The Climate and the Economy

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IIES

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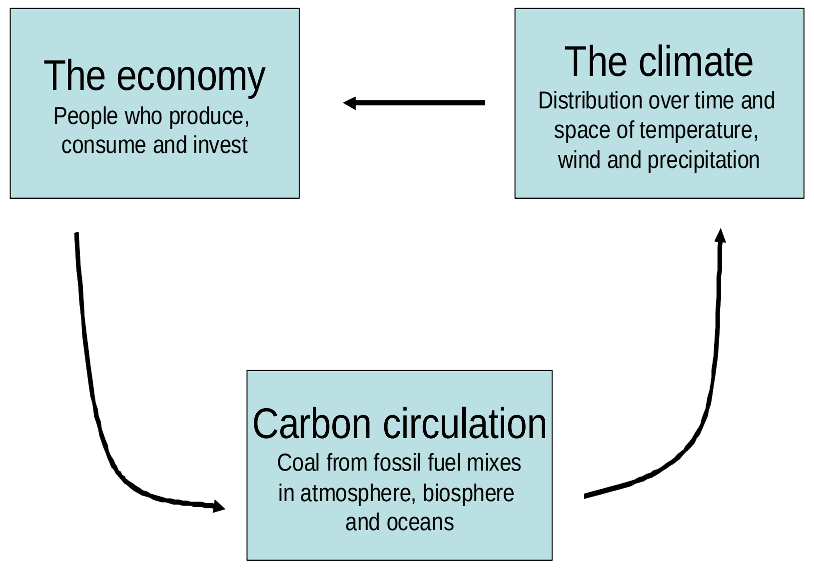
- (first put together by Bill Nordhaus).

A SCHEMATIC IAM: INTERACTIONS



The economy

People who produce, consume and invest

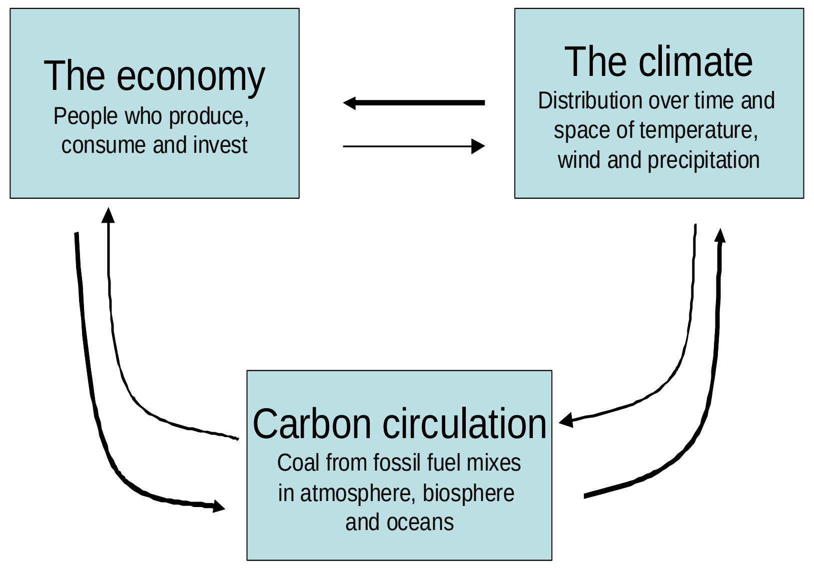
The climate

Distribution over time and space of temperature, wind and precipitation

Carbon circulation

Coal from fossil fuel mixes in atmosphere, biosphere and oceans

A SCHEMATIC IAM: DYNAMICS, BIDIRECTIONAL



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CLIMATE MODELS – FORCING AND THE ENERGY BUDGET

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Climate models – forcing and the energy budget

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  - where the latter is a function of, in particular, temperature and greenhouse gas concentration in earth's atmosphere.
- What happens out of steady state? For example, without greenhouse gases and atmosphere, ground temperature would be -19. How can we understand this statement?

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- In an economy, a steady state involves investment equating the amount of capital depreciated and the distribution of income remaining constant.

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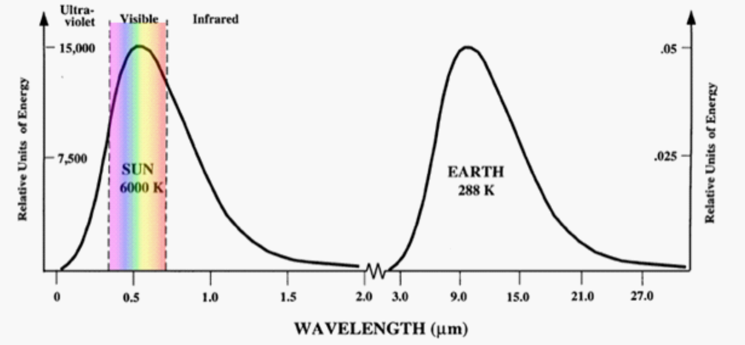
Radiation

- Visible sunlight and infrared heat waves are both electromagnetic radiation, but with different frequencies (freq=Speed of light/wave length).
- Frequency of radiation emitted depends on temperature. Compare with dimmer on halogen lights,



Comparison of the emission spectra of the sun and the earth. Note the huge disparity in the amount of energy emitted by the sun (left-hand scale) and the earth (right-hand scale).

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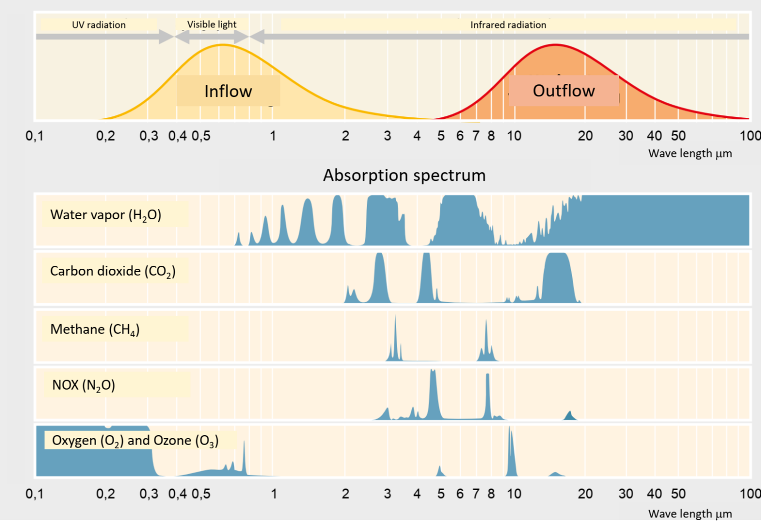
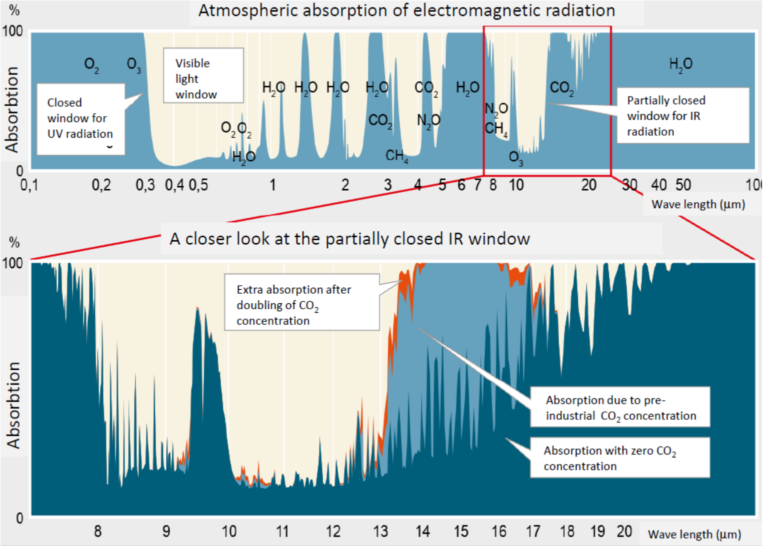
- Compare to a band playing in a bar. The bass guitar can make some objects, e.g. cups and cutlery vibrate, but a high pitched tone from the guitar has no effect.

Absorption of different radiation



More on absorption



Source: Bernes, Claes, (2016), “En varmare värld”, Naturvårdsverket.

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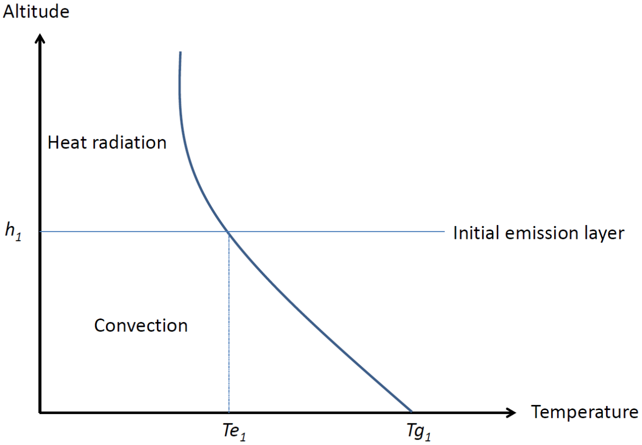
- The accumulation of energy increases the temperature in the atmosphere until the temperature at the emission level again is high enough to imply that the energy flow out in space is the same as the flow into earth.

Heat transfer and temperature gradient



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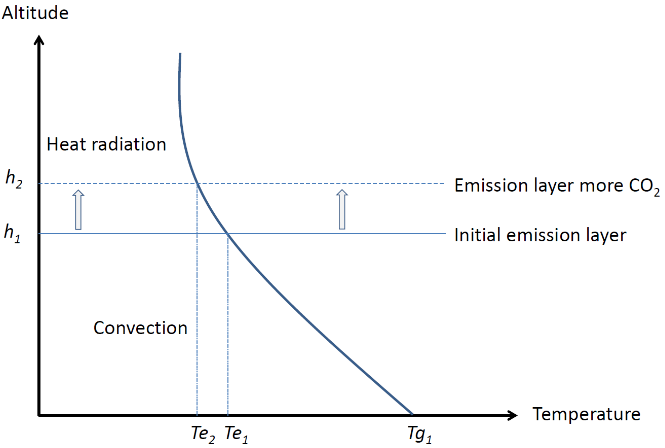

Figure: Lower temp at emission level -> less energy outflow. Surplus in energy budget.

Surplus leads to higher temperatures

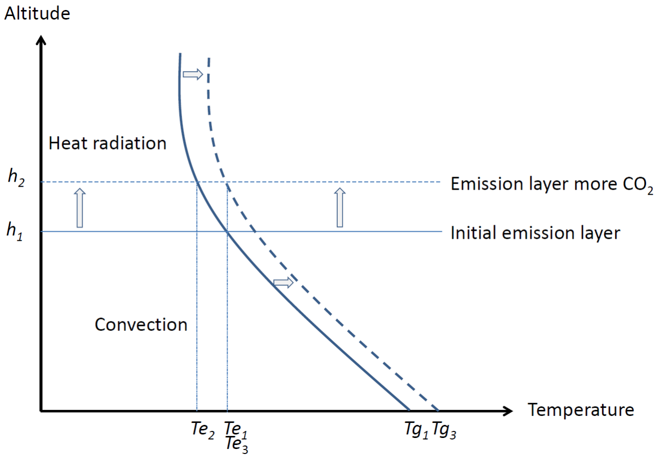

Figure: Heat accumulation gradually increases temperature. Gradient shifts rightwards until temp at h_2 has returned to Te_1 and ground temperature increased to Tg_3.

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- Without the Greenhouse Gas blanket, life as we know it could not have started

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- Now the budget is no longer balanced but in surplus and the system would no longer be in steady state.

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- As the temperature goes up, outgoing flow increases with higher temperature (sometimes called *Planck feedback*).

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- We can approximate the increase in outflow as $O\left(T_t\right) \approx \kappa_{Planck} T_t$ and the energy budget is then $f - O\left(T_t\right)$.

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- Using Stefan-Boltzmann law and temperature at the emision level of -18°C, $\kappa_{Planck} \approx 3.8 \frac{W/m^2}{^{\circ}C}$. Due to feedbacks, actual outflow will likely rise substantially less. A typical value imputed is $\kappa_{Planck} \approx 3.2 \frac{W/m^2}{^{\circ}C}$.

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- Surplus (forcing) 1.7 and 1 $\mathrm{W/m^2}$, respectively, yielding a total value for forcing of 2.7 W/m^2,.

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- As discussed above, more greenhouse gases pushes emission level outwards which creates a surplus in energy budget relative to preindustrial situation.
- Surplus depends on greenhouse gas concentration.
- Most important is water vapor. Second is CO_2.
- Human activities has increased concentration of CO_2 and other greenhouse gases, e.g., methane. We also emit particles and aerosols that have a direct negative effect on reflection and a quite uncertain negative effect on the energy balance via changed cloud formation.
- Surplus (forcing) 1.7 and 1 W/m^2, respectively, yielding a total value for forcing of 2.7 W/m^2,.
- Disregarding the feedbacks, only considering the Planck feedback, we can calculate the long run effect of that on Earth's temperature as

$$\frac{2.7}{3.8} \approx 0.7^{\circ} C$$

Forcing in 2019 relative to 1750

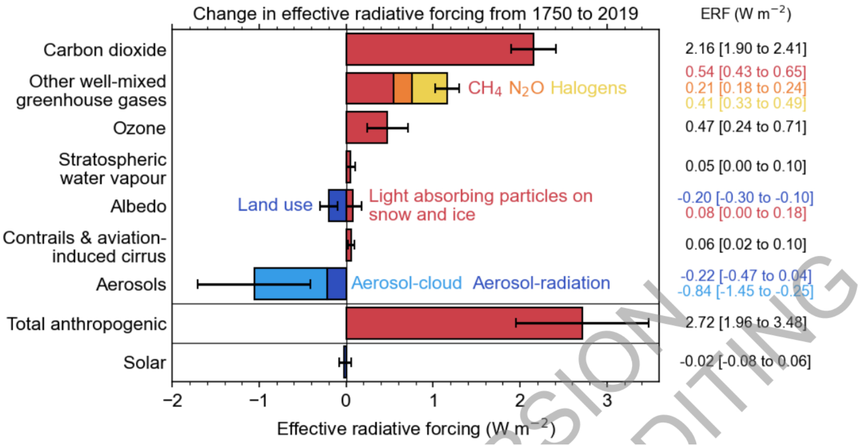
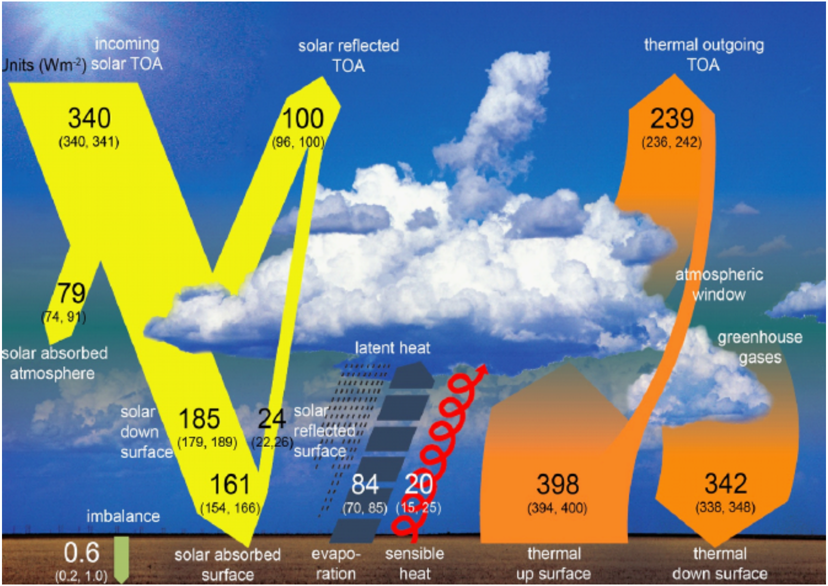


Figure: Fig 7.6 IPCC 6th report page 7-182.

Energy Flows



Orders of magnitude

- Area of Earth's surface is 510 million km^2. This is 510*$10^6 * 1000^2 = 5.1 \times 10^{14} m^2$. Thus, the inflow net of reflection is 240*$5.1 \times 10^{14} = 1.22 \times 10^{17} W$.
- A nuclear power plant is around 1000 MW, i.e., $10^9 W$. Thus, the inflow of solar energy is equivalent to $1.22 \times 10^8 = 122$ million nuclear power plants (NPP). We currently have around 440 in operation.
- The human induced forcing of 2.7W/m^2 is equivalent to $2.7 * 5.1 \times 10^{14}/10^9 = 1.4$ million NPP.
- Global yearly energy use is around 600 million TJ, i.e., $6 * 10^{2+6+12} = 6 * 10^{20} J$. Dividing by the number of seconds per year, we get the average power use. 6*$10^{20}/(365 * 24 * 3600) \approx 1.9 \times 10^{13} W$ or 19000 NPP.
- Thus, solar inflow is $\frac{1.22 \times 10^{17}}{1.9 \times 10^{13}} \approx 6400$ times global energy use. If we could harness 0.1%, it would allow 6 times current energy use.

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FEEDBACKS IN THE ENERGY BALANCE

- Let us include feedbacks in energy budget:

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- Direct effect of CO_2 emission on f, (as well as of κ_{Planck}) fairly certain. Not the case for feedbacks.

Current Feedbacks

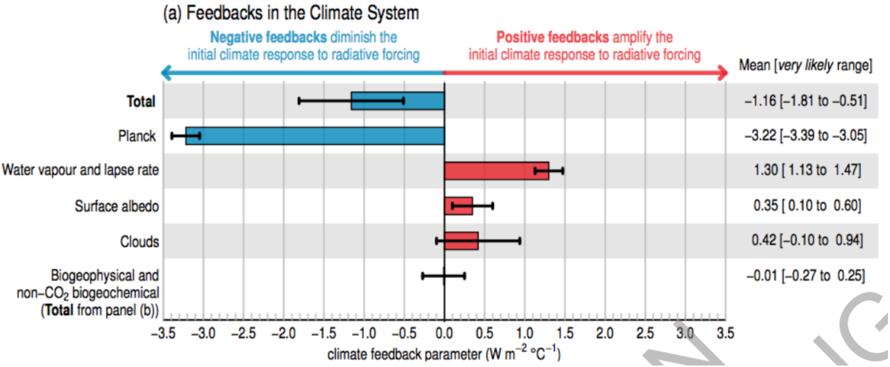

Figure: Figure TS.17 IPCC 6th report.

QUANTIFYING GREENHOUSE EFFECT ON ENERGY BUDGET

- Higher concentration of CO_2in atmosphere reduces outgoing (infra red) energy flow. Well approximated by a logarithmic function (Arrhenius greenhouse law, 1896). A concentration S of CO_2 in the atmosphere and the pre-industrial level S_0, yields

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- IPCC 6th report: ECS is “likely" 2.5 to 4°C, with a "best estimate" of 3. Narrower than the 5th report's 1.5 to 4.5. "Likely" means a 2/3 confidence interval. A 90% interval is 2-5°C.

HEATING OF OCEANS

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- These new terms *do not balance* if temperature is different (in an average sense).
- New law-of-motion for atmosphere

$$\frac{dT_t}{dt} = \sigma_1 \left(f - \left(\kappa_{Planck} - \kappa_{other} - \kappa_{refl} \right) T_t - \sigma_2 \left(T_t - T_t^L \right) \right)$$

where T_t and T_t^L, respectively, denote the atmospheric and ocean temperature in period t.

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- One can (climate scientists do) use even more layers.

SIMULATION

- Make a discrete time approximation. Yields a system of difference equations;

$$\begin{aligned} T_t &= T_{t-1} + \sigma_1 \left(\begin{array}{c} f_{t-1} - (\kappa_{Planck} - \kappa_{other} - \kappa_{refl})\, T_t \\ -\sigma_2 \left(T_{t-1} - T^L_{t-1} \right) \end{array} \right) \\ T^L_t &= T^L_{t-1} + \sigma_3 \left(T_{t-1} - T^L_{t-1} \right) \end{aligned}$$

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- Can easily be simulated in a spread-sheet program.

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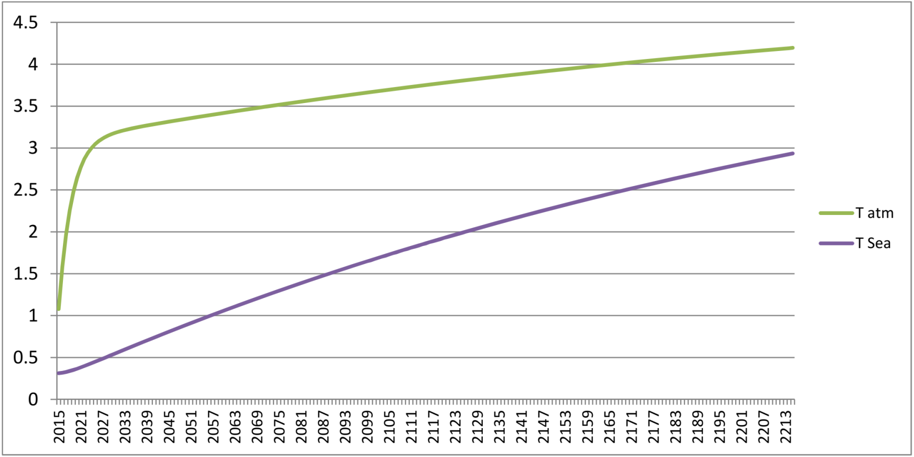
- Note that σ_1 is much larger than σ_3. Atmospheres energy balance settles to a temporary steady state of 0 quickly.

Simulation of a doubling of current forcing



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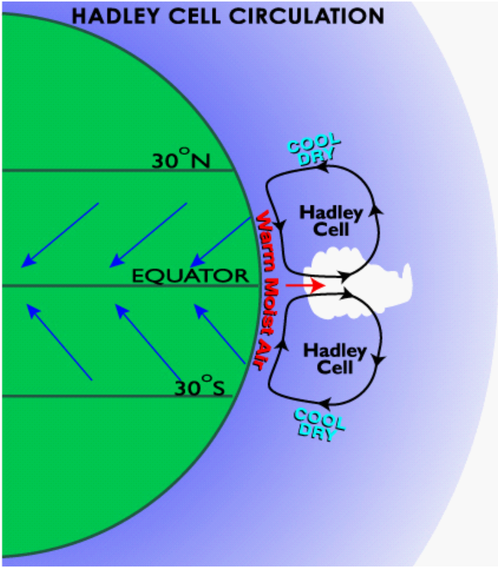
- Energy is not evenly radiated to the earth. Highest around equator.
- Creates systematic flows of air and water.
- Used to forecast weather, but also climate.

CIRCULATION CELLS



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  - But unconditional distribution is stable, e.g., mean and variance of temperature and wind speeds.
  - Best forecast for forecasts beyond a few weeks is unconditional distribution.
- State-of-the-art climate models are build on these principles.

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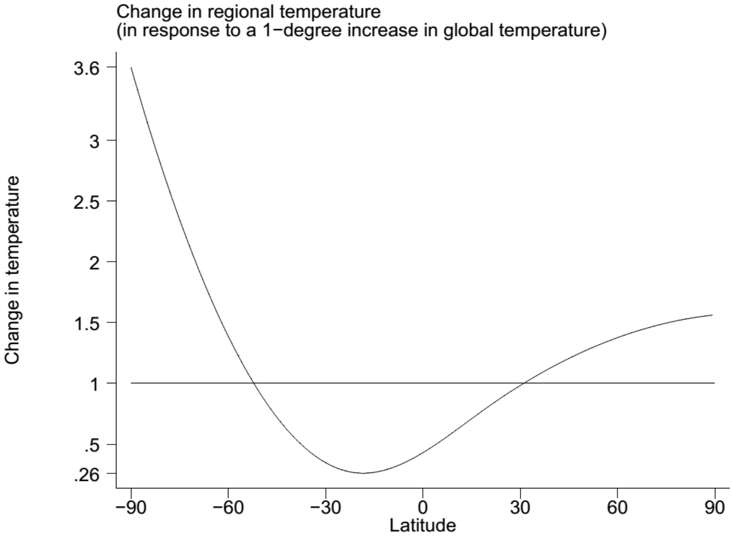
Change in regional temperature
(in response to a 1−degree increase in global temperature)
Change in temperature
3.6
3
2.5
2
1.5
1
.5
.26
−90
−60
−30
0
30
60
90
Latitude

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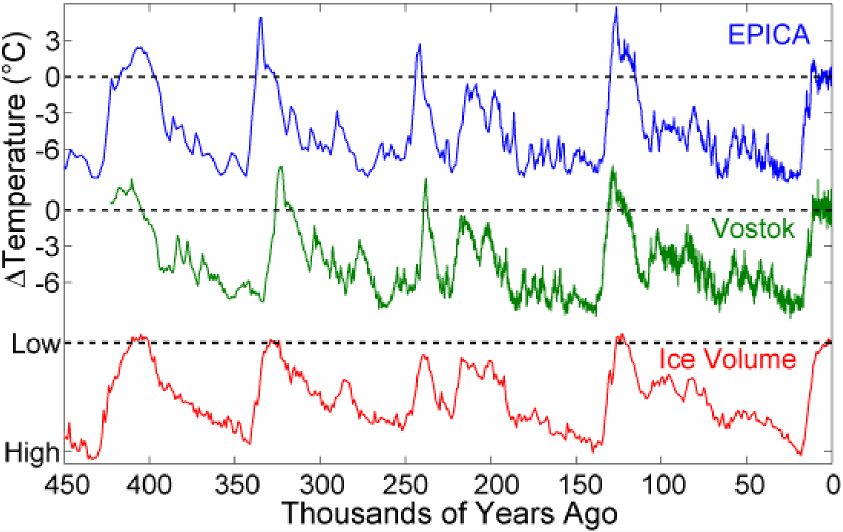
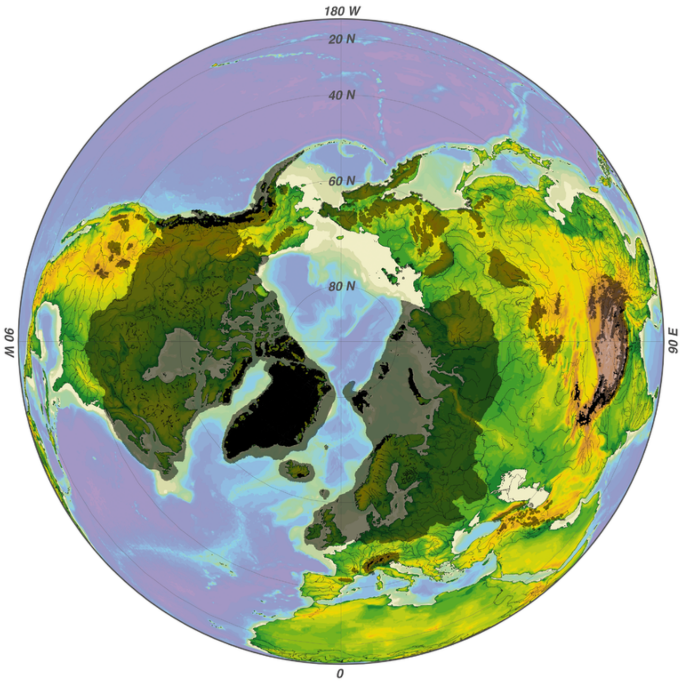
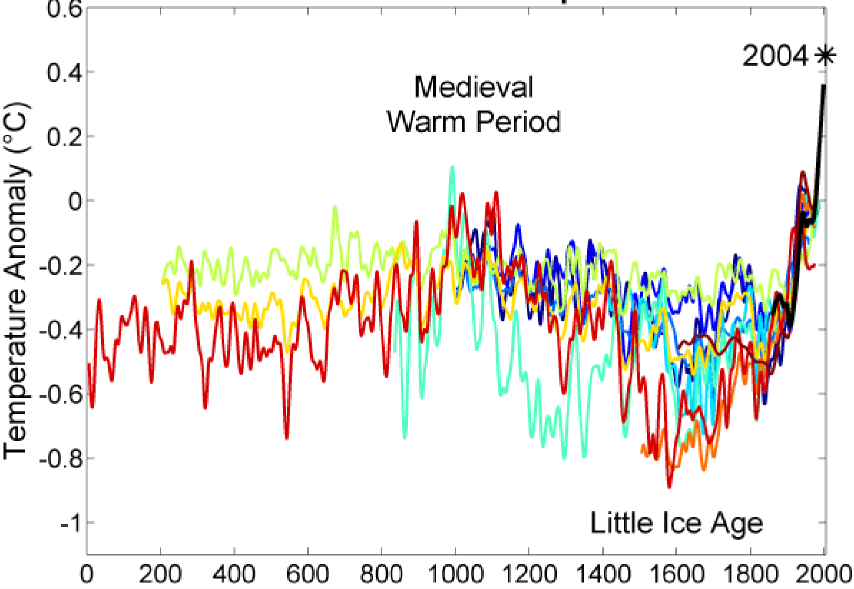
- For more recent climate info, see
  ▸ Link `https://youtu.be/gGOzHVUQCw0`

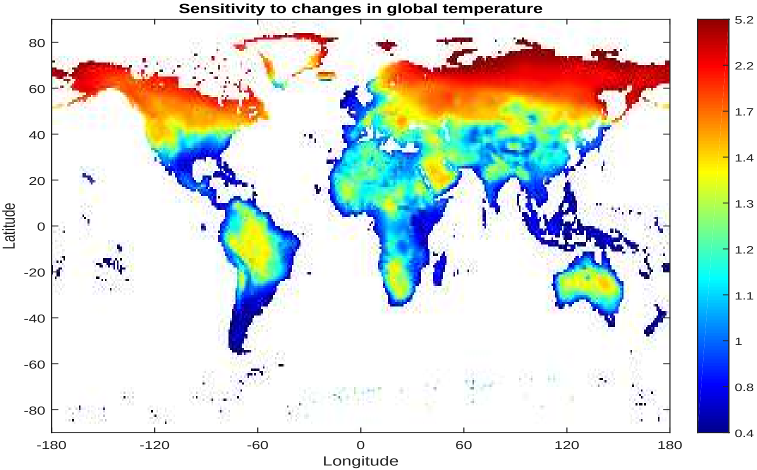
Ice Age Temperature Changes




180 W
20 N
40 N
60 N
80 N
90 W
90 E
0

Reconstructed Temperature




Sensitivity to changes in global temperature
Latitude
80
60
40
20
0
-20
-40
-60
-80
-180
-120
-60
0
60
120
180
Longitude
5.2
2.2
1.7
1.4
1.3
1.2
1.1
1
0.8
0.4

NON-LINEARITIES

- Recall that the equilibrium climate sensitivity is affected by feedbacks

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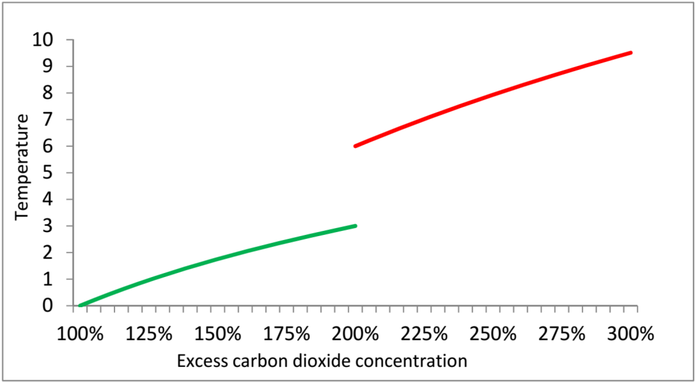
- $\kappa_{other} + \kappa_{refl} = \begin{cases} 2.1 \text{ if } T \text{ ever was larger than } 3^o C \\ 2.72 \text{ else} \end{cases}$

Tipping points

- Suppose $\eta = 3.7$ and $\kappa = 3.3$. and $x = 2.1$ if $T < 3^o C$ and 2.72 else. Then, the relation between CO$_2$ concentration and long-run temperature looks like follows



-

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- IPCC 6th report claims “there is no evidence of such non-linear responses at the global scale in climate projections for the next century, which indicate a near-linear dependence of global temperature on cumulative GHG emissions. (IPCC AR6 WG1, chap. 1 p. 202).

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- Suppose the uncertainty about $\kappa_{other} + \kappa_{refl}$ by a symmetric triangular density function with mode 2.1 and endpoints at 1.35 and 2.85. The mean, and most likely, value of $\kappa_{other} + \kappa_{refl}$ translates into a climate sensitivity of 3.

Feedback uncertainty

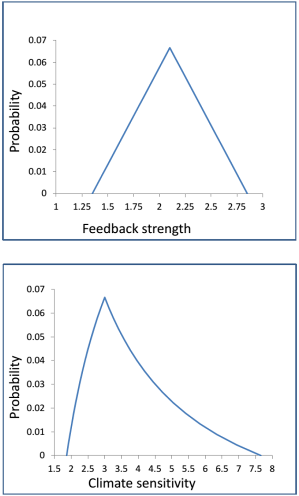

Figure: Example of symmetric uncertainty of feedbacks producing right skewed climate sensitivity.

The carbon cycle

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- For policy analysis as well as for forecasts, we need to know the dynamic mapping from path of emissions to path of CO_2 concentrations.

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- Note difference between measuring emissions in CO_2 and C. A mole of carbon atoms weighs 12 grams and a mole of oxygen weighs 16. Then a kg of carbon produces $\frac{2*16+12}{12} \approx 3.67$ kg CO_2.

EASY STOCK-FLOW CASE IN CONTINUOUS TIME

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- Flow in other direction proportional to S^L, with proportionality factor ϕ_2.

Changes in stocks equal net flows (in minus out), apart from emission inflow E. This gives

$$\begin{aligned}\frac{dS(t)}{dt} &= -\phi_1 S(t) + \phi_2 S^L(t) + E(t)\\ \frac{dS^L(t)}{dt} &= -\phi_2 S^L(t) + \phi_1 S(t).\end{aligned}$$

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With $E(t) = 0$, steady state satisfies

$$\begin{aligned}0 &= -\phi_1 S + \phi_2 S^L \\ 0 &= \phi_1 S - \phi_2 S^L\end{aligned}$$

which cannot be uniquely solved: all solutions satisfy $S = \frac{\phi_2}{\phi_1} S^L$. Why? Note that $S(t) + S^L(t) = \int_{s=0}^{t} E(s)ds$ at all times. The total amount of carbon increases as emissions continue.

Same case with discrete-time approximation

$$\begin{aligned}
S_t - S_{t-1} &= -\phi_1 S_{t-1} + \phi_2 S^L_{t-1} + E_{t-1}. \\
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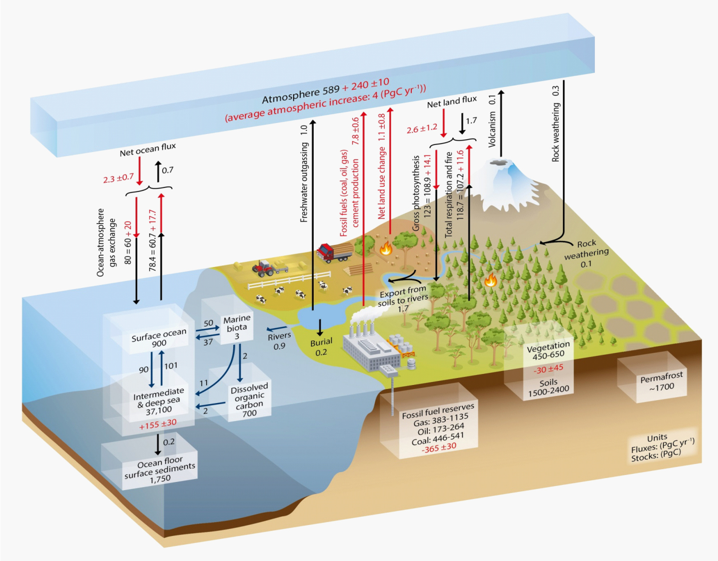
- The law of motion for the stocks follows ($s \geq 0$)

$$\begin{aligned} S_{t+s} &= \tfrac{\phi_2}{\phi_1+\phi_2}\left(S_t + S^L_t\right) - \tfrac{\phi_2 S^L_t - \phi_1 S_t}{\phi_1+\phi_2}\left(1 - \phi_1 - \phi_2\right)^s \\ S^L_{t+s} &= \tfrac{\phi_1}{\phi_1+\phi_2}\left(S_t + S^L_t\right) + \tfrac{\phi_2 S^L_t - \phi_1 S_t}{\phi_1+\phi_2}\left(1 - \phi_1 - \phi_2\right)^s. \end{aligned}$$

The stock-flow approach visually



Global carbon cycle. Stocks in GtC (PgC) and flows GtC/year. Source: Intergovernmental Panel on Climate Change (IPCC) (2013), Figure 6.1.

A MORE GENERAL STOCK-FLOW MODEL: THREE RESERVOIRS

- S_t represents (carbon concentration in) the atmosphere in period t, S_t^U the surface ocean, and finally S_t^L the deep oceans.

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  - Use flow to deep ocean, giving $\phi_{23} = \frac{90}{900} = 0.100$.
  - Finally, the flow from the deep ocean to the surface ocean is the same, giving $\phi_{32} = \frac{90}{37100} \approx 0.00243$.

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- They choose $\phi_{12} = 0.053, \phi_{21} = 0.0536$, $\phi_{23} = 0.0042$ and $\phi_{32} = 0.001422$ when the time step is a year. The initial values of the stocks are $S_{2015} = 850$, $S^U_{2015} = 765$ and $S^L_{2015} = 1799$. Note that in particular the deep oceans is much smaller than in reality. To model it that small makes the dynamics of the model more in line with the (much) more advanced models.

PROPERTIES OF STEADY STATE

- If emissions stop, this system also approaches a steady state. Solve

$$\begin{aligned}
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- i.e., proportions between stocks are always restored. Stocks sum to sum of past emissions.

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- These features can be modeled directly by a depreciation function (rather: "remainder function"), d_s that says how much remains of an emitted unit after s period.

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- $d_1 = 0.5 = 0.2 + (1 - 0.2)\,\varphi_0\,(1 - 0.023)^1\,, \Rightarrow \varphi_0 = 0.38$.

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Gives $\ln(1 - \varphi) = \frac{\ln \frac{1}{2}}{30}$, so $-\varphi \approx= -0.023$.

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- $d_1 = 0.5 = 0.2 + (1 - 0.2)\,\varphi_0\,(1 - 0.023)^1, \Rightarrow \varphi_0 = 0.38$.

CALIBRATION

- Use decades.

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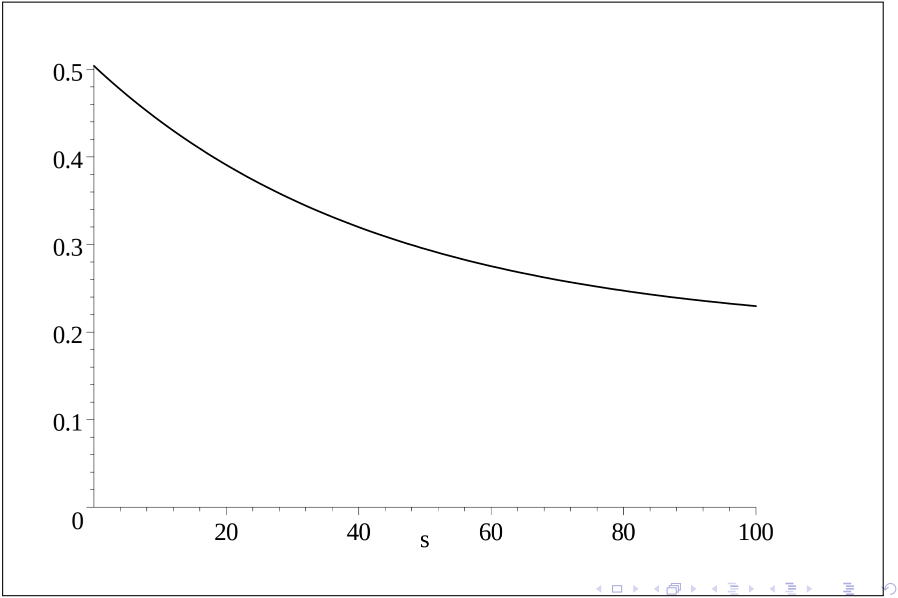
Implies

$$d_s = \left[\varphi_L + (1 - \varphi_L)\,\varphi_0\,(1 - \varphi)^s\right]_{\varphi_L = 0.2, \varphi = 0.023, \varphi_0 = 0.38}.$$

$$\left[\varphi_L + (1-\varphi_L)\,\varphi_0\,(1-\varphi)^s\right]_{\varphi_L=0.2,\varphi=0.023,\varphi_0=0.38}$$



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- According to the latest (6th) IPCC report, σ_{CCR} is "likely" (which should be interpreted as a 2/3 confidence interval) between 1.0 and 2.3 degrees Celsius per 1000 GtC (corresponding to 0.27-0.63$^\circ / TtCO_2$). This constant is called CCR (Carbon Climate Response, sometimes CRE or TCRE).

Carbon budgets

- Given a linear relation between accumulated emissions and temperature, a remaining carbon budget can be calculated.
- The large uncertainty about the CCR coefficient, makes this problematic.
- We have now burnt around 650 GtC. If CCR is 1, we have committed $0.6*1{=}0.65^{\circ}C$ and can emit another 850 GtC before reaching $1.5^{\circ}C$.
- This would take around 85 years with current emission rates.
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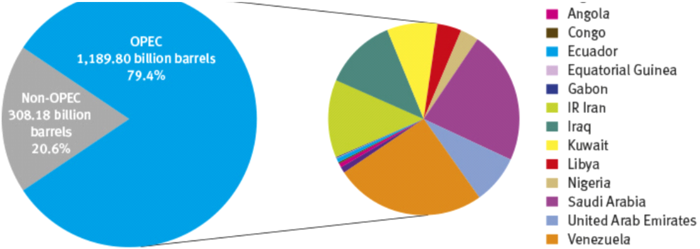
    1. Reserves (recoverable under current economic and technological conditions)
    2. Resources (recoverable under possible future economic and technological conditions).
- Technological developments are and have been fast. Leading to continuos reclassifications.

OPEC's own estimates



OPEC proven crude oil reserves, at end 2018 (billion barrels, OPEC share)

Venezuela	302.81	25.5%	Kuwait	101.50	8.5%	Algeria	12.20	1.0%	Gabon	2.00	0.2%
Saudi Arabia	267.03	22.4%	UAE	97.80	8.2%	Ecuador	8.27	0.7%	Equatorial Guinea	1.10	0.1%
IR Iran	155.60	13.1%	Libya	48.36	4.1%	Angola	8.16	0.7%			
Iraq	145.02	12.2%	Nigeria	36.97	3.1%	Congo	2.98	0.3%			

Source: OPEC Annual Statistical Bulletin 2019.

- Is 1190 billion brls a lot? A barrel is $1/7.33$ tons and oil contains 85% carbon. So this is $138GtC$. Likely gives 0.14-$0.32^{\circ}C$ warming using IPCC's likely CCR coefficient.

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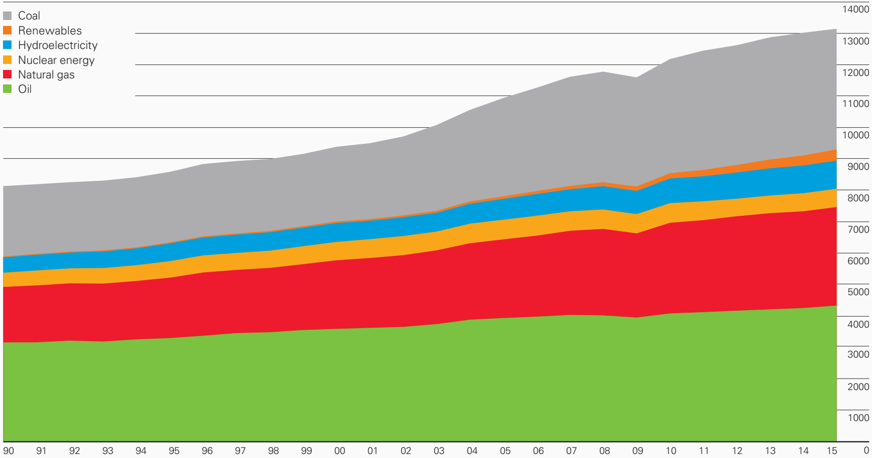
- Note that these are emissions not net contributions to the stock of CO_2.

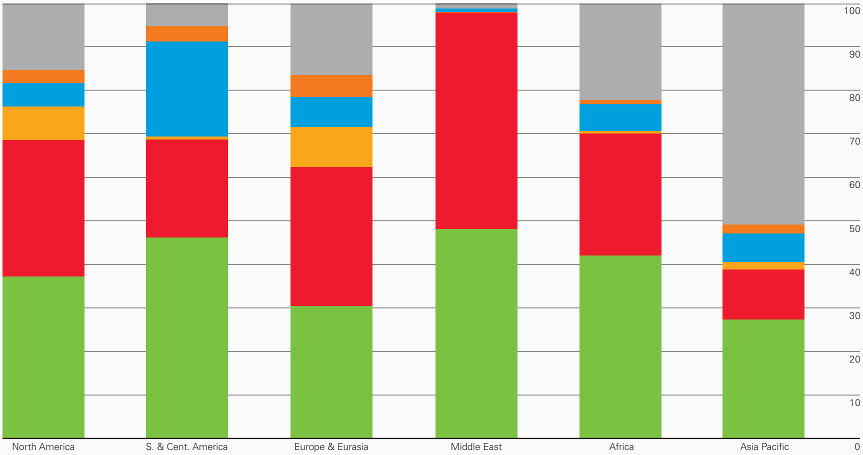
World consumption

Million tonnes oil equivalent



Regional consumption by fuel 2015

Percentage



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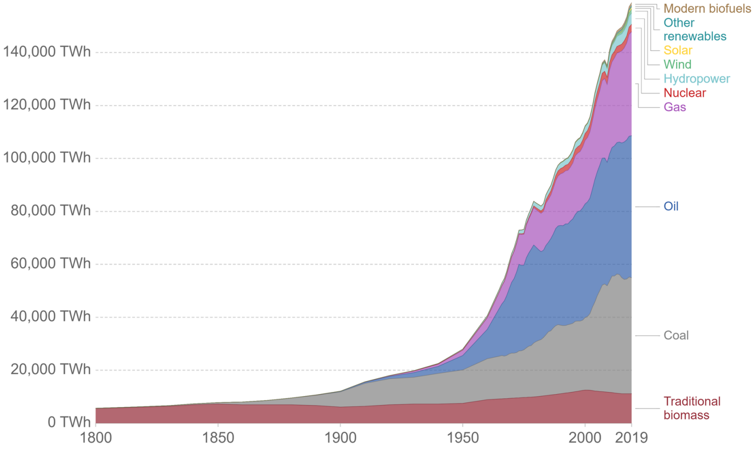
  - Coal is more than 3000 GtC,
- Coal is what really matters.
- Emissions are (now) about 10Gtc/ per year which means that about half of that is the (permanent) net increase of CO_2.

Finite resources

- For the purposes of climate-economy modeling, we need
    - a long-run growth model consistent with data,
    - with a production function using energy as input, and
    - to also model supply of energy.
- Since industrial revolution, energy is largely about fossil fuel, a resource in finite supply.
- Today:
    - discuss supply and demand of finite resources, and
    - a primer on endogenous resource saving technical change.

Global primary energy supply (including conversion losses)



Source: Vaclav Smil (2017) and BP Statistical Review of World Energy

OurWorldInData.org/energy • CC BY

Global shares of different energy sources

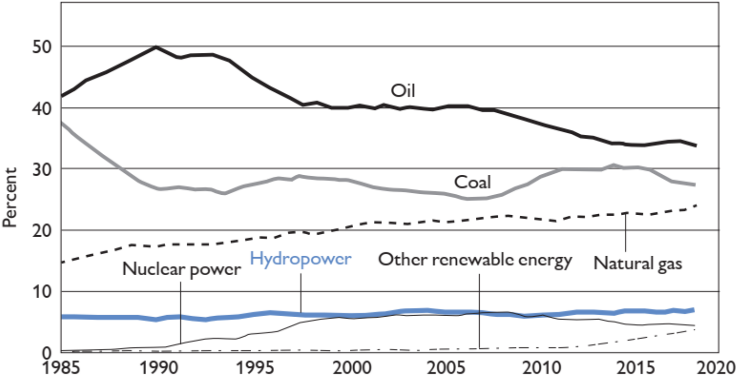


Figure: Source: BP Statistical Review of World Energy 2019

European shares of different energy sources

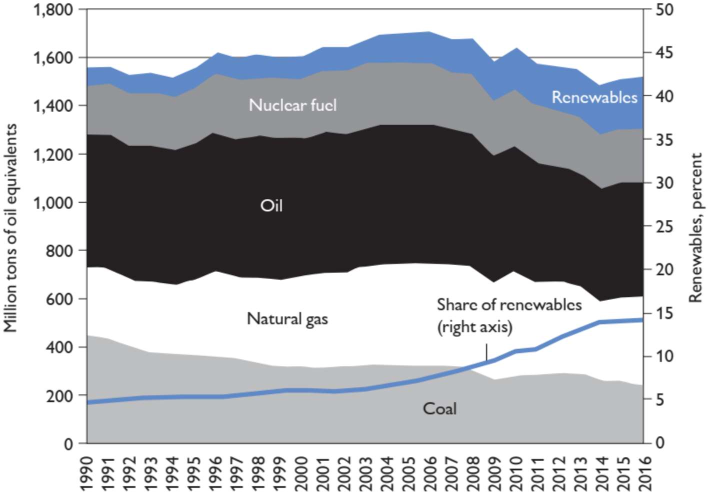
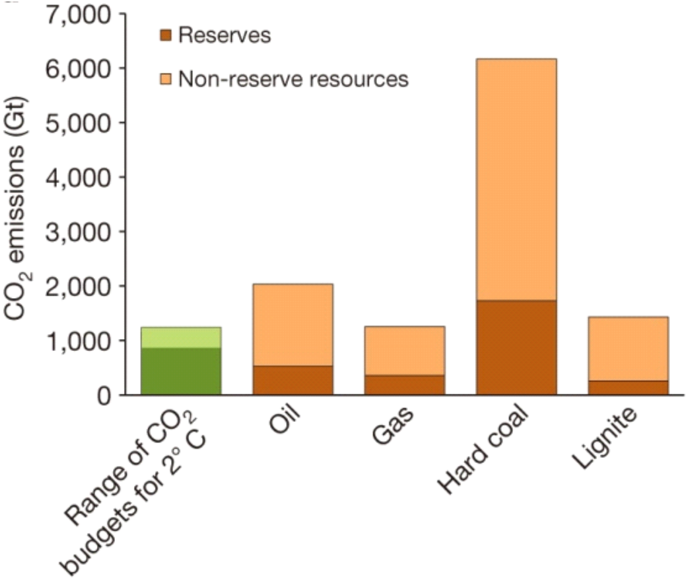
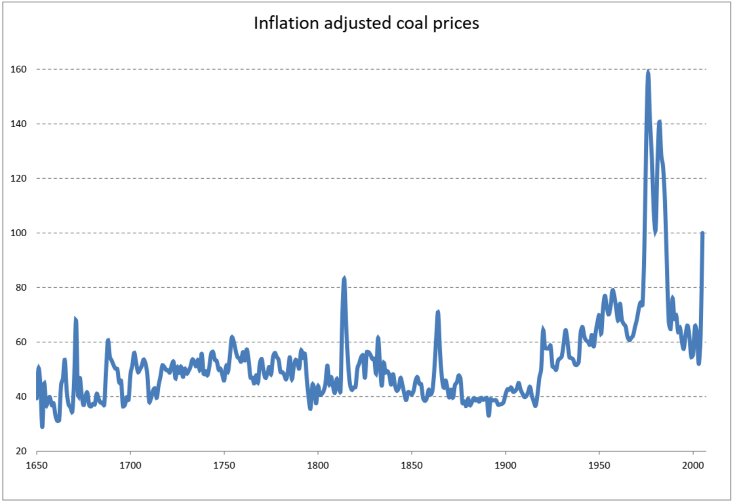


Figure: Source data: European Environment Agency

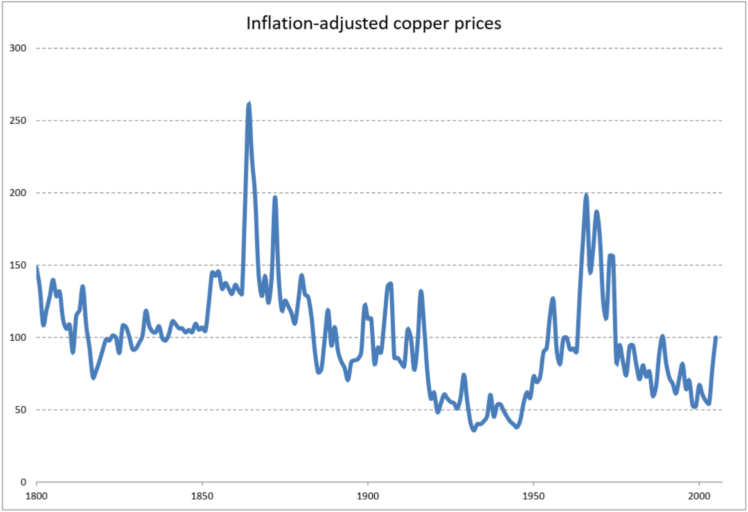
Oil price


7,000
6,000
5,000
4,000
3,000
2,000
1,000
0
CO2 emissions (Gt)
Reserves
Non-reserve resources
Range of CO2 budgets for 2° C
Oil
Gas
Hard coal
Lignite

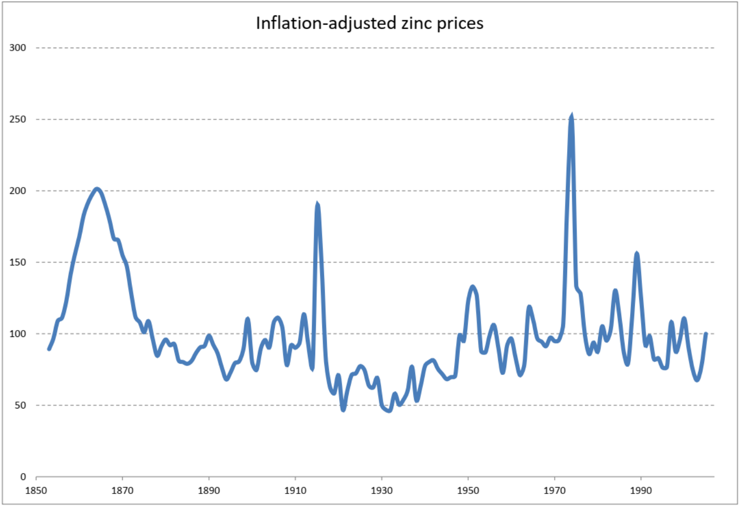
Coal price


Inflation adjusted coal prices
160
140
120
100
80
60
40
20
1650
1700
1750
1800
1850
1900
1950
2000

Copper price



Zinc price


Inflation-adjusted zinc prices
300
250
200
150
100
50
0
1850
1870
1890
1910
1930
1950
1970
1990

Real price composite of 57 minerals and energy sources

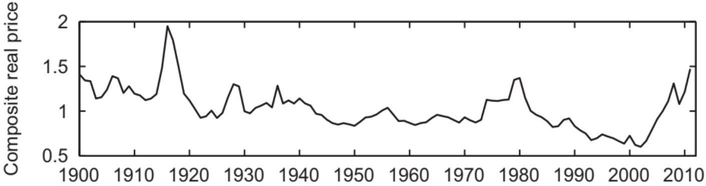
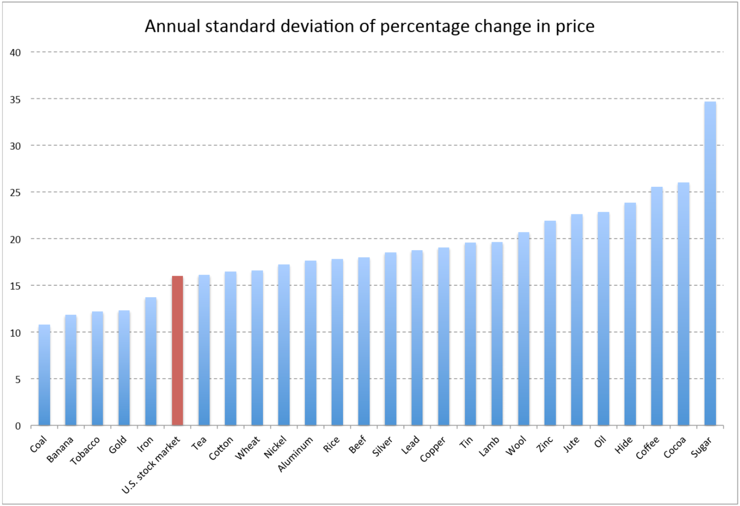


Figure: Source: Daniel Spiro, JEDC (2014).

Price volatility



A simple fossil fuel dichotomy

- Different fossil fuels have quite different supply characteristics. A general rule is that amount increases with cost of extraction.
- One end of spectrum – conventional oil. Exists in limited supply and is very cheap to extract relative to value. High profit margin.
- The other end – coal reserves. Very large quantities and price close to marginal cost. Low if any profit margin.
- But;
  - there are things in between, and
  - technological change shifts the boundaries.
- Still, even a small emission price makes coal unprofitable but this is not the case for conventional oil. (Current ETS price $\approx$ 5 cent per liter gasoline but kills coal power).

Finite Resource Theory: 1 Cake eating

- Consider planning problem under zero extraction costs.

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_t$$

subject to

$$\sum_{t=0}^{\infty} c_t \leq R_0$$

- Euler equation:

$$\begin{aligned} U'(c_t) &= \beta U'(c_{t+1}) \\ \frac{1}{c_t} &= \beta \frac{1}{c_{t+1}} \Rightarrow c_{t+1} = \beta c_t \end{aligned}$$

- Using resource constraint yields $c_t = (1-\beta)R_t$ where $R_{t+1} = R_t - c_t$. Implies $c_t = (1-\beta)\beta^t R_0$.

Finite Resource Theory: 2 Production

- Same problem, now with Cobb-Douglas production and full depreciation of capital Also cake-like.

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_t$$

$$\text{s.t.} \quad : \quad c_t + k_{t+1} = Ak_t^{\alpha} e_t^{\nu} \text{ and } \sum_{t=0}^{\infty} e_t \leq R_0$$

- Saving rate is constant and equal to $\alpha\beta$ immediate to show from Euler equation. Now, two means of saving, the resource and capital. Must have equal return on equilibrium/optimum. Return on capital the marginal product and on the resource the price growth

$$\frac{\alpha Ak_{t+1}^{\alpha} e_{t+1}^{\nu}}{k_{t+1}} = \frac{\alpha Ak_{t+1}^{\alpha} e_{t+1}^{\nu}}{\alpha\beta Ak_t^{\alpha} e_t^{\nu}} = \frac{\nu Ak_{t+1}^{\alpha} e_{t+1}^{\nu} / e_{t+1}}{\nu Ak_t^{\alpha} e_t^{\nu} / e_t} \Rightarrow e_{t+1} = \beta e_t$$

- Again: Solution: $e_t = (1-\beta)R_t$, where $R_{t+1} = R_t - e_t$. Hence $e_t = (1-\beta)\beta^t R_0$.

Finite Resource Theory: 3 Adding tech growth

- Now add technological growth at a gross rate of γ

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_t$$

$$\text{s.t.} \quad : \quad c_t + k_{t+1} = A\gamma^t k_t^{\alpha} e_t^{\nu} \text{ and } \sum_{t=0}^{\infty} e_t \leq R_0$$

- Again savings rate is $\alpha\beta$ and by arbitrage

$$\frac{\alpha\gamma^{t+1} A k_{t+1}^{\alpha} e_{t+1}^{\nu}}{\alpha\beta\gamma^t A k_t^{\alpha} e_t^{\nu}} = \frac{\nu\gamma^{t+1} A k_{t+1}^{\alpha} e_{t+1}^{\nu} / e_{t+1}}{\nu\gamma^t A k_t^{\alpha} e_t^{\nu} / e_t} \Rightarrow e_{t+1} = \beta e_t$$

- In balanced growth, capital and output grows at same rate and resource use fall at gross rate β.Thus $g = \gamma g^{\alpha} \beta^{\nu} = (\gamma\beta^{\nu})^{\frac{1}{1-\alpha}}$. For $\gamma > \beta^{-\nu}$ $g > 0$.
- From Euler equation, $g = \beta(1+r)$, so $1 + r = (\gamma\beta^{\nu})^{\frac{1}{1-\alpha}} / \beta$.Positive real interest rate if $g > \beta$, then resource price grows.

Finite Resource Theory: 3 Adding extraction costs

- Suppose there is a cost of extracting resources. The arbitrage intuition still works (Hotelling (1931)). Return on saving a unit of the resource with price p_t is now

$$\frac{p_{t+1} - mc_{t+1}}{p_t - mc_t}$$

which must equal return on saving in the form of capital. Yields

$$\frac{p_{t+1}}{p_t} = 1 + r_t + \frac{1}{p_t}\left(mc_{t+1} - (1 + r_t)mc_t\right)$$

- If $mc_{t+1} - (1 + r_t)mc_t < 0$. price growth is lower than the interest rate.

Challenges

- Key natural-resource “puzzles”:
  - Why no positive trend in prices?
  - Why so volatile? (And are natural resources different than other “commodities”?
  - Why upward trend in use?
- Related puzzle: why isn’t all the cheap fossil fuel extracted first?

Back to the production function

- With Cobb-Douglas production function, all income shares constant. Not too bad for capital and labor. What about energy?

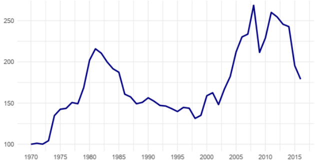


Figure: **The real price of a unit (Btu) of energy, U.S.**

Average real (using a GDP deflator) price of a Btu for the U.S., including all energy sources. **Source:** US Energy Information Administration.

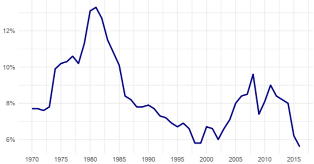


Figure: **The energy share in the U.S.**

A more reasonable production function

- Consider instead CES production function:

$$y \equiv F\left(Ak^{\alpha}l^{1-\alpha}, A_e e\right) = \left[(1-\gamma)\left(Ak^{\alpha}l^{1-\alpha}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left(A_e e\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

  with $\varepsilon < 1$. A is capital-labor augmenting technology and A_e is energy augmenting technology. A special case is Leontief ($\varepsilon = 0$): $y = \min\left\{Ak^{\alpha}l^{1-\alpha}, A_e e\right\}$
- This fits the short-run fluctuations in data really well. But non-explosive price paths is a knife-edge property (require $Ak^{\alpha}l^{1-\alpha}$ and $A_e e$ to grow at same rate). In general, income share of energy would go to zero or one depending on supply and growth rates of A and A_e.
- Need something that makes (relative) growth rates of A and A_e endogenous.

A static example of endogenous technology choice

- Assume

$$\begin{aligned} y &= \left[(1-\gamma)(Ak)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left(A_e e\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ s.t. G(A, A_e) &= \bar{A} \end{aligned}$$

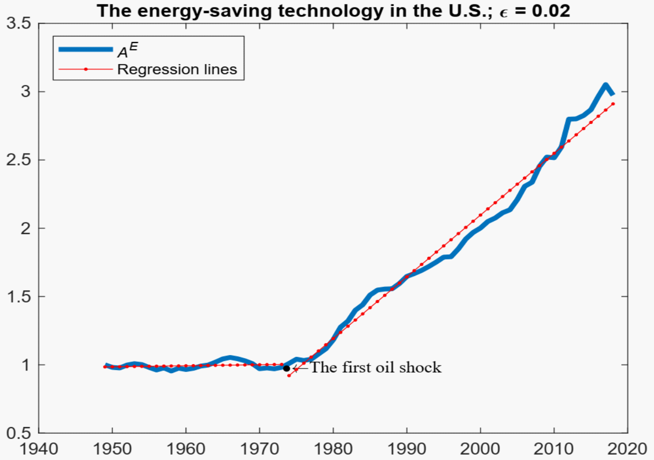
  so with $\bar{A}$ given but A and A_e endogenous: directed technical change.
- Consider a simple case: suppose G is $\lambda \ln A + (1-\lambda)\ln A_e = \ln \bar{A}$
- Lagrangean with shadow value Λ. FOC:

$$\begin{aligned} A; \left[\frac{y}{Ak}\right]^{\frac{1}{\varepsilon}}(1-\gamma)k &= \Lambda\frac{\lambda}{A} \text{ and } A_e; \left[\frac{y}{A_e e}\right]^{\frac{1}{\varepsilon}}\gamma e = \Lambda\frac{1-\lambda}{A_e} \\ &\Rightarrow \frac{\left[\frac{y}{Ak}\right]^{\frac{1}{\varepsilon}}(1-\gamma)Ak}{\left[\frac{y}{A_e e}\right]^{\frac{1}{\varepsilon}}\gamma A_e e} = \frac{MP_k k}{MP_e e} = \frac{\lambda}{1-\lambda} \end{aligned}$$

- Income shares are constant and independent of k and e. Income shares instead depend on the ratio $\frac{\lambda}{1-\lambda}$, how hard it is to improve capital efficiency relative to energy efficiency.

Dynamic directed technology choice

- We can now think of the choice of A and A_e as occurring over medium-run time. Higher energy prices, e.g., lead to higher growth rates of A_e (at the expense of A). On impact, energy income share increases but stabilizes over time.



Summary

- There are puzzles in this area, and big quantitative challenges!
- Distinguish between oil and coal and remember that there are intermediates and technological change.
- Substantial medium run flexibility in production due to directed technical change. Energy and fossil fuel do not have to grow in parallel to output in medium and long-run.
- For long-run analysis, we are comfortable using C-D in energy, despite a very low short-run substitutability between energy and other inputs.
- Similar results can be derived for substitutability between different energy types.

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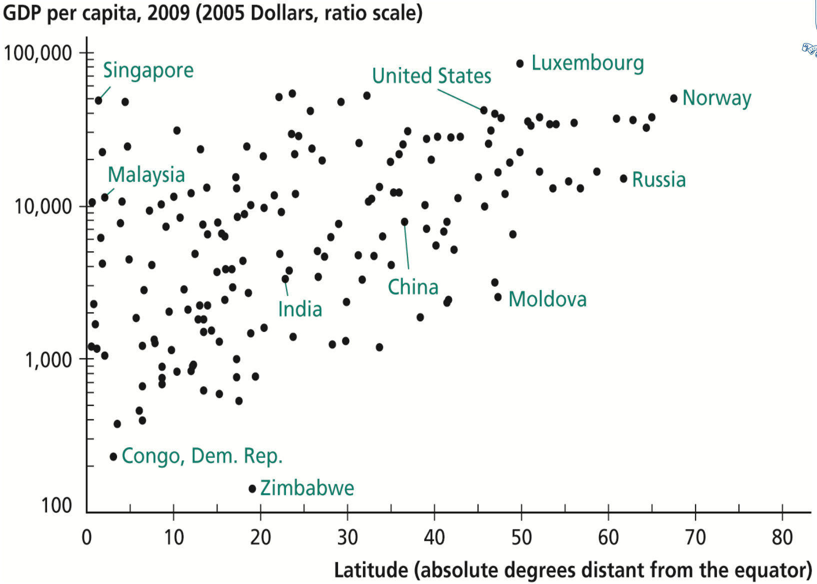
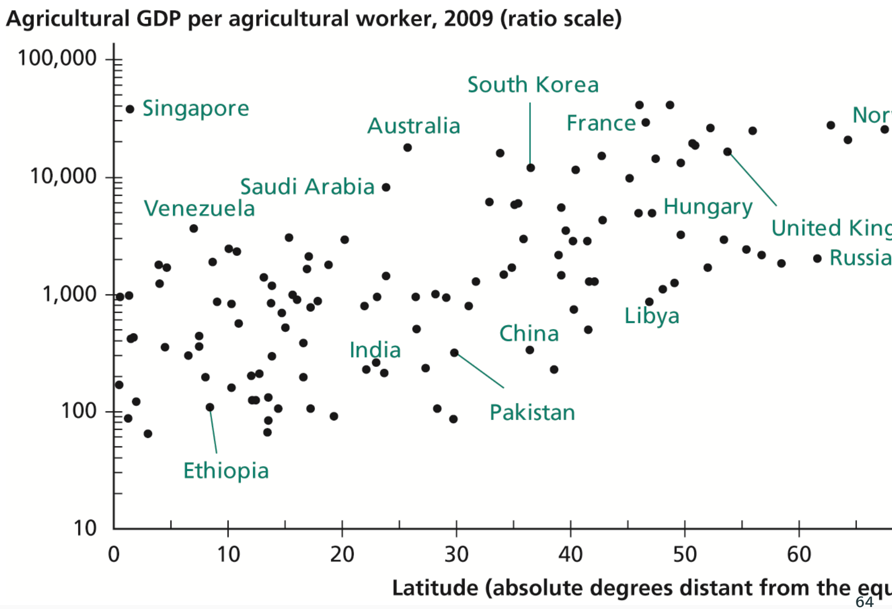
  - reduced form: directly use correlation between natural variation in climate and relevant aggregate outcomes (GDP, mortality, etc.).
- Approaches have different pros and cons. Complementary.


GDP per capita, 2009 (2005 Dollars, ratio scale)
100,000
10,000
1,000
100
0
10
20
30
40
50
60
70
80
Latitude (absolute degrees distant from the equator)
Singapore
Malaysia
United States
Luxembourg
Norway
Russia
India
China
Moldova
Congo, Dem. Rep.
Zimbabwe

LATITUDE VS. AGRICULTURAL GDP PER WORKER



NORDHAUS'S DAMAGES IN RICE: A BOTTOM-UP APPROACH

Nordhaus's DICE model (Dynamic Integrated model of Climate and the Economy) and the later RICE (also dynamic, but with R for Regional) do the bottom part as follows.

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- Use 13 regions: U.S., OECD part of Europe, Eastern Europe, Japan, Russia, China, Africa, India, other high income, other middle, other low middle income, and low- and high-income OPEC.

FUNCTIONAL SPECIFICATION

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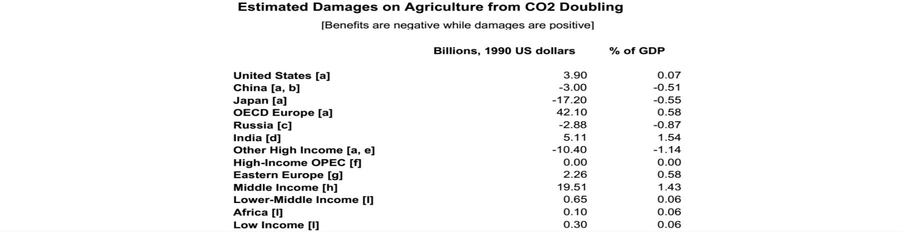

Estimated Damages on Agriculture from CO2 Doubling

[Benefits are negative while damages are positive]

	Billions, 1990 US dollars	% of GDP
United States [a]	3.90	0.07
China [a, b]	-3.00	-0.51
Japan [a]	-17.20	-0.55
OECD Europe [a]	42.10	0.58
Russia [c]	-2.88	-0.87
India [d]	5.11	1.54
Other High Income [a, e]	-10.40	-1.14
High-Income OPEC [f]	0.00	0.00
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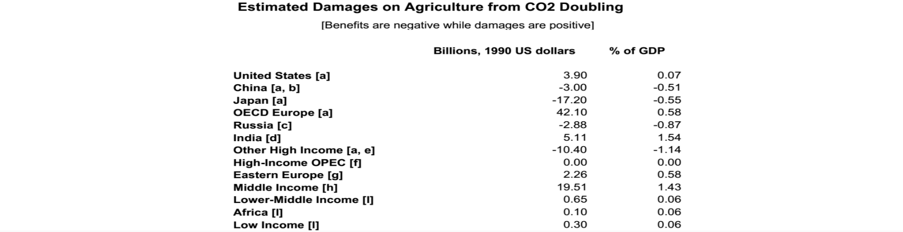

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- Positive effects if initial temperature is below 11.5 degrees. Suggests quadratic damage $\alpha_{ag}^{1}\left(T+T_{0}^{j}\right)+\alpha_{ag}^{2}\left(T+T_{0}^{j}\right)^{2}+\alpha_{ag}^{j}$.

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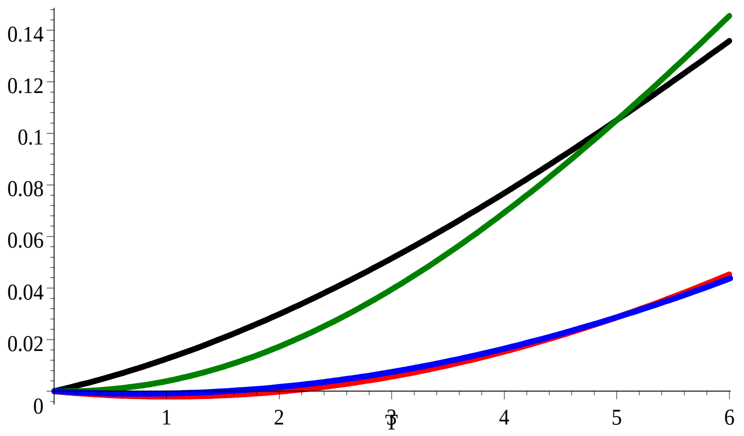
- India twice as willing, the U.S. and China less than half.

NORDHAUS 2000 SUMMARY

- Damages as percent of GDP, described by $D(T) = 1 - \frac{1}{1+\theta_{j,\mathbf{1}}T+\theta_{j,\mathbf{2}}T^{\mathbf{2}}}$ with region-specific $\theta_j s$, giving (Blue-USA, Red-Chi, Green-Eur, Black-LI).



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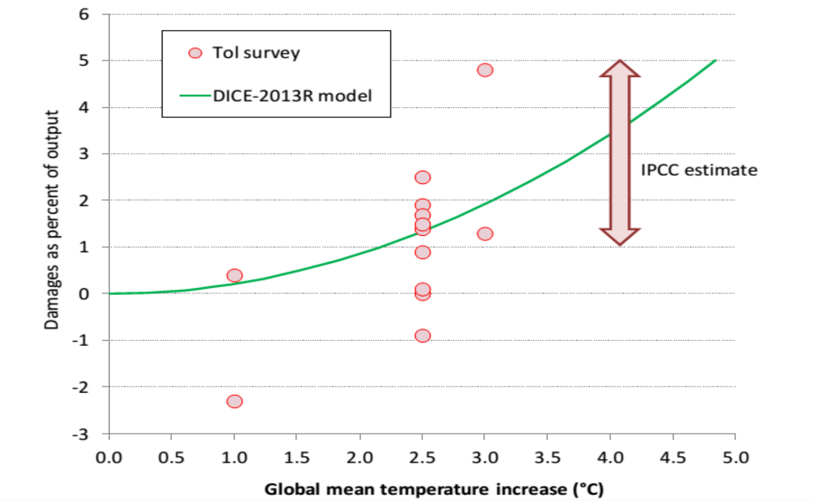
- Nordhaus stresses that the damage function for high temperatures (>3 or 4 degrees?) should not be taken very seriously.

CISCAR ET AL. (PNAS FEB 2011)

- Another bottom-up studie, but for Europe only.
- Sums the impact for 5 types of damages: agricultural production, river floods, coastal effects, tourism (market), and health.
- Uses different high-resolution models 50x50 km and uses distribution of weather outcomes (not only temperature).
- Compares different scenarios for year 2080 to baseline of no climate change.
- For EU as a whole, yearly damages equivalent to 1% of consumption for 5.4 degree heating in EU. Small positive effects on tourism and substantial positive effects on Northern Europe.
- Relative to growth rate over 70 years ($1.02^{70} \approx 4$), these effects seem fairly small.



The solid line is the estimate from the DICE-2013 $D(T) = 1 - \frac{1}{1+0.00267T^2} \approx 0.023\left(\frac{T}{3}\right)^2$.

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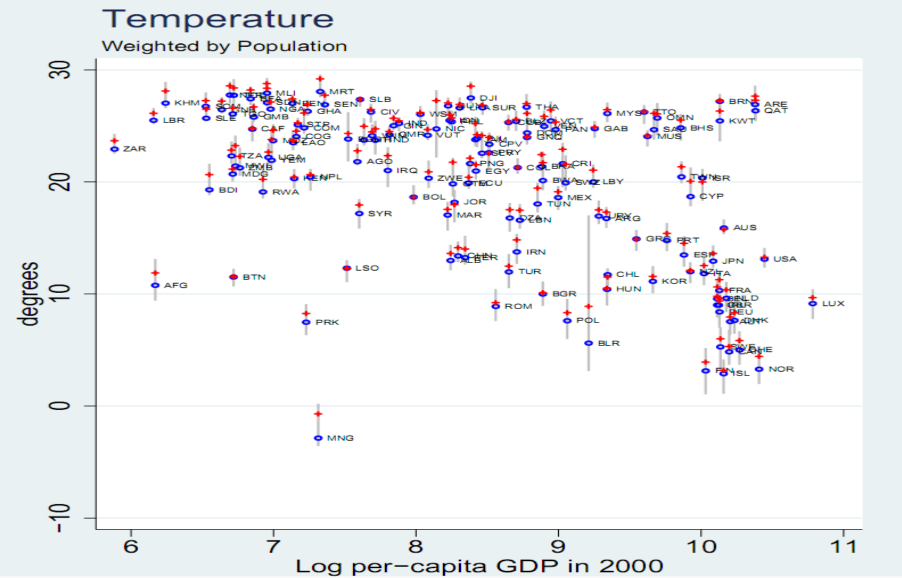
- Use Diff in Diff to obtain reliable estimates. Mostly across time, but also using within country variation.

SUBSTANTIAL NATURAL VARIATION



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- Crucial feature is whether there are growth effects or not.

TEMPERATURE-GDP CORRELATIONS WITH HIGH-RESOLUTION DATA

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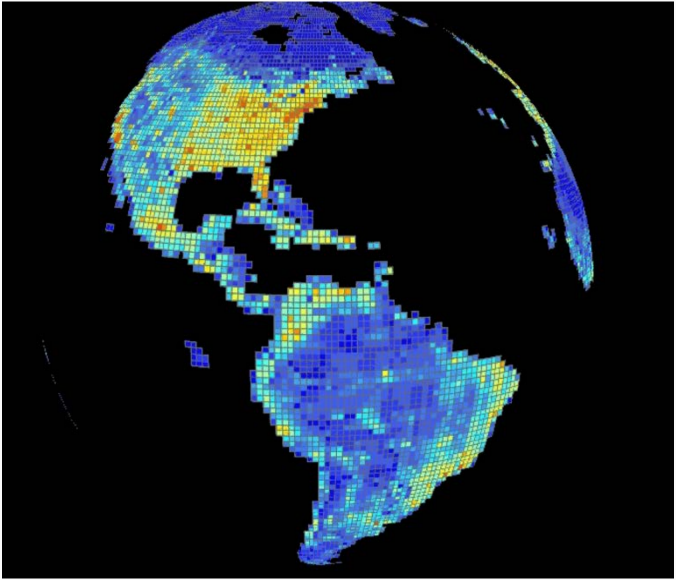
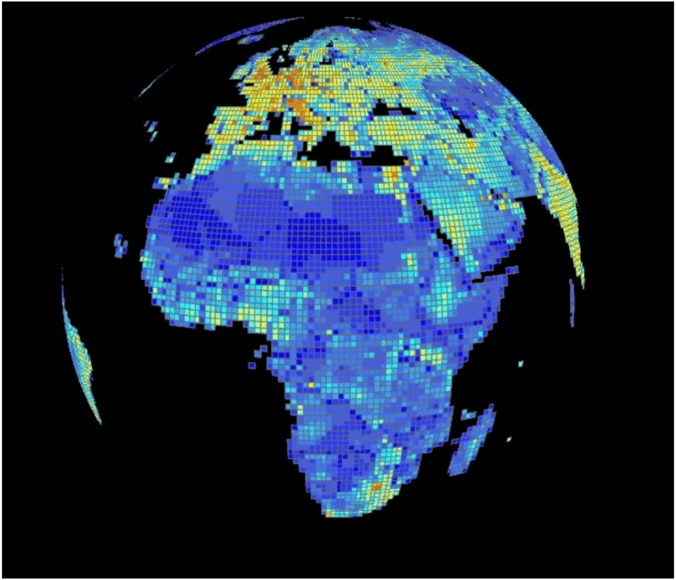
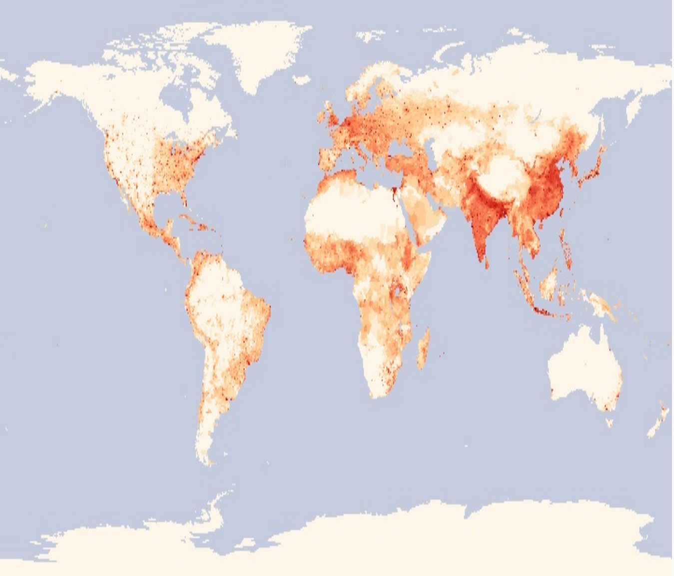
- Produces nice charts!





POPULATION DENSITY



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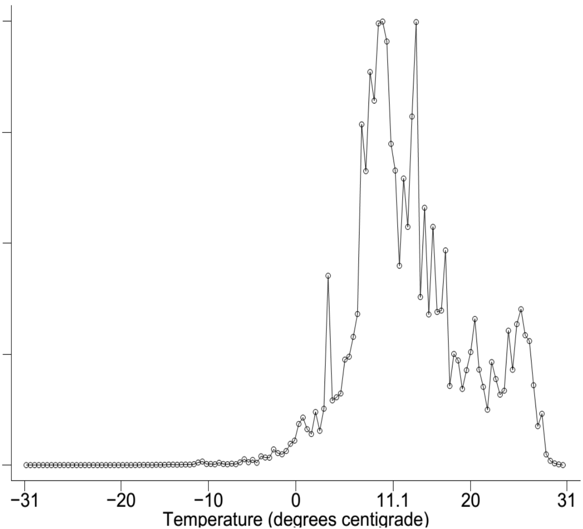
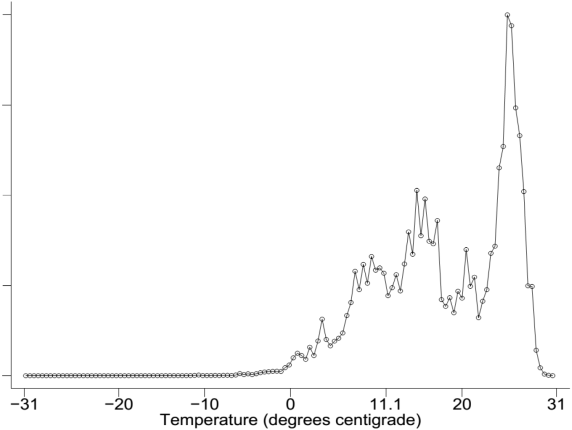
- Obvious *pros* as well as *cons* with this methodology.

SHARE OF GLOBAL GDP VS. YEARLY MEAN TEMP



Population as function of local temperature



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$$D(T_j) = \begin{cases} 1 - \left(p + (1-p)e^{\gamma_h (T_j - T^\star)^2}\right) & \text{if } T_j < T^\star \\ 1 - \left(p + (1-p)e^{\gamma_l (T_j - T^\star)^2}\right) & \text{if } T_j > T^\star \end{cases}$$

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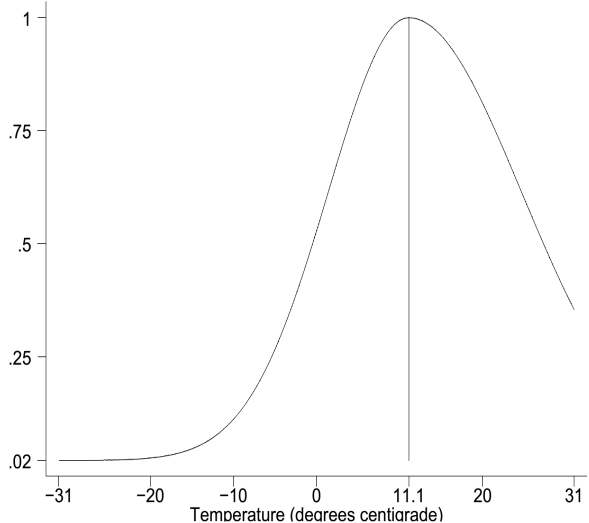
❺ Aggregate damages and choose $(T^\star, \gamma_h, \gamma_l, p)$ to match aggregate damages implied by Nordhaus's DICE damage function.

IMPLIED 1 – (DAMAGE FUNCTION)



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- Consequently, there are also strong indications that there will be significant migration pressures from climate change.

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- Much to be learnt from further research.

What to do?

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How to design a (common) optimal policy?

MAIN GOAL

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- Goal: Derive the optimal policy —here a tax on carbon— so that the externality is internalized.

EXTERNALITY

- Higher levels of carbon dioxide in the atmosphere contributes to global warming, which in turn causes damages like production shortfalls, poor health or deaths, capital destruction and much more.

- Map carbon concentration to climate, and then map climate to damages.

- Expected sum of future damage elasticities: the percentage change in output resulting from a percentage change in the amount of carbon in the atmosphere, caused by emitting a unit of carbon today.

- Discounted because of time preferences and because of carbon depreciating.

THE CARBON CYCLE

Carbon circulation system: carbon is exchanged through various reservoirs such as the atmosphere, the terrestrial biosphere, and different layers of the ocean.

The representation of the carbon cycle in this paper is given by the equation:

$$(1 - d_s) = \phi_L + (1 - \phi_L)\phi_0(1 - \phi)^s$$

- ϕ_L: the share of carbon that stays in the atmosphere forever
- $(1 - \phi_0)$: of the carbon that does not stay in the atmosphere forever, this is the share that exits the atmosphere into the biosphere or ocean within a decade
- the remaining carbon in the atmosphere, $(1 - \phi_L)\phi_0$, decays at a geometric rate ϕ

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❺ Inclusion of Exhaustible Resources that induces savvy economic behavior.

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- Dirty energy has cost constant cost ξ_j. Clean energy has convex cost $\xi_J(E_{J,T})$.

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- Here, $-T$ is defined as the start of industrialization.

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- After that we worry about implementation

Planner's Problem

$$\max_{\substack{\{C_t, N_t, K_{t+\mathbf{1}}, R_{j,t+\mathbf{1}},\\ E_{j,t}, S_t\}_{t=\mathbf{0}}^{\infty} \geq \mathbf{0}}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \qquad \text{s.t.}$$

$$C_t + K_{t+1} = F_t(K_t, N_t, E_t, S_t) + (1-\delta)K_t \qquad \text{FB}$$

$$E_t = \sum_j E_{j,t}\, \alpha^j \qquad \text{AGE}$$

$$R_{j,t+1} = R_{j,t} - E_{j,t} \geq 0 \qquad \text{for all } j \qquad \textit{ExE}$$

$$S_t = \tilde{S}_t \left(\sum_{j=1}^{J_g - 1} E_{j,-T}, \sum_{j=1}^{J_g - 1} E_{j,-T+1}, \ldots, \sum_{j=1}^{J_g - 1} E_{j,t} \right) \qquad \textit{CC}$$

NOTATION FOR THE PLANNER'S PROBLEM

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- α^j Conversion of units of energy of type j from being in terms of carbon emissions to units of energy.

CHARACTERIZATION OF THE SOLUTION

- The marginal externality damage is the same for all j:

$$\Lambda_t^s = \mathbb{E} \sum_{i=0}^{\infty} \beta^i \frac{U'(C_{t+i})}{U'(C_t)} \frac{\partial F_{t+i}}{\partial S_{t+i}} \frac{\partial S_{t+i}}{\partial E_{j,t}}$$

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- Under our specific assumptions, this expression simplifies to:

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- Further, if the planner's problem implies a constant savings rate, then the expression can be written as:

$$\Lambda_t^s = Y_t \left[\mathbb{E} \sum_{i=0}^{\infty} \beta^i \gamma_{t+i}(1 - d_i) \right]$$

CHARACTERIZATION OF THE SOLUTION II

- The FOC of the planner says

$$\alpha_j \frac{\partial F_t}{\partial E_t} - \xi_j - \Lambda_t^s = 0$$

DECENTRALIZED EQUILIBRIUM: CONSUMERS

$$\max_{\{C_t, N_t, K_{t+\mathbf{1}}\}_{t=\mathbf{0}}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$\text{subject to} \qquad \mathbb{E}_0 \sum_{t=0}^{\infty} q_t (C_t + K_{t+1})$$

$$= \mathbb{E}_0 \sum_{t=0}^{\infty} q_t ((1 + r_t - \delta) K_t + w_t N_t + T_t) + \Pi_t.$$

DECENTRALIZED EQUILIBRIUM: FIRMS

$$\Pi_0 = \max_{\{K_t, N_t, E_t\}_{t=\mathbf{0}}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} q_t \Bigg[F_t(K_t, N_t, E_t, S_t) \\ - r_t K_t - w_t N_t - \sum_{j=1}^{J} p_{j,t} E_{j,t} \Bigg]$$

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- This is the optimal first best tax on carbon emissions.
- If there are multiple externalities (for instance an R&D component to the model) then a separate Pigouvian tax is required for each externality.

COMPARING THE OPTIMAL TAX RATES

To understand the magnitude of the optimal tax rates given by this model, they can be compared with estimates from other models, and also with tax rates that are currently being used around the world.

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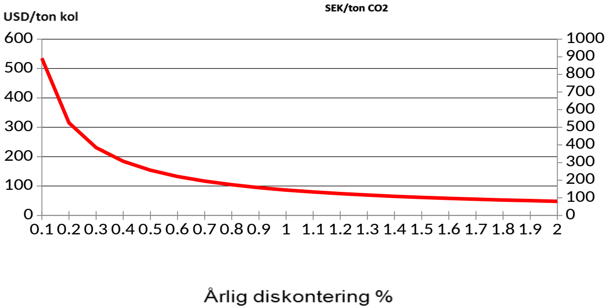
- In Sweden, the current tax on private consumption of carbon exceeds \$600 per ton of carbon, which is larger than the estimates for the optimal tax in this paper. However, these taxes are significantly higher than many other countries, for instance the EU has a tax of around \$77 per ton of carbon.

Sum damages over time => ”optimal” tax!



Sweden has carbon tax ~ 600 USD/tC!

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- => Coal is the main threat!

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    - Taking the climate damage into account, using coal simply isn't worth it.
- So: bad for the coal industry (the world over), no big deal otherwise

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    - In Europe, this is not the case —low world demand and high caps culprits.

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  - No: reduce them where they are least needed/least efficient (e.g., buy emission rights in EU trading system, pay to keep forests, ...)

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- The quantitative magnitudes of feedback are disputed. The "average" view seems to be that feedbacks strengthen the direct warming effect considerably, but there is much uncertainty.

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- What is the appropriate level of the tax? For this, we use standard cost-benefit analysis.

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- Together, the two steps are $D(T(S))$ mapping additional atmospheric carbon to damages. Let's examine the mapping.

A SIMPLER MAPPING

- It turns out that $1 - D\left(T\left(S\right)\right)$, i.e., how much is left after damages as a function of S, is well approximated by the function $e^{-\gamma S}$: for $\gamma = 5.3 * 10^{-5}$ (black), it is quite close to $1 - D\left(T\left(S\right)\right)$ (red dashed), as seen in the figure.

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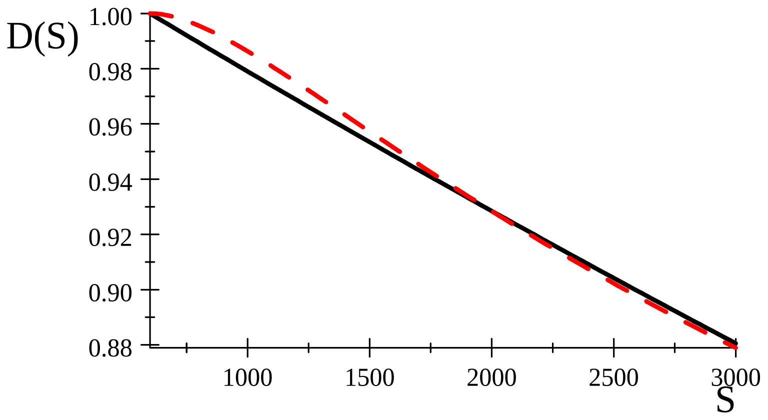

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