6 Introduction to Human Capital

- Education levels are very different across countries.

- Rich countries tend to have higher educational levels than poor countries.

- We have the intuition that education (learning skills) is an important factor in economic growth.
Production Function

- Cobb-Douglas aggregate production function:
  \[ Y = K^\alpha H^\beta (AL)^{1-\alpha-\beta} \]

- Again we have constant returns to scale.

- Human capital and labor enter with a different coefficient.
Inputs Accumulation

- Society accumulates human capital according to:
  \[ \dot{H} = s_h Y - \delta H \]

- Capital accumulation equation:
  \[ \dot{K} = s_k Y - \delta K \]

- Technological progress: \[ \frac{\dot{A}}{A} = g > 0. \]

- Labor force grows at constant rate: \[ \frac{\dot{L}}{L} = n > 0. \]

- Human Capital accumulates with schools. More schools more capital.

- But eventually there are decreasing returns to the combination of human and physical capital.
Alternative Specification (similar but different than Jones)

- Cobb-Douglas aggregate production function:
  \[ Y = K^\alpha (A H)^{1-\alpha-\beta} \]

- Human Capital Accumulates
  \[ \dot{H} = e^{\phi u} L \]

where \( u \) is fraction of time studying, and \( \phi \) is a parameter.

- Then, again it is exactly like the basic model without human capital, at some point it cannot grow any more.

- In Jones there is no actual growth in \( H \), it is just
  \[ H = e^{\phi u} L \]

It generates differences in levels.
Rewriting the Model in Efficiency Units

- Redefine the variables in efficiency units:
  \[ \tilde{x} \equiv \frac{X}{AL} \]

- Then, dividing the production function by \( AL \):
  \[ \tilde{y} = \tilde{k}^\alpha \tilde{h}^\beta \]

- Decreasing returns to scale in per efficiency units.
Human Capital Accumulation

- The evolution of inputs is determined by:

\[
\dot{k} = s_k \tilde{k}^\alpha \tilde{h}^\beta - (n + g + \delta) \tilde{k}
\]

\[
\dot{h} = s_h \tilde{k}^\alpha \tilde{h}^\beta - (n + g + \delta) \tilde{h}
\]

- System of two differential equations.

- Solving it analytically it is bit tricky so we will only look at the BGP.
Phase Diagram

- Solving the system analytically it is bit tricky.

- Alternatives:
  
  1. Use numerical methods.
  
  2. Linearize the system.
  
  3. Phase diagram.
To find the BGP equate both equations to zero:

\[ s_k \tilde{k}^{\alpha} \tilde{h}^{\beta} - (n + g + \delta)\tilde{k}^* = 0 \]
\[ s_h \tilde{k}^{\alpha} \tilde{h}^{\beta} - (n + g + \delta)\tilde{h}^* = 0 \]

From first equation:

\[ \tilde{h}^* = \left( \frac{(n + g + \delta)\tilde{k}^*1-\alpha}{s_k} \right)^{\frac{1}{\beta}} \]
Balanced Growth Path Analysis II

• Plugging it in the second equation

\[ s_h \tilde{k}^\alpha \left( \frac{n + g + \delta}{s_k} \right) \tilde{k}^{1-\alpha} - (n + g + \delta) \left( \frac{n + g + \delta}{s_k} \tilde{k}^{1-\alpha} \right)^{\frac{1}{\beta}} = 0 \Rightarrow \]

\[ \frac{s_h \tilde{k}^*}{s_k} = \left( \frac{n + g + \delta}{s_k} \tilde{k}^{1-\alpha} \right)^{\frac{1}{\beta}} \]

• Work with the expression.
Some Algebra

\[ \frac{s_h \tilde{k}^*}{s_k} = \left( \frac{n + g + \delta}{s_k} \tilde{k}^* \right) \left( 1 - \alpha \right) \frac{1}{\beta} \Rightarrow \]

\[ \tilde{k}^* \frac{1 - \frac{1 - \alpha}{\beta}}{s_k} = \tilde{k}^* \frac{1 - \frac{1 - \alpha - \beta}{\beta}}{s_k} = s_k \left( \frac{n + g + \delta}{s_k} \right) \left( \frac{n + g + \delta}{s_k} \right) \frac{1}{\beta} \Rightarrow \]

\[ \tilde{k}^* = \left( \frac{s_k^{1-\beta} s_h^\beta}{n+g+\delta} \right) \frac{1}{1-\alpha-\beta} \]

\[ \tilde{h}^* = \left( \frac{s_k^\alpha s_h^{1-\alpha}}{n+g+\delta} \right) \frac{1}{1-\alpha-\beta} \]
Evaluating the Model

• The central issue for growth is the value of $\beta$.

• If $\beta = 1/3$ we have that small differences in $\{s_h, s_k, \phi, n, g, \delta\}$ can account for (relatively) large differences in output per capita across countries.

• According to Jones differences in $k$ account for a factor of 2 differences in output, differences in educational attainment account for 2.2 (using educational attainment differentials and the return to schooling). The remainder is still 7 or 8 times that have to be imputed to differences in TFP.