$ is an index of the productivity of the research sector and $\gamma \in \Re_{++}$. The research sector present "private" decreasing returns: when the investment increases, the quality of the technology A_i also increases and the productivity of investment decreases. If follows from (2) and (4) that the share of skilled labor is always smaller than $\overline{\alpha} \equiv \frac{\varphi}{\gamma} + \underline{\alpha}$. It is assumed that $\overline{\alpha} < 1$. Thus the share of skilled labor is always in the set $[\underline{\alpha}, \overline{\alpha}) \subset (0,1)$.

The state of know how Z increases when the average quality index of the technology used is higher than the state of know how:

$$\dot{Z} = \eta\left\{\left[\int_0^1 A_i \, di\right] - Z\right\} \tag{5}$$

The intermediate good is produced with the same production function than the final good (1). That is, the cost of producing one unit of intermediate good is one.

The final good is produced by perfect competitive firms with zero profits, the intermediate goods sector is produced by monopolistic firms.

B. Households

There is a continuum of households indexed in the interval $[0,1]$, each of them has one unit of unskilled labor and h unit of skilled labor, where $h \in (0,1)$. Thus, the amount of per-capita amount of labor has been normalized to the unit. The assumption that $h \in (0,1)$ means that skilled labor is the relatively scarce/expensive factor. Section VI presents a extension of the model in which the supply of skilled labor is endogenous, the results of the paper do not change substantially.

Households maximize their lifetime utility function, which is given by a time-separable utility function with CES felicity function:

$$\begin{array}{l} \int_0^\infty u(c(t))\ e^{-\rho t}dt \\ where \\ u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} & if\ \sigma \in (0,1) \cup (1,+\infty) \\ \ln c & if\ \sigma = 1 \end{cases} \end{array} \tag{6}$$

where $c(t)$ denotes consumption in period t.

III. The Agents' Decisions

A. Final Good Firms' Decisions

The price of the final good is normalized to the unit. Firms producing the final good are competitive, they maximize profits taking the prices as given:

$$\underset{L,\ H,\ x_i}{Max} \int_0^1 \left[A_i H^{\alpha\left(\frac{A_i}{Z}\right)} L^{1-\alpha\left(\frac{A_i}{Z}\right)}\right]^{1-\beta} x_i^\beta di\text{-}wL\text{-}w_H H\text{-}\int_0^1 p_i x_i di \tag{7}$$

where w, w_H denotes respectively the wage of skilled and unskilled labor, and p_i the price of intermediate good i. The first order conditions (FOCs) of the above problem are as follows:

$$\int_0^1 \frac{(1\text{-}\beta)\alpha\left(\frac{A_i}{Z}\right)\left[A_i L^{1-\alpha\left(\frac{A_i}{Z}\right)}\right]^{1-\beta} x_i^\beta}{H^{1-(1-\beta)\alpha\left(\frac{A_i}{Z}\right)}} di = w_H \tag{8}$$

$$\int_0^1 \frac{(1\text{-}\beta)\left[1\text{-}\alpha\left(\frac{A_i}{Z}\right)\right]\left[A_i L^{1-\alpha\left(\frac{A_i}{Z}\right)}\right]^{1-\beta} x_i^\beta}{L^{1-(1-\beta)\left[1-\alpha\left(\frac{A_i}{Z}\right)\right]}} di = w \tag{9}$$

$$\beta\left[\frac{A_i H^{\alpha\left(\frac{A_i}{Z}\right)} L^{1-\alpha\left(\frac{A_i}{Z}\right)}}{x_i}\right]^{1-\beta} = p_i \tag{10}$$

These first order conditions are very typical: the marginal product of each factor is equal to its price. It follows from the first order condition (10) that the demand for intermediate good i is as follows:

$$x_i(p_i) = \frac{\beta^{\frac{1}{1-\beta}} A_i H^{\alpha\left(\frac{A_i}{Z}\right)} L^{1-\alpha\left(\frac{A_i}{Z}\right)}}{p_i^{\frac{1}{1-\beta}}} \tag{11}$$

B. Monopolistic Firms' Decisions in the Short Run

The unit cost of the intermediate good is one. The monopolistic firms set the price that maximizes their profits:

$$\underset{p_i}{Max}\ (p_i - 1)x_i(p_i) \tag{12}$$

It follows from equations (11) and (12) that the price at which monopolistic firms sell their intermediate goods is:

$$p_i = \frac{1}{\beta} \tag{13}$$

It follows from (11) and (13) that the amount of intermediate goods that the monopolistic firms sell and the profits made are:

$$x_i = \beta^{\frac{2}{1-\beta}} A_i H^{\alpha\left(\frac{A_i}{Z}\right)} L^{1-\alpha\left(\frac{A_i}{Z}\right)} \tag{14}$$

$$\pi_i = \left(\frac{1-\beta}{\beta}\right) \beta^{\frac{2}{1-\beta}} A_i H^{\alpha\left(\frac{A_i}{Z}\right)} L^{1-\alpha\left(\frac{A_i}{Z}\right)} \tag{15}$$

where π_i denotes the profits of the monopolistic firm that produce the intermediate good i.

C. *R&D* Decisions

Monopolistic firms chose the investment in research and development that maximize the discounted value of their profits:

$$\underset{I(t)}{Max} \quad \int_0^\infty \left[\pi_i(t) - I(t)\right] e^{-\int_0^t r(\tau)\, d\tau} dt \tag{16}$$

$$s.t. \;\; \dot{A}_i(t) = \psi\left(\frac{A_i(t)}{Z(t)}\right) I_i(t)$$

where r_t denotes the interest rate at period t. The above maximization problem implies the following condition:

$$\pi_i(t)\left[\underbrace{\frac{1}{A_i(t)}}_{\oplus} + \underbrace{\frac{\alpha_1'\left(\frac{A_i(t)}{Z(t)}\right)}{Z(t)} \ln\left(\frac{H(t)}{L(t)}\right)}_{\ominus}\right]\psi\left(\frac{A_i(t)}{Z(t)}\right) + \underbrace{\frac{\psi'\left(\frac{A_i(t)}{Z(t)}\right)}{Z(t)} I_i(t)}_{\ominus}$$
$$= r(t) + \frac{\psi'\left(\frac{A_i(t)}{Z(t)}\right)\frac{A_i(t)}{Z(t)}}{\psi\left(\frac{A_i(t)}{Z(t)}\right)}\left[\frac{\dot{A}_i(t)}{A_i(t)} - \frac{\dot{Z}(t)}{Z(t)}\right] \tag{17}$$

The above equation means that the marginal benefit of technological advance should be equal to its marginal cost. The marginal benefit (the left hand side of the equation) consists in three terms: the first one is positive and is caused by increase in the demand for intermediate goods due to its higher productivity. The second term is typically negative and represents the reduction in the demand for intermediate goods caused by the higher intensity of new technologies in the relatively scarce and expensive factor: skilled labor. The third term is also negative and represents the increase in the productivity of investment due to decreasing returns. The marginal cost of technological advance (the right hand side of the equation) is equal to the interest rate, which is the opportunity cost of the investment, plus (minus) the marginal reductions (increase) in the investment productivity along time. This term is zero along the balanced growth path. It follows from (2), (4) and (17) the interest rate is:

$$r(t) = \frac{\pi_i(t)}{A_i(t)} \left[1 + \varphi \ln \left(\frac{H(t)}{L(t)}\right)\right] \psi \left(\frac{A_i(t)}{Z(t)}\right) - \frac{B\gamma}{\psi\left(\frac{A_i(t)}{Z(t)}\right)} \frac{\dot{Z}(t)}{Z(t)} \tag{18}$$

D. Households

The households optimization problem is as follows:

$$\begin{aligned}
&\textstyle\int_0^\infty u(c(t))\ e^{-\rho t} dt \\
&s.a: \\
&\dot{b}(t) = r(t)b(t) + w(t) + w_H(t)h - c(t)
\end{aligned}$$

where $b(t)$ is the value of the assets of the consumers, which consist in the shares of monopolistic firms. This optimization implies the following familiar Euler Equation and transversality condition:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}(r(t) - \rho) \tag{19}$$

$$\lim_{t\to\infty} c(t)^{-\sigma}\ e^{-\rho t}\ b(t) = 0 \tag{20}$$

E. Labor Market

It follows from the first order conditions (8) and (9) that the "relative demand" demand for skilled labor is:

$$\frac{H}{L} = \frac{\alpha\left(\frac{A(t)}{Z(t)}\right)}{1\text{-}\alpha\left(\frac{A(t)}{Z(t)}\right)} \frac{w(t)}{w_H(t)} \tag{21}$$

Labor market clears when the relative demand (21) for skilled labor is equal to its supply:

$$\frac{\alpha\left(\frac{A(t)}{Z(t)}\right)}{1\text{-}\alpha\left(\frac{A(t)}{Z(t)}\right)} \frac{w(t)}{w_H(t)} = h \tag{22}$$

IV. Balanced Growth Path

Balanced growth path is an equilibrium path in which the variables $\{c(t), A_i(t), Z(t), w(t), w_H(t)\}$ grow at constant rate g and the interest rate $r(t)$ is constant over time. It follows from the fact that $A_i(t)$ grows at constant rate, that the quality index $A_i(t)$ should be the same for any intermediate good i. Such value is denoted by $A(t)$.

A. *R&D* Sector

If follows from (15), (18), and (24) the interest rate along the balanced growth path is:

$$r = \Gamma \left(\frac{H}{L}\right)^{\alpha\left(\frac{A}{Z}\right)} \left[1{+}\varphi \ln\left(\frac{H}{L}\right)\right] \psi\left(\frac{A}{Z}\right) - \frac{B\gamma}{\psi\left(\frac{A}{Z}\right)} g \tag{23}$$

where $\Gamma \equiv \left(\frac{1\text{-}\beta}{\beta}\right)\beta^{\frac{2}{1-\beta}}$. It follows from the accumulation equation of the state of know how (5) that:

$$g = \frac{\dot{Z}}{Z} = \eta\left(\frac{A}{Z} - 1\right) \Leftrightarrow \frac{A}{Z} = 1 + \frac{g}{\eta} \tag{24}$$

Using the Euler Equation (19) and equations (23), (21) and (24), it follows that the growth rate along the balanced growth path is:

$$g = \frac{1}{\sigma}\left[\Gamma\left(\frac{\frac{\alpha\left(1+\frac{g}{\eta}\right)}{1\text{-}\alpha\left(1+\frac{g}{\eta}\right)}}{\frac{w_H}{w}}\right)^{\alpha\left(1+\frac{g}{\eta}\right)}\left[1{+}\varphi\ln\left(\frac{\frac{\alpha\left(1+\frac{g}{\eta}\right)}{1\text{-}\alpha\left(1+\frac{g}{\eta}\right)}}{\frac{w_H}{w}}\right)\right]\psi\left(1{+}\frac{g}{\eta}\right) - \frac{B\gamma}{\psi\left(1{+}\frac{g}{\eta}\right)}g - \rho\right] \tag{25}$$

The above equation is represented in figure 1 by a curve with positive slope called technological change curve. The reason why this curve has positive slope is quite intuitive: since new technologies are more intensive in skilled labor, when the relative skilled wage rises, new technologies are less profitable and the demand for them falls. This reduces the profits, the incentives to do *R&D* and as a consequence the speed of technological change.

B. The Labor Market:

Using the labor market equation (22) together with equation (24), it follows that along the balanced growth path the labor market is in equilibrium when the relative wage of skilled labor is as follows:

$$\frac{w_H}{w} = \frac{\alpha\left(1+\frac{g}{\eta}\right)}{1\text{-}\alpha\left(1+\frac{g}{\eta}\right)}\frac{1}{h} \tag{26}$$

Equation (26) is represented in figure 1 by a curve with positive slope, which is called "labor market curve". Since more advanced technologies are more intensive in skilled labor, the relative demand and the relative wage of skilled labor increases with the speed of technological change g and decreases with the supply of skilled labor h.

C. Growth Rate

If follows from equations (25) and (26) that the growth rate along the balanced growth path is:

$$g = \frac{1}{\sigma}\left[\Gamma\, h^{\alpha\left(1+\frac{g}{\eta}\right)}\,[1+\varphi \ln h]\,\psi\left(1+\frac{g}{\eta}\right) - \frac{B\gamma}{\psi\left(1+\frac{g}{\eta}\right)} g - \rho\right] \tag{27}$$

The growth rate along the balanced growth path is positive when the following assumption is satisfied:

Assumption 1: $\varphi < \frac{1}{\ln h}$ and $\Gamma\, h^{\underline{\alpha}}\,[1+\varphi \ln h]\,B > \rho$

This type of assumptions is always used endogenous growth literature and means that agents should be patient enough (ρ should be small enough) and the productivity of the investment technology (B) should be large enough in order that the model generate permanent growth. In this model, besides these typical factors, the abundance of skilled labor play an important role.

It follows from the Implicit Function Theorem and (25) the following proposition:

Proposition 1 *The growth rate g and the relative skilled wage along the balanced growth path are as follows:*

$$g = g\left(\underset{\ominus}{\varphi},\ \underset{\ominus}{\underline{\alpha}},\ \underset{\oplus}{B},\ \underset{\ominus}{\gamma},\ \underset{\oplus}{\eta},\ \underset{\ominus}{\rho},\ \underset{\ominus}{\sigma}\right) \tag{28}$$

$$\frac{w_H}{w} = \theta\left(\underset{\oplus}{\varphi},\ \underset{\oplus}{\underline{\alpha}},\ \underset{\oplus}{B},\ \underset{\ominus}{\gamma},\ \underset{\ominus}{\eta},\ \underset{\ominus}{\rho},\ \underset{\ominus}{\sigma}\right) \tag{29}$$

where $g(.)$ and $\theta(.)$ are continuous and differentiable function and the signs bellow each variable is the sign of the derivative of the growth rate and relative skilled wage with respect to such variable.

D. The Effect of a Biased Technological change

It follows from the specification of the technology (2) that the share of skilled labor increases with the parameters φ and $\underline{\alpha}$. Thus, an increase of these parameter may be interpreted as a biased technological change. Proposition 1 and figure 1 shows that a biased technological change not only rises wage inequality but also reduces the productivity growth rate. When φ increases, new technologies becomes more intensive in the relatively scarce/expensive factor: skilled labor, reducing its profitability and its demand. This reduces the stimulus to do $R\&D$ and technological change speed. An increase in $\underline{\alpha}$ rises the demand for skilled labor, increasing its relative price and reducing the demand for new technologies which are intensive in skilled labor. This inhibits $R\&D$ and decelerates technological change.

Figure 1.a represents equation (25) by a curve with negative slope called "technological change" and equation(26) by a curve with positive slope called "labor market". As it was explained above, the labor market curve has positive slope because when technical change is fast, technology becomes more intensive in skilled labor and its relative price rises. The technological change curve has negative slope because new technologies are intensive in skilled labor, and the higher its relative price the lower the incentives to do $R\&D$. A biased technological change increases the demand for skilled labor making the labor market curve to shift upward: for the same growth rate the relative skilled wage is higher. A biased technological change makes new technologies less profitable since these technologies are more intensive in the more expensive/less abundant resource: skilled labor. This reduces the demand for new technologies and the incentives to do $R\&D$, making the technological change curve shift to the left: given a relative skilled wage, the growth rate is smaller. Summarizing, a biased technological change (an increases in φ or $\underline{\alpha}$) make both curves shift to the left, making the growth rate to fall and relative skilled wage to rise.

E. The Effect of a Reduction in the standardization Speed

It follows from equation (5) that the standardization speed increases with η. The effect of a reduction in η is the same that the effect of a biased technological change described in figure 1: reduces the growth rate and increases the relative skilled wage. The reduction in the standardization speed makes new technologies more intensive in skilled labor, which is the relatively expensive and less abundant resource. As a consequence, the demand for new technologies drops, inhibiting $R\&D$ and growth.

Thus there are three parameters in the model that may reduce technological change speed and increase wage inequality simultaneously: φ, $\underline{\alpha}$ and η. All of these change may be interpreted as risings technological complexity. Complex technologies are more intensive in skilled labor, thus an increase in φ or $\underline{\alpha}$ may be interpreted as an increase in technological complexity. Complex technology is more difficult to learn and therefore requires more skilled labor. Thus a drop in η may be seem as an augmentation in technological complexity. Summarizing, when technology becomes more complex, the profitability of $R\&D$ falls and the demand for skilled labor increases, consequently technological change slow down and wage inequality rises.

F. The other Parameter

The effect of the other parameters over growth is the predictable one. When B rises or γ drops, the productivity of the research sector increases (see equation 4), making *R&D* more profitable and technological change faster. Since new technologies are intensive in skilled labor, this acceleration of technological change boosts the demand for skilled labor and its relative wage.

Figure 1.b shows that an increase in B or a reduction in γ stimulates *R&D*, moving the technological change curve to the right: for a given relative skilled wage the growth rate is larger. As a consequence both the growth rate and the relative skilled wage go up.

When σ increases or ρ decreases agents have more propensity to save, as a consequence there is more investment in *R&D* and technological change speed up, making relative skilled wages higher. Graphically, the technological change curve shift upwards, making both the growth rate and the relative skilled wage greater.

V. Dynamic Behavior

In order to analyze the dynamic behavior of the economy , the stationary variables $\widetilde{A}(t)$ and $\widetilde{c}(t)$ are defined as follows: $\widetilde{A}(t) \equiv \frac{A(t)}{Z(t)}$, $\widetilde{c}(t) \equiv \frac{c(t)}{Z(t)}$. It is shown in the Appendix that the following system of dynamic equations describes the dynamic behavior of the economy:

$$\begin{aligned} \dot{\widetilde{A}}(t) &= \psi\left(\widetilde{A}(t)\right)\left(\frac{\Gamma}{\beta}\,\widetilde{A}(t)h^{\alpha\left(\widetilde{A}(t)\right)} - \widetilde{c}(t)\right) - \eta\left(\widetilde{A}(t)-1\right)\widetilde{A}(t) \\ \frac{\dot{\widetilde{c}}(t)}{\widetilde{c}(t)} &= \frac{1}{\sigma}\left[\Gamma\, h^{\alpha\left(\widetilde{A}(t)\right)}\left[1+\varphi\ln h\right]\psi\left(\widetilde{A}(t)\right) - \left(\frac{B\gamma}{\psi\left(\widetilde{A}(t)\right)}+\sigma\right)\eta\left(\widetilde{A}(t)\text{-}1\right) - \rho\right] \end{aligned}$$

where $\widetilde{A}(t) \equiv \frac{A(t)}{Z(t)}$ and $\widetilde{c}(t) \equiv \frac{c(t)}{Z(t)}$. Figure 2.a shows the dynamic behavior of such system is the typical of a saddle point: there is a unique path that converges toward the "steady-state". Given the transversality conditions, this is the unique equilibrium path.

Figure 2 also shows the effect of a biased technological change (an increase in φ or $\underline{\alpha}$). A biased technological change increases the relative wage of skilled labor (as figure 2.c shows), reducing the demand for new technologies, which are more intensive in skilled labor. Consequently, the profitability of *R&D* drops, and this involves a shift to the left of the locus $\dot{\widetilde{c}}\ (t) = 0$. The broken line represents the locus before the change and the solid line after that. There is an additional effect: since unskilled labor is relatively more abundant than skilled labor, an increase in the share of skilled labor at the expense of unskilled one reduces the production of the economy: with the same resources the production is smaller. This is reflected in the phase diagram in figure 2 by the shift downward of the locus $\dot{\widetilde{A}}\ (t) = 0$.

The reduction in the profitability of $R\&D$ caused by the biased technological change involves a drop in $R\&D$ and in the speed of technological change ($g^A = \frac{\dot{A}}{A}$) as figure 2.b shows. The technological slows down diminish the distance between the technology used A_t and the standard one Z_t, making the share and the relative wage of skilled labor go down along the transition toward the new steady-state, where the relative wage is higher than at the initial steady-state (see figure 2.c).

Figure 3 represents the effect of a reduction in the standardization speed (a reduction in η). When η drops, both $\dot{\widetilde{A}}\ (t) = 0$ and $\dot{\widetilde{c}}\ (t) = 0$ shift to the right. The reason may be found in the definition of $\widetilde{A}(t)$ and $\widetilde{c}(t)$: $\widetilde{A}(t) \equiv \frac{A(t)}{Z(t)}$, $\widetilde{c}(t) \equiv \frac{c(t)}{Z(t)}$. A reduction in η involve a slow down in $Z(t)$, which generates an acceleration of $\widetilde{A}(t) \equiv \frac{A(t)}{Z(t)}$ and $\widetilde{c}(t) \equiv \frac{c(t)}{Z(t)}$ since the denominator goes slower. However this does not mean that the economy goes faster, it happens just the opposite. The slow down of the standardization process originate an increment in the distant from new technologies to the standard one ($\widetilde{A}(t)$ increases), which reduces the productivity of research ($\psi\left(\widetilde{A}(t)\right)$ goes down) and makes new technologies more intensive in skilled labor and consequently less demanded. This diminish the stimulus to $R\&D$ and consequently the speed of technological change, as figure 3.b shows. The increase in $\widetilde{A}(t)$ also has the effect of increasing the share and the relative wage of skilled labor as figure 3.c shows.

VI. Some Extensions

A. Endogenous Biased Technological Change

Consider the same model than before but now the intensity of the technology in skilled labor not only depends upon the ratio $A(t)/Z(t)$ but also in the technological level itself, in such a way that equation 2 becomes as follows:

$$\alpha\left(\frac{A_i}{Z}\right) = \varphi(A_i)\ln\left(\frac{A_i}{Z}\right) + \underline{\alpha} \tag{30}$$

$$\varphi(A_i) = \overline{\varphi} - \left(\overline{\varphi} - \underline{\varphi}\right)\min\left\{\frac{1}{A_i^{\xi}}, 1\right\} \tag{31}$$

where $\overline{\varphi}$, $\underline{\varphi}$, $\xi \in \Re_+$, and $\overline{\varphi} > \underline{\varphi}$. Phase diagram in figure 4 represents the dynamic behavior of the economy (see Appendix). If $A_t \leq 1$, then $\varphi_t = \underline{\varphi}$, the steady-state that would occur if φ_t were constant and equal to $\underline{\varphi}$ is called "quasi steady-state". When $A_t \leq 1$ the state variables tends toward the surrounding of the "quasi steady-state". Figure 4 shows the transition from the "quasi steady-state" toward the steady-state. Technological change slow down wether wage inequality rises along the transition path. φ_t increases along the transition path making new technologies more intensive in skilled labor, which is the relatively more scare and expensive factor. Thus, the demand for new technologies fall over time, reducing the stimulus to $R\&D$ and the speed of technological change. Since new technologies become increasingly intensive in skilled labor, wage inequality rises.

B. Endogenous Skilled Labor Supply

Consider the same model of section II, but now agents have different skills. There is a continuum of agents indexed by $j \in [0,1]$ each with talent $T(j)$, where $T(.)$ is an strictly increasing continuous and differentiable function. That is, agents are ordered from less to more talented. Agents chose between working as skilled or unskilled workers. If they decide to becomes unskilled worker, they have one efficiency unit of unskilled labor, if they work as skilled workers, they have $H(j,\underline{T}) = T(j) - \underline{T}$ efficiency units of skilled labor, where $\underline{T}$ is the minimum amount of talent required to work as skilled worker. It is assumed that $\overline{\alpha} \geq \frac{1}{1+H\left(\underline{j},\underline{T}\right)}$ where $\underline{j}$ is defined such that $\underline{j} = \int_{\underline{j}}^{1} H(j,\underline{T})$. This assumption means that skilled labor is scare, that is, the ratio of skilled to unskilled labor is smaller than one.

Agents will chose to work as skilled workers if they have higher income doing so. j^* denotes the agent that is indifferent between becoming skilled or unskilled worker:

$$w_H \; H(j^*,\underline{T}) = w \Leftrightarrow H\left(j^*,\underline{T}\right) = \frac{w}{w_H}$$

It follows from Implicit Function Theorem that j^* is an increasing function of the relative unskilled wage and the minimum talent required to work as skilled worker $j^*\left(\frac{w}{w_H},\underline{T}\right)$. Agents with index j higher than j^* will prefer to be skilled workers, the others will be unskilled workers. This involves the following supplies of skilled and unskilled labor:

$$\begin{aligned} H^s &= \int_{j^*}^{1} H(j,\underline{T})dj \\ L^s &= \int_{0}^{j^*} dj = j^* \end{aligned}$$

The equilibrium in the labor market will occur when relative skilled labor supply is equal to relative skilled labor demand:

$$\frac{\int_{j^*}^{1} H(j,\underline{T})dj}{j^*} = \frac{\alpha\left(\frac{A(t)}{Z(t)}\right)}{1\text{-}\alpha\left(\frac{A(t)}{Z(t)}\right)}\frac{w(t)}{w_H(t)}$$

It follows from Implicit Function Theorem that relative unskilled wage is a decreasing function of α and $\underline{T}$: $\frac{w}{w_H}(\alpha,\underline{T})$. It is possible to define skilled and unskilled labor in function of α and $\underline{T}$:

$$H(\alpha,\underline{T}) = \int_{j^*\left(\frac{w}{w_H}(\alpha,\underline{T}),\underline{T}\right)}^{1} H(j,\underline{T})dj$$

$$
\begin{aligned}
L(\alpha,\underline{T}) &= j^*\left(\frac{w}{w_H}(\alpha,\underline{T}),\underline{T}\right) \\
h(\alpha,\underline{T}) &= \frac{H(\alpha,\underline{T})}{L(\alpha,\underline{T})} = \frac{\int_{j^*\left(\frac{w}{w_H}(\alpha,\underline{T}),\underline{T}\right)}^{1} H(j,\underline{T})dj}{j^*\left(\frac{w}{w_H}(\alpha,\underline{T}),\underline{T}\right)}
\end{aligned}
$$

The "Technological Change" and "Labor Market" curves are very similar to the ones presented in Section IV:

$$
g = \frac{1}{\sigma}\left\{\Gamma\left(\frac{\frac{\alpha\left(1+\frac{g}{\eta}\right)}{1\text{-}\alpha\left(1+\frac{g}{\eta}\right)}}{\frac{w_H}{w}}\right)^{\alpha\left(1+\frac{g}{\eta}\right)} L\left(\alpha\left(1+\frac{g}{\eta}\right),\underline{T}\right)\left[1+\varphi\ln\left(\frac{\frac{\alpha\left(1+\frac{g}{\eta}\right)}{1\text{-}\alpha\left(1+\frac{g}{\eta}\right)}}{\frac{w_H}{w}}\right)\right]\psi\left(1+\frac{g}{\eta}\right)\right. \tag{32}
$$

$$
\left.-\frac{B\gamma}{\psi\left(1+\frac{g}{\eta}\right)}-\rho\right\} \tag{33}
$$

$$
\frac{w_H}{w} = \frac{\alpha\left(1+\frac{g}{\eta}\right)}{1\text{-}\alpha\left(1+\frac{g}{\eta}\right)}\frac{1}{h\left(\alpha\left(1+\frac{g}{\eta}\right),\underline{T}\right)} \tag{34}
$$

The behavior of these two curves are exactly as described in section III. The unique difference is that now there is a new parameter: the minimum amount of talent required to work as skilled worker $\underline{T}$. Figure 5 shows the effect of an increase in $\underline{T}$: the technological change curve move to the right because given a relative wage and increase in $\underline{T}$ increases the amount of unskilled labor, and this increases the productivity due to increasing returns to scale. An increase in $\underline{T}$ reduces the supply of skilled labor and increases the supply of unskilled one, as a result the Labor Market curve shifts upwards. An increase in the minimum talent required to works as skilled labor has a similar effect over the balanced growth path that a biased technological: increase the relative skilled wage and reduces technological change speed. The increase in the minimum talent required to work as skilled worker may be interpreted again as an increase in technological complexity. Highly complex technologies requires workers with more "talent" in order to use it.

The growth rate along the balanced growth path would be as follows:

$$
g = \frac{1}{\sigma}\left[\Delta\left(\alpha\left(1+\frac{g}{\eta}\right),\varphi,\underline{T}\right)\psi\left(1+\frac{g}{\eta}\right)-\frac{B\gamma}{\psi\left(1+\frac{g}{\eta}\right)}g-\rho\right]
$$

where

$$
\Delta(\alpha,\varphi,\underline{T}) = \Gamma H(\alpha,\underline{T})^{\alpha}L(\alpha,\underline{T})^{1-\alpha}\left(1+\varphi\ln h(\alpha,\underline{T})\right)
$$

Assumption 1'**:** $\varphi < \overline{\varphi}$ and $\Delta(\underline{\alpha},\varphi,\underline{T})B > \rho$, where $\overline{\varphi}$ is a constant defined in the Appendix.

Assumption 1' is very similar to assumption 1. There is a balanced growth path with positive growth rate if the demand for new technologies is large enough (φ is small), agents are patient enough (ρ is small) and the productivity of investment in $R\&D$ is large enough (B is large and γ small).

Proposition 2 *The growth rate g and the relative skilled wage along the balanced growth path are as follows:*

$$g = g\left(\underset{\ominus}{\varphi}, \underset{\ominus}{\alpha}, \underset{\oplus}{B}, \underset{\ominus}{\gamma}, \underset{\oplus}{\eta}, \underset{\ominus}{\rho}, \underset{\ominus}{\sigma}, \underset{\ominus}{\underline{T}} \right) \tag{35}$$

$$\frac{w_H}{w} = \theta\left(\underset{\oplus}{\varphi}, \underset{\oplus}{\alpha}, \underset{\oplus}{B}, \underset{\ominus}{\gamma}, \underset{\ominus}{\eta}, \underset{\ominus}{\rho}, \underset{\ominus}{\sigma}, \underset{\oplus}{\underline{T}} \right) \tag{36}$$

where $g(.)$ and $\theta(.)$ are continuous and differentiable function and the signs bellow each variable is the sign of the derivative of the growth rate and relative skilled wage with respect to such variable.

Thus the introduction of endogenous skilled labor supply does not change substantially the behavior of the model. The dynamic behavior only change in the evolution of the amount of skilled and unskilled labor, that now behaves according with wage: the number of agents that work as skilled workers increases with the relative wage of skilled workers and thus the amount of skilled labor also increases, whether the unskilled labor behaves in opposite way .The dynamic behavior of the skilled supply of labor when a biased technological change of a reduction of the speed of the standardization process is represented in figure 6.

An increase in the minium talent to work as skilled worker has a similar dynamic effect that a biased technological change (see 2). An increase in $\underline{T}$ reduces the supply of skilled labor making the relative skilled wage higher and the demand for new technologies and the technological change speed lower.

VII. Conclusion

Empirical evidence shows that most developed countries have suffer rising wage inequality and slow down in productivity. This paper has presented a model in which skilled labor is relatively scarce factor and in which new technologies are more intensive in it. In this environment a biased technological change that makes new technologies more intensive in skilled labor, reduces the profitability of new technologies since skilled labor is the factor relatively scare and more expensive. Thus biased technological change reduces the demand for new technologies and the profitability of $R\&D$, making technological change speed to slow down. Since biased technological change increases the demand for skilled labor, it also makes wage distribution more unequal.

There other changes that have the same effect of slowing down technological change and increase wage inequality. A biased technological change that make all technologies (new and standard) more intensive in skilled labor, a reduction in the standardization speed also have similar effect and an increase in the minimum talent required to work as an skilled worker.

All of these changes may be interpreted as an increase in technological complexity. Thus it may be concluded that an increase in technological complexity boosts the demand for skilled labor and diminishes the demand for new technologies, reducing technological change speed and rising wage inequality.

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IX. Appendix

A. Effect of the parameters over steady-state values

$$g = \frac{1}{\sigma}\left[\left(\frac{1\text{-}\beta}{\beta}\right)\beta^{\frac{2}{1-\beta}}h^{\alpha\left(1+\frac{g}{\eta}\right)}\left[1+\varphi\ln h\right]\psi\left(1+\frac{g}{\eta}\right)-\frac{B\gamma}{\psi\left(1+\frac{g}{\eta}\right)}g-\rho\right]$$

The function F is defined as:

$$F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right) = -\sigma g+\Gamma h^{\alpha\left(1+\frac{g}{\eta}\right)}\left[1+\varphi\ln h\right]\psi\left(1+\frac{g}{\eta}\right)-\frac{B\gamma}{\psi\left(1+\frac{g}{\eta}\right)}g-\rho$$

where $\Gamma\equiv\left(\frac{1\text{-}\beta}{\beta}\right)\beta^{\frac{2}{1-\beta}}$. Using equation (32) it follows that along the balanced growth path:

$$F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right) = 0$$

The derivatives of the above function are as follows:

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial g} = -\sigma + \left[\Gamma h^{\alpha\left(1+\frac{g}{\eta}\right)}\left[1+\varphi\ln h\right]\psi\left(1+\frac{g}{\eta}\right)\right]\left[\frac{\alpha'\left(1+\frac{g}{\eta}\right)\ln h}{\eta}+\frac{\psi'\left(1+\frac{g}{\eta}\right)}{\eta\psi\left(1+\frac{g}{\eta}\right)}\right] -\frac{B\gamma}{\psi\left(1+\frac{g}{\eta}\right)}-\frac{\gamma g}{\eta}\frac{\gamma}{\left[1-\gamma\ln\left(1+\frac{g}{\eta}\right)\right]^2\left(1+\frac{g}{\eta}\right)} < 0$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \varphi} = \Gamma h^{\alpha\left(1+\frac{g}{\eta}\right)}\left[1+\varphi\ln h\right]\psi\left(1+\frac{g}{\eta}\right)\ln h\ln\left(1+\frac{g}{\eta}\right)+ \Gamma h^{\alpha\left(1+\frac{g}{\eta}\right)}\ln h\psi\left(1+\frac{g}{\eta}\right) < 0$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \underline{\alpha}} = \Gamma h^{\alpha\left(1+\frac{g}{\eta}\right)}\left[1+\varphi\ln h\right]\psi\left(1+\frac{g}{\eta}\right)\ln h < 0$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial B} = \Gamma h^{\alpha\left(1+\frac{g}{\eta}\right)}\left[1+\varphi\ln h\right]\frac{\psi\left(1+\frac{g}{\eta}\right)}{B} > 0$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial\gamma} =$$

$$-\Gamma h^{\alpha\left(1+\frac{g}{\eta}\right)}\left[1+\varphi\ln h\right]\ln\left(1+\frac{g}{\eta}\right)-\frac{Bg}{\psi\left(1+\frac{g}{\eta}\right)}-\frac{g\ln\left(1+\frac{g}{\eta}\right)}{\left[1-\gamma\ln\left(1+\frac{g}{\eta}\right)\right]^2}$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial\eta} =$$

$$-\left[\Gamma h^{\alpha\left(1+\frac{g}{\eta}\right)}\left[1+\varphi\ln h\right]\psi\left(1+\frac{g}{\eta}\right)\right]\left[\alpha'\left(1+\frac{g}{\eta}\right)\ln h+\frac{\psi'\left(1+\frac{g}{\eta}\right)}{\psi\left(1+\frac{g}{\eta}\right)}\right]\frac{g}{\eta^2}$$

$$-\frac{B\gamma\psi'\left(1+\frac{g}{\eta}\right)}{\psi\left(1+\frac{g}{\eta}\right)^2}\left(\frac{g}{\eta}\right)^2 > 0$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial\rho} = -1 < 0$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial\sigma} = -g < 0$$

Expression (35) in the main text follows from the above derivatives and the Implicit Function Theorem

$$g = \frac{1}{\sigma}\left[\Gamma h^{\alpha\left(1+\frac{g}{\eta}\right)}\left[1+\varphi\ln h\right]\psi\left(1+\frac{g}{\eta}\right)-\frac{B\gamma}{\psi\left(1+\frac{g}{\eta}\right)}g-\rho\right]$$

$$\alpha=\varphi\ln\left(1+\frac{g}{\eta}\right)+\underline{\alpha}\Leftrightarrow\left(1+\frac{g}{\eta}\right)=e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\Leftrightarrow g=\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)\eta$$

The function G is defined as:

$$G\left(\alpha,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right) =$$

$$-\sigma\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)\eta+\Gamma h^{\alpha}\left[1+\varphi\ln h\right]\psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)-\frac{B\gamma\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)\eta}{\psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}-\rho$$

$$\frac{\partial G\left(\alpha,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial\alpha} = -\frac{\sigma}{\varphi}e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\eta+$$

$$\left[\Gamma h^{\alpha}\left[1+\varphi\ln h\right]\psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right]\left[\ln h+\frac{\psi'\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}{\varphi\psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}\right]$$

$$-\frac{\gamma e^{2\frac{\alpha-\underline{\alpha}}{\varphi}}\eta}{\varphi\psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}-\frac{\gamma\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)\eta}{\varphi B}\frac{1-\gamma\ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)+\gamma}{\left[1-\gamma\ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right]^2} < 0$$

$$\frac{\partial G\left(\alpha,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \alpha} = -\frac{\sigma}{\varphi} e^{\frac{\alpha-\underline{\alpha}}{\varphi}} \eta +$$

$$\left[\Gamma h^{\alpha}\left[1+\varphi \ln h\right] \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right]\left[\ln h + \frac{\psi'\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}{\varphi \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}\right]$$

$$-\frac{\gamma e^{2\frac{\alpha-\underline{\alpha}}{\varphi}} \eta}{\varphi \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)} - \frac{\gamma\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)\eta}{\varphi B} \frac{1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)+\gamma}{\left[1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right]^{2}} < 0$$

$$\frac{\partial G\left(\alpha,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \varphi} = +\frac{\sigma}{\varphi^{2}} e^{\frac{\alpha-\underline{\alpha}}{\varphi}} \eta +$$

$$-\quad \left[\Gamma h^{\alpha}\left[1+\varphi \ln h\right] \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right] \frac{\psi'\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}{\varphi^{2} \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}$$

$$+\frac{\gamma e^{2\frac{\alpha-\underline{\alpha}}{\varphi}} \eta}{\varphi^{2} \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)} + \frac{\gamma\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)\eta}{\varphi^{2} B} \frac{1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)+\gamma}{\left[1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right]^{2}} > 0$$

$$\frac{\partial G\left(\alpha,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \underline{\alpha}} = \frac{\sigma}{\varphi} e^{\frac{\alpha-\underline{\alpha}}{\varphi}} \eta$$

$$-\left[\Gamma h^{\alpha}\left[1+\varphi \ln h\right] \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right] \frac{\psi'\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}{\varphi \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}$$

$$+\frac{\gamma e^{2\frac{\alpha-\underline{\alpha}}{\varphi}} \eta}{\varphi \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)} + \frac{\gamma\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)\eta}{\varphi B} \frac{1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)+\gamma}{\left[1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right]^{2}} > 0$$

$$\frac{\partial G\left(\alpha,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \eta} =$$

$$-\sigma\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right) - \frac{\gamma e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)}{\psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)} < 0$$

B. Dynamic Behavior

It follows from equations (1), (3), (5), (14), (15), (18), (19), (20) and (22), that the dynamic behavior of the economy come from the following system of dynamic equations:

$$
\begin{aligned}
\frac{\dot{A}(t)}{A(t)} &= \psi\left(\frac{A(t)}{Z(t)}\right)\left[\frac{\Gamma}{\beta}\,h^{\alpha\left(\frac{A(t)}{Z(t)}\right)} - \frac{c(t)}{A(t)}\right] \\
\frac{\dot{Z}(t)}{Z(t)} &= \eta\left(\frac{A(t)}{Z(t)} - 1\right) \\
\frac{\dot{c}(t)}{c(t)} &= \frac{1}{\sigma}\left[\Gamma\,h^{\alpha\left(\frac{A(t)}{Z(t)}\right)}\,[1+\varphi\ln h]\,\psi\left(\frac{A(t)}{Z(t)}\right) - \frac{B\gamma}{\psi\left(\frac{A(t)}{Z(t)}\right)}\frac{\dot{Z}(t)}{Z(t)} - \rho\right] \\
\lim_{t\to\infty} c(t)^{-\sigma}e^{-\rho t}A(t) &= 0
\end{aligned}
$$

The above dynamic system may be rewritten as follows:

The above dynamic system may be rewritten as follows:

$$
\begin{aligned}
\dot{\widetilde{A}}(t) &= \psi\left(\widetilde{A}(t)\right)\left(\frac{\Gamma}{\beta}\,\widetilde{A}(t)h^{\alpha\left(\widetilde{A}(t)\right)} - \widetilde{c}(t)\right) - \eta\left(\widetilde{A}(t)-1\right)\widetilde{A}(t) \\
\frac{\dot{\widetilde{c}}(t)}{\widetilde{c}(t)} &= \frac{1}{\sigma}\left[\Gamma\,h^{\alpha\left(\widetilde{A}(t)\right)}\,[1+\varphi\ln h]\,\psi\left(\widetilde{A}(t)\right) - \left(\frac{B\gamma}{\psi\left(\widetilde{A}(t)\right)}+\sigma\right)\eta\left(\widetilde{A}(t)\text{-}1\right) - \rho\right]
\end{aligned}
$$

where $\widetilde{A}(t) \equiv \frac{A(t)}{Z(t)}$ and $\widetilde{c}(t) \equiv \frac{c(t)}{Z(t)}$.

C. The dynamic system of the model with endogenous biased technological change

The dynamic system that describes the behavior of the economy is as follows:

$$
\begin{aligned}
\frac{\dot{A}(t)}{A(t)} &= \psi\left(\frac{A(t)}{Z(t)}\right)\left[\frac{\Gamma}{\beta}\,h^{\alpha\left(\frac{A(t)}{Z(t)},\varphi(t)\right)} - \frac{c(t)}{A(t)}\right] \\
\frac{\dot{Z}(t)}{Z(t)} &= \eta\left(\frac{A(t)}{Z(t)} - 1\right) \\
\frac{\dot{c}(t)}{c(t)} &= \frac{1}{\sigma}\left[\Gamma\,h^{\alpha\left(\frac{A(t)}{Z(t)},\varphi(t)\right)}\left[1+\left(\varphi(t)+\xi(\overline{\varphi}\text{-}\varphi(t))\ln\left(\frac{A(t)}{Z(t)}\right)\right)\ln h\right]\psi\left(\frac{A(t)}{Z(t)}\right)\right. \\
&\quad \left. -\frac{B\gamma}{\psi\left(\frac{A(t)}{Z(t)}\right)}\frac{\dot{Z}(t)}{Z(t)} - \rho\right] \\
\dot{\varphi}(t) &= \xi(\overline{\varphi} - \varphi(t))\frac{\dot{A}(t)}{A(t)}
\end{aligned}
$$

$$\lim_{t\to\infty} c(t)^{-\sigma} A(t) = 0$$

where $\varphi(t) = \overline{\varphi} - \left(\overline{\varphi} - \underline{\varphi}\right) \frac{1}{A(t)^{\xi}}$. The above dynamic system may be rewritten as follows:

$$\begin{aligned}
\dot{\widetilde{A}}(t) &= F^{A}\left(\widetilde{A}(t), \widetilde{c}(t), \varphi(t)\right) \\
\frac{\dot{\widetilde{c}}(t)}{\widetilde{c}(t)} &= F^{c}\left(\widetilde{A}(t), \widetilde{c}(t), \varphi(t)\right) \\
\dot{\varphi}(t) &= \xi(\overline{\varphi} - \varphi(t))\psi\left(\widetilde{A}(t)\right)\left[\frac{\Gamma}{\beta}\, h^{\alpha\left(\widetilde{A}(t),\varphi(t)\right)} - \frac{\widetilde{c}(t)}{\widetilde{A}(t)}\right]
\end{aligned}$$

where:

$$\begin{aligned}
F^{A}\left(\widetilde{A}, \widetilde{c}, \varphi\right) &= \left[\psi\left(\widetilde{A}\right)\frac{\Gamma}{\beta}\, h^{\alpha\left(\widetilde{A},\varphi\right)} - \eta\left(\widetilde{A}-1\right)\right]\widetilde{A} - \psi\left(\widetilde{A}\right)\widetilde{c} \\
F^{c}\left(\widetilde{A}, \widetilde{c}, \varphi\right) &= \frac{\widetilde{c}}{\sigma}\left[\Gamma\, h^{\alpha\left(\widetilde{A},\varphi\right)}\left[1+\left(\varphi + \xi(\overline{\varphi}-\varphi)\ln\widetilde{A}\right)\ln h\right]\psi\left(\widetilde{A}\right)\right. \\
&\left. -\left(\frac{B\gamma}{\psi\left(\widetilde{A}\right)} + \sigma\right)\eta\left(\widetilde{A}-1\right) - \rho\right]
\end{aligned}$$

Linearizing the above system in the surrounding of the steady-state:

$$\begin{bmatrix} \dot{\widetilde{A}}(t) \\ \dot{\widetilde{c}}(t) \\ \dot{\varphi}(t) \end{bmatrix} = \begin{bmatrix} a_{11} & \underset{\ominus}{a_{12}} & \underset{\ominus}{a_{13}} \\ \underset{\ominus}{a_{21}} & 0 & \underset{\ominus}{a_{23}} \\ 0 & 0 & \underset{\ominus}{a_{33}} \end{bmatrix} \begin{bmatrix} \widetilde{A}(t) - \widetilde{A}^{ss} \\ \widetilde{c}(t) - \widetilde{c}^{ss} \\ \varphi(t) - \overline{\varphi} \end{bmatrix}$$

where $a_{11} = F_{A}^{A'}\left(\widetilde{A}^{ss}, \widetilde{c}^{ss}, \varphi^{ss}\right)$, $a_{12} = -\psi\left(\widetilde{A}^{ss}\right)$, $a_{13} = F_{\varphi}^{A'}\left(\widetilde{A}^{ss}, \widetilde{c}^{ss}, \varphi^{ss}\right)$, $a_{21} = F_{A}^{c'}\left(\widetilde{A}^{ss}, \widetilde{c}^{ss}, \varphi^{ss}\right)$, $a_{23} = F_{\varphi}^{c'}\left(\widetilde{A}^{ss}, \widetilde{c}^{ss}, \varphi^{ss}\right)$, $a_{33} = -\xi g^{ss}$. the signs of the element of the matrix appears bellow them. The eigenvalues of the above matrix are $\frac{-a_{11}+\sqrt{a_{11}^2+4a_{12}a_{21}}}{2}$, $\frac{-a_{11}+\sqrt{a_{11}^2-4a_{12}a_{21}}}{2}$ and $-a_{33}$. It follows from the fact that a_{12} and a_{21} are both negative, that all the roots are real and there are two negatives roots and a positive one. These roots will be denoted as follows: $\lambda_1 = \max\left\{\frac{-a_{11}+\sqrt{a_{11}^2-4a_{12}a_{21}}}{2}, \frac{-a_{11}+\sqrt{a_{11}^2-4a_{12}a_{21}}}{2}\right\}$, $\lambda_2 = \min\left\{\frac{-a_{11}+\sqrt{a_{11}^2-4a_{12}a_{21}}}{2}, \frac{-a_{11}+\sqrt{a_{11}^2-4a_{12}a_{21}}}{2}\right\}$, $\lambda_3 = -a_{33}$. The solution of the above dynamic system is as follows:

$$\begin{bmatrix} \widetilde{A}(t) - \widetilde{A}^{ss} \\ \widetilde{c}(t) - \widetilde{c}^{ss} \\ \varphi(t) - \overline{\varphi} \end{bmatrix} = \begin{bmatrix} 1 & 1 & b_{13} \\ b_{21} & b_{22} & b_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d_1 e^{\lambda_1 t} \\ d_2 e^{\lambda_2 t} \\ d_3 e^{\lambda_3 t} \end{bmatrix}$$

where $b_{21} = \frac{a_{21}}{\lambda_1}$, $b_{22} = \frac{a_{21}}{\lambda_2}$, $b_{13} = \frac{a_{12}a_{21}-(a_{11}+a_{33})a_{33}}{a_{12}a_{23}-a_{13}a_{33}}$, $b_{23} = -\frac{a_{23}+a_{21}b_{13}}{a_{33}}$, and d_1, d_2, and d_3 are three constants. It follows from transversality conditions that d_1 should be zero. It follows from initial conditions that $d_3 = (\varphi(0) - \overline{\varphi})$, and $d_2 = \left(\widetilde{A}(0) - \widetilde{A}^{ss}\right) - b_{13}(\varphi(0) - \overline{\varphi})$. Then the solution of the dynamic system is:

$$\begin{bmatrix} \widetilde{A}(t) - \widetilde{A}^{ss} \\ \widetilde{c}(t) - \widetilde{c}^{ss} \\ \varphi(t) - \overline{\varphi} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & b_{13} \\ b_{21} & b_{23} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \left[\left(\widetilde{A}(0) - \widetilde{A}^{ss}\right) - b_{13}(\varphi(0) - \overline{\varphi})\right] e^{\lambda_2 t} \\ (\varphi(0) - \overline{\varphi}) e^{\lambda_3 t} \end{bmatrix}$$

D. Endogenous Skilled Supply

Lemma 3 $h < 1$

Proof.

It has been assumed that $\overline{\alpha} \geq \frac{1}{1+H\left(\underline{j},\underline{T}\right)} \Leftrightarrow \frac{1-\overline{\alpha}}{\overline{\alpha}} \geq H(\underline{j},\underline{T})$

Consider than $h \geq 1 \Rightarrow \frac{w}{w_H} = \frac{1-\alpha}{\alpha}h > \frac{1-\overline{\alpha}}{\overline{\alpha}} \geq H(\underline{j},\underline{T}) \Rightarrow \underline{j} < j^* \Rightarrow h(\alpha) = \frac{\int_{j^*}^1 H(j,\underline{T})dj}{j^*} < \frac{\int_{\underline{j}}^1 H(j,\underline{T})dj}{\underline{j}} = 1 \Rightarrow\Leftarrow$ ■

Lemma 4 *There exist a $\overline{\varphi}$ such that if $\varphi \leq \overline{\varphi}$ then $\Delta'_\alpha(\alpha) < 0$.*

Proof.

The definition of $\Delta(\alpha,\varphi,\underline{T})$ is as follows:

$$\Delta(\alpha,\varphi,\underline{T}) = \Gamma H(\alpha,\underline{T})^\alpha L(\alpha,\underline{T})^{1-\alpha}(1+\varphi\ln h(\alpha,\underline{T}))$$

Thus its derivative with respect to α is as follows:

$$\begin{aligned}
&\Delta'_\alpha(\alpha,\varphi,\underline{T}) = \\
&\Delta(\alpha,\varphi,\underline{T})\left\{\ln h(\alpha,\underline{T}) + \frac{\varphi}{1+\varphi\ln h(\alpha,\underline{T})}\frac{1}{h(\alpha,\underline{T})}h'(\alpha,\underline{T})\right\} + \\
&(1+\varphi\ln h(\alpha,\underline{T}))\underbrace{\left(w_H j^*\left(\frac{w}{w_H},\underline{T}\right) - w\right)}_{0} j^{*\prime}_{\frac{w}{w_H}}\left(\frac{w}{w_H},\underline{T}\right)\left(\frac{w}{w_H}\right)'_\alpha(\alpha,\underline{T}) = \\
&\Delta(\alpha,\varphi,\underline{T})\{\ln h(\alpha,\underline{T}) + \\
&\frac{\varphi}{1+\varphi\ln h(\alpha,\underline{T})}\frac{1}{h(\alpha,\underline{T})}\frac{-H(j^*,\underline{T})j^* - \int_{j^*}^1 H(j,\underline{T})dj}{(j^*)^2}\frac{1}{H'_j(j^*,\underline{T})}\frac{\frac{w}{w_H}\frac{1}{(1\text{-}\alpha)^2}}{\frac{-H(j^*,\underline{T})j^* - \int_{j^*}^1 H(j,\underline{T})dj}{(j^*)^2}\frac{1}{H'_j(j^*,\underline{T})} - \frac{\alpha}{1\text{-}\alpha}}
\end{aligned}$$

$$\Delta(\alpha,\varphi,\underline{T})\left\{\ln h(\alpha,\underline{T}) + \frac{\varphi}{1{+}\varphi\ln h(\alpha,\underline{T})}\frac{1}{h(\alpha,\underline{T})}h(\alpha,\underline{T})\frac{1}{(1\text{-}\alpha)\,\alpha}\frac{H(j^*,\underline{T})+h(\alpha,\underline{T})}{H(j^*,\underline{T})+h(\alpha,\underline{T})+\frac{\alpha}{1\text{-}\alpha}H'(j^*,\underline{T}}\right.$$

$$\Delta(\alpha,\varphi,\underline{T})\left\{\ln h(\alpha,\underline{T}) + \frac{\varphi}{1{+}\varphi\ln h(\alpha,\underline{T})}\frac{1}{(1\text{-}\alpha)\,\alpha}\right\}$$

It follows from the restriction that $\overline{\alpha} = \underline{\alpha}{+}\frac{\varphi}{\gamma} < \frac{1}{1+H\left(\underline{j},\underline{T}\right)}$ that $\varphi < \left[1-\underline{\alpha}\left(1+H\left(\underline{j},\underline{T}\right)\right)\right]\gamma$. Define $f : \left[0, \left[1-\underline{\alpha}\left(1+H\left(\underline{j},\underline{T}\right)\right)\right]\gamma\right] \to \Re_+$ as follows:

$$f(\varphi) = \underset{\alpha\in\left\{\underline{\alpha},\, \underline{\alpha}+\frac{\varphi}{\gamma}\right\}}{Max} \left\{\ln h(\alpha,\underline{T})\left[1{+}\varphi\ln h(\alpha,\underline{T})\right](1\text{-}\alpha)\,\alpha+\varphi\right\}$$

It follows from lemma 1 that $f(0) = \ln h(\underline{\alpha}) < 0$. $f(\varphi)$ is an increasing continuous function. If $f\left(\left[1-\underline{\alpha}\left(1+H\left(\underline{j}\right)\right)\right]\gamma\right) \leq 0$ then $\overline{\varphi} = \left[1-\underline{\alpha}\left(1+H\left(\underline{j}\right)\right)\right]\gamma$. If $f\left(\left[1-\underline{\alpha}\left(1+H\left(\underline{j}\right)\right)\right]\gamma\right) > 0$, define $\overline{\varphi}$ such that $f(\overline{\varphi}) = 0$. It follows from the definition of $\overline{\varphi}$ that if $\varphi \leq \overline{\varphi}$ then $\Delta'_\alpha(\alpha,\varphi) < 0$. ■

The effect of the parameter over the growth rate along the balanced growth path may be established applying the Implicit Function Theorem to the following equation:

$$g = \frac{1}{\sigma}\left[\Delta\left(\alpha\left(1{+}\frac{g}{\eta}\right),\varphi,\underline{T}\right)\psi\left(1{+}\frac{g}{\eta}\right) - \frac{B\gamma}{\psi\left(1{+}\frac{g}{\eta}\right)}g-\rho\right]$$

The function F is defined as:

$$F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right) =$$
$$-\sigma g + \Delta\left(\alpha\left(1{+}\frac{g}{\eta}\right),\varphi\right)\psi\left(1{+}\frac{g}{\eta}\right) - \frac{B\gamma}{\psi\left(1{+}\frac{g}{\eta}\right)}g-\rho$$

where $\Gamma \equiv \left(\frac{1\text{-}\beta}{\beta}\right)\beta^{\frac{2}{1-\beta}}$. Using equation (32) it follows that along the balanced growth path:

$$F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right) = 0$$

The derivatives of the above function are as follows:

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial g} = -\sigma + \frac{\Delta'_\alpha\left(\alpha\left(1{+}\frac{g}{\eta}\right),\varphi\right)}{\eta}$$
$$\frac{\Delta\left(\alpha\left(1{+}\frac{g}{\eta}\right),\varphi\right)\psi'\left(1{+}\frac{g}{\eta}\right)}{\eta\psi\left(1{+}\frac{g}{\eta}\right)} - \frac{B\gamma}{\psi\left(1{+}\frac{g}{\eta}\right)} + \frac{B\gamma g\psi'\left(1{+}\frac{g}{\eta}\right)}{\eta\left[\psi\left(1{+}\frac{g}{\eta}\right)\right]^2} < 0$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \varphi} =$$
$$\Delta'_\alpha\left(\alpha\left(1{+}\frac{g}{\eta}\right),\varphi\right)\psi\left(1{+}\frac{g}{\eta}\right)\ln\left(1{+}\frac{g}{\eta}\right) +$$
$$\Delta'_\varphi\left(\alpha\left(1{+}\frac{g}{\eta}\right),\varphi\right)\psi\left(1{+}\frac{g}{\eta}\right) < 0$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \underline{\alpha}} = \Delta'_{\alpha}\left(\alpha\left(1+\frac{g}{\eta}\right),\varphi\right) \ < \ 0$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial B} = \Delta\left(\alpha\left(1+\frac{g}{\eta}\right),\varphi\right)\frac{\psi\left(1+\frac{g}{\eta}\right)}{B} \ > \ 0$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \gamma} = -\Delta\left(\alpha\left(1+\frac{g}{\eta}\right),\varphi\right)B\ln\left(1+\frac{g}{\eta}\right) - \frac{B\gamma g\ln\left(1+\frac{g}{\eta}\right)}{\left[\psi\left(1+\frac{g}{\eta}\right)\right]^2} - \frac{Bg}{\psi\left(1+\frac{g}{\eta}\right)} \ < \ 0$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \eta} = -\left[\Delta\left(\alpha\left(1+\frac{g}{\eta}\right),\varphi\right)\psi\left(1+\frac{g}{\eta}\right)\right]\left[\alpha'\left(1+\frac{g}{\eta}\right)\ln h + \frac{\psi'\left(1+\frac{g}{\eta}\right)}{\psi\left(1+\frac{g}{\eta}\right)}\right]\frac{g}{\eta^2} - \frac{B\gamma\psi'\left(1+\frac{g}{\eta}\right)}{\psi\left(1+\frac{g}{\eta}\right)^2}\left(\frac{g}{\eta}\right)^2 \ > \ 0$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \rho} = -1 \ < \ 0$$

$$\frac{\partial F\left(g,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \sigma} = -g \ < \ 0$$

Expression (35) in the main text follows from the above derivatives and the Implicit Function Theorem

$$g = \frac{1}{\sigma}\left[\Delta\left(\alpha\left(1+\frac{g}{\eta}\right),\varphi\right)\left[1+\varphi\ln h\right]\psi\left(1+\frac{g}{\eta}\right) - \frac{B\gamma}{\psi\left(1+\frac{g}{\eta}\right)}g - \rho\right]$$

$$\alpha = \varphi\ln\left(1+\frac{g}{\eta}\right) + \underline{\alpha} \Leftrightarrow \left(1+\frac{g}{\eta}\right) = e^{\frac{\alpha-\underline{\alpha}}{\varphi}} \Leftrightarrow g = \left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}} - 1\right)\eta$$

The function G is defined as:

$$G\left(\alpha,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right) = -\sigma\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}} - 1\right)\eta + \Delta\left(\alpha,\varphi\right)\psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right) - \frac{\gamma e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}} - 1\right)\eta}{\psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)} - \rho$$

$$
\begin{aligned}
\frac{\partial G\left(\alpha,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \alpha} &= -\frac{\sigma}{\varphi} e^{\frac{\alpha-\underline{\alpha}}{\varphi}} \eta + \\
& \Delta_{\alpha}\left(\alpha,\varphi\right) \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right) + \frac{\Delta\left(\alpha,\varphi\right) \psi'\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}{\varphi} \\
& -\frac{\gamma e^{2\frac{\alpha-\underline{\alpha}}{\varphi}} \eta}{\varphi \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)} - \frac{\gamma\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)\eta}{\varphi B} \frac{1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)+\gamma}{\left[1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right]^{2}} < 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial G\left(\alpha,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \varphi} &= +\frac{\sigma}{\varphi^{2}} e^{\frac{\alpha-\underline{\alpha}}{\varphi}} \eta + \Delta_{\varphi}\left(\alpha,\varphi\right) \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right) \\
& -\Delta_{\varphi}\left(\alpha,\varphi\right) \frac{\psi'\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}{\varphi^{2}\psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)} + \frac{\gamma e^{2\frac{\alpha-\underline{\alpha}}{\varphi}} \eta}{\varphi^{2} \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)} \\
& +\frac{\gamma\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)\eta}{\varphi^{2} B} \frac{1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)+\gamma}{\left[1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right]^{2}} > 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial G\left(\alpha,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \underline{\alpha}} &= \frac{\sigma}{\varphi} e^{\frac{\alpha-\underline{\alpha}}{\varphi}} \eta \\
& -\left[\Delta\left(\alpha,\varphi\right) \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right] \frac{\psi'\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}{\varphi\psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)} \\
& +\frac{\gamma e^{2\frac{\alpha-\underline{\alpha}}{\varphi}} \eta}{\varphi \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)} + \frac{\gamma\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)\eta}{\varphi B} \frac{1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)+\gamma}{\left[1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right]^{2}} > 0
\end{aligned}
$$

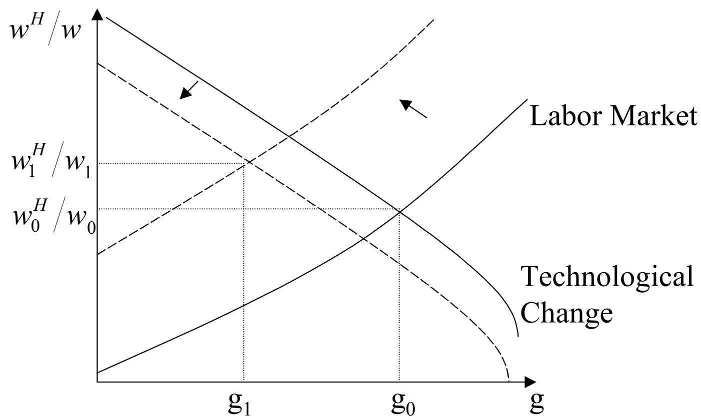
$$
\begin{aligned}
\frac{\partial G\left(\alpha,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \underline{\alpha}} &= \frac{\sigma}{\varphi} e^{\frac{\alpha-\underline{\alpha}}{\varphi}} \eta \\
& -\left[\Delta\left(\alpha,\varphi\right) \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right] \frac{\psi'\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)}{\varphi\psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)} \\
& +\frac{\gamma e^{2\frac{\alpha-\underline{\alpha}}{\varphi}} \eta}{\varphi \psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)} + \frac{\gamma\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)\eta}{\varphi B} \frac{1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)+\gamma}{\left[1-\gamma \ln\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)\right]^{2}} > 0
\end{aligned}
$$

$$
\frac{\partial G\left(\alpha,\ \varphi,\ \underline{\alpha},\ B,\ \gamma,\ \eta,\ \rho,\ \sigma\right)}{\partial \eta} =
$$

$$
-\sigma\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)-\frac{\gamma e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}-1\right)}{\psi\left(e^{\frac{\alpha-\underline{\alpha}}{\varphi}}\right)} < 0
$$

Figure 1

1.a Effect of a biased technological change or a reduction in the learning speed



1.b Effect of a "neutral" technological change or an increase in the consumers' propensity to save

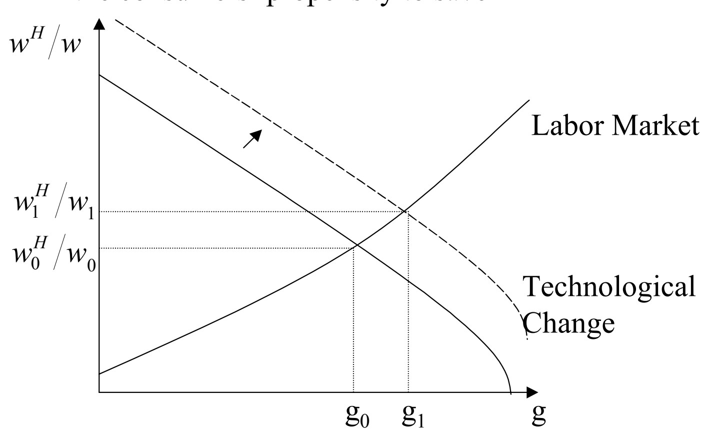


Figure 1:

Figure 2: Dynamic effect of a biased technological change

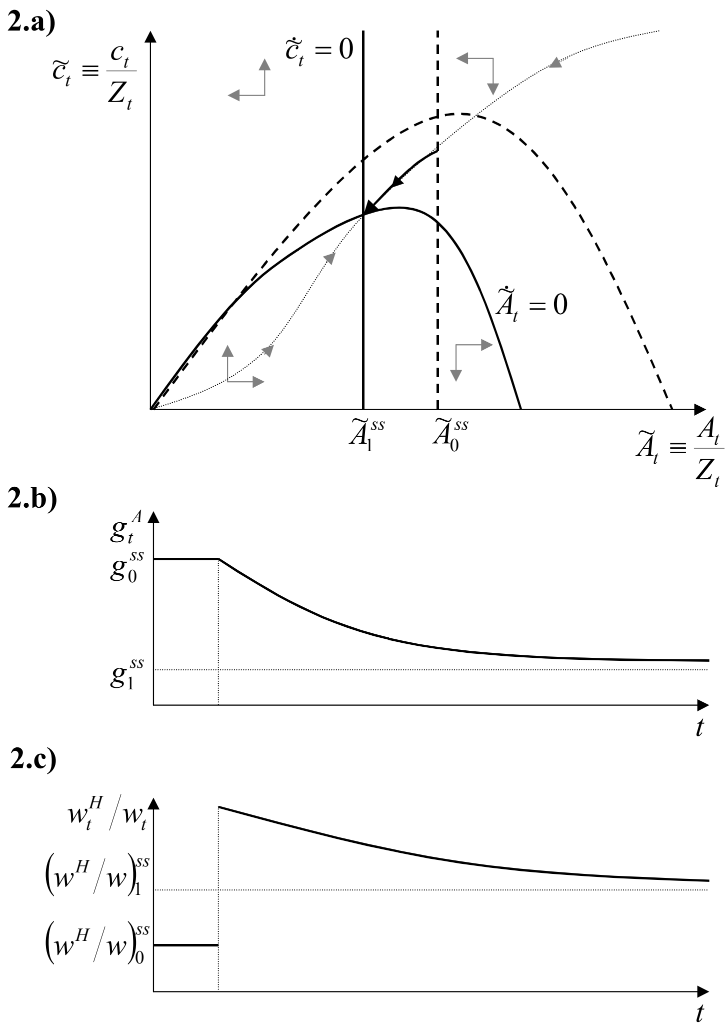


Figure 2:

Figure 3: Slow down of the standardization process

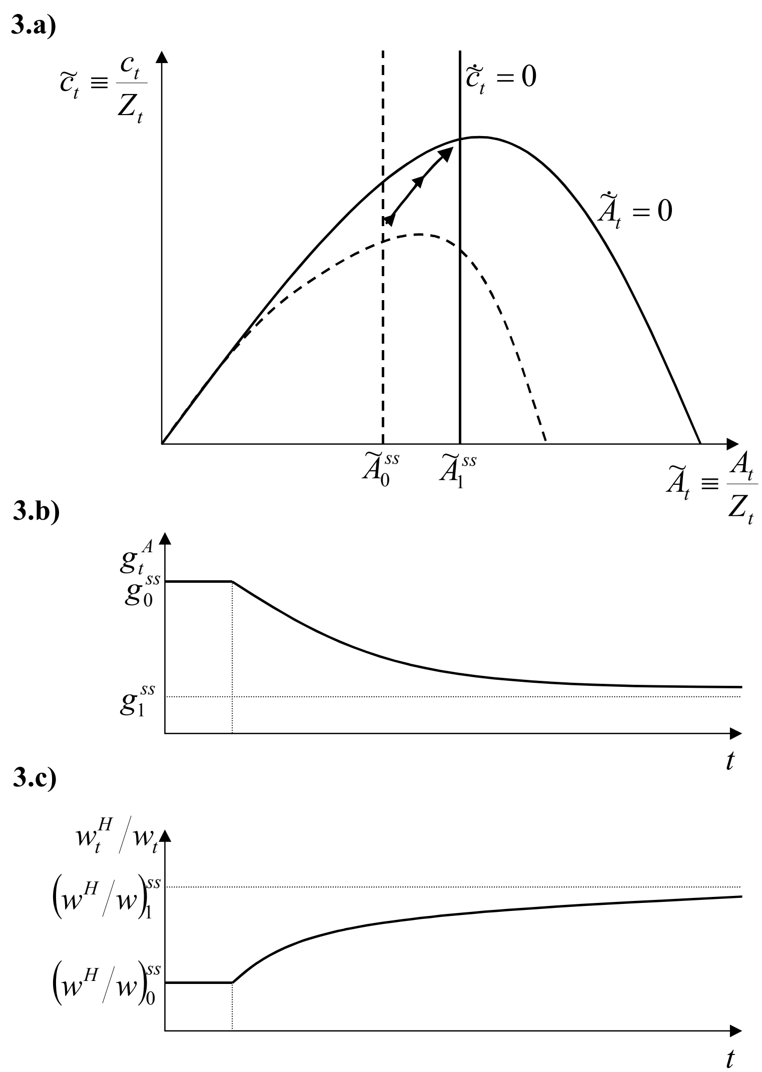


Figure 3:

Figure 4: Transition to the Steady State departing from the initial "quasi steady state"

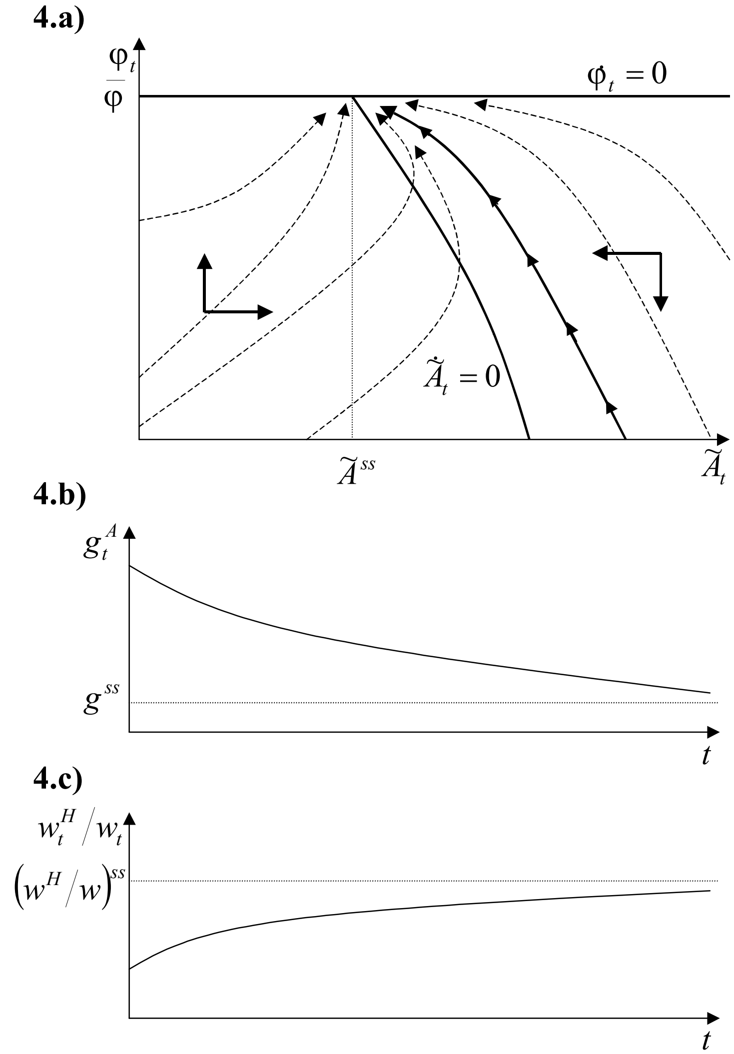


Figure 4:

Figure 5

1.a Effect of an increase in the minimum talent required to work as skilled worker

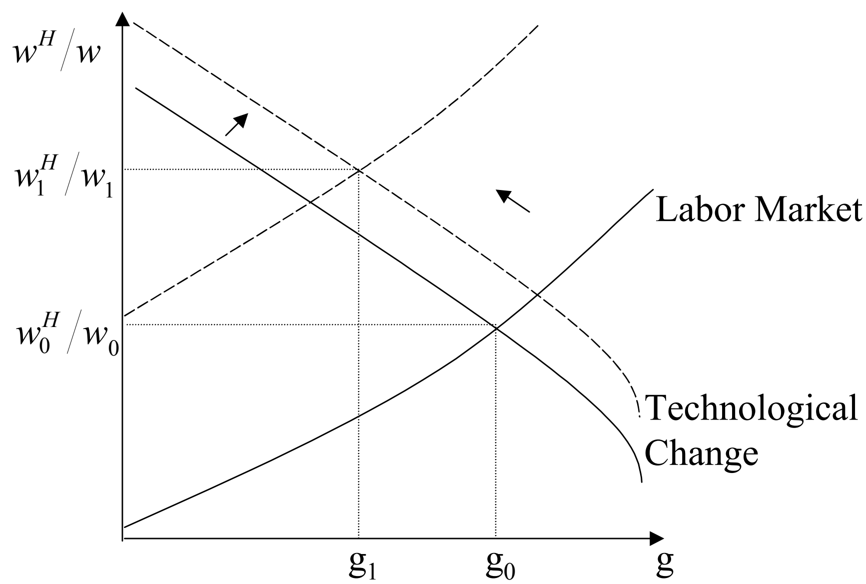
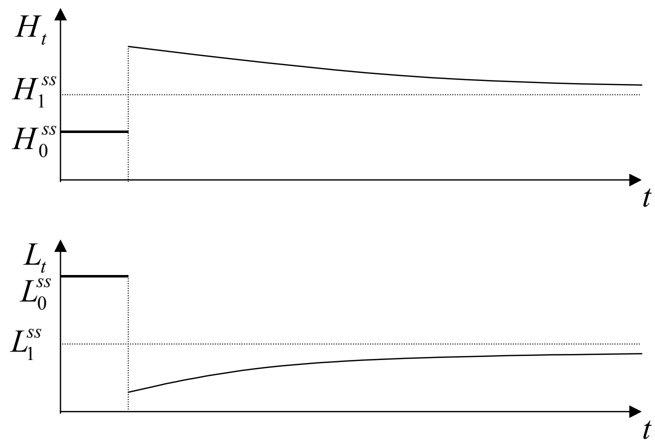


Figure 5:

Figure 6

6.a: Dynamic effect of a biased technological change (an increase in φ or α̲) over skilled/unskilled labor



6.b: Dynamic effect of a reduction in the standardization speed (a reduction in η) over skilled/unskilled labor

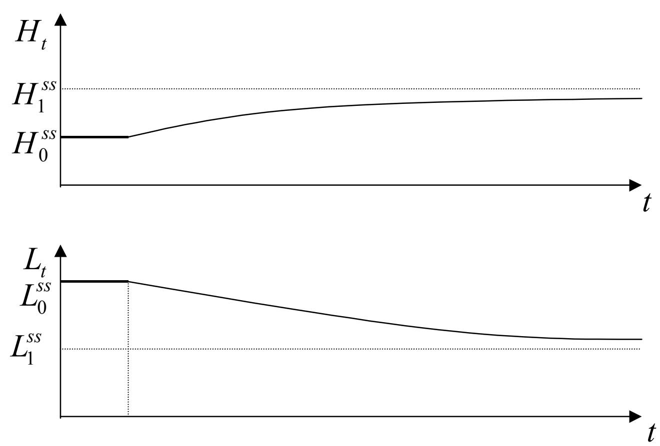


Figure 6: