April 12th, 2007

Overlapping Generations (OLG or OG) Models

So far we been utilizing the NGM with infinite horizon with no demographic details. This wasn't because we didn't have the tools to consider a finite horizon. Many important questions in macroeconomics should be approached within a framework where these details matter. These models contain agents who are born at different dates and have finite lifetimes, even though the economy goes on forever. This induces a natural heterogeneity across individuals at a point in time, as well as nontrivial life-cycle considerations for a given individual across time. These features of the model can also generate differences from models where there is a finite set of time periods and agents, or from models where there is an infinite number of time periods but agents live forever.

Let's know first consider a very basic OLG model: Suppose that the economy goes on forever, and that at every date t there is born a new generation of individuals (t-born agents) who live for two periods. This is the simplest case where the generations overlap. There is also a generation of initial old guys at $t = 1$ who only live for one period. For now, every generation consists of a $[0, 1]$ continuum of homogeneous agents. In the 1st and 2nd periods of life, and let $w_1 = 3$ and $w_2 = 1$ be (time-invariant) endowments in the 1st and 2nd periods of life of a $t-$born agent. Let c_t^y and c_{t+1}^o denote the consumption of a $t-$born agent when he is young in period t and when he is old in period $t + 1$, respectively. The utility function of a t-born agent is $log(c_t^y) + log(c_{t+1}^o$. Initial old guys consume only c_0^o and are endowed $e_2 = 1$.

Definition: A SME for this economy is a list of sequences, $\{p_t^*, c_t^{y^*}, c_{t+1}^{o^*}\}_{t=0}^{\infty}$ s.t.

1. Given prices, $\{c_t^{y^*}, c_{t+1}^{o^*}\}$ solves the t-born agents problem: for $t = 0, 1, 2..$

$$\max_{c_t^y, c_{t+1}^o} \log(c_t^y) + \log(c_{t+1}^o)$$

$$s.t. \qquad p_t^* + 3p_{t+1}^* = c_t^y p_t^* + c_{t+1}^y p_{t+1}^*$$

2. $c_0^o = 1$

3. Markets are clear: $c_t^{y^*} + c_t^{o^*} = 4, \qquad t = 0, 1, ..$

Autarky is the unique equilibrium of this economy: Since only option for the initial olds is to eat their endowment, by market clearance condition every future generation consume their endowment. Then by the FOC of the t-born agents problem, we characterize the prices assuming $p_0^* = 1$:

$$\frac{c_{t+1}^{o^*}}{c_t^{y^*}} = \frac{p_t^*}{p_{t+1}^*} = \frac{1}{3} \qquad t = 0, 1, 2..$$

Is this equilibrium (autarky allocation) Pareto optimal? Obviously not (if at every period youngs give olds $\epsilon \leq 1$ of their endowment, everyone would be better off). Then what is wrong with the First Basic Welfare Theorem? Indeed there is nothing wrong with FBWT, but it just doesn't apply to this environment. Why it doesn't apply to this environment? Because autarky is a SME not an Arrow-Debrou equilibrium.

Now we introduce money (an intrinsically valueless piece of paper) into our model:

Definition: A monetary equilibria is a list of sequences, $\{q_t^*, R_t^*, c_t^{y^*}, c_{t+1}^{o^*}, m_t^*, b_t^*\}_{t=0}^{\infty}$ s.t.

1. Given prices, $\{q_t^*, R_t^*, \}$, $\{c_t^{y^*}, c_{t+1}^{o^*}, b_t^*\}$ solves the t-born agents problem: for $t = 0, 1, 2..$

$$\max_{c_t^y, c_{t+1}^o} \log(c_t^y) + \log(c_{t+1}^o)$$

$$s.t. \qquad c_t^y + q_t^* m_t + b_t = 3$$

$$c_{t+1}^o = q_{t+1}^* m_t + R_t^* b_t + 1$$

2. $c_0^o = 1 + q_0^* M$

3. Markets are clear: $c_t^{y^*} + c_t^{o^*} = 4$, $m_t = M$, $b_t^* = 0 \qquad t = 0, 1, ..$

Definition: A non-monetary equilibria is a monetary equilibria where $q_t^* = 0$, $\forall t$.

Definition: A stationary monetary equilibria is a monetary equilibria where $q_t^* = q^*$, $\forall t$.

Note: Please check the Problem set 9 Q3 for the difference equations that characterize the monetary equilibria.

April 16th, 2007

0.1 Labor Earnings

What is a good theory on ϵ? If we look at the average wage per hour at the different age ($w\epsilon$), the wage per hour increases with age, peaks at around 40, and slowly decreases until the retirement. Since w is assumed to be same for all agents, we need a theory that explains the difference in ϵ to replicate the hump shape of the average wage profile. What kind of theory do we have? There are two ways, in general:

1. Hormones: Take $\{\epsilon\}$ as exogenous; i.e., assuming that the young agents are useless because they are young.

2. Human capital theory. Assume that the difference in capital stock between the young agents and the old agents yields the difference in ϵ. There are three branches:

   (a) **Learning-by-doing:** Assume that agents accumulate human capital, ϵ by working. Agents learn something which enhances their human capital stock while they are working. Imagine interns of doctor. The young doctors learn how to do operations by actually working at hospitals. This idea is represented by:

   $$\epsilon_{i+1} = \phi_i(\epsilon_i; n_i)$$

   where n_i is hours worked of agents of age i. ϕ_i is indexed by i because learning ability can be different depending on age.
   The problem of an agent is :

   $$V_i(a, \epsilon) = \max_{c,n} u(c) + \nu(n) + \beta V_{i+1}(a', \epsilon')$$

   $$c + a' = w\epsilon' n + (1 + r)a$$
   $$\epsilon' = \phi_i(\epsilon, n)$$

   (b) **Learning-by-not-doing:** Assume that agents accumulate human capital by actually learning (which is different from working or enjoying leisure). This idea is represented by:

   $$\epsilon_{i+1} = \phi_(\epsilon_i; l_i)$$

where l_i is the time spent on learning, which is different from working or enjoying leisure. Agents allocate their time in learning to accumulate human capital.
The problem of an agent is:

$$V_i(a, \epsilon) = \max_{c,n} u(c) + \nu(n + l) + \beta V_{i+1}(a', \epsilon')$$

$$c + a' = w\epsilon' n + (1 + r)a$$
$$\epsilon' = \phi_i(\epsilon, l)$$

(c) **Schooling:** the difference from learning models above is that most of education is acquired in the early stage of life. Keane and Ken Wolpin showed that 90% of people's fate is determined before age 16, by using structurally estimated model of the career choice.
Non college guys start earning 4-5 years earlier than college graduates, however college graduates make more than non college guys. Then why some people decide to go to college or some of them not?

Borrowing constraint

Discount factor: College graduates are more patient than non college guys

For non college guys it is more costly to go to college in terms of intelligence

Mostly parents make the college decision.

April 17th, 2007

σ_y, standard deviation of income among people, increases by age. What could be the possible reasons?

- The skill levels are amplified: Someone with better skill level at the beginning augment his skills better than others.

- Luck: It is all luck, luck leads to more variance. Also luck is persistent: ϵ with σ_ϵ then the variance of $\rho\epsilon_1 + \epsilon_2$ has variance of $(1 + \rho^2)\sigma_\epsilon^2$.

OLG in Business Cycles

Agents live I periods, their endowment of labor in period i is ϵ_i. Newborns born with zero asset.
State variables: i (age of the agent),z (Productivity shock), A (Vector of aggregate asset levels of I cohorts), a (individual asset level).

$$V_i(z, A, a) = \max_{c, a'(z')} u(c) + \beta \sum_{z'} \Gamma_{zz'} V_{i+1}(z', A'(z'), a'(z'))$$

$$s.t. \qquad c + \sum_{z'} q_{z'}(z, A) a'(z') = R(z, K)a + w(z, K)\epsilon_i$$

$$A'(z') = G_{z'}(z, A)$$

$$R(z, K) = (1 - \delta) + zF_1(\sum_{i=2}^{I}, \sum_{i=1}^{I} \epsilon_i)$$

$$w(z, K) = zF_2(\sum_{i=2}^{I}, \sum_{i=1}^{I} \epsilon_i)$$

$$\texttt{with solution } a'(z') = g^i_{z'}(z, A, a)$$

Definition: A RCE of this equilibrium is a list of $\{V^{i^*}(.), g^{i^*}(.), G^*(.), q^*(.), R^*(.), w^*(.)\}$ s.t.

1. Given $\{G^*(.), q^*(.), R^*(.), w^*(.)\}$, $\{V^{i^*}(.), g^{i^*}(.)\}$ solves the agent's problem.

2. $G^i_{z'}(z, A) = g^{i-1}_{z'}(z, A, A_{i-1})$

3. There are $n_z - 1$ market clearing conditions:

$$\sum_i G^i_{\hat{z}'}(z, A) = \sum_i G^i_{\tilde{z}'}(z, A) \qquad \forall \hat{z}', \tilde{z}' \in Z$$

4. $\sum_{z'} q_{z'}(z, A) = 1$ (No Arbitrage condition)

Now let's introduce the probability of surviving from one period to next. Let s_i is the probability of surviving from i to $i+1$. (So we know that $s_I = 0$.) Then the probability of living in the i^{th} period is $\mu_i = \prod_{j=0}^{i-1} s_i$.
But what happens to the assets of the death people. There are few options to incorporate this issue into our model:

1. Pharaoh Model: Bury the capital with the death. Then the problem of the agent is:

$$V_i(z, A, a) = \max_{c, a'(z')} u(c) + \beta s_i \sum_{z'} \Gamma_{zz'} V_{i+1}(z', A'(z'), a'(z'))$$

$$s.t. \qquad c + \sum_{z'} q_{z'}(z, A) a'(z') = R(z, K)a + w(z, K)\epsilon_i$$

2. The capital of death is reallocated to cohort j (cohort j is arbitrarily chosen):

$$V_i(z, A, a) = \max_{c, a'(z')} u(c) + \beta s_i \sum_{z'} \Gamma_{zz'} V_{i+1}(z', A'(z'), a'(z'))$$

$$s.t. \qquad c + \sum_{z'} q_{z'}(z, A) a'(z') = R(z, K)a + w(z, K)\epsilon_i + 1_{i=j} B(z, A)$$

$$B(z, A) = \sum_i \mu_i (1 - s_i) A_i$$

3. Perfect Annuity Market: The assets of the death is equally reallocated to all survivors:

$$V_i(z, A, a) = \max_{c, a'(z')} u(c) + \beta s_i \sum_{z'} \Gamma_{zz'} V_{i+1}(z', A'(z'), a'(z'))$$

$$s.t. \qquad c + \sum_{z'} q_{z'}(z, A) a'(z') s_i = R(z, K)a + w(z, K)\epsilon_i$$

April 18th, 2007

What are the important things in life?

- Health
- Money
- Love
- Children

Health

Suppose s is the health condition of an agent which is assumed to be Markov process. Then the agent's problem is:

$$V_i(s,a) = \max_{c,a'} u(c,s) + \gamma^i(s)\beta \sum_{s'} \Gamma_{ss'} V_{i+1}(s',a')$$

$$\Psi(s) + c + a' = (1+r)a + w\phi^i(s)$$

where $\gamma^i(s)$ is the probability of survival of a cohort i agent from i to $i+1$, $\Psi(s)$ is the cost of being in a health condition of s, and $\phi^i(s)$ is the labor endowment function of cohort i agent which depends on health condition.

Love (Marriage)

Value function of a female in cohort i married with a guy in cohort j:

$$V^f_{i,j}(a) = \max_{c,n,f} u^f(c,n^f) + \beta V^f_{i+1,j+1}(a')$$

$$c + a' = \epsilon_j w^m + \epsilon_i n^f w^f + a(1+r)$$

Let's introduce divorce into this function:

$$V^f_{i,j}(a,z) = \max_{c,n,f} u^f(c,n^f) + \beta \sum_{z'} \Gamma_{zz'} V^f_{i+1,j+1}(a',z')$$

$$c + a' = \epsilon_j w^m 1_{\{z=j\}} + \epsilon_i n^f w^f + a(1+r)$$

April 19th, 2007

A model of single people with health under partial control:

$$V(a,h) = \max_{c,c',y} u(c,y) + \beta\{\phi(h')u(c',h') + (1-\phi(h'))u_D\}$$

$$h' = \psi(h,y), \qquad \phi' > 0$$

$$a = c + c'$$

Why people marry?
- Match specific quality, q, of the partner.

- Individual education level.
- q, e are female's attributes, and q^*, e^* are the male's.
- V^{fm} is the value function of married female.
- V^{fs} is the value function of single female.
- V^{mm} is the value function of married male.
- V^{ms} is the value function of single male.

$$V^{fm}(e, e^*, q, q*) - V^{fs}(e) > 0$$

$$V^{mm}(e^*, e, q^*, q) - V^{ms}(e^*) > 0$$