Macro 702, Sp 2007, Midterm.

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In the following there are 15 questions for 105 points. Please answer all questions. Be as BRIEF as you can and good luck. You have 80 minutes.

Growth Models (Catching up with the Jones)

There is an economy with many identical consumers and infinite time. Consumers have preferences

$$E\left\{\sum_{t=0}^{\infty}\beta^t\,u(c_t, n_t, C_{t-1})\right\}$$

where c_t is own consumption at time t, n_t is the fraction of time worked by the agent at time t and C_{t-1} is the economy wide average consumption in period $t-1$. The first partial derivative of u is positive while the others are negative. These agents hate the idea that other people have consumed a lot in the past.

Output can be produced with labor and capital according to a standard neoclassical production function

$$z_t\,F(K_t, N_t)$$

where K_t, is capital. Shocks to productivity z have finite support and follow a Markov chain with transition matrix Γ. Capital depreciates at rate δ. Output can be used either for consumption or for investment purposes.

1. (10 points) Define an Arrow-Debreu competitive equilibrium. Carefully define the commodity space, and the consumption and production possibility sets.

2. (5 points) State the two welfare theorems.

3. (5 points) Briefly describe what may go wrong for the first welfare theorem to hold.

Suppose now that the household owns capital and rents it to firms.

4. (10 points) Define a recursive competitive equilibrium. Make sure that you list the state variables. Briefly describe what is NOT standard in this problem.

Now assume that the government taxes/subsidizes labor at rate τ and returns the proceeds in a lump sum manner.

5. (10 points) Write a formula that links the equilibrium transfer as a function of the τ and the state variables.

6. (10 points) Imagine that the utility function is separable in all its arguments. Does this imply any simplification to your answer to the definition of recursive competitive equilibrium? Explain.

Lucas tree

Assume there is a representative agent economy. Each agent owns a tree that produces fruit d_t which follows a Markov chain with transition matrix Γ. In addition, each agent has a backyard that yields one unit of a special type of fruit that gives the same utility to the agents as the one from the tree but that cannot be traded due to regulations by the Health Department. The agent has preferences given by

$$E\left\{\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}\right\}$$

7. (5 points) Define equilibria recursively.

8. (15 points) How much would an individual agent pay to have the restriction on sales of backyard fruit lifted?

9. (5 points) Write a formula for an option to buy land tomorrow at price p_1 and then reselling it at price p_2 the period after.

10. (5 points) Make any assumptions that you want to ensure that in equilibrium the amount of consumption is constant.

11. (5 points) Under the assumptions of the previous question, characterize tree prices.

Industry Equilibriia

Imagine that the shock that affects a firm's productivity can take 10 values $\{s^1, \cdots, s^{10}\}$ with transition matrix Γ_{ij}. Imagine that the optimal policy is to quit if the shock is ever s^1 or s^2. Imagine that a measure .3 of firms enter each period and that they all enter with shock s^4. Give formulas for

12. (5 points) The transition function that characterizes the evolution of firms. Please verify that what you construct is indeed a transition.

13. (5 points) The updating operator for the distribution of firms in the industry

14. (5 points) What conditions and what kind of object would the stationary distribution of firms be?

15. (5 points) What price would be the one that gave zero profit given a production function, a cost of entry and a wage?