Econ 702, Spring 2007 Problem set 2¹ Due Tuesday Feb. 6th

Problem 1. Characterize the relationship between commodity prices (output/consumption, capital services and labor services) in the valuation equilibrium framework, when the depreciation rate is not equal to 1, i.e., when the problem looks like the following:

$$\max_{x \in X} \sum_{t} \beta^{t} u \left[c_{t}(x) \right]$$

such that

$$\sum_{t} \left[p_{1t} \{ c_t + k_{t+1} - (1 - \delta) k_t \} - p_{2t} k_t - p_{3t} \right] \le 0$$

Problem 2. Consider the sequential market formulation of the deterministic growth model:

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_t \beta^t u[c_t]$$

s.t.

$$c_t + k_{t+1} + \ell_t = \ell_{t-1} R_{t-1}^{\ell} + k_t R_t^k + w_t \quad \forall t$$

Write the definition of a sequential market equilibrium (SME) as completely as you can.

Problem 3. For the stochastic growth model, verify the conditions on L, X and Y (commodity space, consumption possibility set and production possibility set) such that the uniqueness and welfare theorems apply.

Problem 4. Write the definition of SME for the stochastic growth problem.

Problem 5. We saw in class that the sequential budget constraint for the stochastic growth model is:

$$c_t(h_t) + k_{t+1}(h_t) + \ell_t(h_t) + \sum_{z_{t+1}} q_t(h_t, z_{t+1}) b_t(h_t, z_{t+1}) = R_t^k(h_t) k_t(h_{t-1}) + R_t^\ell(h_{t-1}) \ell_{t-1}(h_{t-1}) + w_t(h_t) + b_{t-1}(h_{t-1}, z_t)$$

Characterize equilibrium return rates $(R_t^k(h_t) \text{ and } R_t^{\ell}(h_{t-1}))$ in terms of $q_t(h_t, z_{t+1})$

¹Throughout this problem set, I'm using the simplifying assumption that the aggregate production technology has constant returns to scale, so I can omit firms from the definition of maximization problems... but you shouldn't omit them from equilibrium definitions!

Problem 6. Write a modification to the stochastic growth model with two locations (All remaining/neccessary assumptions are yours to make). Write the commodity space, the consumption possibility set and the production possibility set and pose the maximization problem with both a 'Valuation-equilibrium' budget constraint and a 'sequential market equilibrium' budget constraint.

(note: this question is left open intentionally. There is no correct answer, but CONSIS-TENT answers, i.e., your response must be consistent with your assumptions)