

Course in Heterogeneity and Fluctuations

VIII: Banking Dynamics and Capital Regulation in General Equilibrium

Jose-Victor Rios-Rull

University of Oslo

August 2019

Based on joint work with Tamon Takamura and Yaz Terajima

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- Mutual Funds
 - Manage Loan firms
 - Own Equity firms
 - Open and own banks with transfer difficulties

1 Steady State

PRICES, AGGREGATE VARIABLES, AND OTHER OBJECTS

- Prices
 - Interest rate q for bonds: Safe
 - Interest rate r^ℓ for loans: Unsafe
 - Interest rate for deposits q^D Safe because insured by Gov.
 - Wage function $w(k, C)$ (I am using a guess and verify based on logs)
- Quantities
 - Employment, and Number of Firms/Plants N
 - Capital per Plant K
 - Output, Cons, Inv, $C + \delta NK = Y = NAK^\alpha$ – Intermediate Inputs
 - Loans $L = (1 - \lambda)NK$ **V: (Double check, but similar formula)**
 - Deposits D
 - Bonds B
 - Taxes, Banks Loses T
- Other Elements
 - A Banking Industry with a measure of banks x , new entrants m^E , and dividends C^b
 - Mutual funds that manage/own all firms

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$$(BC) \quad c^b + \ell^n + \xi^{i,n}(\ell^n) + \xi^{i,b}(b') \leq a + q^{i,b}(\ell, \ell^n, b')b' + q^d \xi^{i,d}$$

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$$(KR) \quad \frac{\ell^n + \ell - q^d \xi^{i,d} - q^{i,b}(\ell, \ell^n, b') b'}{\omega^r (n + \ell) + \omega^s \mathbf{1}_{b' < 0} b' q^{i,b}(\ell, \ell^n, b')} \geq \theta$$

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ENTRY AND EXIT OF BANKS

- Some banks go bankrupt when they cannot roll over debt. Let the default set be $M^i(A, L)$
- There is entry of new banks, (m^E is the measure of entrants), occurs as long as the free-entry condition is satisfied:

$$W^E(a^E, \ell^E) = u^b(\kappa^{Eb})$$

- a^E, ℓ^E is the prespecified values of new entrants.
- Function $u^b(\cdot)$ translates units of the good into units of the objective function of banks
- $\kappa^{E,b}$ is the opening cost of a new bank.

INDUSTRY EQUILIBRIA

- The definition is exactly like the one in the other paper. But for our purposes we need to link it with the rest of the model.
- We proceed by specifying what are inputs to the banks
- Given safe interest rate, $1/q$, deposit rate $1/q^d$, loan rate r^ℓ and cost of entry κ^{Eb} , it yields
 - A measure of Banks over their states x , including entrants m^E , and fraction of loans in hands of failing banks d^B .
 - Total Quantity of Bonds B
 - Total Quantity of Deposits D
 - Total Dividends C^b
 - Total Loses T to be covered by government
 - Total resources needed by new entrants $m^E \kappa^{Eb}$

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- Output is

$$Y' = (1 - \delta_1)Y + N^n A k^\alpha.$$

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INVESTMENT AND FIRMS: FINANCING

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- If the Bank does not fail (prob $1 - d^B$), then with probability $1 - \lambda$, the firm continues to be debt-financed and pays interest kr^ℓ ; with probability λ , a loan terminates. With probability γ , the firm chooses refinancing by banks. Otherwise, the mutual fund pays $(1 + r^\ell)k$ at the beginning of next period, and the firm becomes an Equity firm.

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- If the bank fails (prob d^B), we assume that the loan also terminates with prob γ and the Mutual pays the government $k(1 + r^\ell + \zeta^F)$. V: What happens with prob $(1 - \lambda)$?

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- If the bank fails (prob d^B), we assume that the loan also terminates **with prob γ** and the Mutual pays the government $k(1 + r^\ell + \zeta^F)$. **V: What happens with prob $(1 - \lambda)$?**
- d^B is the endogenous fraction of loans held by defaulting banks:

$$d^B = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell) \in D_i} \ell \, dm_i(a, \ell)}{\sum_{i=1}^{N_\xi} \int \ell \, dm_i(a, \ell)}$$

VALUE OF FIRMS: THERE ARE MEASURES $m^0(k, r^\ell)$ AND $m^1(k)$ OF THEM

- Given capital k , the maintenance cost δ_2 , interest rate r^ℓ , wage $w(k)$, and the repayment cost ζ^F when banks default, the value of a loan firm is

$$\begin{aligned}\Pi^0(k, r^\ell) = & Ak^\alpha - w(k) - (r^\ell + \delta_2)k + (1 - d^B)(1 - \lambda)q(1 - \delta_1)\Pi^0(k, r^\ell) \\ & + q(1 - \delta_1) \left\{ \lambda(1 - d^B) + d^B \right\} (1 - \gamma)\Pi^0(k) \\ & + q(1 - \delta_1) \left[\lambda(1 - d^B) + d^B \right] \gamma [-k + \Pi^1(k)] - q(1 - \delta_1)d^B\gamma\zeta^F k\end{aligned}$$

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- Letting $R(k) = Ak^\alpha - w(k)$, $\Pi^0 < \Pi^1$ due to loan repayment costs:

$$\begin{aligned}\Pi^1(k) &= \frac{R(k) - \delta_2 k}{1 - q(1 - \delta_1)} \\ \Pi^0(k, r^\ell) &= \frac{R(k) - \delta_2 k}{1 - q(1 - \delta_1)} - \frac{r^\ell + q(1 - \delta_1)\gamma [\lambda(1 - d^B) + d^B + d^B\zeta^f]}{1 - q(1 - \delta_1) [1 - \gamma \{ \lambda(1 - d^B) + d^B \}]} k\end{aligned}$$

- Given the expected value, a firm chooses the size of capital:

$$k^* = \arg \max_k \{ q \Pi^0(k, r^\ell) - \kappa^{Ef} \}$$

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$$k^* = \left\{ \frac{(1 - \mu)\alpha A}{\frac{r^\ell + q(1 - \delta_1)\gamma[\lambda(1 - d^B) + d^B + d^B \zeta^f][1 - q(1 - \delta_1)]}{1 - q(1 - \delta_1)[1 - \gamma\{\lambda(1 - d^B) + d^B\}] + \delta_2}} \right\}^{\frac{1}{1 - \alpha}}$$

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- Firms enter until profits are zero:

$$\kappa^{E,f} = q \Pi^0(k^*; r^\ell)$$

OUTCOME OF INVESTMENT DECISIONS

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 5. $I = (\delta_1 + \delta_2)kN$

MUTUAL FUNDS

- Households own Mutual Funds which in turn own firms and banks, but do not trade its shares, just passively receive its dividends.

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- Mutual Funds manage Loan-firms and own Equity Firms:

$$\begin{aligned} \pi^f &= Y - \mu Y - (1 - \mu) \bar{b} N - r^\ell K^0 \\ &\quad - (1 - d^B) \lambda K^0 - d^B (1 + \zeta^F) K^0 - \kappa^{E,f} N^n \\ &= \int_{k,r^\ell} [R^0(k, r^\ell) - k r^\ell - (1 - d^B) \lambda k - d^B (1 + \zeta^F) k] dm^0(k, r^\ell) \\ &\quad + \int_k R^1(k) dm^1(k) - \kappa^{E,f} N^n \end{aligned}$$

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 4. Loan Repayment
 5. Profits from Equity Firms

WAGE DETERMINATION

- A bargaining process between the firm and the worker. V : (We may change this to get more wage rigidity and avoid the Shymer puzzle)
- The bargaining process is repeated every period and if unsuccessful neither firm nor worker can partner with anybody else within a period. We assume that the financial obligations to the bank by the firm do not disappear. Let μ be the bargaining weight of the worker and \bar{b} is workers' outside option. Then, we have

$$w^0(k) = w^1(k) = \mu A k^\alpha + (1 - \mu)\bar{b}$$

- Total (per capita) Labor Income paid in the Economy are

$$W N = N \int [\mu A k^\alpha + (1 - \mu)\bar{b}] di = \mu Y + (1 - \mu)\bar{b}N$$

$$v(a) = \max_{c, b', d'} u(c, d') + \beta v(a') \quad \text{s.t.}$$

$$c + q^d d' + q b' = a + W N + (1 - N)\bar{b} + \pi^f + \pi^B - T$$

$$a' = d' + b'$$

where T is the taxes needed to pay for bank losses. FOCs:

$$u_c = \frac{\beta}{q} u'_c$$

$$u_d = q^d u_c - \beta u'_c$$

The cost of deposit insurance is the amount of deposits that defaulting banks owe minus liquidated capital.

$$T = \sum_{i=1}^{N_{\xi}} \xi^{i,d} \int_{(a,\ell) \in D} dm^i(a, \ell) - K^0 d^B (1 - \zeta^B)$$

where ζ^B is the fraction that the government is unable to recover during the liquidation process.

OUTPUT OF HOUSEHOLD PROBLEM

- Given safe interest rate, $1/q$, deposit rate $1/q^d$, Taxes T , wages W , Profits Π , and Bonds B , Employment N we obtain
 1. Consumption C
 2. Deposits D

Deposits

$$D' = \sum_{i=1}^{N_{\xi}} \xi_i^d \int_{(a,\ell) \notin D_i} dm^{bi}(a, \ell) + \xi^{dE} m^E$$

Bonds

$$qB' = \sum_{i=1}^{N_{\xi}} \int_{(a,\ell) \notin D_i} q^{ib}(\ell, \ell^{in}(a, \ell), b^{i'}(a, \ell)) b^{i'}(a, \ell) dm^i(a, \ell) + q^{Eb} b'^E m^E$$

MARKET CLEARING (CONTINUED): V: HOW DOES NIPA TREAT F? INTERMEDIATE GOODS?

New loans

$$k^* N^n + (1 - \gamma) \left\{ \lambda(1 - d^B) + d^B \right\} K^0 = \sum_{i=1}^{N_\xi} \int_{(a, \ell) \notin D_i} \ell_i^n(a, \ell) dm_i(a, \ell) + \ell_E^n m_E$$

Goods

$$\begin{aligned} Y &= C + kN^n + \delta_2 kN + \\ &+ \sum_{i=1}^{N_\xi} \int_{(a, \ell) \notin D_i} \xi_i^n \left(\ell_i^n(a, \ell) \right) dm_i(a, \ell) + \xi_E^n \left(\ell_E^n \right) \quad (\text{Bank loan issuance costs}) \\ &+ \sum_{i=1}^{N_\xi} \int_{(a, \ell) \notin D_i} \xi_b \left(b_i^b(a, \ell) \right) dm_i(a, \ell) + \xi_E^b \left(b_E^b \right) \quad (\text{Bank bond issuance costs}) \\ &+ \kappa_E^b m_E + \kappa_E^f N^n \quad (\text{Entry costs}) \\ &+ d^B (\zeta^B + \zeta^F) K^0 \quad (\text{Bank default costs}) \end{aligned}$$

STEADY STATE CONDITIONS (1)

Households: $u(C, D, N) = \log(C) + \eta^D \log(D)$,

$$q = \beta \quad (1)$$

$$\frac{\eta^D C}{D} = q^d - \beta \quad (2)$$

Firms:

$$k^* = \left\{ \frac{(1 - \mu)\alpha A}{\frac{[r^\ell + q(1 - \delta_1)\gamma\{\lambda(1 - d^B) + d^B + d^B\zeta^f\}][1 - q(1 - \delta_1)]}{1 - q(1 - \delta_1)[1 - \gamma\{\lambda(1 - d^B) + d^B\}]} + \delta_2} \right\}^{\frac{1}{1 - \alpha}} \quad (3)$$

$$\kappa^{Ef} = q \Pi^0 \quad (4)$$

$$\Pi^0 = \frac{(1 - \mu)(A(k^*)^\alpha - \bar{b}) - \delta_2 k^*}{1 - q(1 - \delta_1)} - \frac{r^\ell + q(1 - \delta_1)\gamma[\lambda(1 - d^B) + d^B + d^B\zeta^f]}{1 - q(1 - \delta_1)[1 - \gamma\{\lambda(1 - d^B) + d^B\}]} k^* \quad (5)$$

STEADY STATE CONDITIONS (2)

Wages:

$$w = \mu A(k^*)^\alpha + (1 - \mu)\bar{b} \quad (6)$$

Banks:

$$r^{E,b} = W^E(a^E, \ell^E) \quad (7)$$

$$d^B = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell) \in D^i} \ell \, dm^i(a, \ell)}{\sum_{i=1}^{N_\xi} \int \ell \, dm^i(a, \ell)} \quad (8)$$

STEADY STATE CONDITIONS (3)

Market clearing conditions:

$$D = \sum_{i=1}^{N_{\xi}} \xi^{i,d} \int_{(a,\ell) \notin D_i} dm^i(a, \ell) + \xi^{E,d} m^E \quad (9)$$

$$qB = \sum_{i=1}^{N_{\xi}} \int_{(a,\ell) \notin D_i} q^{i,b} \left(\ell, \ell^{i,n}(a, \ell), b^{i'}(a, \ell) \right) b^{i'}(a, \ell) dm^i(a, \ell) + q^{E,b} b'^E m^E \quad (10)$$

$$k^* N^n + (1 - \gamma) \{ \lambda(1 - d^B) + d^B \} K^0 = \sum_{i=1}^{N_{\xi}} \int_{(a,\ell) \notin D_i} \ell^{i,n}(a, \ell) dm^i(a, \ell) + \ell^{E,n} m^E \quad (11)$$

$$\begin{aligned} Y = C + k^* N^n + \delta_2 k N + \sum_{i=1}^{N_{\xi}} \int_{(a,\ell) \notin D_i} \xi^{i,n} \left(\ell^{i,n}(a, \ell) \right) dm^i(a, \ell) + \xi^{E,n} \left(\ell^{E,n} \right) m^E \\ + \sum_{i=1}^{N_{\xi}} \int_{(a,\ell) \notin D_i} \xi^{i,b} \left(b^{i'}(a, \ell) \right) dm^i(a, \ell) + \xi^{E,b} \left(b'^E \right) m^E \\ + \kappa^{E,b} m^E + \kappa^{E,f} N^n + d^B (\zeta^B + \zeta^F) K^0 \quad (12) \end{aligned}$$

STEADY STATE CONDITIONS (4)

Laws of motion:

$$Y = \frac{A(k^*)^\alpha N^n}{\delta_1} \quad (13)$$

$$N = \frac{N^n}{\delta_1} \quad (14)$$

$$K^0 = \frac{k^* N^n}{1 - (1 - \delta_1) [1 - \gamma \{ \lambda(1 - d^B) + d^B \}]} \quad (15)$$

Aggregate endogenous variables:

$$C, D, B, k^*, K^0, N^n, N, Y, d^B, m(a, \ell), m^E, q, q^d, r^\ell, w$$

Parameters:

$$\text{HHs: } \beta, \mu, \bar{b}, \eta^D$$

$$\text{Firms: } \alpha, A, \kappa^{E,f}, \zeta^F$$

$$\text{Banks: } u^b(), \beta^B, \lambda, \xi^{i,d}, \xi^{i,n}, \xi^{i,b}, F(\delta'), \kappa^{E,b}, \zeta^B$$

ALGORITHM TO GET STEADY STATE

- Set Parameters of Banking: $u^b(), \beta^B, \lambda, \xi^{i,d}, \xi^{i,n}, \xi^{i,b}, F(\delta')$ and prices r^ℓ, q^d, q . **V: (may come back to this)**
- Compute the banking industry equilibrium. Get loans L , deposits D bank dividends C^b , losses T , resources for new entrants $m^E \kappa^{Eb}$.
- Set HH preference parameters β, \bar{b}, η_D , and the bargaining power μ so that they are consistent with q , the observed consumption-to-deposit ratio and the labor share of 2/3.
- Set Technology A, α as well as δ_2 and ζ^F to solve the firms' problem. Given α and δ_2 , adjust A to make sure that all markets clear.
V: (I think that λ doesn't matter much so we should set this to get the equity/debt ratio of the nonfinancial sector and a normalize)
- Generate key moments of interest.

SETTING THE BARGAINING PARAMETERS

- We target labor share and the outside option for workers $\bar{b} = \phi_b w$:

$$LS = \mu + (1 - \mu) \frac{\bar{b}N}{Y} = \mu + (1 - \mu) \frac{\phi_b w}{A(k^*)^\alpha}$$

$$w = \mu A(k^*)^\alpha + (1 - \mu) \bar{b} = \mu A(k^*)^\alpha + (1 - \mu) \phi_b w$$

- Solving the two conditions simultaneously,

$$\mu = \frac{(1 - \phi_b)LS}{1 - \phi_b LS}$$

$$w(k^*) = \frac{\mu}{1 - (1 - \mu)\phi_b} A(k^*)^\alpha$$

$$\bar{b} = \phi_b w(k^*)$$

- $LS = 2/3$ and $\phi_b = 0.9$ imply $\mu = 1/6$.

- $\beta = q$ by (1)
- $N^n = \delta_1 \bar{N}$ by (14), where $\bar{N} = 0.9$.
- The banking industry equilibrium gives L^n : back out k^* from (11).
- Set A so that the loan demand (3) is equal to the loan supply.
- $Y = Ak^* \bar{N}$ and $I = (\delta_1 + \delta_2)k^* N$.
- Compute K^0 from (15)
- C is determined as a residual in (12)

- For simplicity, we ignore various intermediate costs for now
- Consumption-deposit ratio:

$$\begin{aligned} \frac{C}{D} &= \frac{C}{L^n} \frac{L^n}{D} = \frac{Y - I}{k^* \delta_1 \bar{N} \left[1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]} \right]} \frac{L^n}{D} \\ &= \frac{A(k^*)^{\alpha-1} - \delta_1 - \delta_2}{\delta_1 \left[1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]} \right]} \frac{L^n}{D} \\ &= \left[\frac{1}{\frac{K}{Y} \delta_1 \left[1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]} \right]} - \frac{\delta_1 + \delta_2}{\delta_1 \left[1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]} \right]} \right] \frac{L^n}{D} \end{aligned}$$

- With $K/Y = 3$, $L^n/D = 0.9$, $\delta_1 = 0.02$, $\delta_2 = 0.08$, $\gamma = \lambda = 0.5$, $d^B = 0$, consumption-deposit ratio is about 5.4.

2 Equilibrium in Terms of Sequences

EXISTING BANK'S PROBLEM GIVEN $V_{t+1}, r_t^\ell, q_t^d, \theta_t$

$$V_t^i(a, \ell) = \max \{0, W_t^i(a, \ell)\}$$

$$W_t^i(a, \ell) = \max_{\ell^n \geq 0, c \geq 0, b'} u^b(c^b) + \beta^b \sum_i \Gamma_{i,i'} \sum_{\delta'} \left\{ \pi_t(\delta') V_{t+1}^{i'}[a'(\delta'), \ell'(\delta')] \right\} \text{ s.t.}$$

$$(TL) \quad \ell' = (1 - \lambda)(1 - \delta')\ell + (1 - \bar{\delta})\ell^n$$

$$(TA) \quad a' = (\lambda + r_t^\ell)(1 - \delta')\ell + \lambda(1 - \bar{\delta})\ell^n - \xi^{i,d} - b'$$

$$(BC) \quad c^b + \ell^n + \xi^{i,n}(\ell^n) + \xi^{i,b}(b') \leq a + q_t^{i,b}(\ell, \ell^n, b')b' + q_t^d \xi^{i,d}$$

$$(KR) \quad \frac{\ell^n + \ell - q_t^d \xi^{i,d} - q_t^{i,b}(\ell, \ell^n, b')b'}{\omega_t^r(n + \ell) + \omega_t^s \mathbf{1}_{b' < 0} b' q_t^{i,b}(\ell, \ell^n, b')} \geq \theta_t$$

π_t is an exogenous aggregate shock.

θ_t is exogenous. A feedback rule to be considered in the next step.

- Entry condition:

$$W_t^E(a^E, \ell^E) = u^b(\kappa^{E,b}) \quad (16)$$

- A fraction of loans destroyed by bank default:

$$d_{t-1}^B = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell) \in M_t^i} \ell \, dm_{t-1}^i(a, \ell)}{\sum_{i=1}^{N_\xi} \int \ell \, dm_{t-1}^i(a, \ell)} \quad (17)$$

- The value is

$$\Pi_t^1(k) = A_t k^\alpha - w_t(k) - \delta_2 k + q_t(1 - \delta)\Pi_{t+1}^1(k) \quad (18)$$

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- The wage is given by

$$w_t(k) = \mu A_t k^\alpha + (1 - \mu)\bar{b} \quad (19)$$

- The value of bank-financed firm is

$$\begin{aligned} \Pi_t^0(k) = & Ak^\alpha - w(k) - (r_t^\ell + \delta_2)k + (1 - d_t^B)(1 - \lambda)q_t(1 - \delta_1)\Pi_{t+1}^0(k) \\ & + q_t(1 - \delta_1) \{ \lambda(1 - d_t^B) + d_t^B \} (1 - \gamma)\Pi_{t+1}^0(k) \\ & + q_t(1 - \delta_1) [\lambda(1 - d_t^B) + d_t^B] \gamma [-k + \Pi_{t+1}^1(k)] \\ & - q_t(1 - \delta_1)d_t^B\gamma\zeta^F k \quad (20) \end{aligned}$$

- Given q_t and Π_{t+1}^0 , entrants choose k_t^* :

$$k_t^* = \arg \max_k \{ q_t \Pi_{t+1}^0(k) - \kappa^{E,f} \} \quad (21)$$

- Entry occurs until firms break even *ex-ante*

$$q_t \Pi_{t+1}^0(k_t^*) = \kappa^{E,f} \quad (22)$$

- Aggregate output:

$$Y_t = A_t(k_t^*)^\alpha N_t^n + (1 - \delta_1)Y_{t-1} \quad (23)$$

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- Aggregate capital:

$$K_t = k_t^* N_t^n + (1 - \delta_1)K_{t-1} \quad (25)$$

- Aggregate capital held by bank-financed firms:

$$\begin{aligned} K_t^0 &= k_t^* N_t^n \\ &+ \left[(1 - d_{t-1}^B)(1 - \lambda) + (1 - \gamma) \left\{ \lambda(1 - d_{t-1}^B) + d_{t-1}^B \right\} \right] (1 - \delta_1)K_{t-1}^0 \end{aligned} \quad (26)$$

- Consumption Euler equation:

$$u_{c,t} = \beta \frac{u_{c,t+1}}{q_t} \quad (27)$$

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- Consumption-deposit marginal condition:

$$u_{d,t} = q_t^d u_{c,t} - \beta u_{c,t+1} \quad (28)$$

MARKET CLEARING CONDITIONS

$$D_t = \sum_{i=1}^{N_\xi} \xi^{i,d} \int_{(a,\ell) \notin M_{t-1}^i} dm_{t-1}^i(a, \ell) + \xi^{E,d} m_t^E \quad (29)$$

$$\begin{aligned} k_t^* N_t^n + (1 - \gamma) \{ \lambda(1 - d_{t-1}^B) + d_{t-1}^B \} K_{t-1}^0 \\ = \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin M_t^i} \ell_t^{i,n}(a, \ell) dm_{t-1}^i(a, \ell) + \ell_t^{E,n} m_t^E \end{aligned} \quad (30)$$

$$\begin{aligned} Y_t = C_t + k_t^* N_t^n + \delta_2 K_{t-1} \\ + \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin M_t^i} \xi^{i,n} \left(\ell_t^{i,n}(a, \ell) \right) dm_{t-1}^i(a, \ell) + \xi^{E,n} \left(\ell_t^{E,n} \right) m_t^E \\ + \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin M_t^i} \xi^{i,b} \left(b_t^{i,b}(a, \ell) \right) dm_{t-1}^i(a, \ell) + \xi^{E,b} \left(b_t^{E,b} \right) m_t^E \end{aligned} \quad (31)$$

- Aggregate prices: r_t^l , q_t , q_t^d

EQUILIBRIUM OBJECTS TO BE COMPUTED

- Aggregate prices: r_t^ℓ , q_t , q_t^d
- Endogenous aggregate states: Y_{t-1} , K_{t-1} , K_{t-1}^0 , $m_{t-1}^i(a, \ell)$, d_{t-1}^B

EQUILIBRIUM OBJECTS TO BE COMPUTED

- Aggregate prices: r_t^ℓ, q_t, q_t^d
- Endogenous aggregate states: $Y_{t-1}, K_{t-1}, K_{t-1}^0, m_{t-1}^i(a, \ell), d_{t-1}^B$
- Other endogenous aggregate variables: $I_t, C_t, D_t, N_t^n, k_t^*, m_t^E$

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- Other endogenous aggregate variables: $I_t, C_t, D_t, N_t^n, k_t^*, m_t^E$
- Banking industry decisions:
 $\{c_t^{i,b}(a, \ell), \ell_t^{i,n}(a, \ell), b_t^{i'}(a, \ell), M_t^i, c_t^E, \ell_t^{E,n}, b_t'^E, q_t^{i,b}(\ell, \ell^n, b')\}$

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- Exogenous aggregate variables: θ_t, A_t, π_t

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- Banking industry decisions:
 $\{c_t^{i,b}(a, \ell), \ell_t^{i,n}(a, \ell), b_t^{i,i}(a, \ell), M_t^i, c_t^E, \ell_t^{E,n}, b_t^{i,E}, q_t^{i,b}(\ell, \ell^n, b')\}$
- Exogenous aggregate variables: θ_t, A_t, π_t
- B_t can be computed once we know the equilibrium path.

SOLUTION ALGORITHM: OUTLINE

- The economy is in steady state in $t = 1$ and $t \geq T$
- Banks' problem do not depend on endogenous aggregate quantities, but firms' problem depend on d_t^B . [This isn't the case if a policy rule reacts to, say, aggregate output. But, we can still use what we do here to generate an initial guess.]
- Firm-entry conditions determine r_t^ℓ , given q_t : This process is inexpensive, as opposed to finding q_t^d given q_t and r_t^ℓ from the bank-entry condition
- Thus, our approach is to guess $\{q_t\}_{t=1}^T$, $\{q_t^d\}_{t=1}^T$, $\{d_t^B\}_{t=1}^T$, $\{m_t^E\}_{t=1}^T$ and $\{N_t^n\}_{t=1}^T$, and gradually adjust these objects to meet market-clearing conditions

SOLUTION ALGORITHM: SOLVING BACKWARDS

Guess $\{q_t, q_t^d, d_t^B\}_{t=1}^{T-1}$ and start with V_T , Π_T^0 and Π_T^1 . For $t = T - 1, \dots, 2$,

1. Given r_t^ℓ , q_t , d_t^B , Π_{t+1}^0 and Π_{t+1}^1 , compute firms' value functions, (18) and (20), where r_t^ℓ is pinned down by the entry condition (22) given q_{t-1} :

$$q_{t-1} \Pi_t^0(k_{t-1}^*; r_t^\ell) = \kappa^{E,f}$$
$$k_{t-1}^* = \arg \max_k \Pi_t^0(k; r_t^\ell)$$

2. Solve the bank's problem given q_t^d , r_t^ℓ , q_t and V_{t+1}
3. Using (21), compute k_t^* given q_t and Π_{t+1}^0
4. Using (27) and (28), compute C_t and D_t given q_t and q_t^d

SOLUTION ALGORITHM: INTEGRATING FORWARD

In each h -th iteration, do the following for $t = 2, \dots, T - 1$, given Y_1, K_1, K_1^0 , the decision rules of HHs, banks and firms, and $\{m_t^{E,(h)}, N_t^{n,(h)}\}_{t=1}^T$:

1. Aggregate banks' decisions using $m_{t-1}^i(a, \ell)$
2. Aggregate output: $Y_t = A_t(k_t^*)^\alpha N_t^{n,(h)} + (1 - \delta_1)Y_{t-1}$
3. Using the goods MCC (31), compute $N_t^{n,*}$:

$$Y_t = C_t + k_t^* N_t^{n,*} + \delta_2 K_{t-1} + \text{loan issuance costs given } m_t^{E,(i)} \\ + \text{WSF issuance costs given } m_t^{E,(i)}$$

4. Given $N_t^{n,*}$, compute $m_t^{E,*}$ using the loan MCC (30):

$$k_t^* N_t^{n,*} + (1 - \gamma) \{ \lambda(1 - d_{t-1}^B) + d_{t-1}^B \} K_{t-1}^0 \\ = \sum_{i=1}^{N_\xi} \int_{(a, \ell) \notin M_t^i} \ell_t^{i,n}(a, \ell) dm_{t-1}^i(a, \ell) + \ell_t^{E,n} m_t^{E,*}$$

5. Update the distribution of banks $m_t^i(a, \ell)$, based on banks' decisions and $m_{t-1}^i(a, \ell)$: we can get $d_t^{B,*}$ in this process

- The deposit MCC (29) implies excess demand for deposits:

$$X_t^d = \sum_i \xi^{i,d} \int_{(a,\ell) \notin M_t^i} dm_{t-1}^i(a,\ell) + \xi^{E,d} m_t^{E,(h)} - D_t \quad (32)$$

- For $\lambda^d < 0$, the updating algorithm for q_t^d is:

$$q_t^{d,(h+1)} = (1 + \lambda^d X_t^d) q_t^{d,(h)} \quad (33)$$

- An intuition here is to make deposit more expensive when its demand exceeds supply

SOLUTION ALGORITHM: UPDATING THE DISCOUNT PRICE OF RISK-FREE ASSETS

- From (16), the excess bank-entry condition is:

$$X_t^v = W_t^E(a^E, \ell^E) - \kappa^{E,b} \quad (34)$$

- For $\lambda^v < 0$, the updating algorithm for q_t is:

$$q_t^{(h+1)} = (1 + \lambda^v X_t^d) q_t^{(h)} \quad (35)$$

- An intuition here is to make an entry more costly when the net value of entry is positive

SOLUTION ALGORITHM: UPDATING OTHER GUESSES

- Updating of d_t^B :

$$d_t^{B,(h+1)} = \gamma^q d_t^{B,*} + (1 - \gamma^q) d_t^{B,(h)}$$

- Updating of the measure of bank and firm entry :

$$\begin{aligned} m_t^{E,(h+1)} &= \gamma^m m_t^{E,*} + (1 - \gamma^m) m_t^{E,(h)} \\ k_t^* N_t^{n,(h+1)} + (1 - \gamma) \left\{ \lambda (1 - d_{t-1}^{B,(h+1)}) + d_{t-1}^{B,(h+1)} \right\} K_{t-1}^{0,(h+1)} \\ &= \sum_{i=1}^{N_\xi} \int_{(a,\ell) \notin M_t^i} \ell_t^{i,n}(a, \ell) dm_{t-1}^i(a, \ell) + \ell_t^{E,n} m_t^{E,(h+1)} \end{aligned}$$