April 4th to April 12th, 2006 (Intensive Week)

1 Optimal unemployment insurance

1.1 (with observable effort)

Consider an economy where the probability of finding a job $p(a)$ is a function of effort $a \in [0,1]$. And we assume that once the agent gets a job, she will have wage w for ever. Thus, the individual problem is

$$\max_{a_t} E \sum_t \beta^t \left[u(c_t) - a_t\right]$$

There are two cases: when the agent has got a job, she will pay no effort and enjoy w for ever. The life long utility is

$$V^E = \sum_t \beta^t u(w) = \frac{u(w)}{1-\beta} \tag{1}$$

When the agent is still unemployed, she will have nothing to consumer. Her problem is

$$V^u = \max_a \left\{u(0) - a + \beta\left[p(a) V^E + (1 - p(a) V^{u\prime})\right]\right\} \tag{2}$$

If the optimal solution of a is interior, $a \in (0,1)$, then the first order condition gives

$$-1 + \beta p'(a)\left(V^E - V^u\right) = 0 \tag{3}$$

And since the V^u is stationary,

$$V^u = \max_a \left\{u(0) - a + \beta\left[p(a) V^E + (1 - p(a) V^u)\right]\right\} \tag{4}$$

Solving (3)(4) gives the optimal a and V^u. Another way is to successively substitute a and obtain a solution because the problem defines a contraction mapping operator. We can fix V_0^u, then solve (4) to get $a(V_0^u)$ and obtain V_1^u. Keeping going until $V_n^u = V_{n+1}^u$. In a word, optimal effort level a^* solves (4) with $V^u = V^{u\prime}$.

The probability of finding a job $p(a)$ is called hazard rate. If agents did not find a job with effort level a^*, next period, she will still execute the same effort level a^*. Why? Because the duration of unemployment is not state variable in agent's problem. (If agents do not have enough realization about the difficulty of getting a job. With learning, their effort a will increase as they revise their assessment of the difficulty. But such revision of belief is not in this model.)

Now suppose resource is given to people who is unemployed to relive her suffering by a benevolent planner. This planner has to decide the minimal cost of warranting agent a utility level V, $\Psi(V)$. To warrant utility level V, the planner tells the agent how much to consume, how much effort to exert and how much utility she will get if she stay unemployed next period. Obviously, the cost function $\Psi(V)$ is increasing in V.

The cost minimization problem of the planner can be written in the following recursive problem:

$$\Psi(V) = \min_{c,a,V^u} c + [1 - p(a)] \frac{1}{1+r} \Psi(V^u) \tag{5}$$

subject to

$$V = u(c) - a + \beta \left[p(a) V^E + (1 - p(a)) V^u \right] \tag{6}$$

To solve the problem, construct Lagrangian function

$$\mathcal{L} = c + [1 - p(a)] \beta \Psi(V^u) + \theta \left[V - u(c) + a - \beta \left[p(a) V^E + (1 - p(a)) V^u \right] \right]$$

FOC: (c)

$$\theta = \frac{1}{u_c} \tag{7}$$

(a)

$$\Psi(V^u) = \theta \left[\frac{1}{\beta p'(a)} - \left(V^E - V^u \right) \right] \tag{8}$$

(V^u)

$$\Psi'(V^u) = \theta \tag{9}$$

Envelope condition

$$\Psi'(V) = \theta \tag{10}$$

We will work on some implication of these conditions:

1. Compare (8) and (3), we can see that the substitution between consumption and effort is different from the one in agent's problem without unemployment insurance. This is because the cost of effort is higher for work that it is from the viewpoint of planner.

2. (9) tells us that the marginal cost of warranting an extra unit of utility tomorrow is θ.,provided that tomorrow V^u is optimally chosen when today's promise is V. And (10) tells us that the marginal cost of warranting an extra unit of V today is θ.

3. Given that Ψ is strictly concave, $V = V^u$.

4. Regardless of unemployment duration, $V = V^u$. So, effort required the planner is the same over time. Hazard rate is still constant.

Next , we will study the case when effort is not observable. Planner can only choose consumption and V^u. Effort level is chosen optimally by worker and it is unobservable.

1.2 (Problem with UN-observable effort)

When a is not observable, planner can only choose c and V^u. And households choose a optimally. Now it becomes a principle-agent problem. We will solve the problem backward.

If given c and V^u, the agent will solve

$$\max_a u(c) - a + \beta \left[p(a) V^E + (1 - p(a)) V^u \right] \tag{11}$$

FOC is

$$\left[p'(a) \beta \right]^{-1} = V^E - V^u \tag{12}$$

This FOC gives an implicit function of a as a function of V^u: $a = g(V^u)$. (Because c and a are separate in the utility function, a is not a function of c).

Then, the planner solve her cost minimization problem, in which the optimality condition is also one constraint.

$$\Psi(V) = \min_{c,a,V^u} c + [1 - p(a)] \beta \Psi(V^u)$$

subject to

$$\begin{aligned} V &= u(c) - a + \beta \left[p(a) V^E + (1 - p(a)) V^u \right] & (13) \\ 1 &= \left[p'(a) \beta \right] \left[V^E - V^u \right] & (14) \end{aligned}$$

Lagrangian is

$$\begin{aligned} &c + [1 - p(a)] \beta \Psi(V^u) + \theta \left[V - u(c) + a - \beta \left[p(a) V^E + (1 - p(a)) V^u \right] \right] \\ &+ \eta \left[1 - \left[p'(a) \beta \right] \left[V^E - V^u \right] \right] \end{aligned}$$

FOC: (c)

$$\theta^{-1} = u_c$$

(a)

$$\Psi(V^u) = \theta \left[\frac{1}{\beta p'(a)} - \left(V^E - V^u \right) \right] - \eta \frac{p''(a)}{p'(a)} \left(V^E - V^u \right) \tag{15}$$

(V^u)

$$\Psi'(V^u) = \theta - \eta \frac{p'(a)}{1 - p(a)} \tag{16}$$

Envelope condition

$$\Psi'(V) = \theta \tag{17}$$

Again, (16) tells the marginal cost to warrant additional amount of delayed promise. (17) gives the marginal cost to increase today's utility. The Lagrangian multiplier associated with constraint (14) is positive, $\eta > 0$, which means that the constraint is binding. So,

$$\eta \frac{p'(a)}{1 - p(a)} > 0$$

Therefore, we have

$$\Psi'(V^u) < \Psi'(V) \Rightarrow V^u < V$$

from the strict concavity of $\Psi(.)$. The delayed promised utility decreases over time.

Let $\theta^u = \theta - \eta \frac{p'(a)}{1-p(a)}$, then $\theta^u < \theta$,which tells us about the consumption path. Consumption decreases over time because $\theta^{-1} = u_c$.

Overall, we get the following model implications: optimal unemployment insurance says that longer unemployment period the agent stays, the less insurance she will be insured for. In this way, the planner induces the higher effort level. Although you cannot let people do what is optimal, such behavior can be achieved by giving out less consumption and promised utility over time. This model implies that time-varying unemployment insurance plan is optimal, under which the replacement rate θ goes down over time.

2 Models with one sided lack of commitment

We now study an economy where the agent cannot commit to the contract that the Social Planner offers her. This means that as long as what the Planner offers is better than what the agent can do alone she stays around. If in some period the shock that the agent receives is so good that her value under autarky is higher than what the planner is offering she will be willing to walk away and be on her own (at no cost). On the other hand, if the planner wants to keep her around, he now has to offer even more. In this model there is no storage or financial markets, and so the agent consumes the fruit (or fish) which is just the shock that she receives in each period.

Consider a village of agents, each of them receive $s \in \{s_1, s_2, ..., \bar{s}\}$ in every period. We assume that s is iid. The probability that a certain s is realized is Π_s. h_t is a history of shocks up to period t.

First, if the agent stays in autarky, she will enjoy total utility,

$$V_{AUT} = \sum_{t=0}^{\infty} \beta^t \sum_s \Pi_s u(s) = \frac{\sum_s \Pi_s u(s)}{1-\beta}$$

Note that here V^A is the utility of the agent before endowment shock is realized.

Now we assume that the Insurer offers a contract to the agent, which transfer resources and provide insurance to her. The Insurer can commit. But the agent may leave the insurer (Commitment problem). Thus, this model has one-sided commitment model: an agent can walk away from a contract but the other cannot. Therefore, the contract should be always in the interest of the agent for her to stay.

We define a contract $f_t : H_t \to c \in [0, \tau]$. We will see next class that incentives compatibility constraint requires that at each node of history H_t, the contract should guarantee a utility which is higher than that in autarky.

Notice that the problem is different from Lucas tree model because of the shock realization timing. In Lucas tree model, shock is state variable because action takes place after shock is realized. Thus, action is indexed by shock. Here action is chosen before shock realization. Therefore, shock is not a state variable and action is state contingent.

In Lucas tree model, $V(s) = \max_c u(c) + \beta \sum_{s'} \Pi_{ss'} V(s')$. Here, if we write the problem recursively, it is $V = \max_{c_s} \sum_s \Pi_s u(c_s) + \beta V$.

Remember, the Insurer will make a deal with the agent. They sign a contract to specify what to do in each state. $h_t \in H_t$. Contract is thus a mapping $f_t(h_t) \rightarrow c(h_t)$. With this contract, the agent gives y_t to the Insurer and receives $c_t = f_t(h_{t-1}, y_t)$. But if the agent decided not to observe the contract, she consumes y_t this period and cannot enter a contract in the future, i.e. she has to live in autarky in the future.

For the Insurer to keep the agent around, the contract has to be of interest to the agent (commitment issue). There are two possible outcome if this contract is broken. One is that the agent goes away with current and future endowment. The other is that they renegotiate. We ignore the second possibility as no renegotiation is allowed, since it would introduce a lot of messy algebra.

The first best outcome is to warrant a constant consumption c_t to the agent who is risk averse. But because of the one-side lack of commitment, the first best is not achievable. The contract should always be attractive to the agent, otherwise, when she gets lucky with a high endowment y_s, she will feel tempted to leave. So, this is a dynamic contract problem which the Insurer will solve in order to induce good behavior from the agent. The contract is dynamic because nature keeps moving.

We say the contract $f_t(h_t)$ is incentive compatible or satisfies participation constraint if for all h_t,

$$u(f_t(h_t)) + \sum_{\tau=1}^{\infty} \beta^{\tau} \sum_s \Pi_s u(f_{t+\tau}(h_{t+\tau})) \geq u(y_s(h_t)) + \beta V^A \tag{18}$$

The left hand side is utility guaranteed in the contract. And the right hand side is the utility that the agent can get by herself. The participation constraint is not binding if y_s is low. And when y_s is high, PC is binding.

2.1 Problem of the Insurer

In this model, problem of the Insurer is to find an optimal contract that maximizes the value of such a contract of warranting V to her. We define the problem using recursive formula. Firstly, let's define the value of contract to the Insurer if she promised V to the agent. $\Phi(V)$ can be defined recursively as the following:

$$\Phi(V) = \max_{\{c_s, \omega_s\}_{s=1}^{S}} \sum_s \Pi_s[(s - c_s) + \beta \Phi(\omega_s)] \tag{19}$$

subject to

$$u(c_s) + \beta \omega_s \geq u(s) + \beta V^A \quad \forall s \tag{20}$$

$$\sum_s \Pi_s[u(c_s) + \beta \omega_s] \geq V \tag{21}$$

Notice that there are $1+S$ constraints. The choice variables c_s, ω_s are state-contingent where ω_s is the promised utility committed to the agent in each state. In the objective function, $\sum_s \Pi_s(s-c_s)$ is the expected value of net transfer.

There are two sets of constraints. (20) is IC and (21) is promise keeping constraint.

The First Order Conditions to the problem are:

(c_s)

$$\Pi_s = (\theta_s + \lambda\Pi_s)u'(c_s) \tag{22}$$

(ω_s)

$$-\Pi_s\Phi'(\omega_s) = \lambda\Pi_s + \theta_s \tag{23}$$

(μ)

$$\sum_s \Pi_s[u(c_s) + \beta\omega_s] = V \tag{24}$$

(λ)

$$u(c_s) + \beta\omega_s \geq u(s) + \beta V^A \tag{25}$$

In addition, Envelope Theorem tells that:

$$\Phi'(v) = -\lambda \tag{26}$$

Interpreting the first order conditions:

1. (22) tells that in an optimal choice of c_s, the benefit of increasing one unit of c equals the cost of doing so. The benefit comes from two parts: first is $\lambda\Pi_s u'(c_s)$ as increasing consumption helps the Insurer to fulfill her promise and the second part is $\theta_s u'(c_s)$ since increase in consumption helps alleviated the participation constraint. And the cost is the probability of state s occurs.

2. (23) equates the cost of increasing one unit of promised utility and the benefit. The cost to the Insurer is $-\Pi_s\Phi'(\omega_s)$ and the benefit is $\mu\Pi_s + \lambda_s$ which helps the Insurer deliver promise utility and alleviate the participation constraint.

How about the contract value $\Phi(V)$. First, $\Phi(V)$ can be positive or negative.

Claim: There exits V such that $\Phi(V) > 0$.

What's the largest V we will be concerned with? When PC will be binding for sure. If PC binds for the best endowment shock s, then PC holds for all the shock s. When the agent gets the best shock, the best autarky value is then

$$V_{AM} = u(\bar{s}) + \beta V_A$$

And the cheapest way to guarantee V_{AM} is to give constant consumption $\overline{c_S}$, such that

$$V_{AM} = \frac{u(c_{\bar{s}})}{1-\beta}$$

From this case, we can see that because of lack of commitment, the Insurer will have to give more consumption in some states. While when there is no lack of commitment, strict concavity of $u(.)$ implies that constant stream of consumption beats any $\{c_t\}$ that have the same present value, as there is no PC.

2.2 Characterizing the Optimal Contract

We will characterize the optimal contract by considering the two cases: (i) $\theta_s > 0$ and (ii) $\theta_s = 0$.

Firstly, if $\theta_s = 0$, we have the following equations from FOC and EC:

$$\Phi'(\omega_s) = -\mu \tag{27}$$

$$\Phi'(V) = -\mu \tag{28}$$

Therefore, for s where PC is not binding,

$$V = \omega_s$$

c_s is the same for all s. For all s such that the Participation Constraint is not binding, the Insurer offers the same consumption and promised future value.

Let's consider the second case, where $\theta_s > 0$. In this case, the equations that characterize the optimal contract are:

$$u'(c_s) = \frac{-1}{\Phi'(\omega_s)} \tag{29}$$

$$u(c_s) + \beta\omega_s = u(s) + \beta V^A \tag{30}$$

Note that this is a system of two equations with two unknowns (c_s and ω_s). So these two equations characterize the optimal contract in case $\theta_s > 0$. In addition, we can find the following properties by carefully observing the equations:

1. The equations don't depend on V. Therefore, if a Participation Constraint is binding, promised value does not matter for the optimal contract.

2. From the first order condition with respect to ω_s, $\Phi'(\omega_s) = \Phi'(V) - \frac{\theta_s}{\Pi_s}$, where $\frac{\theta_s}{\Pi_s}$ is positive. Besides, we know that Φ is concave. This means that $V < \omega_s$. In words, if a Participation Constraint is binding, the moneylender promises more than before for future.

Combining all the results we have got, we can characterize the optimal contract as follows:

1. Let's fix V_0. We can find a $s^*(V_0)$, such that $\forall s < s^*(V_0)$, the participation constraint is not binding and $\forall s \geq s^*(V_0)$, the constraint is binding, i.e. $\theta_s > 0$.

2. The optimal contract that the Insurer offers to an agent is the following:

   If $s_t \leq s^*(V_0)$, the Insurer gives $(V_0, c(V_0))$. Both of them are the same as in the previous period. In other words, the Insurer offers the agent the same insurance scheme as before.

If at some point in time $s_t > s^*(V_0)$, the moneylender gives $(V_1, c(V_1))$, where $V_1 > V_0$ and c doesn't depend on V_0. In other words, the moneylender promises larger value to the agent to keep her around.

So the path of consumption and promised value for an agent is increasing with steps.