Economics 244: Macro Modeling
Dynamic Fiscal Policy
PARTS III and IV

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A large chunk of this material was developed by Dirk Krueger

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Part III

The Life Cycle Model

- Generalization of the two-period model to multiple periods

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- Friedman's permanent income hypothesis focuses on impact of timing and characteristics of uncertain income on consumption choices.

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- In each period t household earns after-tax income y_t and consumes c_t.
- May have initial wealth $A \geq 0$

- Period budget constraint

$$c_t + s_t = y_t + (1 + r)s_{t-1}$$

Here r denotes interest rate, s_t denotes financial assets carried over from period t to period $t + 1$ and s_{t-1} denotes assets from period $t - 1$ carried to period t.

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- Period 1 budget constraint

$$c_1 + s_1 = A + y_1.$$

$$U(c_1, c_2, \dots, c_T) = u(c_1) + \beta u(c_2) + \beta^2 u(c_3) + \dots + \beta^{T-1} u(c_T)$$

or

$$U(c) = \sum_{t=1}^{T} \beta^{t-1} u(c_t)$$

where $c = (c_1, c_2, \dots, c_T)$ denotes the lifetime consumption profile

Rewrite the period-by-period budget constraints as a single intertemporal budget constraint: note that

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- and plug into first equation, to obtain

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- which can be rewritten as

$$c_1 + \frac{c_2}{1+r} + \frac{s_2}{1+r} = A + y_1 + \frac{y_2}{1+r}$$

- Repeat this procedure: from third period budget constraint

$$c_3 + s_3 = y_3 + (1+r)s_2$$

we can solve for

$$s_2 = \frac{c_3 + s_3 - y_3}{1+r}$$

and plug in to obtain

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} + \frac{s_3}{(1+r)^2} = A + y_1 + \frac{y_2}{1+r} + \frac{y_3}{(1+r)^2}$$

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- Continue the process T times, to arrive at the intertemporal budget constraint

$$\begin{aligned} & c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} + \ldots + \frac{c_T}{(1+r)^{T-1}} + \frac{s_T}{(1+r)^{T-1}} \\ = \quad & A + y_1 + \frac{y_2}{1+r} + \frac{y_3}{(1+r)^2} \ldots + \frac{y_T}{(1+r)^{T-1}} \end{aligned}$$

- s_T denotes saving from period T to $T+1$. Household lives only for T periods, so she has no use for saving in period $T+1$. We don't allow $s_T < 0$. Thus $s_T = 0$ and

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or

$$\sum_{t=1}^{T} \frac{c_t}{(1+r)^{t-1}} = A + \sum_{t=1}^{T} \frac{y_t}{(1+r)^{t-1}}$$

Present discounted value of lifetime consumption $(c_1, \ldots, c_T)$ equals the present discounted value of lifetime income $(y_1, \ldots, y_T)$ plus initial bequests.

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- Household maximizes utility subject to budget constraint

- In order to solve this problem, need to use Lagrangian method.

1. Rewrite all constraints of the problem in the form

$$\textit{stuff} = 0$$

For our problem

$$\begin{aligned} & A + y_1 + \frac{y_2}{1+r} + \frac{y_3}{(1+r)^2} \dots + \frac{y_T}{(1+r)^{T-1}} \\ & -c_1 - \frac{c_2}{1+r} - \frac{c_3}{(1+r)^2} - \dots - \frac{c_T}{(1+r)^{T-1}} \\ = \quad & 0 \end{aligned}$$

- Write down the Lagrangian: take the objective function and add all constraints, each pre-multiplied by a so-called Lagrange multiplier. This entity λ can be treated as a constant number. Lagrangian becomes

$$
\begin{aligned}
& \mathcal{L}(c_1, \ldots, c_T) \\
= \quad & u(c_1) + \beta u(c_2) + \beta^2 u(c_3) + \ldots + \beta^{T-1} u(c_T) + \\
& \lambda \left(\begin{array}{c} A + y_1 + \frac{y_2}{1+r} + \frac{y_3}{(1+r)^2} \ldots + \frac{y_T}{(1+r)^{T-1}} \\ -c_1 - \frac{c_2}{1+r} - \frac{c_3}{(1+r)^2} \ldots - \frac{c_T}{(1+r)^{T-1}} \end{array} \right) \\
= \quad & \sum_{t=1}^{T} \beta^{t-1} u(c_t) + \lambda \left(A + \sum_{t=1}^{T} \frac{y_t}{(1+r)^{t-1}} - \sum_{t=1}^{T} \frac{c_t}{(1+r)^{t-1}} \right)
\end{aligned}
$$

- Take first order conditions with respect to all choice variables and set them equal to 0. For example chose variables are $(c_1, \ldots, c_T)$

$$u'(c_1) - \lambda = 0 \qquad \text{or} \qquad u'(c_1) = \lambda.$$

Doing the same for c_2 yields

$$\beta u'(c_2) - \lambda \frac{1}{1+r} = 0 \qquad \text{or} \qquad (1+r)\beta u'(c_2) = \lambda$$

and for an arbitrary c_t we find $(1+r)^{t-1}\beta^{t-1}u'(c_t) = \lambda$. Combining

$$\begin{aligned} u'(c_1) &= (1+r)\beta u'(c_2) \\ &= \ldots = [(1+r)\beta]^{t-1} u'(c_t) = [(1+r)\beta]^{t} u'(c_{t+1}) \\ &= \ldots = [(1+r)\beta]^{T-1} u'(c_T) \end{aligned}$$

These equations determine relative consumption levels across periods, that is, the ratios $\frac{c_2}{c_1}, \frac{c_3}{c_2}$ and so forth. For absolute consumption levels need to use the budget constraint.

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  - Similar argument for consuming one unit more today and saving one unit less leads to
  $$-u'(c_1) + (1+r)\beta u'(c_2) \geq 0.$$
  - Combining the two equations leads to the Euler equation.

- Suppose the market discounts income at the same rate $\frac{1}{1+r}$ as the household discounts utility, β. In this case $\beta = \frac{1}{1+r}$ or $\beta(1+r) = 1$. Euler equation becomes

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- Since utility function is strictly concave (i.e. $u''(c) < 0$) we have that

$$c_1 = c_2 = \ldots = c_t = \ldots = c_T = \bar{c}$$

Consumption is constant over a households' lifetime; the timing of income and consumption is completely de-coupled.

- Consumption level: from the intertemporal budget

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- Since $c_t = \bar{c}$ for all times t we have:

$$\begin{aligned} \bar{c}\sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} &= I \\ \bar{c} &= \frac{1}{\sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}}} * I \end{aligned}$$

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- Thus, if households are infinitely lived:

$$\bar{c} = c_1 = c_t = \frac{r}{1+r} I$$

An Example

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- We assume that the interest rate is $r = 0$ and $\beta = 1$.

- From previous discussion we know that consumption over the households' lifetime is constant

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- Level of consumption? Total discounted lifetime value of income.

$$\begin{aligned} & y_1 + \frac{y_2}{1+r} + \frac{y_3}{(1+r)^2} \ldots + \frac{y_{60}}{(1+r)^{T-1}} \\ = \quad & y_1 + y_2 + y_3 \ldots + y_{60} = y_1 + y_2 + y_3 \ldots + y_{45} \\ = \quad & 45 * \$40,000 = \$1,800,000 \end{aligned}$$

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- Total discounted lifetime cost of consumption

$$\begin{aligned} & c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} + \ldots + \frac{c_{60}}{(1+r)^{59}} \\ = \quad & c_1 + c_2 + \ldots + c_{60} = 60 * c \end{aligned}$$

- Equating lifetime income and cost of lifetime consumption yields

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- In all working years the household consumes $\$10,000$ less than income and puts the money aside for consumption in retirement.

- Savings in all working periods is

$$\begin{aligned} sav_t &= y_t + rs_{t-1} - c_t \\ &= y_t - c_t \\ &= \$40,000 - \$30,000 \\ &= \$10,000 \end{aligned}$$

whereas for all retirement periods

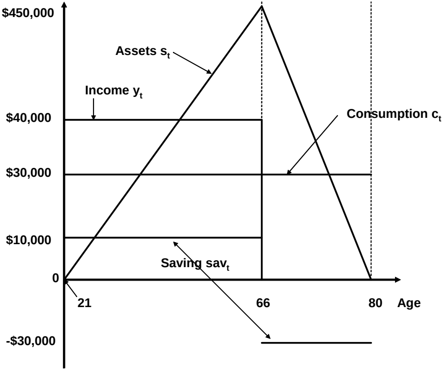
$$\begin{aligned} sav_t &= y_t + rs_{t-1} - c_t \\ &= -c_t \\ &= -\$30,000 \end{aligned}$$

- Asset position of the household. Remember that

$$\begin{aligned} sav_t &= s_t - s_{t-1} \text{ or} \\ s_t &= s_{t-1} + sav_t \end{aligned}$$

Since the household starts with 0 bequests, $s_0 = 0$. Thus

$$\begin{aligned} s_1 &= s_0 + sav_1 \\ &= \$0 + \$10,000 = \$10,000 \\ s_2 &= s_1 + sav_2 = \$10,000 + \$10,000 = \$20,000 \\ s_{45} &= s_{44} + sav_{45} = \$440,000 + \$10,000 = \$450,000 \\ s_{46} &= s_{45} + sav_{46} = \$450,000 - \$30,000 = \$420,000 \\ s_{60} &= s_{59} + sav_{60} = \$30,000 - \$30,000 = \$0 \end{aligned}$$



Life Cycle Profiles, Model

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- Combining this with the intertemporal budget constraint

$$c_1 + \frac{c_2}{1+r} = A + y_1 + \frac{y_2}{1+r}$$

yields

$$\begin{aligned} c_1 &= \frac{I}{1+\beta} \\ c_2 &= \frac{(1+r)\beta}{1+\beta} I \end{aligned}$$

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- This implies $\frac{u'(c_1)}{u'(c_2)} = (1+r)\beta.$ and thus

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- Since $u'(c)$ is a strictly decreasing function we have $c_1 < c_2$.

- Similarly

$$
\begin{aligned}
\left[(1+r)\beta\right]^{t-1} u'(c_t) &= \left[(1+r)\beta\right]^{t} u'(c_{t+1}) \\
\frac{u'(c_t)}{u'(c_{t+1})} &= \frac{\left[(1+r)\beta\right]^{t}}{\left[(1+r)\beta\right]^{t-1}} = (1+r)\beta > 1
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so that $c_{t+1} > c_t$.

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so that $c_{t+1} > c_t$.

- Now suppose that $\beta < \frac{1}{1+r}$ or $\beta(1+r) < 1$.

- Identical argument to the one above shows that now

$$c_1 > c_2 > \ldots > c_t > \ldots > c_T.$$

- Consider the specific CRRA period utility function

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- In this case

$$u'(c) = c^{-\sigma}$$

- Euler equations

$$(c_1)^{-\sigma} = (1+r)\beta(c_2)^{-\sigma} = [(1+r)\beta]^{t-1}(c_t)^{-\sigma} = [(1+r)\beta]^{t}(c_{t+1})^{-\sigma}$$

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- Thus for any period t

$$\begin{aligned} [(1+r)\beta]^{t-1}(c_t)^{-\sigma} &= [(1+r)\beta]^t(c_{t+1})^{-\sigma} \\ (c_t)^{-\sigma} &= [(1+r)\beta](c_{t+1})^{-\sigma} \\ \left(\frac{c_{t+1}}{c_t}\right)^{\sigma} &= (1+r)\beta \\ \frac{c_{t+1}}{c_t} &= [(1+r)\beta]^{\frac{1}{\sigma}} \end{aligned}$$

- Consumption levels: note that

$$
\begin{aligned}
c_{t+1} &= [(1+r)\beta]^{\frac{1}{\sigma}} c_t = [(1+r)\beta]^{\frac{2}{\sigma}} c_{t-1} = \ldots [(1+r)\beta]^{\frac{t}{\sigma}} c_1 \text{ or} \\
c_t &= [(1+r)\beta]^{\frac{t-1}{\sigma}} c_1
\end{aligned}
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- Intertemporal budget constraint

$$\sum_{t=1}^{T} \frac{c_t}{(1+r)^{t-1}} = I$$

- Plugging in for c_t yields

$$\sum_{t=1}^{T} \frac{[(1+r)\beta]^{\frac{t-1}{\sigma}} c_1}{(1+r)^{t-1}} = I$$

- Solving this out for c_1 yields

$$\begin{aligned} c_1 &= \frac{1-\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}-1}}{1-\left[\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}-1}\right]^T} * I \\ c_t &= [(1+r)\beta]^{\frac{t-1}{\sigma}} * \frac{1-\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}-1}}{1-\left[\beta^{\frac{1}{\sigma}}(1+r)^{\frac{1}{\sigma}-1}\right]^T} * I \end{aligned}$$

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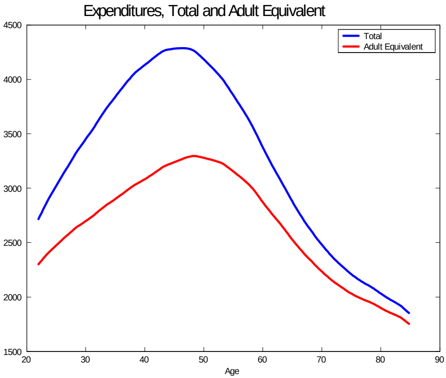

Consumption over the Life Cycle

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- How can we account for the difference?

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- Life cycle model only asserts that marginal utility of consumption should be smooth over the life cycle, not necessarily consumption expenditures themselves.
- But: if adjust data by household equivalence scales, still 50% of the hump persists

- Households spend resources to be able to work:

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    - Working Clothes

- Households can either produce a good at home or buy it. When there is more time available they produce it:

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- As a consequence they purchase consumption goods cheaper
- So expenditures may be lower but consumption is actually not lower

Potential Explanations: Consumption and Labor Supply

- Same predictions as before if consumption and leisure are separable in the utility function,

$$U(c, l) = \sum_{t=1}^{T} \beta^{t-1} \left[u(c_t) + v(l_t) \right]$$

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$$U(c, l) = \sum_{t=1}^{T} \beta^{t-1} \left[u(c_t) + v(l_t) \right]$$

- But if consumption and leisure are substitutes, then if labor supply is hump-shaped over the live cycle (because labor productivity is), then households may find it optimal to have a hump-shaped labor supply and consumption profile over the life cycle.

Potential Explanations: Missing Data at the End of Life

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- In the absence of Annuities Closeness to Death Makes End of Life Scary.

- Declining consumption profile over the life cycle can be explained by $\beta(1+r) < 1$.

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- If young households can't borrow against their future labor income, then best thing they can do is to consume whatever income whey have when young. Since income is increasing in young ages, so is consumption.

- As households age they want to start saving and the borrowing constraints lose importance. But now the fact that $\beta(1 + r) < 1$ kicks in and induces consumption to fall.

Potential Explanations: Uncertain Income and Lifetime.

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- Combination of changes in household size and income and lifetime uncertainty can generate a hump in consumption over the life cycle of similar magnitude and timing as in the data (see Attanasio et al., 1999).

- Now assume that incomes $\{y_1, y_2, \ldots, y_T\}$ are risky.

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- Thus, $E_t y_{t+1}$ is expectation in period t of income in period $t+1$, $E_t y_{t+2}$ is period t expectation of income in period $t+2$ etc. Timing convention: when expectations are taken in t, y_t is known.

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$$\max_{s_t} \quad u(c_t) + E_t\left\{u\left[c_{t+1}(\omega_{t+1})\right]\right\} = u(c_t) + \sum_{\omega_{t+1}} p(\omega_{t+1})\ u\left[c_{t+1}(\omega_{t+1})\right] \quad \text{s.t.}$$

$$c_t = y_t - s_t$$

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- With First Order Condition

$$u'(c_t) = \sum_{\omega_{t+1}} p(\omega_{t+1})\ u'\left[c_{t+1}(\omega_{t+1})\right] = E_t u'\left[c_{t+1}(\omega_{t+1})\right]$$

- Assume interest rate r is not random. Also assume lifetime horizon of the household is infinite, $T = \infty$. Generalization of Euler equation

$$u'(c_t) = \beta(1+r)E_t u'(c_{t+1})$$

- Since income in period $t+1$ is risky from the perspective of period t, so is consumption c_{t+1}.

- Main problem for analysis: in general cannot pull the expectation into the marginal utility function, since in general

$$E_t u'(c_{t+1}) \neq u'(E_t c_{t+1})$$

- But now assume that the utility function is quadratic:

$$u(c_t) = -\frac{1}{2}(c_t - \bar{c})^2$$

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- For all consumption levels $c_t < \bar{c}$ we have

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- Recall a household with strictly concave utility function is risk averse

- Euler equation becomes

$$-(c_t - \bar{c}) = -E_t(c_{t+1} - \bar{c})$$

Thus

$$E_t c_{t+1} = c_t$$

- Households arrange consumption such that, in expectation, it stays constant between today and tomorrow.

- But: in presence of income risk realized consumption c_{t+1} in period $t+1$ might deviate from this plan.

- In order to determine the level of consumption we need the intertemporal budget constraint:

$$E_t \sum_{s=0}^{\infty} \frac{c_{t+s}}{(1+r)^s} = (1+r)s_{t-1} + E_t \sum_{s=0}^{\infty} \frac{y_{t+s}}{(1+r)^s}$$

- Euler equation implies (by law of iterated expectations) that

$$\begin{aligned} E_t c_{t+1} &= c_t \\ E_t c_{t+2} &= E_t E_{t+1} c_{t+2} = E_t c_{t+1} = c_t \\ E_t c_{t+s} &= c_t \end{aligned}$$

- Left hand side of intertemporal budget constraint:

$$\begin{aligned} E_t \sum_{s=0}^{\infty} \frac{c_{t+s}}{(1+r)^s} &= \sum_{s=0}^{\infty} \frac{E_t c_{t+s}}{(1+r)^s} = \\ c_t \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} &= \frac{1}{1-\frac{1}{1+r}} c_t = \frac{1+r}{r} c_t . \end{aligned}$$

- Optimal consumption rule:

$$c_t = \frac{r}{1+r} \left((1+r)s_{t-1} + E_t \sum_{s=0}^{\infty} \frac{y_{t+s}}{(1+r)^s} \right)$$

- For period 1, thus consumption becomes

$$c_1 = \frac{r}{1+r}\left(A + E_1 \sum_{s=0}^{\infty} \frac{y_{1+s}}{(1+r)^s}\right) = \frac{r}{1+r} E_1 I$$

- Compare this to the certainty case

$$c_1 = \frac{r}{1+r} I$$

- Both expressions: optimal consumption rules are exactly alike: in both cases the household consumes permanent income!

- Surprising result: despite presence of income risk the household makes the same planned consumption choices as in the absence of risk. Called certainty equivalence behavior

- Households do *not* engage in *precautionary savings behavior* by saving more in the presence than in the absence of future income risk: only expected future income matters for planned consumption, not income risk.

- This is true despite household risk aversion.

- *Realized* consumption in period $t+1$ will in general deviate from $E_t c_{t+1} = c_t$

- Realized change in consumption between period t and $t+1$ is given by

$$c_{t+1} - c_t = \frac{r}{1+r} \sum_{s=0}^{\infty} \frac{E_{t+1} y_{t+1+s} - E_t y_{t+1+s}}{(1+r)^s}$$

- Realized change in consumption given by annuity value $\frac{r}{1+r}$ of the sum of discounted *revisions in expectations* about future income in periods $t+1+s$, that is, $E_{t+1} y_{t+1+s} - E_t y_{t+1+s}$.

- How large are realized changes in consumption? Depends crucially on type of income shock the household experiences between the two periods.

- Consider two examples: perfectly permanent shock (unexpected but permanent promotion) and fully transitory shock (unexpected one-time bonus).

- Permanent promotion: extra income p for rest of households' life. Since unexpected in period t, for *all* future periods

$$E_{t+1}y_{t+1+s} - E_t y_{t+1+s} = p.$$

- Thus

$$c_{t+1} - c_t = \frac{r}{1+r}\sum_{s=0}^{\infty}\frac{p}{(1+r)^s} = p\frac{r}{1+r}\frac{1}{1-\frac{1}{1+r}} = p$$

- Consumption goes up by full amount of the unexpected but permanent income increase between period t and $t+1$.

- Now consider a one time unexpected bonus b in period $t+1$. Then

$$E_{t+1}y_{t+1} - E_t y_{t+1} = b$$

and for all future periods beyond $t+1$

$$E_{t+1}y_{t+1+s} - E_t y_{t+1+s} = 0.$$

- Then

$$c_{t+1} - c_t = \frac{r}{1+r}\sum_{s=0}^{\infty}\frac{b}{(1+r)^0} = \frac{r}{1+r}b$$

- Realized consumption change

$$c_{t+1} - c_t = \frac{r}{1+r} b$$

- Increase in consumption is only $\frac{r}{1+r}$ of the bonus (of about 2% if the real interest rate is $r = 2\%$).

- Instead, most of the bonus is saved and used to increase consumption in *all* future periods by a small bit.

- Example: Two periods. Income in first is 1. Income in second is $1+\ell$ with probability .5 and $1-\ell$ with probability .5. Log utility. $1+r=1$.

$$\max_{c_1, s, c_{2g}, c_{2b}} \log c_1 + \frac{1}{2}\log c_{2g} + \frac{1}{2}\log c_{2b}$$

$$\begin{aligned} c_1 + s &= 1 \\ c_{2g} &= s + 1 + \ell \\ c_{2b} &= s + 1 - \ell \end{aligned}$$

- Rewriting after substitution

$$\max_{c_1, s, c_{2g}, c_{2b}} \log(1-s) + \frac{1}{2}\log(s+1+\ell) + \frac{1}{2}\log(s+1-\ell)$$

- First order conditions (absent algebra errors)

$$\frac{-1}{1-s}+\frac{1}{2}\frac{1}{s+1+\ell}+\frac{1}{2}\frac{1}{s+1-\ell}=0$$

- Simplifying

$$\frac{1}{1-s}=\frac{1+s}{s^2+2s+1-\ell^2}$$

$$(s+1)^2-\ell^2=(1+s)(1-s)$$

$$2s^2+2s-\ell^2=0$$

$$s=\frac{-2+\sqrt{4+8\ell^2}}{4}$$

- Note that if $\ell=0$ then $s=0$ while if $\ell=1$ then $s=.36$.
- The higher the variance (here ℓ) the higher the savings

- In $u'''(c) > 0$ then agents have precautionary savings, this is they save more the higher the risk that they save.
- When $\beta(1 + r) = 1$ then $c_t < E[c_{t+1}]$
- People save extra not so much for a rainy day but for wild weather.
- People are more like this than like quadratic preferences.

Part IV

Positive Theory of Government Activity

- So far: analysis of individual household behavior

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- Now: introduction of government activity: taxation, transfers, government spending, issuing and repaying debt

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- Question 1: What are the constraints the government faces?

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- Now: introduction of government activity: taxation, transfers, government spending, issuing and repaying debt
- Question 1: What are the constraints the government faces?
- Question 2: How do government policies affect private household decisions?

2023 (Estimate) Federal Budget (in billion $)	
Receipts	4,638.2
Individual Income Taxes	2,345.2
Corporate Income Taxes	500.9
Social Insurance Receipts	1,509.9
Excise Taxes	90.7
Other	191.5
Outlays	5,792.0 (1,186.7 off budget)
National Defense	808.6
International Affairs	63.4
Health	782.4
Medicare	854.5
Income Security	688.2
Social Security	1,318.7
Net Interest	395.5
Other	880.7
Surplus	–1,153.8

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- Interest on government debt: rB_{t-1} =Net Interest

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- For simplicity we assume that all government bonds have a maturity of one period.

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- Note that

$$def_t = B_t - B_{t-1}$$

- For $t = 2$, budget constraint reads as

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- Plug this into budget constraint for period 1 to get

$$\begin{aligned} G_1 &= T_1 + \frac{T_2 + B_2 - G_2}{1+r} \\ G_1 + \frac{G_2}{1+r} &= T_1 + \frac{T_2}{1+r} + \frac{B_2}{1+r} \end{aligned}$$

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- Continue this process to

$$\begin{aligned} & G_1 + \frac{G_2}{1+r} + \frac{G_3}{(1+r)^2} + \ldots + \frac{G_T}{(1+r)^{T-1}} \\ = \quad & T_1 + \frac{T_2}{1+r} + \frac{T_3}{(1+r)^2} + \ldots + \frac{T_T}{(1+r)^{T-1}} + \frac{B_T}{(1+r)^{T-1}} \end{aligned}$$

- Assume that even the government cannot die in debt:

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or more compactly

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- Present discounted value of total government expenditures equals present discounted value of total taxes.

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- Two principal ways to levy revenues for a government
  - Tax in the current period
  - Issue government debt, the interest and principal of which has to be paid via taxes in the future.
- What are the macroeconomic consequences of using these different instruments, and which instrument is to be preferred from a normative point of view?

- Ricardian Equivalence: it makes no difference. A switch from taxing today to issuing debt and taxing tomorrow does not change real allocations and prices in the economy.

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- His question: how to finance a war with annual expenditures of $20 millions. Asked whether it makes difference to finance the $20 millions via current taxes or to issue government bonds with infinite maturity (so-called consols) and finance the annual interest payments of $1 million in all future years by future taxes (at an assumed interest rate of 5%).

- His conclusion was (in “Funding System”) that

> *in the point of the economy, there is no real difference in either of the modes; for twenty millions in one payment [or] one million per annum for ever … are precisely of the same value*

- Ricardo formulates and explains the equivalence hypothesis, but is sceptical about its empirical validity

    ...but the people who pay the taxes never so estimate them, and therefore do not manage their affairs accordingly. We are too apt to think, that the war is burdensome only in proportion to what we are at the moment called to pay for it in taxes, without reflecting on the probable duration of such taxes. It would be difficult to convince a man possessed of $\$20,000$*, or any other sum, that a perpetual payment of* $\$50$ *per annum was equally burdensome with a single tax of* $\$1,000$.

- Ricardo doubts that agents are as rational as they should, according to “in the point of the economy”, or that they rationally believe not to live forever and hence do not have to bear part of the burden of the debt. Since Ricardo didn’t believe in the empirical validity of the theorem, he has a strong opinion about which financing instrument ought to be used to finance the war

  war-taxes, then, are more economical; for when they are paid, an effort is made to save to the amount of the whole expenditure of the war; in the other case, an effort is only made to save to the amount of the interest of such expenditure.

Formal Derivation of Ricardian Equivalence

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- Government has to finance a war in the first period. The war costs G_1 pounds. Assume that government does not do any spending in the second period, so that $G_2 = 0$.

- Question: does it makes a difference whether the government collects taxes for the war in period 1 or issues debt and repays the debt in period 2?

- Budget constraints for the government

$$
\begin{aligned}
G_1 &= T_1 + B_1 \\
(1+r)B_1 &= T_2
\end{aligned}
$$

where we used the fact that $G_2 = 0$ and $B_2 = 0$

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- Note that both policies satisfy the intertemporal government budget constraint

$$G_1 = T_1^i + \frac{T_2^i}{1+r} \qquad \text{for} \quad i \in \{A, B\}$$

- Individual behavior: Household maximizes utility

$$u(c_1) + \beta u(c_2)$$

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- Let

$$\begin{aligned} y_1 &= e_1 - T_1 \\ y_2 &= e_2 - T_2 \end{aligned}$$

where e_1, e_2 are the pre-tax earnings of the household and T_1, T_2 are taxes paid by the household.

- Government policies only affect after tax incomes. But

$$
\begin{aligned}
c_1 + \frac{c_2}{1+r} &= e_1 - T_1 + \frac{e_2 - T_2}{1+r} + A \\
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- Household spends present discounted value of pre-tax income $e_1 + \frac{e_2}{1+r} + A$ on present discounted value of consumption $c_1 + \frac{c_2}{1+r}$ and present discounted value of income taxes.

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- Two tax-debt policies that imply the same present discounted value of lifetime taxes therefore lead to exactly the same lifetime budget constraint and thus exactly the same individual consumption choices.

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- Period by period budget constraints

$$\begin{aligned} c_1 + s &= e_1 - T_1 \\ c_2 &= e_2 - T_2 + (1+r)s \end{aligned}$$

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- Savings choice s^{*B} denote the policy B saving policy. Thus

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- Under policy B the household saves exactly T_1^A more than under the first policy, the full extent of the tax reduction from the second policy. This extra saving T_1^A yields $(1+r)T_1^A$ extra income in the second period, exactly enough to pay the taxes T_2^B levied in the second period by the government to repay its debt.

Theorem

(Ricardian Equivalence) A policy reform that does not change government spending $(G_1, \ldots, G_T)$*, and only changes the timing of taxes, but leaves the present discounted value of taxes paid by each household in the economy has no effect on aggregate consumption in any time period.*

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- Key Assumption 1: No Borrowing Constraint

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- Key Assumption 1: No Borrowing Constraint

- Key Assumption 2: No Redistribution of the Burden of Taxes

- Key Assumption 3: Lump Sum Taxation

- Binding borrowing constraints can lead a household to change her consumption choices, even if a change in the timing of taxes does not change her discounted lifetime income.
- Proof by example: French British war; costs \$100 per person.
- Utility function

$$\log(c_1) + \log(c_2)$$

  and pre-tax income of $\$1,000$ in both periods of their life.
- For simplicity $r = 0$.
- Policy 1: tax \$100 in the first period
- Policy 2: incur \$100 in government debt, to be repaid in the second period. Since $r = 0$, government has to repay \$100 in the second period

- Without borrowing constraints we know from general theorem that the two policies have identical consequences. Under both policies discounted lifetime income is $\$1,900$ and

$$c_1 = c_2 = \frac{1,900}{2} = 950$$

- With borrowing constraints: policy 1

$$c_1 = y_1 = 900 \text{ and } c_2 = y_2 = 1000$$

- Second policy

$$c_1 = c_2 = 950$$

- If households are borrowing constrained, current taxes have stronger effects on current consumption than the issuing of debt, since postponing taxes to the future relaxes borrowing constraints.

No Redistribution of Tax Burden

- If change in timing of taxes involves redistribution of the tax burden across generations, then, unless these generations are linked together by operative, altruistically motivated bequest motives Ricardian equivalence fails.

- Example: as before, but now interest rate of 5%

- Policy A: levy the $\$100$ cost per person by taxing everybody $\$100$ in period 1

- Policy B: issue government debt of $\$100$ and to repay simply the interest on that debt. Under that households face taxes of $T_2^B = \$5$, $T_3^B = \$5$ and so forth.

- For a person born in period 1: under policy A, her present discounted value of lifetime income is

$$I = \$1000 - \$100 + \frac{\$1000}{1.05} = 1852.38$$

and under policy B it is

$$I = \$1000 + \frac{\$995}{1.05} = 1947.6$$

- Under policy A consumption equals

$$\begin{aligned} c_1^A &= 926.2 \\ c_2^A &= 972.5 \end{aligned}$$

and under policy B it equals

$$\begin{aligned} c_1^B &= 973.8 \\ c_2^B &= 1022.5 \end{aligned}$$

- Under policy B, part of the cost of the war is borne by future generations that inherit the debt from the war, at least the interest on which has to be financed via taxation.

- Ricardian equivalence was thought to be an empirically irrelevant theorem because timing of taxes always shifts tax burden across generations.
- Robert Barro (1974) resurrected debate.
- Step 1: if households live forever, Ricardian equivalence holds.
- Consider two arbitrary government tax policies. Since we keep G_t fixed in every period, the intertemporal budget constraint

$$\sum_{t=1}^{\infty} \frac{G_t}{(1+r)^{t-1}} = \sum_{t=1}^{\infty} \frac{T_t}{(1+r)^{t-1}}$$

requires that the two tax policies have the same present discounted value.

- Without borrowing constraints only the present discounted value of lifetime after-tax income matters for a household's consumption choice. But since the present discounted value of taxes is the same under the two policies it follows that present discounted value of after-tax income is unaffected by the switch from one tax policy to the other. Private decisions thus remain unaffected, therefore all other economic variables in the economy remain unchanged by the tax change. Ricardian equivalence holds.

- Step 2: argue that households live forever. Key: bequests.
- Suppose that people live for one period and have utility function

$$U(c_1) + \beta V(b_1)$$

where V is the maximal lifetime utility of children with bequests b.

- Now parameter β measures intergenerational altruism. A value of $\beta > 0$ indicates that you are altruistic, a value of $\beta < 1$ indicates that you love your children not as much as you love yourself.
- Budget constraint

$$c_1 + b_1 = y_1$$

- Bequests are constrained to be non-negative, that is $b_1 \geq 0$.

- Utility function of child is given by

$$U(c_2) + \beta V(b_2)$$

  and the budget constraint is

$$c_2 + b_2 = y_2 + (1 + r)b_1$$

- Note that $V(b_1)$ equals the maximized value of $U(c_2) + \beta V(b_2)$
- Economy with one-period lived people that are linked by altruism and bequests is identical to economy with people that live forever and face borrowing constraints (since we have that bequests $b_1 \geq 0, b_2 \geq 0$ and so forth).
- But: binding borrowing constraints invalidate Ricardian equivalence.
- Conclusion: in Barro model with one-period lived individuals Ricardian equivalence holds if a) individuals are altruistic ($\beta > 0$) and bequest motives are operative.

- A lump-sum tax is a tax that does not change the relative price between two goods that are chosen by private households.
- Demonstrate that timing of taxes is not irrelevant if the government does not have access to lump-sum taxes by example
- Utility function

$$\log(c_1) + \log(c_2)$$

- Income before taxes of \$1000 in each period and $r = 0$. The war costs \$100.
- Policy A: levy a \$100 tax on first period labor income.
- Policy B: issue \$100 in debt, repaid in the second period with proportional consumption taxes at rate τ.

- Under policy A optimal consumption choice is

$$\begin{aligned} c_1^A &= c_2^A = \$950 \\ s^A &= \$900 - \$950 = -\$50 \end{aligned}$$

- The two budget constraints under policy B read as

$$\begin{aligned} c_1^B + s &= \$1000 \\ c_2^B(1+\tau) &= \$1000 + s \end{aligned}$$

which can be solved for using $c_1^B + (1+\tau)c_2 = \$2000$

- Maximizing utility subject to the lifetime budget constraint yields

$$\begin{aligned} c_1^B &= \$1000 \\ c_2^B &= \frac{\$1000}{1+\tau} \end{aligned}$$

- Under policy B the households consumes strictly more than under the first policy. Reason: tax on second period consumption makes consumption in the second period more expensive, relative to consumption in the first period. Households substitute away from the now more expensive good.

- Fact that the tax changes the effective relative price between the two goods qualifies this tax as a non-lump-sum tax.

- Government must levy $\$100$ in taxes. Tax revenues are given by

$$\tau c_2^B = \frac{\tau 1000}{1+\tau} = 100$$

- Thus

$$\begin{aligned} \tau &= \frac{0.1}{0.9} = 0.1111 \\ c_2^B &= 900 \\ s^B &= 0 \end{aligned}$$

- Households prefer the lump-sum way of financing the war to the distortionary way:

$$\log(950) + \log(950) > \log(1000) + \log(900).$$

- Report by Jagadeesh Gokhale from 2013, *Spending Beyond our Means: How We are Bankrupting Future Generations*
- The book is `https://object.cato.org/sites/cato.org/files/pubs/pdf/spending-beyond-our-means.pdf`
- A very short description and required reading is `https://www.cato.org/sites/cato.org/files/articles/gokhale-generational_accounting.pdf`

- Fiscal Imbalance:

$$FI_t = PVE_t^{cfp} + B_t - PVR_t^{cfp}$$

where PVE_t^{cfp} is the present discounted value of projected expenditures under current fiscal policy, PVR_t^{cfp} is present discounted value of all projected receipts and B_t is government debt at the end of period t.

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- In terms of our previous notation

$$PVE_t = \sum_{\tau=t+1}^{\infty} \frac{G_\tau}{(1+r)^{\tau-t}}$$

and

$$PVR_t = \sum_{\tau=t+1}^{\infty} \frac{T_\tau}{(1+r)^{\tau-t}}$$

as well as

$$B_t = \sum_{\tau=1}^{t} \frac{G_\tau}{(1+r)^{\tau-t}} - \sum_{\tau=1}^{t} \frac{T_\tau}{(1+r)^{\tau-t}}$$

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- But

$$PVE_t^{cfp} \neq PVE_t$$

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- Which means that either PVE_t^{cfp} or PVR_t^{cfp} will change to adjust to reality.

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- GI_t is that part of the fiscal imbalance FI_t that results from transactions of the government with past (through B_t) and living generations.
- Difference $FI_t - GI_t$ denotes the projected part of fiscal imbalance due to future generations.

- Real interest rate (discount rate for the present value calculations) of 3.68% per annum (average yield on a 30 year Treasury bond in recent years).

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- Growth of health care costs? Account for fact that the expenditure (in per capita terms) growth rate in Medicare is projected to be significantly above the projections from growth rates of wages for immediate future. Beyond 2035 this gap is assumed to gradually shrink to zero.

1. Baseline policy scenario corresponds to current fiscal policy

$$PVE_t^{cfp}, \; PVR_t^{cfp}$$

- In order to compute GI, one needs to break down taxes paid and outlays received by generations.

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$$PVE_t^{cfp},\ PVR_t^{cfp}$$

2. Alternative policy scenario factors in likely policy changes.

$$PVE_t^{as},\ PVR_t^{as}$$

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Fiscal Imbalance, Baseline (Billion of 2012 Dollars)	Current Fiscal Projections		
Part of the Budget	2012	2017	2022
FI in Social Insurance	$64,853$	$70,961$	$82,564$
FI in Rest of Federal Government	$-10,502$	$-10,687$	$-11,742$
Total *FI*	$54,675$	$60,274$	$70,822$

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Fiscal Imbalance, Alternative Fiscal Scenario (Billion of 2012 Dollars)

Part of the Budget	2012	2017	2022
FI in Social Insurance (SS+Med.)	65, 934	72, 036	83, 606
FI in Social Security	20, 077	22, 272	26, 660
FI in Medicare	45, 857	49, 764	56, 946
FI in Rest of Federal Government	25, 457	29, 826	36, 660
Total *FI*	91, 391	101, 862	120, 266

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FI in Social Security	20,077	22,272	26,660
FI in Medicare	45,857	49,764	56,946
FI in Rest of Federal Government	25,457	29,826	36,660
Total *FI*	91,391	101,862	120,266

Fiscal Imbal., Alternative Fiscal Scenario (% of Pres. Val. GDP)

Part of the Budget	2012	2017	2022
FI in Social Insurance (SS+Med.)	6.5%	6.5%	6.8%
FI in Rest of Federal Government	2.5%	2.7%	3.0%
Total *FI*	9.0%	9.1%	9.8%

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4. Largest part (about 1/2) of FI is due to Medicare. Comes from a) fast increases of medical goods prices and b) population aging.

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2. This is before the 2017 TCJA that introduced a further tax reduction and the pandemic induced enormous additional deficits.

3. *FI* grows over time at a gross rate of $(1 + r) = 1.0368$ per year.

4. Largest part (about 1/2) of *FI* is due to Medicare. Comes from a) fast increases of medical goods prices and b) population aging.

5. *FI* dwarfs official government debt by a factor of 5.

Generational Imbalance, Alternative (Bill of 2012 Dollars)

Part of the Budget	2012	2017	2022
FI in Social Insurance	65,934	72,036	83,606
FI in Social Security	20,077	22,272	26,660
GI in Social Security (incl. Trust Fund)	19,586	21,726	26,032
FI − *GI* in Social Security	491	546	628
FI in Medicare	45,857	49,764	56,946
GI in Medicare	34,487	38,311	44,693
FI − *GI* in Medicare (incl. Trust Fund)	11,370	11,453	12,253

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Interpretation

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3. Magnitude of numbers depends on: growth rate of wages, discount rate applied to future revenues and outlays, temporary differential between expenditure growth in Medicare and the economy.

4. **But** conclusion robust: large spending cuts or tax increases required to restore fiscal balance. Medicare and Social Security key.

The U.S. Federal Income Tax Code: Brief History

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- Individuals with earnings between \$600 – \$10000 had to pay an income tax of 3%; higher rates for people with income above \$10000.

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- Modern federal income tax was permanently introduced in the U.S. in 1913 through the 16-th Amendment to the Constitution. Gave Congress legal authority to tax income of both individuals and corporations.

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- The Reagan tax reforms reduced income tax rates by individuals drastically (with a total reduction amounting to the order of $\$500 - 600$ billion), partially offset by an increase in tax rates for corporations and moderate increases of taxes for the very wealthy.

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  - Overall as of July 2022 (see Auerbach, Kotlikoff and Koehler 2022) the effects some to be quite neutral on progressivity.

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- A tax code is defined by a tax function $T(y)$, which for each possible taxable income y gives the amount of taxes that are due to be paid.

- Example: if $y = \$100,000$ and $T(y) = \$25,000$, then every person with taxable income of $\$100,000$ in 2013 owes the government $\$25,000$ in taxes.

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 whenever $T'(y)$ is well-defined (that is, whenever $T'(y)$ is differentiable.

- Interpretation: average tax rate $t(y)$ indicates what *fraction* of her taxable income a person with income y has to deliver to the government as tax. Marginal tax rate $\tau(y)$ measures how high the tax rate is on the last dollar earned, for a total taxable income of y.

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where the equality follows from the fundamental theorem of calculus.

3. Current U.S. federal personal income tax code is defined by a collection of marginal tax rates.

- A tax code is progressive if the function $t(y)$ is strictly increasing in y for all income levels y . It is progressive over an income interval (y_l, y_h) if $t(y)$ is strictly increasing for all income levels $y \in (y_l, y_h)$. It is also progressive if it is proportional over all income intervals but the proportion is higher in each successive interval.

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- A tax code is proportional if the function $t(y)$ is constant y for all income levels y. It is proportional over an income interval (y_l, y_h) if $t(y)$ is constant for all income levels $y \in (y_l, y_h)$.

- Head tax or poll tax

  $$T(y) = T$$

  where $T > 0$ is a number. This tax is regressive since

  $$t(y) = \frac{T}{y}$$

  is a strictly decreasing function of y. Also note that the marginal tax $\tau(y) = 0$ for all income levels.

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Note that

$$t(y) = \tau(y) = \tau$$

that is, average and marginal tax rates are constant in income and equal to the tax rate τ. This tax system is proportional.

$$T(y) = \begin{cases} 0 & \text{if } y < d \\ \tau(y-d) & \text{if } y \geq d \end{cases}$$

Flat tax with deduction

$$T(y) = \begin{cases} 0 & \text{if } y < d \\ \tau(y-d) & \text{if } y \geq d \end{cases}$$

x where $d, \tau \geq 0$ are parameters. Household pays no taxes if her income does not exceed the exemption level d, and then pays a fraction τ in taxes on every dollar earned above d. Average tax rates

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Marginal tax rates

$$\tau(y) = \begin{cases} 0 & \text{if } y < d \\ \tau & \text{if } y \geq d \end{cases}$$

Tax system is progressive for all income levels above d; for all income levels below it is proportional.

Tax code with step-wise increasing marginal tax rates

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The tax code is characterized by the three marginal rates (τ_1, τ_2, τ_3) and income cutoffs (b_1, b_2) that define the income tax brackets.

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$$\begin{aligned} T(y) &= \int_0^{b_1} \tau_1 dy + \int_{b_1}^{b_2} \tau_2 dy + \int_{b_2}^y \tau_3 dy \\ &= \tau_1 b_1 + \tau_2(b_2 - b_1) + \tau_3(y - b_2) \end{aligned}$$

- Average tax rates are given by

$$t(y) = \begin{cases} \tau_1 & \text{if } 0 \leq y < b_1 \\ \frac{\tau_1 b_1}{y} + \tau_2 \left(1 - \frac{b_1}{y}\right) & \text{if } b_1 \leq y < b_2 \\ \frac{\tau_1 b_1 + \tau_2 (b_2 - b_1)}{y} + \tau_3 \left(1 - \frac{b_2}{y}\right) & \text{if } b_2 \leq y < \infty \end{cases}$$

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- Current U.S. tax code resembles the last example closely, but consists of seven marginal tax rates and six income cut-offs that define the income tax brackets. The income cut-offs vary with family structure.

Theorem

A differentiable tax code $T(y)$ is progressive, that is, $t(y)$ is strictly increasing in y (i.e. $t'(y) > 0$ for all y) if and only if the marginal tax rate $T'(y)$ is higher than the average tax rate $t(y)$ for all income levels $y > 0$, that is

$$T'(y) > t(y)$$

Proof: By definition

$$t(y) = \frac{T(y)}{y}$$

Using the definition the rule for differentiating a ratio of two functions we obtain

$$t'(y) = \frac{yT'(y) - T(y)}{y^2}$$

This expression is positive if and only if

$$yT'(y) - T(y) > 0$$

or

$$T'(y) > \frac{T(y)}{y} = t(y)$$

QED.

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- A similar result can be stated and proved for a regressive or proportional tax system.

$$\begin{aligned}\text{Gross Income} \quad &= \quad \text{Wages and Salaries}\\ &\quad +\text{Interest Income and Dividends}\\ &\quad +\text{Net Business Income}\\ &\quad +\text{Net Rental Income}\\ &\quad +\text{Other Income}\end{aligned}$$

- Other income includes unemployment insurance benefits, alimony, income from gambling, income from illegal activities. Not included: child support, gifts below a certain threshold, interest income from state and local bonds (so-called Muni's), welfare and veterans benefits, employer contributions for health insurance and retirement accounts.

$$
\begin{aligned}
\text{Adjusted Gross Income } (AGI) \quad = \quad & \text{Gross Income} \\
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Note

$$
\begin{aligned}
\text{Taxes due upon filing} &= T(y) \\
&\quad -\text{Tax witholdings} \\
&\quad -\text{Tax credits}
\end{aligned}
$$

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- Still there are large incentives that matter (Kato 2022)

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  - Especially, given the existing upper bound on social security contributions

Tax Rates for 2013, Singles

Income	$T'(y)$	$T(y)$
$0 \leq y < \$8,925$	10%	$0.1y$
$\$8,925 \leq y < \$36,250$	15%	$\$892 + 0.15(y - 8,925)$
$\$36,250 \leq y < \$87,850$	25%	$\$4,991 + 0.25(y - 36,250)$
$\$87,850 \leq y < \$183,250$	28%	$\$17,891 + 0.28(y - 87,850)$
$\$183,250 \leq y < \$398,350$	33%	$\$44,603 + 0.33(y - 183,250)$
$\$398,350 \leq y < \$400,000$	35%	$\$115,586 + 0.35(y - 398,350)$
$\$400,000 \leq y < \infty$	39.6%	$\$116,164 + 0.396(y - 400,000)$

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Tax Rates for 2013, Married Filing Jointly

Income	$T'(y)$	$T(y)$
$0 \leq y < \$17,850$	10%	$0.1y$
$\$17,850 \leq y < \$72,500$	15%	$\$1,785 + 0.15(y - 17,850)$
$\$72,500 \leq y < \$146,400$	25%	$\$9,982 + 0.25(y - 72,500)$
$\$146,400 \leq y < \$223,050$	28%	$\$28,457 + 0.28(y - 146,400)$
$\$223,050 \leq y < \$398,350$	33%	$\$49,919 + 0.33(y - 223,050)$
$\$398,350 \leq y < \$450,000$	35%	$\$107,768 + 0.35(y - 398,350)$
$\$450,000 \leq y < \infty$	39.6%	$\$125,846 + 0.396(y - 450,000)$

Large Marriage Penalty for incomes above $\$146,000$ in 2013

Tax Brackets and Tax Rates, 2019 by Family Type

Tax Rate	For Unmarried Individuals	For Married Individuals Filing Joint Returns	For Heads of Households
	Taxable Income Over		
10%	$0	$0	$0
12%	$9,700	$19,400	$13,850
22%	$39,475	$78,950	$52,850
24%	$84,200	$168,400	$84,200
32%	$160,725	$321,450	$160,700
35%	$204,100	$408,200	$204,100
37%	$510,300	$612,350	$510,300

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2019 Standard Deduction and Personal Exemption

Filing Status	Deduction Amount
Single	$12,200
Married Filing Jointly	$24,400
Head of Household	$18,350

Marriage Penalty Almost Gone

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1. Progressive, as defined above

2. Satisfies across family equity: families with equal household incomes pay equal taxes (independent of how much of that income is earned by different members of each household)

3. Marriage-neutral: a given family pays the same taxes independent of whether the partners of the family are married or not.

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- No privately designed substitute (prenups are not)

Under the new law is a good deal.

Alternative Minimum Tax (AMT) I

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- AMT exemptions phase out at 25 cents per dollar earned once taxpayer AMTI hits a certain threshold. In 2019, the exemption will start phasing out at $510,300 in AMTI for singles and $1,020,600 for married

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- TCJA shields almost all upper-middle and high-income taxpayers from the reach of the AMT.
- The AMT is now most likely to hit those at the top of the income scale who are engaged in certain sheltering activities.
- It has quite less bite than before.

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- The maximum Earned Income Tax Credit in 2019 for single and joint filers is \$529, if the filer has no children (Table 5). The maximum credit is \$3,526 for one child, \$5,828 for two children, and \$6,557 for three or more children. All these are relatively small increases from 2018.

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- Small Potatoes

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- There is a Qualified Business Income Deduction. The TCJA includes a 20% deduction for pass-through businesses against up to \$160,700 (singles)
- **Annual Exclusion for Gifts** In 2019, the first $\$15,000$ of gifts to any person are excluded from tax.

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  - Compare social welfare under progressive tax system with a proportional tax system.

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- Modern Monetary Theory claims without support that Government Expenditures can be expanded a lot without problem as it and can ultimately be paid for by printing new money.

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- Bitcoins privately appropriate the seignorage.

- Hypothetical progressive tax system

$$\tau(y) = \begin{cases} 0\% & \text{if } 0 \leq y < 15000 \\ 10\% & \text{if } 15000 \leq y < 50000 \\ 20\% & \text{if } 50000 \leq y < \infty \end{cases}$$

- Under this tax system total tax revenues from the two agents are

$$
\begin{aligned}
& T(15,000) + T(100,000) \\
= \quad & 0.1 * (20000 - 15000) \\
& +0.1 * 35000 + 0.2(100000 - 50000) \\
= \quad & \$500 + \$13500 \\
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and consumption for the households are

$$
\begin{aligned}
c_1 &= 20000 - 500 = 19500 \\
c_2 &= 100000 - 13500 = 86500
\end{aligned}
$$

- Determine proportional tax rate τ such that revenues are same under hypothetical proportional tax system as under progressive system:

$$\begin{aligned} 14000 &= \tau * 20,000 + \tau * 100,000 = \tau * 120,000 \\ \tau &= \frac{14,000}{120,000} = 11.67\% \end{aligned}$$

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- Which tax system is better? Hard question! Use social welfare function

$$W(u(c_1), \ldots, u(c_N))$$

Examples of Social Welfare Functions: Dictator

- Household i is a "dictator"

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- If dictator is $i = 1$, prefer U.S. system. If dictator is $i = 2$, prefer proportional tax system.

- Utilitarian social welfare function given by

$$W(u(c_1), \ldots, u(c_N)) = u(c_1) + \ldots + u(c_N)$$

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- Comparison for example

$$\begin{aligned} W^{\text{prog}}(u(c_1), u(c_2)) &= \log(19500) + \log(86500) = 21.2461 \\ W^{\text{prop}}(u(c_1), u(c_2)) &= \log(17667) + \log(88333) = 21.1683 \end{aligned}$$

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- Interpersonal Comparisons are difficult (need same utility)

- Rawlsian social welfare function

$$W(u(c_1), \ldots, u(c_N)) = \min_i \{u(c_1), \ldots, u(c_N)\}$$

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- For example

$$\begin{aligned} W^{\text{prog}}(u(c_1), u(c_2)) &= \min\{\log(c_1), \log(c_2)\} = \log(19500) \\ W^{\text{prop}}(u(c_1), u(c_2)) &= \min\{\log(c_1), \log(c_2)\} = \log(17667) \\ &< W^{\text{prog}}(u(c_1), u(c_2)) \end{aligned}$$

Suppose that taxable incomes are not affected by the tax code and suppose that u is strictly concave and the same for every household. Then under Rawlsian and Utilitarian social welfare function it is optimal to have complete income redistribution:

$$c_1 = c_2 = \ldots = c_N = \frac{y_1 + y_2 + \ldots + y_N - G}{N} = \frac{Y - G}{N}$$

where G is total required tax revenue and $Y = y_1 + y_2 + \ldots + y_N$ Tax code that achieves this is given by

$$T(y_i) = y_i - \frac{Y - G}{N}$$

i.e. tax income at a 100% and then rebate $\frac{Y-G}{N}$ back to everybody.

- Suppose that $N = 2$ and $c_2 > c_1$ as the result of tax code. This cannot be optimal!
- Take way a little from household 2 and give it to household 1
- Under Rawlsian social welfare function this improves societal welfare since the poorest person has been made better off.
- Under Utilitarian social welfare function, loss of agent 2, $u'(c_2)$ is smaller than the gain of agent 1, $u'(c_1)$, since by concavity $c_2 > c_1$ implies

$$u'(c_1) > u'(c_2).$$

- But: assumption that changes in the tax system do not change a households' incentive to work, save and thus generate income is a very strong one. Therefore now want to analyze how income and consumption taxes change the economic incentives of households to work, consume and save.

- Utilitarism takes utilities more seriously than it should.
    - Monotonic transformations of Utilities are yield the same allocations but not necessarily the same welfare.
- Rawlsian has the issue of only caring about the worst. But this cannot be taking literally: All societies are in terrible shape as long as there is any infant mortality.
- The idea of the veil of ignorance is an excellent one. It separates our circumstances from our assessment.
- It allows us to pick some particular utility function. It ends up yielding very egalitarian results.
- Still it is difficult to use to assess changes as the veil of ignorance does not apply. We know where we were before the policy change

- Household problem

$$\max_{c_1 c_2, s, \ell} \log(c_1) + \theta \log(1 - \ell) \quad + \quad \beta \log(c_2)$$

$$s.t. \qquad (1 + \tau_{c_1})c_1 + s \quad = \quad (1 - \tau_\ell)w\ell$$

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- Parameter θ measures how much households value leisure, relative to consumption.

- Intertemporal budget constraint. Solving second budget constraint yields

$$s = \frac{(1 + \tau_{c_2})c_2 - b}{(1 + r(1 - \tau_s))}$$

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and thus

$$(1+\tau_{c_1})c_1 + \frac{(1+\tau_{c_2})c_2}{(1+r(1-\tau_s))} = (1-\tau_\ell)w\ell + \frac{b}{(1+r(1-\tau_s))}$$

- Rewrite this. Note that $\ell = 1 - (1 - \ell)$. Then

$$\begin{aligned}
& (1+\tau_{c_1})c_1 + \frac{(1+\tau_{c_2})c_2}{(1+r(1-\tau_s))} = \\
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- Rewrite this. Note that $\ell = 1 - (1-\ell)$. Then

$$\begin{aligned} & (1+\tau_{c_1})c_1 + \frac{(1+\tau_{c_2})c_2}{(1+r(1-\tau_s))} = \\ = \quad & (1-\tau_\ell)w * (1-(1-\ell)) + \frac{b}{(1+r(1-\tau_s))} \end{aligned}$$

$$\begin{aligned} & (1+\tau_{c_1})c_1 + \frac{(1+\tau_{c_2})c_2}{(1+r(1-\tau_s))} + (1-\ell)(1-\tau_\ell)w = \\ = \quad & (1-\tau_\ell)w + \frac{b}{(1+r(1-\tau_s))} \end{aligned}$$

- Interpretation: household has potential income from social security $\frac{b}{(1+r(1-\tau_s))}$ and from supplying *all* her time to the labor market, $(1-\tau_\ell)w$.

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- Buys three goods

    - Consumption c_1 in first period, at effective price $(1+\tau_{c_1})$

    - Consumption c_2 in second period, at effective price $\frac{(1+\tau_{c_2})}{(1+r(1-\tau_s))}$

    - Leisure $1-\ell$ at effective price $(1-\tau_\ell)w$, equal to the opportunity cost of not working.

- Lagrangian

$$
\begin{aligned}
L \quad = \quad & \log(c_1) + \theta \log(1-\ell) + \beta \log(c_2) \\
& + \lambda \left\{ (1-\tau_\ell) w + \frac{b}{(1+r(1-\tau_s))} \right. \\
& \left. -(1+\tau_{c_1}) c_1 - \frac{(1+\tau_{c_2}) c_2}{(1+r(1-\tau_s))} - (1-\ell)(1-\tau_\ell) w \right\}
\end{aligned}
$$

- First order conditions:

$$
\begin{aligned}
\frac{1}{c_1} - \lambda(1+\tau_{c_1}) &= 0 \\
\frac{\beta}{c_2} - \lambda\frac{(1+\tau_{c_2})}{(1+r(1-\tau_s))} &= 0 \\
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\end{aligned}$$

- Rewriting

$$\begin{aligned}
\frac{1}{c_1} &= \lambda(1+\tau_{c_1}) \\
\frac{\beta}{c_2} &= \lambda\frac{(1+\tau_{c_2})}{(1+r(1-\tau_s))} \\
\frac{\theta}{1-\ell} &= \lambda(1-\tau_\ell)w
\end{aligned}$$

- Intertemporal optimality condition

$$\frac{\beta c_1}{c_2} = \frac{(1+\tau_{c_2})}{(1+\tau_{c_1})} * \frac{1}{(1+r(1-\tau_s))}$$

Interpretation: marginal rate of substitution

$$\frac{\beta u'(c_2)}{u'(c_1)} = \frac{\beta c_1}{c_2}$$

should equal relative price between consumption in the second to consumption in the first period, $\frac{1}{(1+r(1-\tau_s))}$. With differential consumption taxes, the relative price has to be adjusted by relative taxes $\frac{(1+\tau_{c_2})}{(1+\tau_{c_1})}$.

- Comparative statics

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1. Increase in capital income tax rate τ_s reduces after-tax interest rate $1 + r(1 - \tau_s)$ and induces households to consume more in first period, relative to second period (ratio $\frac{c_1}{c_2}$ increases).

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2. Increase in consumption taxes in first period τ_{c_1} induces households to consume less in first period, relative to second period (ratio $\frac{c_1}{c_2}$ decreases).

3. Increase in consumption taxes in second period τ_{c_2} induces households to consume more in first period, relative to second period (ratio $\frac{c_1}{c_2}$ increases).

- Intratemporal optimality condition

$$\frac{\theta c_1}{1-\ell} = \frac{(1-\tau_\ell)w}{(1+\tau_{c_1})}.$$

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- Interpretation: marginal rate of substitution between current period leisure and current period consumption,

$$\frac{\theta u'(1-\ell)}{u'(c_1)} = \frac{\theta c_1}{1-\ell}$$

should equal after-tax wage, adjusted by first period consumption taxes $\frac{(1-\tau_\ell)w}{(1+\tau_{c_1})}$.

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Proposition

Proposition: *Suppose we start with tax system with no labor income taxes, $\tau_\ell = 0$ and uniform consumption taxes $\tau_{c_1} = \tau_{c_2} = \tau_c$. Denote by c_1, c_2, ℓ, s the optimal consumption, savings and labor supply decision. Then there exists a labor income tax τ_ℓ and a lump sum tax T such that for $\tau_c = 0$ households find it optimal to make exactly the same consumption choices as before.*

Proof: If consumption tax is uniform, it drops out of the intertemporal optimality condition. Rewrite intratemporal optimality condition as

$$\frac{\theta c_1}{(1-\ell)w} = \frac{(1-\tau_\ell)}{(1+\tau_c)}$$

Right hand side, for $\tau_\ell = 0$, is equal to

$$\frac{1}{(1+\tau_c)}$$

Set $\widehat{\tau}_\ell = \frac{\tau_c}{1+\tau_c}$ and $\widehat{\tau}_c = 0$. Then

$$\frac{(1-\widehat{\tau}_\ell)}{(1+\widehat{\tau}_c)} = 1 - \frac{\tau_c}{1+\tau_c} = \frac{1}{(1+\tau_c)},$$

and household faces the same intratemporal optimality condition as before. Appropriate lump-sum tax T guarantees that tax payments remain the same.

- Intratemporal optimality condition yields

$$c_1 = \frac{(1-\tau_\ell)(1-\ell)w}{(1+\tau_{c_1})\theta}$$

- Intertemporal optimality condition yields

$$c_2 = \beta c_1(1+r(1-\tau_s))\frac{(1+\tau_{c_1})}{(1+\tau_{c_2})} = \frac{(1-\tau_\ell)(1-\ell)w}{\theta}\frac{\beta(1+r(1-\tau_s))}{(1+\tau_{c_2})}$$

- Plugging into budget constraint yields

$$\begin{aligned}
\frac{(1-\tau_\ell)(1-\ell)w}{\theta} + \beta\frac{(1-\tau_\ell)(1-\ell)w}{\theta} &= (1-\tau_\ell)w\ell + \frac{b}{(1+r(1-\tau_s))} \\
(1+\beta)\frac{(1-\tau_\ell)(1-\ell)w}{\theta} &= (1-\tau_\ell)w\ell + \frac{b}{(1+r(1-\tau_s))}
\end{aligned}$$

- Solve for ℓ to obtain

$$\ell^* = \frac{1+\beta}{1+\beta+\theta} - \frac{b}{(1+r(1-\tau_s))\theta w(1-\tau_\ell)(1+\beta+\theta)}$$

- If $b = 0$, then
  - $\ell^* = \frac{1+\beta}{1+\beta+\theta} \in (0,1)$
  - Interpretation: the more the household values leisure (the higher is θ), the less she finds it optimal to work. With $b > 0$, higher social security benefits in retirement reduce labor supply in the working period If b gets really big, then the optimal $\ell^* = 0$.
  - Rest of solution

$$\begin{aligned} c_1 &= \frac{(1-\tau_\ell)}{(1+\tau_{c_1})(1+\beta+\theta)} w \\ c_2 &= \frac{\beta(1-\tau_\ell)(1+r(1-\tau_s))}{(1+\beta+\theta)(1+\tau_{c_2})} w \\ s &= \frac{\beta(1-\tau_\ell)w}{1+\beta+\theta} \end{aligned}$$

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- Households live only one period and maximize

$$\log c + \theta \ \log(1 - \ell_i)$$

subject to:

$$(1 + \tau_{ci})c = \ell \ w_i(1 - \tau_{\ell i}) + T_i$$

- $\max_{\ell} \quad \log \frac{\ell \, w_i (1-\tau_{\ell i}) + T_i}{(1+\tau_{ci})} + \theta \, \log(1-\ell)$

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- The FOC

$$\frac{w_i\,(1-\tau_{\ell i})}{\ell_i\, w_i(1-\tau_{\ell i}) + T_i} = \frac{\theta}{1-\ell_i}$$

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- Getting rid of the denominators

$$(1-\ell_i)\, w_i\,(1-\tau_{\ell i}) = \theta[\ell_i\, w_i(1-\tau_{\ell i}) + T_i]$$

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- Isolating the term with labor

$$w_i \,(1-\tau_{\ell i}) - \theta T_i = (1+\theta)[\ell_i \, w_i \,(1-\tau_{\ell i})]$$

- Which yields

$$\ell_i = \frac{1}{1+\theta} \; \frac{w_i(1-\tau_{\ell i}) - \theta \; T_i}{w_i \; (1-\tau_{\ell i})}$$

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- A very important feature: We work as much as our great grandparents despite having wages that are much higher (this is a straight implication of the preferences that we have posed).

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- A very important feature: We work as much as our great grandparents despite having wages that are much higher (this is a straight implication of the preferences that we have posed).

- It is not historically true, but almost.

With Taxes we had

$$\ell_i = \frac{1}{1+\theta} \; \frac{w_i(1-\tau_{\ell i}) - \theta \; T_i}{w_i \; (1-\tau_{\ell i})} = \frac{1}{1+\theta} \; \frac{w_i(1-\tau_{\ell i}) - \theta \; \xi_i \; (\tau_{\ell i}\ell_i w_i + \tau_{ci} c_i)}{w_i \; (1-\tau_{\ell i})}$$

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- Because of $$T_i = \xi_i \; (\tau_{\ell i}\ell_i w_i + \tau_{ci} c_i)$$
- With $\tau_{ci} = 0$,
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WITH TAXES WE HAD

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- With $\tau_{ci} = 0$,

$$\ell_i = \frac{1}{1+\theta}\;\frac{(1-\tau_{\ell i}) - \tau_{\ell i}\ \theta\ \xi_i \ell_i}{(1-\tau_{\ell i})}$$

- $$\ell_i = \frac{1-\tau_{\ell i}}{(1+\theta)(1-\tau_{\ell i}) + \tau_{\ell i}\theta\ \xi_i} < \frac{1}{1+\theta}, \qquad \text{if } \theta\xi_i \text{ is not far from 2}$$

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- We will use such an expression to actually compare across countries.

- Key for labor supply: tax wedge $\frac{(1-\tau_{\ell i})}{(1+\tau_{ci})}$ in the intratemporal optimality condition

$$\frac{\theta c_i}{1-\ell_i} = \frac{(1-\tau_{\ell i})}{(1+\tau_{ci})} w_i$$

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- Wages w_i? Recall neoclassical production function operated by typical firm in the economy.

$$Y_i = A_i K_i^{\alpha} L_i^{1-\alpha}$$

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- Profit maximization

$$\max_{(K_i, L_i)} A_i K_i^{\alpha} L_i^{1-\alpha} - w_i L_i - \rho_i K_i.$$

New Expression for Hours Worked using Consumption

- Taking FOC with respect to L and setting it equal to 0 yields

$$\begin{aligned}
(1-\alpha)A_i K_i^{\alpha} L_i^{-\alpha} &= w_i \\
\frac{(1-\alpha)A_i K_i^{\alpha} L_i^{1-\alpha}}{L_i} &= w_i \\
(1-\alpha)\frac{Y_i}{L_i} &= w_i
\end{aligned}$$

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- Labor share equals $1-\alpha$, capital share equals α.

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- Labor share equals $1-\alpha$, capital share equals α.
- Use $w_i = (1-\alpha)\frac{Y_i}{L_i}$ and equilibrium in labor market $L_i = \ell_i$ to obtain

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$$

- Solving relates hours to taxes and Consumption to Output ratios.

$$
\ell_i = \frac{1-\alpha}{1-\alpha+\frac{\theta(1+\tau_{ci})}{(1-\tau_{\ell i})}\frac{c_i}{Y_i}} \quad \in (0,1)
$$

DATA: 1990'S

Country	GDP p.p.	Hours	GDP p.h.
Germany	74	75	99
France	74	68	110
Italy	57	64	90
Canada	79	88	89
United Kingdom	67	88	76
Japan	78	104	74
United States	100	100	100

Country	GDP p.p.	Hours	GDP p.h.
Germany	75	105	72
France	77	105	74
Italy	53	82	65
Canada	86	94	91
United Kingdom	68	110	62
Japan	62	127	49
United States	100	100	100

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- Early 70's due to lower productivity.
- In mid 90's: not due to lower productivity, but rather due to lower hours worked.

- Why do Europeans now work so much less than Americans? Proposed answer by Prescott (2004): taxes.

Question

- Why do Europeans now work so much less than Americans? Proposed answer by Prescott (2004): taxes.

- Use

$$\ell_{it} = \frac{1-\alpha}{1-\alpha+\frac{\theta(1+\tau_{cit})}{(1-\tau_{\ell it})}\frac{c_{it}}{Y_{it}}}$$

  to assess whether answer makes quantitative sense.

- $\frac{c_{it}}{Y_{it}}$ from NIPA accounts.

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  - Assume that all but military government spending is yielding private consumption.
  - Indirect consumption taxes part of NIPA consumption, but not part of c in model.
  - This is what we meant by ξ.
- τ_{cit} is set to ratio between total indirect consumption taxes and total consumption expenditures in data.

- Labor income taxes

$$\tau_\ell = \tau_{ss} + \tau_{inc}$$

For τ_{ss} take payroll tax rates (currently 15.3%, shared by employers and employees). To compute marginal income tax rate τ_{inc}, compute average income taxes. by dividing total direct taxes by national income. Multiply by 1.6, to capture progressivity of tax code.

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- Specify parameter values, θ and α.

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  - Since α equals the capital share, set $\alpha = 0.3224$, the average across countries and time.

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- Specify parameter values, θ and α.
    - Since α equals the capital share, set $\alpha = 0.3224$, the average across countries and time.
    - Parameter θ determines fraction of time worked. Choose θ such that in model number of hours spent working equals the average hours (across countries) in the data, which requires 1.54.

- Combined labor income and consumption tax rate relevant for the labor supply decision.

$$\frac{(1-\tau_\ell)}{(1+\tau_c)} = 1-\tau$$

where $\tau = \frac{\tau_\ell+\tau_c}{1+\tau_c}$.

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$$\frac{(1-\tau_\ell)}{(1+\tau_c)} = 1-\tau$$

where $\tau = \frac{\tau_\ell+\tau_c}{1+\tau_c}$.

- A person wanting to spend one dollar on consumption needs to earn x dollars as labor income, where x solves

$$\begin{aligned} x(1-\tau) &= 1 \text{ or} \\ x &= \frac{1}{1-\tau} \end{aligned}$$

Country	Tax Rate τ	$\frac{c}{Y}$	Hours per Person per Week	
			Actual	Predicted
Germany	0.59	0.74	19.3	19.5
France	0.59	0.74	17.5	19.5
Italy	0.64	0.69	16.5	18.8
Canada	0.52	0.77	22.9	21.3
United Kingdom	0.44	0.83	22.8	22.8
Japan	0.37	0.68	27.0	29.0
United States	0.40	0.81	25.9	24.6

Country	Tax Rate τ	$\frac{c}{Y}$	Hours per Person per Week	
			Actual	Predicted
Germany	0.52	0.66	24.6	24.6
France	0.49	0.66	24.4	25.4
Italy	0.41	0.66	19.2	28.3
Canada	0.44	0.72	22.2	25.6
United Kingdom	0.45	0.77	25.9	24.0
Japan	0.25	0.60	29.8	35.8
United States	0.40	0.74	23.5	26.4

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- Large part of the difference in hours worked between the U.S. and Europe (but not all of it) is explained by tax differences.

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Main Findings II

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- Does predict that in the early 70's Germans and French did not work so much less than Americans, precisely because tax rates on labor were lower then than in the 90's in these countries.
- Two big failures of the model: Japan and Italy. What other than taxes depressed labor supply in these countries in this time period.

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- Various major forces responsible for the introduction of social security at that time.

- U.S. economy had undergone a dramatic transition for an agrarian to an industrialized economy.

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- Why was farm life less likely to leave the elders impoverished? Elders could perform less physically demanding tasks on family farms. Also, elders tended to own the farms. Second, employment opportunities in agriculture were less volatile than in the rest of the economy.

- Great depression in 1929-1932, the most severe recession in U.S. economic history, reduced unemployment opportunities of the elderly.

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- Destroyed most of retirement wealth: September 1, 1929, value of stocks listed at NYSE was \$89.7 billion; in middle of 1932 it was \$15.6 billion, a decline of over 80%. In 1930 and 1931 over $3,000$ banks suspended operations, deposits being lost were more than \$2 billion. Prices of wheat and cotton dropped by 66% and 75%, respectively, with it incomes and asset values in agricultural sector.

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- Consequently, the great depression left an entire generation impoverished.

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- Several public programs arose out of this idea, one of which was social security. Designed to deal with the specific problems of the impoverished elders.

- The Elderly Population had started to grow as a result of increased life expectancy

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- It is hard to coexist with large numbers of very poor elderly.

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- A fully funded Social Security as in Chile is a pile of assets that various groups want to steal
- Hence unfunded (pay as you go) social security systems become the norm

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Early History

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- By 1939 it become clear that the widespread poverty of the old could needed more than a funded system: It was changed to pay-as-you go.

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- U.S. social security system has accumulated the so-called trust fund, but with the expressed purpose of handling the retirement of the massive baby boom generation without having to increase payroll taxes.

Current system is defined by

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- A benefit formula that calculates social security benefits as a function of the labor earnings over your lifetime.

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- Maximum amount an employee has to pay in 2021 is

$$0.062 * \$142,800 = \$8,853.60$$

Year	Max. Taxable Ear.	Tax Rate
1937	$3,000	2.00%
1950	$3,000	3.00%
1960	$4,800	6.00%
1970	$7,800	8.40%
1980	$29,700	10.16%
1990	$51,300	12.40%
1998	$68,400	12.40%
2007	$97,500	12.40%
2012	$110,100	12.40%
2017	$127,200	12.40%
2019	$132,900	12.40%
2021	$142,800	12.40%

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1. For each year t define qualified earnings as

$$\hat{y}_t = \min\{y_t, \bar{y}_t\}$$

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2. Adjust for inflation. Let P_{1977} denote CPI in 1977 and P_{2021} CPI in 2021. Then $\frac{P_{2021}}{P_{1977}}$ is the relative price of a typical basket of consumption goods in 2021, relative to 1977. Thus we take

$$\begin{aligned} \tilde{y}_{1977} &= \hat{y}_{1977} * \frac{P_{2021}}{P_{1977}} \\ \tilde{y}_t &= \hat{y}_t * \frac{P_{2021}}{P_t} \end{aligned}$$

1. 3 Adjust by average wage growth. Define as the gross growth rate of average wages between 1977 and 2021

$$
\begin{aligned}
G_{1977,2021} &= \frac{\bar{w}_{2021}}{\bar{w}_{1977}} \\
G_{t,2021} &= \frac{\bar{w}_{2021}}{\bar{w}_{t}}
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In addition to inflation earnings in early years of a persons's life are therefore adjusted in the following fashion

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2. 4 We arrive at 45 numbers, $\{Y_{1977}, Y_{1978}, \ldots, Y_{2021}\}$. AIME equals the average of the 35 highest entries from the list (so if worked less than 35 years the AIME will be calculated with zero earnings for those years).

- Benefit formula (monthly)

$$b = \begin{cases} 0.9AIME & \text{if } AIME \leq \$996 \\ 896 + 0.32(AIME - 996) & \text{if } \$996 < AIME \leq \$6,002 \\ 2,498 + 0.15(AIME - 6,002) & \text{if } \$6,002 < AIME \end{cases}$$

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- This gives household's benefits in 2022. From that point on benefits are indexed by inflation. Benefits are paid until death.

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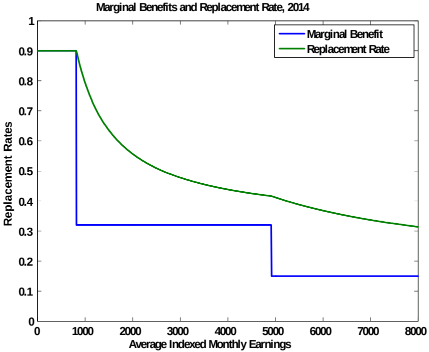
- Define the replacement rate as

$$rr(AIME) = \frac{b(AIME)}{AIME}$$


Marginal Benefits and Replacement Rate, 2014
Marginal Benefit
Replacement Rate
Replacement Rates
Average Indexed Monthly Earnings
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1000
2000
3000
4000
5000
6000
7000
8000

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    - Life expectancy increased
    - Fertility rates decreased
    - Higher (predicted) dependency ratio (the ratio of people above 65 to the population aged 16-65)

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What Reforms?

- Increase social security tax rates further
- Reduce benefits (e.g. increase the retirement age). Retirement age will gradually increase to age 67.
- Limit the scope of the program by reducing benefits and giving incentives to complement public pensions by private retirement accounts.

- Does a Pay-As-You-Go social security system reduce private savings rates?

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- Under what conditions is the introduction of a Pay-As-You-Go social security system a good idea.

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- Social Security as Insurance against Longevity Risk

- Household maximizes

$$\begin{aligned}\max_{c_1, c_2, s} \quad & \log(c_1) + \beta \log(c_2) \qquad \text{s.t.} \\ c_1 + s &= (1-\tau)y \\ c_2 &= (1+r)s + b\end{aligned}$$

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- Population grows at rate n, technical progress at rate g
- Social security system balances its budget

$$b = (1+n)(1+g)\tau y$$

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- Rewrite the budget constraints of the household as

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- Consolidate

$$c_1 + \frac{c_2}{1+r} = (1-\tau)y + \frac{(1+n)(1+g)\tau y}{1+r} = I(\tau)$$

$$c_1 = \frac{I}{1+\beta}$$

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$$s = (1-\tau)y - \frac{I}{1+\beta}$$

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\begin{aligned}
s &= (1-\tau)y - \frac{I}{1+\beta} \\
&= \frac{\beta y}{1+\beta} - \frac{(1+n)(1+g)+\beta(1+r)}{(1+r)(1+\beta)} * \tau y
\end{aligned}
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- Because of its pay-as-you go nature of the system the social security system itself does not save, so total savings in the economy unambiguously decline with an increase in the size of the system as measured by τ.

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- Under which $I(\tau)$ is strictly increasing in τ?

$$\begin{aligned} I(\tau) &= y - \tau y + \frac{(1+g)(1+n)\tau y}{1+r} \\ &= y + \left[\frac{(1+g)(1+n)}{1+r} - 1\right]\tau y \end{aligned}$$

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- Pay-as-you go social security system is welfare improving if and only if $(1+n)(1+g) > 1+r$.
- As good approximation

$$n + g > r$$

- If people save by themselves for their retirement, the return on their savings equals $1 + r$. If they save via a social security system (are forced to do so), their return to this forced saving consists of $(1 + n)(1 + g)$.

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- May help to understand why in some countries the reform away from a PAYGO system is underway, in others not.
- But transition problem: there is one missing generation (since initial generation received benefits without paying taxes). If we abolish the system, either the currently young pay double, or we just default on the promises for the old.

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- People that live (unexpectedly) longer receive more over their lifetime than those that die prematurely.
- But: could also be done by private annuities.

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- Intertemporal budget constraint: $c_1 + \frac{c_2}{1+r} = y$
- Solution

$$
\begin{aligned}
c_1 &= \frac{1}{1+p} y \\
c_2 &= \frac{p(1+r)}{1+p} y
\end{aligned}
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- With social security: budget constraints

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- Consolidating household budget constraints and substituting for b yields

$$c_1 + \frac{c_2}{1+r} = y + \tau y \left(\frac{(1+n)(1+g)}{p(1+r)} - 1 \right)$$

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- Two reasons for social security
    - If $(1+n)(1+g) > 1+r$, the implicit return on social security is higher than the return on private assets, even absent the insurance aspect.
    - As long as $p < 1$, even if $(1+n)(1+g) \leq 1+r$ social security may be good, since the surviving individuals are implicitly insured by their dead brethren: the implicit return on social security is $\frac{(1+n)(1+g)}{p} > (1+n)(1+g)$.

- Focus on the insurance aspect and assume

$$(1+n)(1+g) = 1+r$$

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- Private insurance via annuities. An annuity is a contract where the household pays \$1 today, for the promise of the insurance company to pay you $\$(1+r_a)$ as long as you live, from tomorrow on.

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- Return on the annuity equals return via social security, as long as $(1 + n)(1 + g) = 1 + r$. Insurance against longevity can equally be provided by a social security system or by private annuity markets.

- Most countries provide this insurance publicly, with social security system? Why?

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    - Adverse selection: individuals have better information about their life expectancy than insurance companies

- A variety of public insurance programs

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- Goal: insuring citizens against the major risks of life

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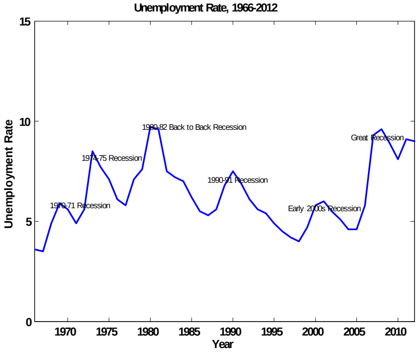
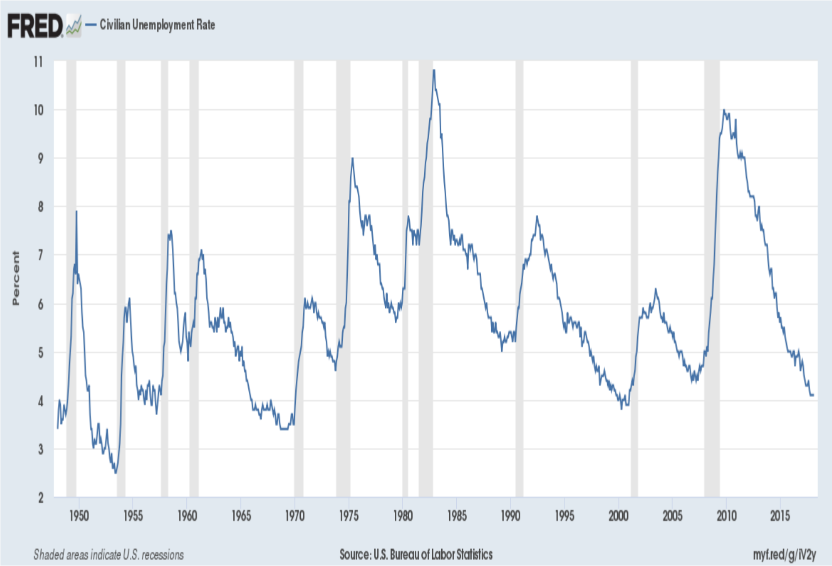
- Examples: unemployment insurance, welfare, food stamps, social security, public health insurance



The U.S. Unemployment Rate

U.S. Unemployment Rate 1950–2018



Unemployment Spell	2006	2010
$<$ 5 weeks	37%	19%
5 - 14 weeks	30%	22%
15 - 26 weeks	15%	16%
$>$ 26 weeks	18%	43%

	Unemployment (%)				≥ 1 Year		
	2000	2008	2011	2017	1999	2006	2011
France	9.0	7.8	9.7	9.0	38.7	41.9	41.4
Germany	8.0	7.5	5.9	3.7	51.7	56.4	48.0
Spain	11.7	11.3	21.6	15.6	46.3	21.7	41.6
Italy	10.1	6.7	8.4	11.0	61.4	49.6	51.9
Greece	11.2	7.7	17.7	20.7	55.3	54.3	49.6
Portugal	4.0	7.7	12.9	9.0	41.2	50.2	48.2
Sweden	5.6	6.2	7.5	6.3	30.1	13.0	17.2
UK	5.4	5.7	8.0	4.4	29.6	22.3	33.4
US	4.0	5.8	9.7	4.1	6.8	10.0	31.7
Tot. OECD	6.1	6.0	5.6	8.0	32.2	31.4	33.6

	Single			With Dependent Spouse		
	1. Y.	2.-3. Y.	4.-5. Y.	1. Y.	2.-3. Y.	4.-5. Y.
Belgium	79	55	55	70	64	64
France	79	63	61	80	62	60
Germany	66	63	63	74	72	72
Netherlands	79	78	73	90	88	85
Spain	69	54	32	70	55	39
Sweden	81	76	75	81	100	101
UK	64	64	64	75	74	74
US	34	9	9	38	14	14

European Unemployment Dilemma of 1980's and 1990's: A Potential Explanation

- Ljungqvist & Sargent: unemployment benefits & increased turbulence

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- Ljungqvist & Sargent: unemployment benefits & increased turbulence

- Increased turbulence in the 80's: laid-off workers faced a higher risk of losing their skills when becoming unemployed.

- Newly laid off worker in Europe has access to high and long-lasting unemployment compensation; on other hand, he may have lost his skill and thus is not offered new jobs that are attractive enough. Decides to stay unemployed, rather than accept a bad job. European unemployment dilemma.

- Why U.S. unemployment dilemma in/after Great Recession: Mitman and Rabinovich (2014) point to extension of unemployment benefits.

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- Europeization of U.S. labor market.

- Household lives for two periods. First period: job for sure, wage y_1.

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- Interest rate $r = 0$.

- Utility function

$$\log(c_1) + p\log(c_2^e) + (1-p)\log(c_2^u)$$

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- Suppose that $y_1 = y$ and $y_2^e = y_2^u = y_1 = y$.

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$$\max_{c_1, c_2^e, c_2^u, s} \quad \log(c_1) + p \log(c_2^e) + (1-p)\log(c_2^u)$$

$$\text{s.t.}$$

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- Income is perfectly smooth and $\beta(1+r) = 1$, so consumption simply equals income in every period.

- Let $y_1 = y$ and $p = 0.5$ and $y_2 = 2y_1 = 2y$. Mean-preserving spread, since

$$0.5 * 2y + 0.5 * 0 = y$$

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- Maximization problem

$$\max \log(c_1) + 0.5 \log(c_2^e) + 0.5 \log(c_2^u)$$

$$\begin{aligned} c_1 + s &= y \\ c_2^e &= 2y + s \\ c_2^u &= s \end{aligned}$$

- Lagrangian

$$\begin{aligned} L &= \log(c_1) + 0.5\log(c_2^e) + 0.5\log(c_2^u) + \lambda_1\,(y - c_1 - s) \\ &\quad +\lambda_2\,(2y + s - c_2^e) + \lambda_3\,(s - c_2^u) \end{aligned}$$

- First order conditions with respect to (c_1, c_2^e, c_2^u, s) yields

$$\begin{aligned} \frac{1}{c_1} - \lambda_1 &= 0 \\ \frac{0.5}{c_2^e} - \lambda_2 &= 0 \\ \frac{0.5}{c_2^u} - \lambda_3 &= 0 \\ -\lambda_1 + \lambda_2 + \lambda_3 &= 0 \end{aligned}$$

- Rewriting

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\begin{aligned}
\frac{1}{c_1} &= \lambda_1 \\
\frac{0.5}{c_2^e} &= \lambda_2 \\
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\lambda_2 + \lambda_3 &= \lambda_1
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- Rewriting

$$\begin{aligned} \frac{1}{c_1} &= \lambda_1 \\ \frac{0.5}{c_2^e} &= \lambda_2 \\ \frac{0.5}{c_2^u} &= \lambda_3 \\ \lambda_2 + \lambda_3 &= \lambda_1 \end{aligned}$$

- Substituting the first three equations into the last yields

$$\frac{0.5}{c_2^e} + \frac{0.5}{c_2^u} = \frac{1}{c_1}$$

- Use the budget constraints to obtain

$$\frac{0.5}{2y+s} + \frac{0.5}{s} = \frac{1}{(y-s)}$$

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- Bringing the equation to one common denominator, $s*(2y+s)*(y-s)$, yields

$$\frac{0.5s(y-s)}{s\,(2y+s)\,(y-s)}+\frac{0.5\,(2y+s)\,(y-s)}{s\,(2y+s)\,(y-s)}=\frac{s\,(2y+s)}{s\,(2y+s)\,(y-s)}$$

or

$$\frac{0.5s(y-s)+0.5\,(2y+s)\,(y-s)-s\,(2y+s)}{s\,(2y+s)\,(y-s)}=0$$

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or

$$\frac{0.5s(y-s) + 0.5\,(2y+s)\,(y-s) - s\,(2y+s)}{s\,(2y+s)\,(y-s)} = 0$$

- But this can only be 0 if the numerator is 0, or

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- Multiplying things out and simplifying a bit yields

$$s^2+ys-\frac{1}{2}y^2=0$$

- Quadratic equation, has two solutions:

$$
\begin{aligned}
s_1 &= -\frac{y}{2} - \left[\left(\frac{3}{4}\right) y^2\right]^{0.5} = -\frac{1}{2} y \left(1 + 3^{0.5}\right) < 0 \\
s_2 &= -\frac{y}{2} + \left[\left(\frac{3}{4}\right) y^2\right]^{0.5} = \frac{1}{2} y \left(3^{0.5} - 1\right) > 0
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- Optimal consumption and savings choices with uncertainty satisfy

$$\begin{aligned} \hat{s} &= \frac{1}{2} y \left(3^{0.5} - 1\right) > 0 \\ \hat{c}_1 &= y - \frac{1}{2} y \left(3^{0.5} - 1\right) = \frac{1}{2} y \left(3 - 3^{0.5}\right) < y \\ \hat{c}_2^e &= 2y + \hat{s} = \frac{1}{2} y \left(3 + 3^{0.5}\right) \\ \hat{c}_2^u &= \frac{1}{2} y \left(3^{0.5} - 1\right) \end{aligned}$$

- Note:

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\hat{c}_1 &= \frac{1}{2}y\left(3-3^{0.5}\right) < y = c_1 \\
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- Strict concavity of u (that is, risk-aversion, $u'' < 0$) is not enough for this result. If utility quadratic, $u(c) = -\frac{1}{2}(c - 100,000)^2$ then consumption and savings choice in first period identical to no uncertainty case.

- Suppose

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- In this case the first order conditions become

$$\begin{aligned}
-(c_1 - 100,000) &= \lambda_1 \\
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-0.5(c_2^u - 100,000) &= \lambda_3 \\
\lambda_2 + \lambda_3 &= \lambda_1
\end{aligned}$$

- Inserting the first three equations into the fourth yields

$$-(c_1 - 100,000) = -0.5(c_2^e - 100,000) - 0.5(c_2^u - 100,000)$$

or

$$c_1 = 0.5(c_2^e + c_2^u)$$

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- Now using the budget constraints one obtains

$$\begin{aligned} y - s &= 0.5(2y + s + s) \\ y - s &= y + s \\ 2s &= 0 \end{aligned}$$

and thus the optimal savings choice with quadratic utility is $s = 0$, as in the case with no uncertainty.

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- It is easy to verify that with quadratic utility $u''' = 0$.

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- Budget constraints in the second period of life become

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- Then maximization problem of household becomes

$$\begin{aligned} & \max \log(c_1) + 0.5\log(c_2) + 0.5\log(c_2) \\ = & \max \log(c_1) + \log(c_2) \\ & \text{s.t.} \\ c_1 + s = & \; y \\ c_2 = & \; y + s \end{aligned}$$

with obvious solution

$$\begin{aligned} c_1 &= c_2 = y \\ s &= 0. \end{aligned}$$

- Exactly as without income uncertainty. When the government completely insures unemployment risk, private households make exactly the same choices as if there was no income uncertainty.

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$$\log\left(\frac{1}{2}y\left(3-3^{0.5}\right)\right) + 0.5\log\left(\frac{1}{2}y\left(3+3^{0.5}\right)\right) + 0.5\log\left(\frac{1}{2}y\left(3^{0.5}-1\right)\right)$$

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which exactly equals the lifetime utility without income uncertainty.

- Without unemployment insurance lifetime utility is $V^{no} =$

$$\log\left(\frac{1}{2}y\left(3-3^{0.5}\right)\right) + 0.5\log\left(\frac{1}{2}y\left(3+3^{0.5}\right)\right) + 0.5\log\left(\frac{1}{2}y\left(3^{0.5}-1\right)\right)$$

- Easy to calculate that $V^{ins} > V^{no}$.

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- But they prefer more insurance to less, absent any adverse selection or moral hazard problem.

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Why not Full Insurance in Real World

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- Trade-off between insurance and economic incentives. If the government could perfectly monitor individuals things would be easy: simply condition payment of benefits on good behavior. But with private information the complicated trade-off between efficiency and insurance arises.