Profitable Unproductive Innovations*

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Abstract

The present paper studies resource allocation in an expanding product variety framework à la Romer (1990). We consider the limiting case where net returns to specialization are zero and, therefore, R&D is completely useless, socially. Nevertheless, the market equilibrium involves allocation of ressources to R&D, and this wasteful allocation may take place even in a steady state.

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1 Introduction

The paper studies resource allocation in an expanding product variety framework à la Romer (1990), but with the parameters for returns to specialization, market power, and share of capital, respectively, disentangled from each other. We examine the limiting case where, at the aggregate level, indirect negative effects of specialization completely offset the immediate private benefits of innovation so that returns to specialization are zero.

Let manufacturing output be

$$Y = A^{\eta} X^{\alpha} {N_Y}^{1-\alpha},$$

where A indicates the level of technical knowledge in society, N_Y is labor input, and X is a CES aggregate of quantities, x_i, of specialized capital goods with elasticity of substitution equal to $1/(1-\varepsilon)$. The parameter η captures "net returns to specialization", i.e., the degree to which society benefits from specializing production in an increasing number of branches. The original Romer (1990) article had implicitly the three parameters linked by $\eta = 1-\alpha$ and $\alpha = \varepsilon$.[1] As a result, the model had the particular feature that the amount of R&D is always insufficient compared to the social optimum. In later articles Benassy (1998), Groot and Nahuis (1998), and Alvarez and Groth (2002) showed that disentangling one or both of Romer's parameter links could lead to too much R&D in the market economy. But none of these papers considered the possibility that $\eta = 0$, that is, the case where, at the aggregate level, indirect, negative effects of specialization completely offset the immediate positive effects of an enlarged spectrum of input varieties. We are going to show that in spite of R&D being, in this case, completely useless from a social point of view, there may exist a market equilibrium involving allocation of resources to R&D, indeed this is so even in a steady state.

2 The model

The economy is populated by a constant number, L, of households each of which supplies one unit of labor inelastically and has the utility function

$$U_0 = \int_0^{\infty} e^{-\rho t} \frac{c^{1-\theta}-1}{1-\theta} dt, \quad \rho > 0, \theta > 0, \tag{2.1}$$

[1]That is, $Y = (\int_0^A {x_i}^{\alpha} di) N_Y^{1-\alpha}$, ignoring that Romer considered *two* kinds of labor.

where $c = c(t)$ is consumption at time t.[2]

There are two production sectors, here called the basic-goods sector and the specialized capital-goods sector. In the *basic-goods sector* the production function is as above, though with $\eta = 0$:

$$Y = X^{\alpha} {N_Y}^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{2.2}$$

There is a continuum of specialized capital goods, measured on the interval $[0, A]$, where A indicates the stock of engineering principles that grows through research. The composite factor X is given by

$$X = A(\frac{1}{A}\int_0^A {x_i}^{\varepsilon} di)^{\frac{1}{\varepsilon}}, \qquad 0 < \varepsilon < 1. \tag{2.3}$$

The output of basic goods is used for consumption, $C \equiv cL$, and investment in "raw" capital. The stock of raw capital K changes according to

$$\dot{K} = Y - C - \delta K, \qquad \delta \geq 0, \quad K(0) = K_0 > 0. \tag{2.4}$$

In the *specialized capital-goods sector,* which is also the *innovative sector,* a unit of raw capital can immediately be transformed to a specialized capital good on the basis of a given technical design. The number of new designs created per time unit is assumed proportional to the existing stock of knowledge, as in Romer (1990),

$$\dot{A} = \gamma N_A A, \qquad \gamma > 0, \quad A(0) = A_0 > 0, \tag{2.5}$$

where γ is a productivity parameter, and N_A is aggregate research work.

Because of the strict concavity of X in x_i and the symmetric cost structure, static efficiency requires $x_i = x$ for all $i \in [0, A]$.[3] Hence, assuming static efficiency, $X = Ax$ from (2.3), and when demand for raw capital equals supply we have

$$X = Ax = K. \tag{2.6}$$

Inserting into (2.2) gives output of basic goods as

$$Y = K^{\alpha} {N_Y}^{1-\alpha}. \tag{2.7}$$

A feasible path $(K, A, C, Y, N_Y, N_A)_{t=0}^{\infty}$ is called a *steady state* if K, C, and Y are strictly positive and grow at constant (though not necessarily equal or positive)

[2] In case $\theta = 1$, the expression $\frac{c^{1-\theta}-1}{1-\theta}$ should be interpreted as $\ln c$.

[3] Therefore, obsolescence of old capital goods never occurs.

rates. Let the rate of growth of a strictly positive variable x be denoted g_x, i.e., $g_x \equiv \dot{x}/x$. Let u be the fraction of total labor supply employed in the basic-goods sector, i.e., with full employment, $N_Y \equiv uL$, $0 \leq u \leq 1$. By (2.5), $g_A \geq 0$ always.

Lemma 1 *(i) In a steady state* $g_c = g_Y = g_K = 0$. *(ii) Further,* g_A *is constant and satisfies* $0 \leq g_A = \gamma(1-u)L < \gamma L$.

Proof. (i) Consider a steady state. By definition of a steady state, $Y > 0$; hence, from (2.7), $N_Y \equiv uL > 0$ and

$$g_Y = \alpha g_K + (1-\alpha) g_u. \tag{2.8}$$

Therefore, since in a steady state, g_Y and g_K are constant, g_u is also constant, implying $g_u = 0$ in view of $0 < u \leq 1$ for all $t \geq 0$. Now (2.8) gives $g_Y = g_K = 0$. Then, by (2.4), cL/K is constant, hence $g_c = g_K = 0$. (ii) From (2.5), $g_A = \gamma(1-u)L$ is constant, since u is constant. Further, since $0 < u \leq 1$, we have $0 \leq g_A < \gamma L$. ■

Now, we will embed this economic system in a market economy. Apart from allowing $\varepsilon \neq \alpha$ and $\eta = 0$, the set-up is similar to Romer (1990).

The representative firm in the basic goods sector rents labor at the wage w and specialized capital goods at the rental rate R_i, $i \in [0, A]$. Using basic goods as our *numeraire,* profit maximization under perfect competition yields

$$\frac{\partial Y}{\partial N_Y} = (1-\alpha)\frac{Y}{N_Y} = w, \tag{2.9}$$

$$\frac{\partial Y}{\partial x_i} = \alpha \frac{Y}{X}\frac{\partial X}{\partial x_i} = R_i. \tag{2.10}$$

From (2.10) we can express the demand for the specialized capital good i conditional on a given X as:

$$x_i = \frac{X}{A}\left(\frac{R_i}{R}\right)^{-\frac{1}{1-\varepsilon}}, \qquad i = 1, 2, ..., A, \tag{2.11}$$

where $R = (A^{-1}\int_0^A R_i^{\frac{\varepsilon}{\varepsilon-1}})^{\frac{\varepsilon-1}{\varepsilon}}$ is the minimum cost per unit of X.

The supply of capital good i is decided by the firm that invented its design. The firm gets compensated for the sunk research cost through retention of monopoly power which is supported by patents of infinite duration[4]. Given design i, to deliver $x(i)$ units of capital good i, it takes $x(i)$ units of raw capital. At each instant of time, firm i, facing the demand curve (2.11) and taking X and R as given, sets the

[4]At initial time, $t = 0$, the number of firms, $A(0)$, is large enough so that each firm's action is negligible in the aggregate economy.

rental rate R_i so that current profit $\pi_i \equiv R_i x_i - (r+\delta)x_i$ is maximized, i.e.,

$$R_i = \frac{1}{\varepsilon}(r+\delta), \tag{2.12}$$

where r is the real rate of interest.

Since, by (2.12), all firms in the specialized capital-goods sector set the same rental price, they supply the same quantity, x, and they earn the same profit

$$\pi = (\frac{1}{\varepsilon} - 1)(r+\delta)x. \tag{2.13}$$

The equilibrium value, p, of a patent satisfies the no-arbitrage condition

$$\frac{\pi + \dot{p}}{p} = r. \tag{2.14}$$

There is free entry to research activity. Research is done by new firms wanting to enter the specialized capital-goods sector. Profit maximization subject to (2.5) entails, in equilibrium,

$$w \geqq p\gamma A, \qquad \text{with} = \text{if } N_A > 0. \tag{2.15}$$

By increasing A, research has a positive external effect on the productivity of future research[5]. But since aggregate returns to specialization are zero, research does not contribute to total factor productivity in manufacturing (in contrast to the endogenous growth literature).

Households consume and save, and savings can be either in capital or in shares of the monopoly firms. Financial wealth of the representative household is $v \equiv (K + pA)/L$. The household makes a plan $(c)_{t=0}^{\infty}$ to maximize U_0 subject to $\dot{v} = w + rv - c$, $v(0) = v_0$, and the standard no-Ponzi-game condition. Necessary and sufficient conditions for a solution are that the Keynes-Ramsey rule,

$$g_c = \frac{1}{\theta}(r - \rho), \tag{2.16}$$

and the transversality condition, $\lim_{\tau\to\infty} ve^{-\int_t^\tau r ds} = 0$, hold for all $t \geq 0$.

Given the clearing conditions, $K = X = xA$, $L = N_A + N_Y$, and the definitions $k \equiv K/N_Y$ and $u \equiv N_Y/L$ we have, from (2.9), (2.10), and (2.12),

$$w = (1-\alpha)\frac{Y}{uL} = (1-\alpha)k^{\alpha}, \tag{2.17}$$

$$\frac{1}{\varepsilon}(r+\delta) = \frac{\partial Y}{\partial K} = \alpha\frac{Y}{K} = \alpha k^{\alpha-1}. \tag{2.18}$$

[5]Each research firm is small and therefore perceives, correctly, its contribution to aggregate $\dot{A}$ to be practically negligible.

If $u < 1$, i.e., $N_A > 0$, then w also equals the value of the marginal product of labor in research so that (2.15) reduces to $w = p\gamma A$. This together with (2.17) gives the market value of a patent as

$$p = \frac{1-\alpha}{\gamma A} k^{\alpha}. \tag{2.19}$$

An *interior equilibrium* is an equilibrium such that for all $t \geq 0$, $0 < u < 1$ (there is positive employment in both sectors). In an interior equilibrium, by (2.19), $g_p = \alpha g_k - g_A$. Inserting this together with (2.13), (2.18), and (2.19) into (2.14), using the fact that $\gamma u L = \gamma L - g_A$, from (2.5), gives the market interest rate as

$$r = \frac{1-\varepsilon}{1-\alpha} \alpha(\gamma L - g_A) + \alpha g_k - g_A. \tag{2.20}$$

3 The main result

By Lemma 1, in a steady state, $g_c = 0$, implying $r = \rho$, from (2.16); and since also $g_k = 0$ in a steady state, inserting into (2.20) gives

$$g_A = \frac{(1-\varepsilon)\alpha\gamma L - (1-\alpha)\rho}{1-\varepsilon\alpha} > 0, \tag{3.1}$$

presupposing the parameter restriction

$$\rho < \frac{1}{1-\alpha}(1-\varepsilon)\alpha\gamma L. \tag{A1}$$

This restriction comes from the interiority condition $u < 1$. If A1 is violated, then impatience is so large that R&D activity cannot be supported in a steady state equilibrium. In that case the steady state solution of the model is like that of a one-sector model without technical change, and $g_A = 0$.

Defining *uniqueness* of a steady state to be present if, given the parameters $(\alpha, \varepsilon, \theta, \rho, \delta, \gamma, L)$, there exists only one pair $(g_c, Y/K)$ which is consistent with the steady state conditions, we have:

Proposition 1 *(i) There exists a unique steady state in the market economy; it has* $g_c = 0$, *and if A1 holds, then*

$$N_A = \frac{(1-\varepsilon)\alpha\gamma L - (1-\alpha)\rho}{(1-\varepsilon\alpha)\gamma} > 0, \tag{3.2}$$

and $g_A = \gamma N_A > 0$, *while* $N_A = g_A = 0$ *otherwise. In any case* $Y/K = (\rho+\delta)/(\varepsilon\alpha)$. *(ii) In contrast, the unique steady state of the social optimum has* $N_A^* = g_A^* = g_c^* = 0$, *and* $Y/K^* = (\rho+\delta)/\alpha$.

Proof. (i) Given A1, (3.2) follows from (2.5) and (3.1). With $r = \rho$, (2.16) and (2.18) give $Y/K = (\rho + \delta)/(\varepsilon\alpha)$, whether or not A1 holds. (ii) Since increasing A is a waste of resources, the social planner's problem reduces to a standard one-sector Ramsey problem with no technical progress. Hence, in steady state, $N_A^* = g_A^* = g_c^* = 0$, and, from the Keynes-Ramsey rule, $g_c^* = (\alpha Y/K^* - \delta - \rho)/\theta = 0$, we get $Y/K^* = (\rho + \delta)/\alpha$. ■

Though from a social point of view resources applied to R&D are wasted, the firms bearing the research costs are able to recover them by taking advantage of their market power (granted by the patent legislation). As soon as a new specialized capital good is invented and supplied to the market, there is a demand for it due to the symmetry and the strict concavity of Y in x_i, $i \in [0, A]$. This demand reflects the productive effect of redistribution of a given amount of raw capital to an enlarged spectrum of varieties – the "direct effect" of increased specialization. But at the aggregate level this effect is completely outweighed by a concomitant decrease in the productivity of the old capital goods, since we have $\eta = 0$, i.e., net returns to specialization are zero. This "indirect effect" of increased specialization (a negative externality) may be interpreted as a result of higher complexity in a more specialized world[6].

As to the comparative statics of (3.2) we have

$$\begin{aligned} \frac{\partial N_A}{\partial \varepsilon} &= -\frac{\alpha(1-\alpha)(\gamma L + \rho)}{\gamma(1-\varepsilon\alpha)^2} < 0, \\ \frac{\partial N_A}{\partial \alpha} &= \frac{(1-\varepsilon)(\gamma L + \rho)}{\gamma(1-\varepsilon\alpha)^2} > 0. \end{aligned}$$

From this together with Proposition 1 it follows that both Y/K and N_A in the market economy depend on the substitution parameter ε while the allocation in the social optimum does not. When specialized capital-goods are close substitutes (ε high), the markup over marginal cost in the specialized capital-goods sector becomes low, making inventions of new designs less profitable, thereby reducing R&D. Similarly, the capital share parameter α affects resource allocation in the market economy, but not in the social optimum. Indeed, the higher is α, the lower is the wage share, $1 - \alpha$, which, given the factor prices, implies less room for profitable employment

[6]The decomposition of the effects of specialization into direct and indirect effects is studied, in a more general context, in Alvarez and Groth (2003). The indirect effect can be seen as a reminiscence of the "creative destruction" effect in the quality ladder models, cf. Aghion and Howitt (1998).

in the basic-goods sector[7]. Thereby more of the fixed labor force is available for employment in research.

Whether or not $N_A > 0$, the steady state of the market economy has a "too high" output/capital ratio. This is because monopoly pricing implies a wedge between the price of the services of specialized capital goods and the marginal cost of providing them.[8] Correction of the market failures may be accomplished by a subsidy $\sigma = 1 - \varepsilon$ to buyers of capital services and a tax $\tau = 1 - \frac{\varepsilon}{1-\varepsilon}\frac{1-\alpha}{\alpha}\frac{\rho}{\gamma L}$ on monopoly profits, supplemented by a lump-sum tax (or transfer) to cover the possible shortage (surplus) of public revenue.

4 Final remarks

Since our result emerges without ambiguity in a straightforward way it may well hold in a more general setting as to technologies in the two sectors, population growth, slightly *negative* net returns to specialization, etc.

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[7] (2.12), (2.17), and (2.18) give $uL = \frac{1-\alpha}{\alpha}K\frac{R}{w}$.

[8] Notice, that, given $A1$, along the steady state path of the market economy, variety expands at the rate $g_A > 0$, while market size, x, and profits, π, of each single monopolist contracts at the same rate (in view of (2.13) where $x = K/A$).