# Macroeconomics 702, Spring 2006, First Midterm Exam:

**José-Víctor Ríos-Rull**

September 26, 2005

In the following there are 11 questions for 100 points. Please answer all questions. Be as BRIEF as you can and good luck. You have 80 minutes.

If there is any question or issue that is not clear in this exam and you do not get a clear answer on the spot, please describe the issue in the exam itself.

## Growth Models

There is an economy with many consumers that can be of two types $i \in \{1, 2\}$ and infinite time. All consumers have preferences

$$E\left\{\sum_{t=0}^{\infty} \beta^t \, u(c_t, n_t)\right\}$$

where $c_t$ is own consumption at time $t$, and $n_t$ is the fraction of time worked by the agent at time $t$. The first partial derivative of $u$ is positive while the second is negative. Consumers differ in their endowent of efficiency units of labor $\epsilon^i$, and half of them are of each type. The efficiency of type 1 consumers is constant while that of type 2 is subject to shocks with finite support, $z_t$ and transition matrix $\Gamma$.

Output can be produced with labor and capital according to a modified neoclassical production function

$$K_t^{\theta} \, L_t^{1-\theta} = \, K_t^{\theta} \, \left(\frac{1}{2}\,\epsilon^1 + \, z_t \, \frac{1}{2}\,\epsilon^2\right)^{1-\theta}$$

where $K_t$, is capital, and $L_t$ is effective units of labor. Capital depreciates at rate $\delta$. Output can be used either for consumption or investment.

1. (5 points) Briefly define an Arrow-Debreu equilibrium. Define the consumption possibility set.

2. (10 points) Briefly define a sequence of market competitive equilibrium. Make sure that it has sufficient markets.

3. (10 points) Show that any Arrow-Debreu equilibrium allocation can be supported as a a sequence of market competitive equilibrium.

4. (10 points) Define recursive competitive equilibria with firms owning capital and hiring workers and sufficient markets.

5. (5 points) Define recursive competitive equilibria with firms owning capital and hiring workers, and consumers buying shares of the firm (and perhaps other things).

6. (5 points) State the second welfare theorem. Does it apply to this economy?

Imagine now that there state contingent trades are outlawed.

7. (5 points) Briefly define how recursive competitive equilibrium is different from before.

8. (15 points) Pose a tax transfer scheme that may be Pareto improving. Make sure that you describe what conditions are needed for equilibrium.

9. (5 points) Given your answer to question 8, and if the government were to use a tax rate that is a function of the shock alone $\tau(z)$. How would you recommend it to be? Discuss.

## Lucas Trees

Assume there is a representative agent economy. There is one tree in the economy that produces two different fruits, apricots and bananas. Consumers have preferences

$$E\left\{\sum_{t=0}^{\infty}\beta^t\,\frac{\left(\alpha\;a_t^{\rho}\;+(1-\alpha)\;b_t^{1-\rho}\right)^{1-\sigma}}{1-\sigma}\right\}$$

where $a_t$ and $b_t$ stand for apricots and bananas respectively. Fruit follows a Markov process with joint transition $\Gamma$.

10. (15 points) Define equilibria recursively. Make sure that you define the state space and the equilibrium conditions.

11. (15 points) Write a formula for an option to buy one share of the tree at price $p_1$ tomorrow and again the day after tomorrow.