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1 Growth Through Externalities in Capital Accumulation

We have seen in the AK model that the growth rate is endogenous and determined solely by model primitives. Still, it is not directly or indirectly determined by the agents' choices in our model. In Lucas' human capital model, the growth rate is determined by the choice of agents, specifically by the optimal ratio of human and physical capital. The source of growth in Lucas' model is reproducibility of human capital. In this next model, Romer introduces the notion of externality generated by the aggregate capital stock to go through the problem of diminishing marginal returns to aggregate capital. In this model, the source of growth will be the aggregate capital accumulation, which is possible with a linear aggregate technology in capital as we saw in the AK model. The firms in our model will not be aware of this externality and will have the usual CRTS technology and observe the source of growth coming from the TFP parameter. As usual with externalities, the equilibrium outcome will not be optimal. Each firm has the following technology,

$$y_t = AK_t^{1-\alpha} k_t^{\alpha} n_t^{1-\alpha} \tag{1}$$

but since the firms are not aware of the positive externality they are facing they are solving the problem with the following technology.

$$y_t = \overline{A}_t k_t^{\alpha} n_t^{1-\alpha} \tag{2}$$

(3)

where

$$\overline{A}_t = A_t K_t^{1-\alpha}$$

We can see that the social planner in fact is solving an AK model in per-capita terms. So does the de-centralized version of this economy have a BGP and if it does, how would it look like? Assuming CRRA preferences without leisure we can derive the BGP condition and pin down the growth rate from the euler equation of a typical household,

$$1 = \beta\gamma^{-\sigma}(1+r) \tag{4}$$

where $\gamma = \frac{c_{t+1}}{c_t}$ is the growth rate at the balanced path as usual and r=MP$_k$. So to find out the marginal product of capital for the firm we differentiate the technology w.r.t. k_t,

$$1 + r_t = \alpha AK_t^{1-\alpha} k_t^{\alpha-1} n_t^{1-\alpha} + (1-\delta) \tag{5}$$

and since the prices are determined by aggregate state variables $K_t = k_t$ gives,

$$A\alpha - \delta = r \tag{6}$$

and substituting this into the euler equation we get the growth rate of consumption.

$$[(A\alpha - \delta + 1)\beta]^{\frac{1}{\sigma}} = \gamma \tag{7}$$

Solving the AK problem the SP faces we can verify the optimal growth rate for consumption is,

$$[(A - \delta + 1)\beta]^{\frac{1}{\sigma}} = \gamma^{sp}. \tag{8}$$

The important properties of the decentralized model are,

1. It is sub-optimal due to firms' unawareness of the externality they are facing and thus have lower growth rate.
2. Once again, the rental rate does not depend on the capital stock (due to the aggregate linear technology, the states variables drop out from the euler equation) and there is no transitional dynamics generated by the model.

To sum up what we have done so far, we have started with models that had exogenous growth and saw that we can make these models look and behave like our NGM after appropriate transformation. Then we went on to look at models that generate growth endogenously and saw that a prerequisite for growth in these models is linearity of the technology in reproducible factors. We looked at the simple AK model, where the technology is linear in capital stock and analyzed the BGP of such an economy. Then we looked at Lucas' human capital model, in which we had two forms of capital, human and physical, both of which are reproducible in terms of output. Then we analyzed the model by Romer, which again has linearity in the reproducible factor at the aggregate level (capital stock), but firms were facing the CRTS technology with diminishing marginal return on capital and not aware of the positive externality they face. Next we will see another model by Romer with monopolistic competition and a R&D sector which can generate endogenous growth.

2 Monopolistic Competition, Endogenous Growth and R&D

Romer's monopolistic competition model has three production sectors, the final goods production, intermediate goods production and R&D i.e. variety production. Our usual TFP parameter in production function will represent the 'variety' in production inputs and as we will see, the growth of varieties through research and development firms will make sure a balanced growth path is sustainable. The production function in this economy is,

$$Y_t = L_{1t}^{\alpha} \int_0^{A_t} x_t(i)^{1-\alpha} di \tag{9}$$

where $x_t(i)$ is the type i intermediate good and there is a measure A_t of different intermediate goods and L_{1t} is the amount of labor allocated to the final good production. The production function exhibits CRTS. The intermediate goods are produced with the following linear technology,

$$\int_0^{A_t} \eta x_t(i) di = K_t \tag{10}$$

Now suppose the variety of goods grows at rate γ,that is $A_{t+1} = \gamma A_t$. Is long run sustainable growth possible? The answer to this question will depend whether our final goods production technology is linear in growing terms. We do know that by the curvature of the technology, optimality implies equal amount of each variety will be used in production, $x_t(i) = x_t$,then we have,

$$A_t \eta x_t = K_t \tag{11}$$

and our output at this equal variety becomes,

$$Y_t = L_{1t}^{\alpha} A_t {x_t}^{1-\alpha} \tag{12}$$

then substituting for x_t we have,

$$Y_t = \frac{L_{1t}^{\alpha}}{\eta^{1-\alpha}} A_t^{\alpha} K_t^{1-\alpha} \tag{13}$$

thus if both A_t and K_t are growing at rate γ, then production function is linear in growing terms and long run balanced growth is feasible. Note that this model becomes very similar to our previous exogenous labor productivity growth under these assumptions. The purpose of this model is to determine γ endogenously. *What will be the source of growth, where does γ come from?* As we will see, there will be incentives for R&D firms to produce new 'varieties' because there will be a demand for them. These new varieties will be patented to intermediate good production firms, where a patent will mean exclusive rights to produce that intermediate good. So we will have monopolistic competition in the intermediate goods production. Now suppose the law of motion for 'varieties', which is the technology in R&D sector, is given by

$$A_{t+1} = (1 + L_{2t}\zeta) A_t \tag{14}$$

where L_{2t} is the labor employed in R&D sector. Note that this is not a regular law of motion in the sense that every new variety produced helps the production of further new varieties. Hence, there is a positive externality to variety production. Also, assume leisure is not valued and we have an aggregate feasibility condition for labor

$$L_{2t} + L_{1t} = 1 \tag{15}$$

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The period t problem of a firm in the competitive final good production sector is

$$\max_{x_t(i), L_{1t}} \{L_{1t}^{\alpha} \int_0^{A_t} x_t(i)^{1-\alpha} di - w_t L_{1t} - \int_0^{A_t} q_t(i) x_t(i) di\} \tag{16}$$

and since we have CRTS with perfect competition we have zero profit with following FOCs,

$$w_t = \alpha L_{1t}^{\alpha-1} \int_0^{A_t} x_t(i)^{1-\alpha} di \tag{17}$$

$$q_t(i) = (1-\alpha) L_{1t}^{\alpha} x_t(i)^{-\alpha} \tag{18}$$

notice that the inverse demand function for good of variety i is,

$$\left(\frac{q_t(i)}{(1-\alpha) L_{1t}^{\alpha}} \right)^{\frac{-1}{\alpha}} = x_t(i) \tag{19}$$

The intermediate goods industry will show monopolistic competition, in which there is only one firm, that is one patent holder, producing each variety. Each firm takes the demand of its variety and prices as given, and solves the following problem each period

$$\Pi_t(i) = \max_{x_t(i), K_t(i)} \{ q_t(i) x_t(i) - R_t K_t(i) \} \tag{20}$$

$$s.t. \quad x_t(i) = \frac{K_t(i)}{\eta}$$

plugging in the inverse demand function and the technology constraint, the FOC is,

$$(1-\alpha)^2 x_t(i)^{-\alpha} L_{1t} = R_t \eta \tag{21}$$

and because of the symmetry we mentioned ($x_t(i) = x_t = \frac{K_t}{\eta A_t}$) we can write this FOC as,

$$(1-\alpha)^2 (\frac{K_t}{\eta A_t})^{-\alpha} L_{1t} = R_t \eta \tag{22}$$

i.e. the rental price of capital is not equal to it's marginal product and there is opportunities for positive profit. But also remember there is a fixed cost of entering this industry, namely the price paid for the patent. Then as we will see, the relation between the two will be one of our equilibrium conditions. Now lets look at the problem of R&D firms,

$$\max_{A_{t+1}, L_{2t}} \{ p_t^P (A_{t+1} - A_t) - w_t L_{2t} \} \tag{23}$$

$$s.t. A_{t+1} = (1 + L_{2t} \zeta) A_t$$

where p_t^P is the patent of the price. Free entry is assumed, thus there will be zero profits in equilibrium. Notice also the R&D firm is solving a static problem without realizing the positive externality this period's decision creates on next periods production. As we will see, this and the monopoly power of the patent owners will be the sources of sub-optimality in the decentralized solution.The FOC is,

$$p_t^P = \frac{w_t}{\zeta A_t} \tag{24}$$

where the wage (w_t) is determined in the final goods market and given this price, equilibrium quantity will come from the demand function. As we mentioned before, one equilibrium condition will be that at any point in time, total profit a patent generates will be equal to price of it such that there will also be zero profit in the intermediate goods market.

$$p_t^P = \sum_{\tau=t}^{\infty} \frac{\Pi_t(i)}{(1+r)^{\tau-t}} \tag{25}$$

These conditions with constant growth equations for the growing variables is sufficient to characterize the equilibrium growth rate of this economy.

3 Economies with Frictions

In this part of the course, we will discuss some type of models with frictions. We are interested in mainly two types of frictions:

1. Hidden Actions

2. Lack of Commitment

The first model we study, labeled the 'optimal unemployment insurance' problem, deals with the first type of friction. In the next section, we deal directly with problems of commitment (only ONE-sided)