Saving for a Sunny Day: An Alternative Theory of Precautionary Savings

Luigi Briglia (CEMFI)
Satyajit Chatterjee (FRB Philadelphia)
Dean Corbae (Wisconsin)
Kyle Dempsey (The Ohio State University)
José Víctor Ríos Rull (Penn UCL CAERP)

January 31, 2022

Econ 712 Spring

Introduction

- **Motivation:** Heterogeneous Agents Models of the BIHA (?, ?, ?, ?) variety are rooted in uninsurable earnings risk

- **Motivation:** Heterogeneous Agents Models of the BIHA (?, ?, ?, ?) variety are rooted in uninsurable earnings risk
  - $\beta(1+r) < 1 \implies$ **save for a rainy day** to smooth consumption

- **Motivation:** Heterogeneous Agents Models of the BIHA (?, ?, ?, ?) variety are rooted in uninsurable earnings risk
  - $\beta(1+r) < 1 \implies$ **save for a rainy day** to smooth consumption
- **This paper** poses a theory of precautionary savings for consumption levels that occasionally provide additional joy or utility

- **Motivation:** Heterogeneous Agents Models of the BIHA (?, ?, ?, ?) variety are rooted in uninsurable earnings risk
  - $\beta(1+r) < 1 \implies$ **save for a rainy day** to smooth consumption
- **This paper** poses a theory of precautionary savings for consumption levels that occasionally provide additional joy or utility
  - **save for a sunny day** to cash in on these opportunities

- **Motivation:** Heterogeneous Agents Models of the BIHA (?, ?, ?, ?) variety are rooted in uninsurable earnings risk
  - $\beta(1+r) < 1 \implies$ **save for a rainy day** to smooth consumption
- **This paper** poses a theory of precautionary savings for consumption levels that occasionally provide additional joy or utility
  - **save for a sunny day** to cash in on these opportunities
- **Model:** extreme value (EV) shocks provide a convenient approach

- **Motivation:** Heterogeneous Agents Models of the BIHA (?, ?, ?, ?) variety are rooted in uninsurable earnings risk
  - $\beta(1+r) < 1 \implies$ **save for a rainy day** to smooth consumption
- **This paper** poses a theory of precautionary savings for consumption levels that occasionally provide additional joy or utility
  - **save for a sunny day** to cash in on these opportunities
- **Model:** extreme value (EV) shocks provide a convenient approach
  - widely used, but not in this way – new theoretical insights

- **Motivation:** Heterogeneous Agents Models of the BIHA (?, ?, ?, ?) variety are rooted in uninsurable earnings risk
  - $\beta(1+r) < 1 \implies$ **save for a rainy day** to smooth consumption
- **This paper** poses a theory of precautionary savings for consumption levels that occasionally provide additional joy or utility
  - **save for a sunny day** to cash in on these opportunities
- **Model:** extreme value (EV) shocks provide a convenient approach
  - widely used, but not in this way – new theoretical insights
- **Why bother?** Strong predictions about consumption behavior that:

- **Motivation:** Heterogeneous Agents Models of the BIHA (?, ?, ?, ?) variety are rooted in uninsurable earnings risk
  - $\beta(1+r) < 1 \implies$ **save for a rainy day** to smooth consumption
- **This paper** poses a theory of precautionary savings for consumption levels that occasionally provide additional joy or utility
  - **save for a sunny day** to cash in on these opportunities
- **Model:** extreme value (EV) shocks provide a convenient approach
  - widely used, but not in this way – new theoretical insights
- **Why bother?** Strong predictions about consumption behavior that:
  1. are borne out in the data PSID

A new take on popular models

- **Motivation:** Heterogeneous Agents Models of the BIHA (?, ?, ?, ?) variety are rooted in uninsurable earnings risk
  - $\beta(1+r) < 1 \implies$ **save for a rainy day** to smooth consumption
- **This paper** poses a theory of precautionary savings for consumption levels that occasionally provide additional joy or utility
  - **save for a sunny day** to cash in on these opportunities
- **Model:** extreme value (EV) shocks provide a convenient approach
  - widely used, but not in this way – new theoretical insights
- **Why bother?** Strong predictions about consumption behavior that:
  1. are borne out in the data PSID
  2. can discipline key EV parameters

What we do

- **Model:** iid shocks to the utility of specific consumption levels induce deviations from consumption choices predicted by the Euler equation.

What we do

- **Model:** iid shocks to the utility of specific consumption levels induce deviations from consumption choices predicted by the Euler equation.
  - **structural:** shocks give opportunities

What we do

- **<u>Model</u>:** iid shocks to the utility of specific consumption levels induce deviations from consumption choices predicted by the Euler equation.
  - **structural:** shocks give opportunities
  - easily **coexist** with other types of shocks (i.e. earnings risk)

What we do

- **Model:** iid shocks to the utility of specific consumption levels induce deviations from consumption choices predicted by the Euler equation.
  - **structural:** shocks give opportunities
  - easily **coexist** with other types of shocks (i.e. earnings risk)
  - Households **internalize** them: new rationale for precautionary savings

What we do

- **Model:** iid shocks to the utility of specific consumption levels induce deviations from consumption choices predicted by the Euler equation.
    - **structural:** shocks give opportunities
    - easily **coexist** with other types of shocks (i.e. earnings risk)
    - Households **internalize** them: new rationale for precautionary savings
- **Empirics:** measure predicted consumption in PSID.

What we do

- **Model:** iid shocks to the utility of specific consumption levels induce deviations from consumption choices predicted by the Euler equation.
    - **structural:** shocks give opportunities
    - easily **coexist** with other types of shocks (i.e. earnings risk)
    - Households **internalize** them: new rationale for precautionary savings
- **Empirics:** measure predicted consumption in PSID.
    - **empirical fact:** $\uparrow$ wealth, $\uparrow$ deviations from predicted onsumption

What we do

- **Model:** iid shocks to the utility of specific consumption levels induce deviations from consumption choices predicted by the Euler equation.
  - **structural:** shocks give opportunities
  - easily **coexist** with other types of shocks (i.e. earnings risk)
  - Households **internalize** them: new rationale for precautionary savings
- **Empirics:** measure predicted consumption in PSID.
  - **empirical fact:** $\uparrow$ wealth, $\uparrow$ deviations from predicted onsumption
  - **quantitative analysis** proceeds in 2 phases

What we do

- **Model:** iid shocks to the utility of specific consumption levels induce deviations from consumption choices predicted by the Euler equation.
  - **structural:** shocks give opportunities
  - easily **coexist** with other types of shocks (i.e. earnings risk)
  - Households **internalize** them: new rationale for precautionary savings
- **Empirics:** measure predicted consumption in PSID.
  - **empirical fact:** $\uparrow$ wealth, $\uparrow$ deviations from predicted onsumption
  - **quantitative analysis** proceeds in 2 phases
    1. Can our model replicate this? Can others?

What we do

- **Model:** iid shocks to the utility of specific consumption levels induce deviations from consumption choices predicted by the Euler equation.
  - **structural:** shocks give opportunities
  - easily **coexist** with other types of shocks (i.e. earnings risk)
  - Households **internalize** them: new rationale for precautionary savings
- **Empirics:** measure predicted consumption in PSID.
  - **empirical fact:** $\uparrow$ wealth, $\uparrow$ deviations from predicted onsumption
  - **quantitative analysis** proceeds in 2 phases
    1. Can our model replicate this? Can others?
    2. What are the implications for precautionary savings?

What we find

- **Model:** our economies with extreme value shocks have

- **Model:** our economies with extreme value shocks have
  - well-defined, well-behaved continuous and ∞-horizon limits

What we find

- **Model:** our economies with extreme value shocks have
  - well-defined, well-behaved continuous and ∞-horizon limits
  - additional (and intuitive) facets of marginal value of wealth

What we find

- **Model:** our economies with extreme value shocks have
  - well-defined, well-behaved continuous and ∞-horizon limits
  - additional (and intuitive) facets of marginal value of wealth
  - fanning out of consumption "errors": different from shocks to MRS

What we find

- **Model:** our economies with extreme value shocks have
  - well-defined, well-behaved continuous and ∞-horizon limits
  - additional (and intuitive) facets of marginal value of wealth
  - fanning out of consumption "errors": different from shocks to MRS
- **Quantitative analysis:** compare our EV shock economies to ones with earnings risk, marginal utility risk, combinations of all

What we find

- **Model:** our economies with extreme value shocks have
  - well-defined, well-behaved continuous and ∞-horizon limits
  - additional (and intuitive) facets of marginal value of wealth
  - fanning out of consumption "errors": different from shocks to MRS
- **Quantitative analysis:** compare our EV shock economies to ones with earnings risk, marginal utility risk, combinations of all
  - Only EV model can replicate fanning out of consumption errors

What we find

- **Model:** our economies with extreme value shocks have
  - well-defined, well-behaved continuous and ∞-horizon limits
  - additional (and intuitive) facets of marginal value of wealth
  - fanning out of consumption "errors": different from shocks to MRS
- **Quantitative analysis:** compare our EV shock economies to ones with earnings risk, marginal utility risk, combinations of all
  - Only EV model can replicate fanning out of consumption errors
  - Simple empirical moments discipline EV shocks

What we find

- **Model:** our economies with extreme value shocks have
  - well-defined, well-behaved continuous and ∞-horizon limits
  - additional (and intuitive) facets of marginal value of wealth
  - fanning out of consumption "errors": different from shocks to MRS
- **Quantitative analysis:** compare our EV shock economies to ones with earnings risk, marginal utility risk, combinations of all
  - Only EV model can replicate fanning out of consumption errors
  - Simple empirical moments discipline EV shocks
    - We use slope of consumption error variance w.r.t. cash on hand

What we find

- **Model:** our economies with extreme value shocks have
  - well-defined, well-behaved continuous and ∞-horizon limits
  - additional (and intuitive) facets of marginal value of wealth
  - fanning out of consumption "errors": different from shocks to MRS
- **Quantitative analysis:** compare our EV shock economies to ones with earnings risk, marginal utility risk, combinations of all
  - Only EV model can replicate fanning out of consumption errors
  - Simple empirical moments discipline EV shocks
    - We use slope of consumption error variance w.r.t. cash on hand
    - Implied noise is equivalent to increasing earnings risk by 26%

What we find

- **Model:** our economies with extreme value shocks have
  - well-defined, well-behaved continuous and ∞-horizon limits
  - additional (and intuitive) facets of marginal value of wealth
  - fanning out of consumption "errors": different from shocks to MRS
- **Quantitative analysis:** compare our EV shock economies to ones with earnings risk, marginal utility risk, combinations of all
  - Only EV model can replicate fanning out of consumption errors
  - Simple empirical moments discipline EV shocks
    - We use slope of consumption error variance w.r.t. cash on hand
    - Implied noise is equivalent to increasing earnings risk by 26%
  - **limitation:** our mechanism acts evenly over wealth distribution (both poor and rich save for a sunny day)

What we find

- **Model:** our economies with extreme value shocks have
  - well-defined, well-behaved continuous and ∞-horizon limits
  - additional (and intuitive) facets of marginal value of wealth
  - fanning out of consumption "errors": different from shocks to MRS
- **Quantitative analysis:** compare our EV shock economies to ones with earnings risk, marginal utility risk, combinations of all
  - Only EV model can replicate fanning out of consumption errors
  - Simple empirical moments discipline EV shocks
    - We use slope of consumption error variance w.r.t. cash on hand
    - Implied noise is equivalent to increasing earnings risk by 26%
  - **limitation:** our mechanism acts evenly over wealth distribution (both poor and rich save for a sunny day)
    - Extensions to explain top wealth inequality?

- **Rare periods of high consumption:** durable goods (see ? for a review) or its modern cousin, memorable goods (?)

What we contribute to the literature ▸ Full literature

- **Rare periods of high consumption:** durable goods (see ? for a review) or its modern cousin, memorable goods (?)
  - No durability of enjoyment (no need to track the stock)

- **Rare periods of high consumption:** durable goods (see ? for a review) or its modern cousin, memorable goods (?)
  - No durability of enjoyment (no need to track the stock)
  - Not triggered by temporary earnings

What we contribute to the literature ▸ Full literature

- **Rare periods of high consumption:** durable goods (see ? for a review) or its modern cousin, memorable goods (?)
  - No durability of enjoyment (no need to track the stock)
  - Not triggered by temporary earnings
- **Standard utility functions as insufficient to accumulate wealth beyond consumption smoothing:** (as in ? or more recently ?) want wealth in the utility function

What we contribute to the literature ▸ Full literature

- **Rare periods of high consumption:** durable goods (see ? for a review) or its modern cousin, memorable goods (?)
    - No durability of enjoyment (no need to track the stock)
    - Not triggered by temporary earnings
- **Standard utility functions as insufficient to accumulate wealth beyond consumption smoothing:** (as in ? or more recently ?) want wealth in the utility function
    - Rationale for additional value of wealth with empirical discipline

What we contribute to the literature

▸ Full literature

- **Rare periods of high consumption:** durable goods (see ? for a review) or its modern cousin, memorable goods (?)
  - No durability of enjoyment (no need to track the stock)
  - Not triggered by temporary earnings
- **Standard utility functions as insufficient to accumulate wealth beyond consumption smoothing:** (as in ? or more recently ?) want wealth in the utility function
  - Rationale for additional value of wealth with empirical discipline
- **Dynamic discrete choice:** ?, ?, all of IO...

What we contribute to the literature ‣ Full literature

- **Rare periods of high consumption:** durable goods (see ? for a review) or its modern cousin, memorable goods (?)
  - No durability of enjoyment (no need to track the stock)
  - Not triggered by temporary earnings
- **Standard utility functions as insufficient to accumulate wealth beyond consumption smoothing:** (as in ? or more recently ?) want wealth in the utility function
  - Rationale for additional value of wealth with empirical discipline
- **Dynamic discrete choice:** ?, ?, all of IO...
  - Extend EV shocks into realm of fundamentals; change ex ante behavior rather than provide tractable error structure

Simplest Dynamic Model: A two period savings model

Preferences

- Today: Consuming $c \in [0, \overline{c}]$ (Non-binding $\overline{c}$) yields

$$u(c) + \epsilon^c, \qquad \epsilon^c \text{ random variables, one for each } c.$$

Preferences

- Today: Consuming $c \in [0, \overline{c}]$ (Non-binding $\overline{c}$) yields

$$u(c) + \epsilon^c, \qquad \epsilon^c \text{ random variables, one for each } c.$$

- Tomorrow: $u(c')$.

Preferences

- Today: Consuming $c \in [0, \overline{c}]$ (Non-binding $\overline{c}$) yields

$$u(c) + \epsilon^c, \qquad \epsilon^c \text{ random variables, one for each } c.$$

- Tomorrow: $u(c')$.
- u increasing, strictly concave, differentiable.

Preferences

- Today: Consuming $c \in [0, \overline{c}]$ (Non-binding $\overline{c}$) yields

$$u(c) + \epsilon^c, \qquad \epsilon^c \text{ random variables, one for each } c.$$

- Tomorrow: $u(c')$.
- u increasing, strictly concave, differentiable.
- No borrowing, no interest, no income, given wealth a,

$$u(c) + \epsilon^c + u(a - c)$$

Preferences

- Today: Consuming $c \in [0, \overline{c}]$ (Non-binding $\overline{c}$) yields

$$u(c) + \epsilon^c, \qquad \epsilon^c \text{ random variables, one for each } c.$$

- Tomorrow: $u(c')$.
- u increasing, strictly concave, differentiable.
- No borrowing, no interest, no income, given wealth a,

$$u(c) + \epsilon^c + u(a - c)$$

- Two approaches:

Preferences

- Today: Consuming $c \in [0, \overline{c}]$ (Non-binding $\overline{c}$) yields

$$u(c) + \epsilon^c, \qquad \epsilon^c \text{ random variables, one for each } c.$$

- Tomorrow: $u(c')$.
- u increasing, strictly concave, differentiable.
- No borrowing, no interest, no income, given wealth a,

$$u(c) + \epsilon^c + u(a - c)$$

- Two approaches:
  1. Think of the continuum as a convenient approximation to a discrete problem (?). Derivatives give information.

Preferences

- Today: Consuming $c \in [0, \overline{c}]$ (Non-binding $\overline{c}$) yields

$$u(c) + \epsilon^c, \qquad \epsilon^c \text{ random variables, one for each } c.$$

- Tomorrow: $u(c')$.
- u increasing, strictly concave, differentiable.
- No borrowing, no interest, no income, given wealth a,

$$u(c) + \epsilon^c + u(a - c)$$

- Two approaches:
  1. Think of the continuum as a convenient approximation to a discrete problem (?). Derivatives give information.
  2. Pose structure in ϵ^c that yields well behaved probl ?.

Preferences

- Today: Consuming $c \in [0, \overline{c}]$ (Non-binding $\overline{c}$) yields

$$u(c) + \epsilon^c, \qquad \epsilon^c \text{ random variables, one for each } c.$$

- Tomorrow: $u(c')$.
- u increasing, strictly concave, differentiable.
- No borrowing, no interest, no income, given wealth a,

$$u(c) + \epsilon^c + u(a - c)$$

- Two approaches:
  1. Think of the continuum as a convenient approximation to a discrete problem (?). Derivatives give information.
  2. Pose structure in ϵ^c that yields well behaved probl ?.
- Today we follow the first approach

Construction of the Finite Choice Economies

- Economies indexed by N : Cardinality of choices.

- Economies indexed by N : Cardinality of choices.

- Equally spaced grid. $c^i = i\ c^1 \qquad i \in \{1, \cdots, N\}$

Construction of the Finite Choice Economies

- Economies indexed by N : Cardinality of choices.
- Equally spaced grid. $c^i = i \; c^1 \qquad i \in \{1, \cdots, N\}$
- Convenient Normalization: Choose the N Economies so that

- Economies indexed by N : Cardinality of choices.
- Equally spaced grid. $c^i = i\ c^1 \qquad i \in \{1, \cdots, N\}$
- Convenient Normalization: Choose the N Economies so that
  - Consuming 1 is on the grid $c^{M(N)} = 1$.

Construction of the Finite Choice Economies

- Economies indexed by N : Cardinality of choices.
- Equally spaced grid. $c^i = i\ c^1 \qquad i \in \{1, \cdots, N\}$
- Convenient Normalization: Choose the N Economies so that
  - Consuming 1 is on the grid $c^{M(N)} = 1$.
  - We are close to the upper bound: $c^N \geq \overline{c} \geq c^{N-1}$.

Construction of the Finite Choice Economies

- Economies indexed by N : Cardinality of choices.
- Equally spaced grid. $c^i = i\ c^1 \qquad i \in \{1, \cdots, N\}$
- Convenient Normalization: Choose the N Economies so that
  - Consuming 1 is on the grid $c^{M(N)} = 1$.
  - We are close to the upper bound: $c^N \geq \overline{c} \geq c^{N-1}$.
- Then take limits as $N \to \infty$ to get continuous objects.

Preferences in the finite choice economies

- Consumption level c^i associated to η^i, $i \in \{1, \cdots, N\}$.

- Consumption level c^i associated to η^i, $i \in \{1, \cdots, N\}$.
- η^i iid type 1 Extreme Value (no need to bring back ϵ^c). We get

$$u(c_i) + u(a - c_i) + \eta_i$$

- Consumption level c^i associated to η^i, $i \in \{1, \cdots, N\}$.
- η^i iid type 1 Extreme Value (no need to bring back ϵ^c). We get

$$u(c_i) + u(a - c_i) + \eta_i$$

- Assume that η^i are iid, Gumbel: $\eta^i \sim G(\mu^N, \alpha)$.

- Consumption level c^i associated to η^i, $i \in \{1, \cdots, N\}$.
- η^i iid type 1 Extreme Value (no need to bring back ϵ^c). We get

$$u(c_i) + u(a - c_i) + \eta_i$$

- Assume that η^i are iid, Gumbel: $\eta^i \sim G(\mu^N, \alpha)$.
- 2 key parameters: μ_N (location / mean) and α (scale / variance)

- Consumption level c^i associated to η^i, $i \in \{1, \cdots, N\}$.
- η^i iid type 1 Extreme Value (no need to bring back ϵ^c). We get

$$u(c_i) + u(a - c_i) + \eta_i$$

- Assume that η^i are iid, Gumbel: $\eta^i \sim G(\mu^N, \alpha)$.
- 2 key parameters: μ_N (location / mean) and α (scale / variance)
- Note $\alpha = 0$ is the standard model without shocks.

- Consumption level c^i associated to η^i, $i \in \{1, \cdots, N\}$.
- η^i iid type 1 Extreme Value (no need to bring back ϵ^c). We get

$$u(c_i) + u(a - c_i) + \eta_i$$

- Assume that η^i are iid, Gumbel: $\eta^i \sim G(\mu^N, \alpha)$.
- 2 key parameters: μ_N (location / mean) and α (scale / variance)
- Note $\alpha = 0$ is the standard model without shocks.

Preferences in the finite choice economies

- Consumption level c^i associated to η^i, $i \in \{1, \cdots, N\}$.
- η^i iid type 1 Extreme Value (no need to bring back ϵ^c). We get

$$u(c_i) + u(a - c_i) + \eta_i$$

- Assume that η^i are iid, Gumbel: $\eta^i \sim G(\mu^N, \alpha)$.
- 2 key parameters: μ_N (location / mean) and α (scale / variance)
- Note $\alpha = 0$ is the standard model without shocks.

Normalization: expected max of η_i shocks over a unit interval is 0:

- define $\overline{\eta} \equiv \max_{i=1,\ldots,M(N)} \ \eta_i$ and normalize $\mathbb{E}[\overline{\eta}] = 0$

Preferences in the finite choice economies

- Consumption level c^i associated to η^i, $i \in \{1, \cdots, N\}$.
- η^i iid type 1 Extreme Value (no need to bring back ϵ^c). We get

$$u(c_i) + u(a - c_i) + \eta_i$$

- Assume that η^i are iid, Gumbel: $\eta^i \sim G(\mu^N, \alpha)$.
- 2 key parameters: μ_N (location / mean) and α (scale / variance)
- Note $\alpha = 0$ is the standard model without shocks.

Normalization: expected max of η_i shocks over a unit interval is 0:

- define $\overline{\eta} \equiv \max_{i=1,\ldots,M(N)} \; \eta_i$ and normalize $\mathbb{E}[\overline{\eta}] = 0$
- math: $\mu_N = -\alpha(\gamma_E + \ln M(N))$ imposes this; only α left

Preferences in the finite choice economies

- Consumption level c^i associated to η^i, $i \in \{1, \cdots, N\}$.
- η^i iid type 1 Extreme Value (no need to bring back ϵ^c). We get

$$u(c_i) + u(a - c_i) + \eta_i$$

- Assume that η^i are iid, Gumbel: $\eta^i \sim G(\mu^N, \alpha)$.
- 2 key parameters: μ_N (location / mean) and α (scale / variance)
- Note $\alpha = 0$ is the standard model without shocks.

Normalization: expected max of η_i shocks over a unit interval is 0:

- define $\overline{\eta} \equiv \max_{i=1,\ldots,M(N)} \ \eta_i$ and normalize $\mathbb{E}[\overline{\eta}] = 0$
- math: $\mu_N = -\alpha(\gamma_E + \ln M(N))$ imposes this; only α left
- economics: utility **bonus** of a unit interval budget set is 0

- Household chooses

$$\max_{c^i \in \{c^1, \cdots, c^N\}} \quad u(c^i) + \eta^i + u(a - c^i),$$
$$\text{s.t.} \quad c^i \leq a.$$

- Household chooses

$$\max_{c^i \in \{c^1, \cdots, c^N\}} \quad u(c^i) + \eta^i + u(a - c^i),$$
$$\text{s.t.} \quad c^i \leq a.$$

- Or $\max_{i \in \{1, \cdots, J(N)\}} \; u(c^i) + \eta^i + u(a - c^i)$, when $J(N) = \arg\max_{i=1,\ldots,N}\{c_i \leq a\}$.

- Household chooses

$$\max_{c^i \in \{c^1, \cdots, c^N\}} \quad u(c^i) + \eta^i + u(a - c^i),$$
$$\text{s.t.} \quad c^i \leq a.$$

- Or $\max_{i \in \{1, \cdots, J(N)\}} \; u(c^i) + \eta^i + u(a - c^i)$, when $J(N) = \arg\max_{i=1,\ldots,N}\{c_i \leq a\}$.
  - ratio $\frac{J(N,a)}{M(N)} = c_{J(N,a)}$ holds by construction; $\lim_{N \to \infty} \frac{J(N,a)}{M(N)} = a$

- Household chooses

$$\max_{c^i \in \{c^1, \cdots, c^N\}} \quad u(c^i) + \eta^i + u(a - c^i),$$
$$\text{s.t.} \quad c^i \leq a.$$

- Or $\max_{i \in \{1, \cdots, J(N)\}} \; u(c^i) + \eta^i + u(a - c^i)$, when $J(N) = \arg\max_{i=1,\ldots,N}\{c_i \leq a\}$.
    - ratio $\frac{J(N,a)}{M(N)} = c_{J(N,a)}$ holds by construction; $\lim_{N\to\infty} \frac{J(N,a)}{M(N)} = a$
    - **key:** size of budget set (a) determines the number of alternatives and therefore the number of shocks received, $J(N, a)$

- Household chooses

$$\max_{c^i \in \{c^1, \cdots, c^N\}} \quad u(c^i) + \eta^i + u(a - c^i),$$
$$\text{s.t.} \quad c^i \leq a.$$

- Or $\max_{i \in \{1, \cdots, J(N)\}} \ u(c^i) + \eta^i + u(a - c^i)$, when $J(N) = \arg\max_{i=1,\dots,N}\{c_i \leq a\}$.
  - ratio $\frac{J(N,a)}{M(N)} = c_{J(N,a)}$ holds by construction; $\lim_{N\to\infty} \frac{J(N,a)}{M(N)} = a$
  - **key:** size of budget set (a) determines the number of alternatives and therefore the number of shocks received, $J(N, a)$
  - More options increases expected value

- Household chooses

$$\max_{c^i \in \{c^1, \cdots, c^N\}} \quad u(c^i) + \eta^i + u(a - c^i),$$
$$\text{s.t.} \quad c^i \leq a.$$

- Or $\max_{i \in \{1, \cdots, J(N)\}} \; u(c^i) + \eta^i + u(a - c^i)$, when $J(N) = \arg\max_{i=1,\ldots,N}\{c_i \leq a\}$.
  - ratio $\frac{J(N,a)}{M(N)} = c_{J(N,a)}$ holds by construction; $\lim_{N \to \infty} \frac{J(N,a)}{M(N)} = a$
  - **key:** size of budget set (a) determines the number of alternatives and therefore the number of shocks received, $J(N, a)$
  - More options increases expected value
  - Options have cardinal interpretation and shocks are factored in **ex-ante**

- The ex-ante value

$$v^N(a) \;=\; \int \max_{c^i \in \{c^{\mathbf{1}}, \cdots, c^{J(N,a)}\}} \{u(c^i) + \eta^i + u(a - c^i)\} \; dF(\eta^1, \cdots, \eta^N),$$

- The ex-ante value

$$v^N(a) \;=\; \int \max_{c^i \in \{c^{\mathbf{1}},\cdots,c^{J(N,a)}\}} \{u(c^i) + \eta^i + u(a - c^i)\} \; dF(\eta^1, \cdots, \eta^N),$$

- The density

$$h^N(a,i) \;=\; P\left(\underset{j \in \{1,\cdots,J(N,a)\}}{\operatorname{argmax}} \; \{u(c^j) + \eta^j + u(a - c^j)\} = n \right),$$

Ex-Ante Value and Decision Rules

- The ex-ante value

$$v^N(a) \;=\; \int \max_{c^i \in \{c^1, \cdots, c^{J(N,a)}\}} \{u(c^i) + \eta^i + u(a - c^i)\}\, dF(\eta^1, \cdots, \eta^N),$$

- The density

$$h^N(a, i) \;=\; P\left(\operatorname*{argmax}_{j \in \{1, \cdots, J(N,a)\}} \left\{u(c^j) + \eta^j + u(a - c^j)\right\} = n \right),$$

- The cdf

$$H^N(a, a') \;=\; P\left(\operatorname*{argmax}_{c^i \in \{c^1, \cdots, c^{J(N,a)}\}} \left\{u(c^i) + \eta^i + u(a - c^i)\right\} \leq a' \right),$$

- The value satisfies

$$v^N(a) = \alpha \ \ln \left(\frac{1}{J(N,a)} \sum_{i=1}^{J(N,a)} \exp\left\{ \frac{u(c^i) + u(a - c^i)}{\alpha} \right\} \right) + \alpha \ \ln c^{J(N,a)}.$$

- The value satisfies

$$v^N(a) = \alpha \ \ln \left(\frac{1}{J(N,a)} \sum_{i=1}^{J(N,a)} \exp\left\{ \frac{u(c^i) + u(a - c^i)}{\alpha} \right\} \right) + \alpha \ \ln c^{J(N,a)}.$$

  - First term is sort of weighted average of the standard utilities of all choices (notice the log and the exp)

- The value satisfies

$$v^N(a) = \alpha \ \ln \left(\frac{1}{J(N,a)} \sum_{i=1}^{J(N,a)} \exp\left\{ \frac{u(c^i) + u(a - c^i)}{\alpha} \right\} \right) + \alpha \ \ln c^{J(N,a)}.$$

  - First term is sort of weighted average of the standard utilities of all choices (notice the log and the exp)
  - Last term, acts as a *utility bonus of wealth*, a form of option value.

- The value satisfies

$$v^N(a) = \alpha \ \ln \left(\frac{1}{J(N,a)} \sum_{i=1}^{J(N,a)} \exp \left\{ \frac{u(c^i) + u(a - c^i)}{\alpha} \right\} \right) + \alpha \ \ln c^{J(N,a)}.$$

  - First term is sort of weighted average of the standard utilities of all choices (notice the log and the exp)
  - Last term, acts as a *utility bonus of wealth*, a form of option value.
- The probability of each choice i is

$$h^N(a,i) = \frac{\exp\left\{\frac{u(c^i)+u(a-c^i)}{\alpha}\right\}}{\sum_{j=1}^{J(N,a)} \exp\left\{\frac{u(c^j)+u(a-c^j)}{\alpha}\right\}}.$$

Standard results in Extreme Value Theory yield formulas

- The value satisfies

$$v^N(a) = \alpha \ \ln\left(\frac{1}{J(N,a)}\sum_{i=1}^{J(N,a)} \exp\left\{\frac{u(c^i)+u(a-c^i)}{\alpha}\right\}\right) + \alpha \ \ln c^{J(N,a)}.$$

  - First term is sort of weighted average of the standard utilities of all choices (notice the log and the exp)
  - Last term, acts as a *utility bonus of wealth*, a form of option value.
- The probability of each choice i is

$$h^N(a,i) = \frac{\exp\left\{\frac{u(c^i)+u(a-c^i)}{\alpha}\right\}}{\sum_{j=1}^{J(N,a)} \exp\left\{\frac{u(c^j)+u(a-c^j)}{\alpha}\right\}}.$$

- The cdf $H^N(a,a')$ satisfies

$$H^N(a,a') = \frac{\sum_{i=1}^{n(a')} \exp\left\{\frac{u(c^i)+u(a-c^i)}{\alpha}\right\}}{\sum_{i=1}^{J(N,a)} \exp\left\{\frac{u(c^i)+u(a-c^i)}{\alpha}\right\}}.$$

- The Value converges to (because it is essentially a Riemann integral)

$$v(a) = \alpha \ \ln\left(\int_0^a \exp\left\{\frac{u(c)+u(a-c)}{\alpha}\right\} dc\right) + \alpha \ \ln a =$$

$$\alpha \ \ln\left(\int_0^a \exp\left\{\frac{u(a-a')+u(a')}{\alpha}\right\} da'\right) + \alpha \ \ln a$$

- The Value converges to (because it is essentially a Riemann integral)

$$v(a) = \alpha \ \ln\left(\int_0^a \ \exp\left\{\frac{u(c)+u(a-c)}{\alpha}\right\} \ dc\right) + \alpha \ \ln a =$$
$$\alpha \ \ln\left(\int_0^a \ \exp\left\{\frac{u(a-a')+u(a')}{\alpha}\right\} \ da'\right) + \alpha \ \ln a$$

- The CDF converges to

$$H(a,a') = \frac{\int_0^{a'} \ \exp\left\{\frac{u(c)+u(a-c)}{\alpha}\right\} \ dc}{\int_0^{a} \ \exp\left\{\frac{u(c)+u(a-c)}{\alpha}\right\} \ dc}.$$

(multiply and Divide by $J(N,a)$).

- The Value converges to (because it is essentially a Riemann integral)

$$v(a) = \alpha \ \ln \left(\int_0^a \exp\left\{ \frac{u(c) + u(a-c)}{\alpha} \right\} \, dc \right) + \alpha \ \ln a =$$
$$\alpha \ \ln \left(\int_0^a \exp\left\{ \frac{u(a-a') + u(a')}{\alpha} \right\} \, da' \right) + \alpha \ \ln a$$

- The CDF converges to

$$H(a, a') = \frac{\int_0^{a'} \exp\left\{ \frac{u(c)+u(a-c)}{\alpha} \right\} \, dc}{\int_0^{a} \exp\left\{ \frac{u(c)+u(a-c)}{\alpha} \right\} \, dc}.$$

(multiply and Divide by $J(N, a)$).

- Note that these are differentiable functions.

- The Value converges to (because it is essentially a Riemann integral)

$$v(a) = \alpha \ \ln\left(\int_0^a \ \exp\left\{\frac{u(c)+u(a-c)}{\alpha}\right\} \ dc\right) + \alpha \ \ln a =$$
$$\alpha \ \ln\left(\int_0^a \ \exp\left\{\frac{u(a-a')+u(a')}{\alpha}\right\} \ da'\right) + \alpha \ \ln a$$

- The CDF converges to

$$H(a,a') = \frac{\int_0^{a'} \ \exp\left\{\frac{u(c)+u(a-c)}{\alpha}\right\} \ dc}{\int_0^{a} \ \exp\left\{\frac{u(c)+u(a-c)}{\alpha}\right\} \ dc}.$$

(multiply and Divide by $J(N,a)$).

- Note that these are differentiable functions.
- Main insights go through whether discrete or continuous case; in remainder, we'll go with continuous.

Taking Derivatives yields the density of the decision rule

- We can obtain

$$\frac{\partial H(a,a')}{\partial a} = h(a,a') = \frac{\exp\left\{\frac{u(a-a')+u(a')}{\alpha}\right\}}{\int_0^a \exp\left\{\frac{u(c)+u(a-c)}{\alpha}\right\} dc}.$$

- The derivative of the value function $v_a(a)$ satisfies

$$v_a(a) = \int_0^a \left[u'(a-c) - u'(a - c^*(a))\right] h(c;a)dc + u'(a - c^*(a)) + \frac{\alpha}{a}$$

where $c^*(a) = a/2$ is the fundamental / Euler equation solution ($\alpha = 0$).

- The derivative of the value function $v_a(a)$ satisfies

$$v_a(a) = \int_0^a \left[u'(a-c) - u'(a - c^*(a))\right] h(c; a) dc + u'(a - c^*(a)) + \frac{\alpha}{a}$$

  where $c^*(a) = a/2$ is the fundamental / Euler equation solution ($\alpha = 0$).

  - **1st term:** average deviation of marginal utility from optimum

- The derivative of the value function $v_a(a)$ satisfies

$$v_a(a) = \int_0^a \left[u'(a-c) - u'(a-c^*(a))\right] h(c;a)dc + u'(a-c^*(a)) + \frac{\alpha}{a}$$

where $c^*(a) = a/2$ is the fundamental / Euler equation solution ($\alpha = 0$).

  - **1st term:** average deviation of marginal utility from optimum
    - positive by Jensen's inequality given prudence ($u'(a-c)$ convex in c)

- The derivative of the value function $v_a(a)$ satisfies

$$v_a(a) = \int_0^a \left[u'(a-c) - u'(a-c^*(a))\right] h(c;a)dc + u'(a-c^*(a)) + \frac{\alpha}{a}$$

where $c^*(a) = a/2$ is the fundamental / Euler equation solution ($\alpha = 0$).

  - **1st term:** average deviation of marginal utility from optimum
    - positive by Jensen's inequality given prudence ($u'(a-c)$ convex in c)
  - **2nd term:** standard marginal utility effect at optimum

- The derivative of the value function $v_a(a)$ satisfies

$$v_a(a) = \int_0^a \left[u'(a-c) - u'(a - c^*(a))\right] h(c; a) dc + u'(a - c^*(a)) + \frac{\alpha}{a}$$

where $c^*(a) = a/2$ is the fundamental / Euler equation solution ($\alpha = 0$).

  - **1st term:** average deviation of marginal utility from optimum
    - positive by Jensen's inequality given prudence ($u'(a-c)$ convex in c)
  - **2nd term:** standard marginal utility effect at optimum
  - **3rd term:** increased expected bonus coming from more shocks

- The derivative of the value function $v_a(a)$ satisfies

$$v_a(a) = \int_0^a \left[u'(a-c) - u'(a - c^*(a))\right] h(c;a) dc + u'(a - c^*(a)) + \frac{\alpha}{a}$$

where $c^*(a) = a/2$ is the fundamental / Euler equation solution ($\alpha = 0$).

  - **1st term:** average deviation of marginal utility from optimum
    - positive by Jensen's inequality given prudence ($u'(a-c)$ convex in c)
  - **2nd term:** standard marginal utility effect at optimum
  - **3rd term:** increased expected bonus coming from more shocks
  - all 3 terms are positive $\Longrightarrow$ MVW positive

The marginal value of wealth

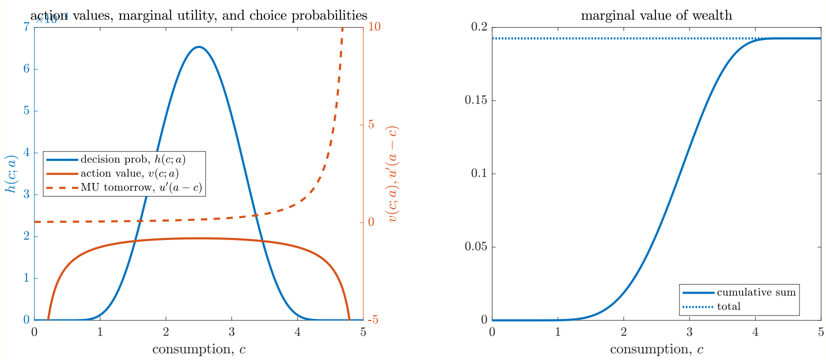
- The derivative of the value function $v_a(a)$ satisfies

$$v_a(a) = \int_0^a \left[u'(a-c) - u'(a - c^*(a))\right] h(c;a) dc + u'(a - c^*(a)) + \frac{\alpha}{a}$$

where $c^*(a) = a/2$ is the fundamental / Euler equation solution ($\alpha = 0$).

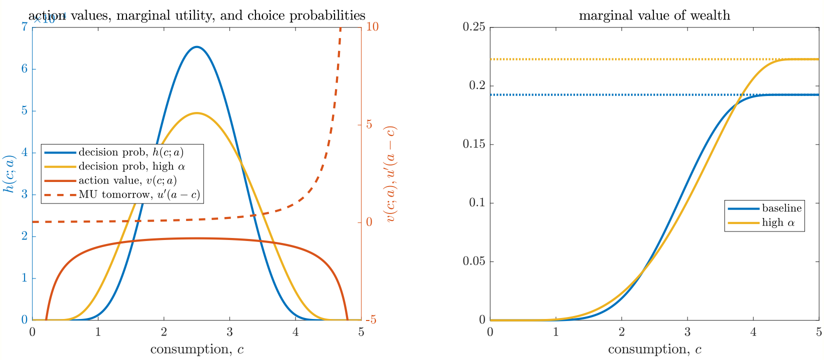
  - **1st term:** average deviation of marginal utility from optimum
    - positive by Jensen's inequality given prudence ($u'(a-c)$ convex in c)
  - **2nd term:** standard marginal utility effect at optimum
  - **3rd term:** increased expected bonus coming from more shocks
  - all 3 terms are positive $\implies$ MVW positive
  - harder to show: MVW increasing in α (but it is!)

Illustrating the marginal value of wealth



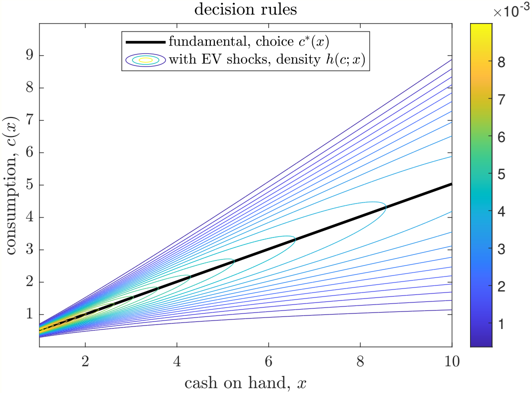
Beyond pure utility bonus, the marginal value of wealth comes from **not** being constrained upon choosing c that lead to low a' (analogy to rainy day).

Marginal value of wealth increases with noise (α)



Higher α fans out $h(c; a) \implies$ more weight on high future MU states $\implies$ MVW increases due to convexity of $u'(a - c)$ (also pure bonus term).

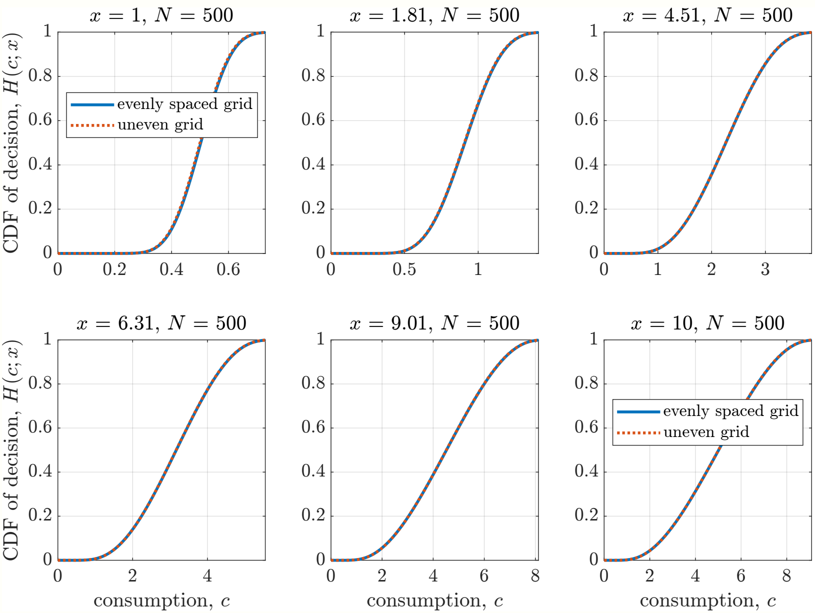
Decision rule contours



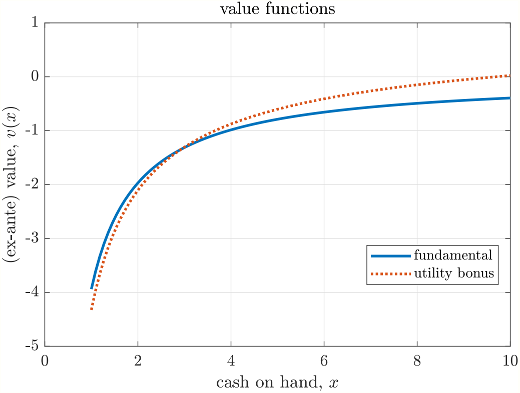
Consumption choices fan out with wealth.

- violations of Euler equation grow
- potential driver of right tail of wealth?

Wealth Disregards Euler Equation: Fanning wide of Consm



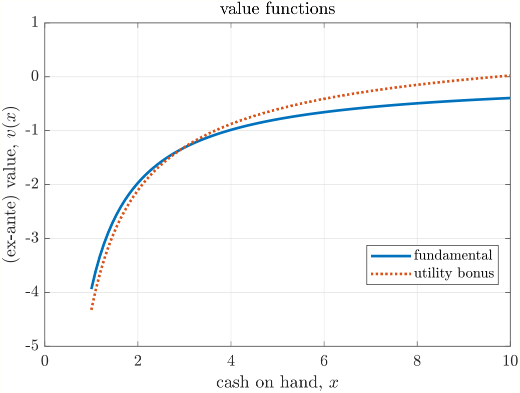
There is a fanning out of choices



Value function becomes **steeper** with EV shocks

- $V_{a|\alpha>0}(a) > V_{a|\alpha=0}(a)$: recall earlier argument

There is a fanning out of choices



Value function becomes **steeper** with EV shocks

- $V_{a|\alpha>0}(a) > V_{a|\alpha=0}(a)$: recall earlier argument

Interestingly, no change in concavity!

- $V_{aa|\alpha>0}(a) = V_{aa|\alpha=0}(a)$: is this the property that cooks us on wealth?

The infinitely-lived savings problem

Extending the environment: $T = 2 \to T = \infty$

Almost everything is the same as the 2-period case, except:

- now assume flow utility and shocks occur each period: $u(c_t) + \epsilon_{c,t}$

Almost everything is the same as the 2-period case, except:

- now assume flow utility and shocks occur each period: $u(c_t) + \epsilon_{c,t}$
- Households discounts future at rate β, takes interest rate r as given

Almost everything is the same as the 2-period case, except:

- now assume flow utility and shocks occur each period: $u(c_t) + \epsilon_{c,t}$
- Households discounts future at rate β, takes interest rate r as given
  - assume and verify that $\beta(1 + r) < 1$

Almost everything is the same as the 2-period case, except:

- now assume flow utility and shocks occur each period: $u(c_t) + \epsilon_{c,t}$
- Households discounts future at rate β, takes interest rate r as given
  - assume and verify that $\beta(1 + r) < 1$
- Define action-specific value as

$$v_t(c_i; a) = u(c_i) + \beta V_{t+1}((a - c_i)(1 + r)) + \eta^i$$

Almost everything is the same as the 2-period case, except:

- now assume flow utility and shocks occur each period: $u(c_t) + \epsilon_{c,t}$
- Households discounts future at rate β, takes interest rate r as given
  - assume and verify that $\beta(1+r) < 1$
- Define action-specific value as

$$v_t(c_i; a) = u(c_i) + \beta V_{t+1}((a - c_i)(1+r)) + \eta^i$$

- First consider a finite number of periods, then take limit as $T \to \infty$

- Proceeding as before, we have

$$
\begin{aligned}
V_t(a) &= \alpha \ln \int_0^a \exp\left(\frac{v_t(c_i;a)}{\alpha}\right) dc + \alpha \ln a \\
h_t(a) &= \frac{\exp\left(\frac{v_t(c_i;a)}{\alpha}\right)}{\int_0^a \exp\left(\frac{v_t(c_i;a)}{\alpha}\right) dc}
\end{aligned}
$$

- Proceeding as before, we have

$$V_t(a) = \alpha \ln \int_0^a \exp\left(\frac{v_t(c_i; a)}{\alpha}\right) dc + \alpha \ln a$$

$$h_t(a) = \frac{\exp\left(\frac{v_t(c_i;a)}{\alpha}\right)}{\int_0^a \exp\left(\frac{v_t(c_i;a)}{\alpha}\right) dc}$$

- Given this, it is straightforward to show that:

- Proceeding as before, we have

$$V_t(a) = \alpha \ln \int_0^a \exp\left(\frac{v_t(c_i;a)}{\alpha}\right) dc + \alpha \ln a$$

$$h_t(a) = \frac{\exp\left(\frac{v_t(c_i;a)}{\alpha}\right)}{\int_0^a \exp\left(\frac{v_t(c_i;a)}{\alpha}\right) dc}$$

- Given this, it is straightforward to show that:
  - $V_t(a) = T(V_{t+1}, a)$ as described above is a contraction

- Proceeding as before, we have

$$V_t(a) = \alpha \ln \int_0^a \exp\left(\frac{v_t(c_i;a)}{\alpha}\right) dc + \alpha \ln a$$

$$h_t(a) = \frac{\exp\left(\frac{v_t(c_i;a)}{\alpha}\right)}{\int_0^a \exp\left(\frac{v_t(c_i;a)}{\alpha}\right) dc}$$

- Given this, it is straightforward to show that:
  - $V_t(a) = T(V_{t+1}, a)$ as described above is a contraction
  - $\implies$ $V_t(a)$ is strictly concave and differentiable

- Proceeding as before, we have

$$
\begin{aligned}
V_t(a) &= \alpha \ln \int_0^a \exp\left(\frac{v_t(c_i; a)}{\alpha}\right) dc + \alpha \ln a \\
h_t(a) &= \frac{\exp\left(\frac{v_t(c_i;a)}{\alpha}\right)}{\int_0^a \exp\left(\frac{v_t(c_i;a)}{\alpha}\right) dc}
\end{aligned}
$$

- Given this, it is straightforward to show that:
  - $V_t(a) = T(V_{t+1}, a)$ as described above is a contraction
  - $\implies V_t(a)$ is strictly concave and differentiable
  - infinite horizon limits exist $V(a)$, h and takes analogous forms

A special case: Log Utility

Thanks to Hanbaek Lee

If $u(c) = \ln c$, we can show (through a laborious guess and verify) that

$$V(a) = \frac{1+2\alpha}{1-\beta} \ln a + B$$

where B is a complicated function of (α, β, r) but independent of a

- usually (i.e. with $\alpha = 0$), get $\widetilde{V}(a) = \frac{1}{1-\beta} \ln a + \widetilde{B}$

▸ Details ▸ Decision rule

If $u(c) = \ln c$, we can show (through a laborious guess and verify) that

$$V(a) = \frac{1+2\alpha}{1-\beta} \ln a + B$$

where B is a complicated function of (α, β, r) but independent of a

- usually (i.e. with $\alpha = 0$), get $\widetilde{V}(a) = \frac{1}{1-\beta} \ln a + \widetilde{B}$
- what about extra 2α then?

Details Decision rule

If $u(c) = \ln c$, we can show (through a laborious guess and verify) that

$$V(a) = \frac{1+2\alpha}{1-\beta} \ln a + B$$

where B is a complicated function of (α, β, r) but independent of a

- usually (i.e. with $\alpha = 0$), get $\widetilde{V}(a) = \frac{1}{1-\beta} \ln a + \widetilde{B}$
- what about extra 2α then?
    - first comes from pure utility bonus term

▸ Details ▸ Decision rule

A special case: Log Utility
Thanks to Hanbaek Lee

If $u(c) = \ln c$, we can show (through a laborious guess and verify) that

$$V(a) = \frac{1+2\alpha}{1-\beta} \ln a + B$$

where B is a complicated function of (α, β, r) but independent of a

- usually (i.e. with $\alpha = 0$), get $\widetilde{V}(a) = \frac{1}{1-\beta} \ln a + \widetilde{B}$
- what about extra 2α then?
  - first comes from pure utility bonus term
  - second comes integral marginal utility deviation term (details?)

▸ Details ▸ Decision rule

If $u(c) = \ln c$, we can show (through a laborious guess and verify) that

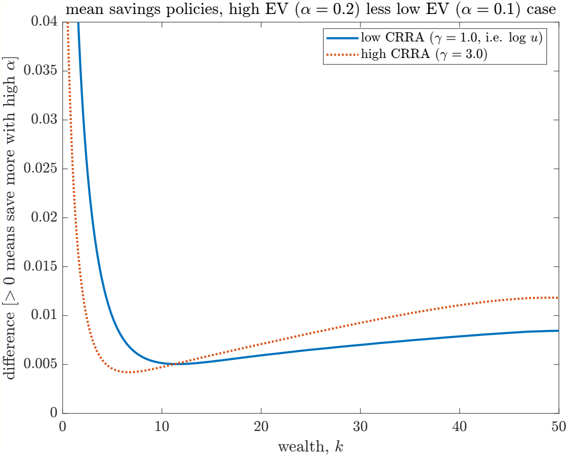
$$V(a) = \frac{1+2\alpha}{1-\beta} \ln a + B$$

where B is a complicated function of (α, β, r) but independent of a

- usually (i.e. with $\alpha = 0$), get $\widetilde{V}(a) = \frac{1}{1-\beta} \ln a + \widetilde{B}$
- what about extra 2α then?
  - first comes from pure utility bonus term
  - second comes integral marginal utility deviation term (details?)
  - net effect: extra patience (unfortunately uniform across wealth dist)

▸ Details ▸ Decision rule

Interaction of EV noise with risk aversion / IES



Fix prices, compare savings across levels of **noise (α)** for different **risk aversion (γ)**.

- **both lines > 0:** more savings with more noise
- **crossing:** effect more pronounced at low wealth for log preferences, high wealth for higher risk aversion

A Comparison of Various Aiyagari type Economies

The Economies to be Compared

- An Ec. with iid Earnings Risk Only (Coefficient of Variation .3)

The Economies to be Compared

- An Ec. with iid Earnings Risk Only (Coefficient of Variation .3)
- An Ec. with iid Marginal Utility Shocks (Coefficient of Variation $\sim$ .5)

The Economies to be Compared

- An Ec. with iid Earnings Risk Only (Coefficient of Variation .3)
- An Ec. with iid Marginal Utility Shocks (Coefficient of Variation $\sim$ .5)
- An Ec. with EVS to utility ($\alpha = .08375$ Variance of shocks)

The Economies to be Compared

- An Ec. with iid Earnings Risk Only (Coefficient of Variation .3)
- An Ec. with iid Marginal Utility Shocks (Coefficient of Variation $\sim$ .5)
- An Ec. with EVS to utility ($\alpha = .08375$ Variance of shocks)
- Otherwise identical. Calibrated to have the same interest rate.

The Economies to be Compared

- An Ec. with iid Earnings Risk Only (Coefficient of Variation .3)
- An Ec. with iid Marginal Utility Shocks (Coefficient of Variation $\sim$ .5)
- An Ec. with EVS to utility ($\alpha =$ .08375 Variance of shocks)
- Otherwise identical. Calibrated to have the same interest rate.
- So same size of *Precautionary Savings*

The Economies to be Compared

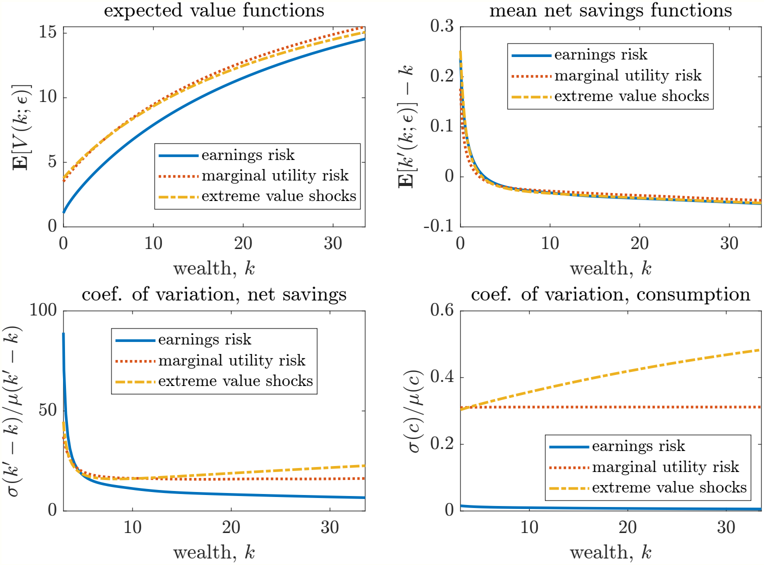
- An Ec. with iid Earnings Risk Only (Coefficient of Variation .3)
- An Ec. with iid Marginal Utility Shocks (Coefficient of Variation $\sim$ .5)
- An Ec. with EVS to utility ($\alpha = .08375$ Variance of shocks)
- Otherwise identical. Calibrated to have the same interest rate.
- So same size of *Precautionary Savings*
- Gives an idea to their strength at generating inequality.

Increased Fanning Out

But Only in Extreme Value Shocks Economy

Increased Fanning Out

But Only in Extreme Value Shocks Economy



Why don't consumption errors fan out with MU shocks?

- Consider the Euler equation for the MUR model, given cash on hand y:

$$\theta c^{-\gamma} = \beta(1+r)V_y[(1+r)(y-c)]$$

Why don't consumption errors fan out with MU shocks?

- Consider the Euler equation for the MUR model, given cash on hand y:

$$\theta c^{-\gamma} = \beta(1+r)V_y[(1+r)(y-c)]$$

- Can show (guess and verify) that $V(y,\theta) = A(\theta)\frac{y^{1-\gamma}}{1-\gamma}$ for a set of constants $A(\theta)$ with mean $\overline{A} = \sum_\theta \pi(\theta)A(\theta)$. Then, solving the Euler equation yields

$$c^*(y,\theta) = \Lambda(\theta)y$$

- Consider the Euler equation for the MUR model, given cash on hand y:

$$\theta c^{-\gamma} = \beta(1+r)V_y[(1+r)(y-c)]$$

- Can show (guess and verify) that $V(y,\theta) = A(\theta)\frac{y^{1-\gamma}}{1-\gamma}$ for a set of constants $A(\theta)$ with mean $\overline{A} = \sum_\theta \pi(\theta)A(\theta)$. Then, solving the Euler equation yields

$$c^*(y,\theta) = \Lambda(\theta)y$$

- for a set of constants $\Lambda(\theta)$. Taking logs and differencing, we obtain

$$\ln c^*(y,\theta) - \ln c^*(y,\theta') = \ln\Lambda(\theta) - \ln\Lambda(\theta')\text{: independent of } x!$$

What about the Data?

- Key testable prediction

Empirical content and discipline for our theory

- Key testable prediction

- Key testable prediction

  Size of log consumption errors is increasing in cash on hand

- Using PSID data, we find that this prediction is borne out.

Empirical content and discipline for our theory

- Key testable prediction

  Size of log consumption errors is increasing in cash on hand

- Using PSID data, we find that this prediction is borne out.
  - variance of log consumption errors increases with cash on hand

- Key testable prediction

  Size of log consumption errors is increasing in cash on hand

- Using PSID data, we find that this prediction is borne out.
  - variance of log consumption errors increases with cash on hand
    - measure by predicted consumption and computing deviations

Empirical content and discipline for our theory

- Key testable prediction

  Size of log consumption errors is increasing in cash on hand

- Using PSID data, we find that this prediction is borne out.
  - variance of log consumption errors increases with cash on hand
    - measure by predicted consumption and computing deviations
    - then group into quantiles of cash on hand and average within bin

Empirical content and discipline for our theory

- Key testable prediction

  Size of log consumption errors is increasing in cash on hand

- Using PSID data, we find that this prediction is borne out.
  - variance of log consumption errors increases with cash on hand
    - measure by predicted consumption and computing deviations
    - then group into quantiles of cash on hand and average within bin
  - **back to theory:** this trend can be used to pin down α

- Key testable prediction

  Size of log consumption errors is increasing in cash on hand

- Using PSID data, we find that this prediction is borne out.
  - variance of log consumption errors increases with cash on hand
    - measure by predicted consumption and computing deviations
    - then group into quantiles of cash on hand and average within bin
  - **back to theory:** this trend can be used to pin down α
    - regardless of other shocks, slope increases in α

- Key testable prediction

  Size of log consumption errors is increasing in cash on hand

- Using PSID data, we find that this prediction is borne out.
  - variance of log consumption errors increases with cash on hand
    - measure by predicted consumption and computing deviations
    - then group into quantiles of cash on hand and average within bin
  - **back to theory:** this trend can be used to pin down α
    - regardless of other shocks, slope increases in α
  - **key:** shocks to marginal utility cannot explain / be disciplined by this

Goal: flexible prediction model of consumption expenditures from PSID

Methodology: proceed in 2 steps

1. adapt Kaplan and Violante (2010) to measure log income

Goal: flexible prediction model of consumption expenditures from PSID

Methodology: proceed in 2 steps

1. adapt Kaplan and Violante (2010) to measure log income
   - 3 components: (i) permanent; (ii) AR(1); and (iii) transitory

Goal: flexible prediction model of consumption expenditures from PSID

Methodology: proceed in 2 steps

1. adapt Kaplan and Violante (2010) to measure log income
   - 3 components: (i) permanent; (ii) AR(1); and (iii) transitory
2. estimate consumption function $\ln c = g(x_{it}, \eta_{it}, Z_{it})$ where x_{it} is cash on hand, η_{it} is a transitory shock, and Z_{it} is a control vector

Empirical approach: predicting consumption

Goal: flexible prediction model of consumption expenditures from PSID

Methodology: proceed in 2 steps

1. adapt Kaplan and Violante (2010) to measure log income
   - 3 components: (i) permanent; (ii) AR(1); and (iii) transitory
2. estimate consumption function $\ln c = g(x_{it}, \eta_{it}, Z_{it})$ where x_{it} is cash on hand, η_{it} is a transitory shock, and Z_{it} is a control vector

Goal: flexible prediction model of consumption expenditures from PSID

Methodology: proceed in 2 steps

1. adapt Kaplan and Violante (2010) to measure log income
   - 3 components: (i) permanent; (ii) AR(1); and (iii) transitory
2. estimate consumption function $\ln c = g(x_{it}, \eta_{it}, Z_{it})$ where x_{it} is cash on hand, η_{it} is a transitory shock, and Z_{it} is a control vector

Key measurement: define residual $\xi_{it} = \ln c_{it} - \hat{g}(x_{it}, \eta_{it}, Z_{it})$, then compute variance within deciles

- implementing analogous measure in-model is trivial

▸ Figure: empirical results

ind. var.	cash on hand: decile mean			cash on hand: decile rank**		
moment	intercept	slope*	required α	intercept	slope	required α
PSID data	0.1091	0.0048	-	0.0980	0.0845	-
	(0.0057)	(0.0007)		(0.0096)	(0.0167)	
model with EVS shocks						
EVS only	0.0742	0.0048	0.1824	0.1265	0.0845	0.3562
add in earnings risk:						
iid	0.0637	0.0048	0.1635	0.1118	0.0845	0.3237
STY (2004)	0.0483	0.0048	0.1143	0.0444	0.0845	0.1441

Notes: Slopes match data to numerical precision by design. Actual regressors for decile rank regressions are 0.05 for decile 1, 0.15 for decile 2, etc.

Why can't shocks to marginal utility match the data?

- Predicted consumption in the MUR model (recall earlier analysis) is

$$\overline{c}(x) = \sum_{\theta} \pi(\theta) c^*(x, \theta) = \overline{\Lambda} x, \text{ where } \overline{\Lambda} = \sum_{\theta} \pi(\theta) \Lambda(\theta)$$

- Predicted consumption in the MUR model (recall earlier analysis) is

$$\overline{c}(x) = \sum_{\theta} \pi(\theta) c^*(x, \theta) = \overline{\Lambda} x, \text{ where } \overline{\Lambda} = \sum_{\theta} \pi(\theta) \Lambda(\theta)$$

- Our measurement of deviations from predicted consumption for x are

$$\begin{aligned} \xi(x) &\equiv \sum_{\theta} \pi(\theta) \left[\ln c(x, \theta) - \ln \overline{c}(x)\right]^2 \\ &= \sum_{\theta} \pi(\theta) \left[\ln \left(\frac{\Lambda(\theta)}{\overline{\Lambda}} \right) \right]^2 : \text{ again, independent of } x! \end{aligned}$$

Why can't shocks to marginal utility match the data?

- Predicted consumption in the MUR model (recall earlier analysis) is

$$\overline{c}(x) = \sum_{\theta} \pi(\theta) c^*(x, \theta) = \overline{\Lambda} x, \text{ where } \overline{\Lambda} = \sum_{\theta} \pi(\theta) \Lambda(\theta)$$

- Our measurement of deviations from predicted consumption for x are

$$\begin{aligned} \xi(x) &\equiv \sum_{\theta} \pi(\theta) \left[\ln c(x, \theta) - \ln \overline{c}(x)\right]^2 \\ &= \sum_{\theta} \pi(\theta) \left[\ln \left(\frac{\Lambda(\theta)}{\overline{\Lambda}}\right)\right]^2 : \text{ again, independent of } x! \end{aligned}$$

- For the EVS model, we have $\xi(x) = \int_0^x h(c; x) \left[\ln c - \ln \overline{c}(x)\right]^2$; variance always increases as bounds shift out with cash on hand!

What does $\alpha = 0.1143$ mean? Consider the following exercise:

- solve the EVS + STY (04) economy from the last row above

What does $\alpha = 0.1143$ mean? Consider the following exercise:

- solve the EVS + STY (04) economy from the last row above
- solve an Aiyagari economy with only STY (04) earnings risk

What does $\alpha = 0.1143$ mean? Consider the following exercise:

- solve the EVS + STY (04) economy from the last row above
- solve an Aiyagari economy with only STY (04) earnings risk
  - then, increase variance of risk until economy has r^* from EVS case

What does $\alpha = 0.1143$ mean? Consider the following exercise:

- solve the EVS + STY (04) economy from the last row above
- solve an Aiyagari economy with only STY (04) earnings risk
  - then, increase variance of risk until economy has r^* from EVS case
- increase in the unconditional variance of earnings risk rel. to STY (04) baseline is a measure of the contribution of EV shocks to savings

What does $\alpha = 0.1143$ mean? Consider the following exercise:

- solve the EVS + STY (04) economy from the last row above
- solve an Aiyagari economy with only STY (04) earnings risk
  - then, increase variance of risk until economy has r^* from EVS case
- increase in the unconditional variance of earnings risk rel. to STY (04) baseline is a measure of the contribution of EV shocks to savings

What does $\alpha = 0.1143$ mean? Consider the following exercise:

- solve the EVS + STY (04) economy from the last row above
- solve an Aiyagari economy with only STY (04) earnings risk
  - then, increase variance of risk until economy has r^* from EVS case
- increase in the unconditional variance of earnings risk rel. to STY (04) baseline is a measure of the contribution of EV shocks to savings

Result: the variance of earnings risk must increase by **26-33%.**

- related exercise: with mean 1 iid normally distributed marginal utility shocks, need a standard deviation of θ of 0.465.

Extensions

Extensions

- This framework can be easily extended in many directions with different empirical goals in mind. For example,

Extensions

- This framework can be easily extended in many directions with different empirical goals in mind. For example,

    1. multiple consumption goods; some with EV shocks, others not

- This framework can be easily extended in many directions with different empirical goals in mind. For example,

    1. multiple consumption goods; some with EV shocks, others not
    2. different shape or correlation structures on EV shocks

Extensions

- This framework can be easily extended in many directions with different empirical goals in mind. For example,

  1. multiple consumption goods; some with EV shocks, others not
  2. different shape or correlation structures on EV shocks
  3. others?

- This framework can be easily extended in many directions with different empirical goals in mind. For example,

    1. multiple consumption goods; some with EV shocks, others not
    2. different shape or correlation structures on EV shocks
    3. others?

- Today, we'll consider (1).

Suppose the decision problem of the household is

$$V(a) = \max_{i \in \{1,\dots,N\}, c_2} u_1(c_{1i}) + u_2(c_2 - \underline{c}_2) + u_2(a - c_{1i} - c_2) + \eta_i$$

subject to $c_{1i} + c_2 \leq a, \; c_2 \geq \underline{c}_2$

- **good 1 ("EVS good / luxury"):** subject to EV shocks as in baseline

Suppose the decision problem of the household is

$$V(a) = \max_{i \in \{1,\dots,N\}, c_2} u_1(c_{1i}) + u_2(c_2 - \underline{c}_2) + u_2(a - c_{1i} - c_2) + \eta_i$$

subject to $c_{1i} + c_2 \leq a, \; c_2 \geq \underline{c}_2$

- **good 1 ("EVS good / luxury"):** subject to EV shocks as in baseline
  - low risk aversion

A simple example: 2 goods, 2 periods

Suppose the decision problem of the household is

$$V(a) = \max_{i \in \{1,\ldots,N\}, c_2} u_1(c_{1i}) + u_2(c_2 - \underline{c}_2) + u_2(a - c_{1i} - c_2) + \eta_i$$

subject to $c_{1i} + c_2 \leq a, \; c_2 \geq \underline{c}_2$

- **good 1 ("EVS good / luxury"):** subject to EV shocks as in baseline
  - low risk aversion
- **good 2 ("non-EVS good / necessity"):** no EV shocks

Suppose the decision problem of the household is

$$V(a) = \max_{i \in \{1,\dots,N\}, c_2} u_1(c_{1i}) + u_2(c_2 - \underline{c}_2) + u_2(a - c_{1i} - c_2) + \eta_i$$

subject to $c_{1i} + c_2 \leq a, \; c_2 \geq \underline{c}_2$

- **good 1 ("EVS good / luxury"):** subject to EV shocks as in baseline
  - low risk aversion
- **good 2 ("non-EVS good / necessity"):** no EV shocks
  - high risk aversion, potentially minimum threshold $\underline{c}_2$

Suppose the decision problem of the household is

$$V(a) = \max_{i \in \{1,\dots,N\}, c_2} u_1(c_{1i}) + u_2(c_2 - \underline{c}_2) + u_2(a - c_{1i} - c_2) + \eta_i$$

subject to $c_{1i} + c_2 \leq a, \;\; c_2 \geq \underline{c}_2$

- **good 1 ("EVS good / luxury"):** subject to EV shocks as in baseline
  - low risk aversion
- **good 2 ("non-EVS good / necessity"):** no EV shocks
  - high risk aversion, potentially minimum threshold $\underline{c}_2$
- separable utility, future value depends only on remaining wealth

A simple example: 2 goods, 2 periods

Suppose the decision problem of the household is

$$V(a) = \max_{i \in \{1,\ldots,N\}, c_2} u_1(c_{1i}) + u_2(c_2 - \underline{c}_2) + u_2(a - c_{1i} - c_2) + \eta_i$$

subject to $c_{1i} + c_2 \leq a, \;\; c_2 \geq \underline{c}_2$

- **good 1 ("EVS good / luxury"):** subject to EV shocks as in baseline
  - low risk aversion
- **good 2 ("non-EVS good / necessity"):** no EV shocks
  - high risk aversion, potentially minimum threshold $\underline{c}_2$
- separable utility, future value depends only on remaining wealth
- relative price of 1 for now; trivial to change

Decision rules: 2 goods, 2 periods

- **"Fundamental:"** equate all margins and use up budget:

$$u_1'(c_1) = u_2'(c_2 - \underline{c}_2) = u_2'(a - c_1 - c_2 - \underline{c}_2)$$

Details

Decision rules: 2 goods, 2 periods

- **"Fundamental:"** equate all margins and use up budget:

$$u_1'(c_1) = u_2'(c_2 - \underline{c}_2) = u_2'(a - c_1 - c_2 - \underline{c}_2)$$

- **EVS:** trade off c_2, a' **residually** **for each** c_1:

$$v_i(a) \equiv u_1(c_{1i}) + \max_{\underline{c}_2 \leq c_2 \leq a - c_{1i} - \underline{c}_2} u_2(c_2) + u_2(a - c_{1i} - c_2)$$

$$\Longrightarrow u_2'\left(c_2^*(c_1)\right) = u_2'\left(a - c_{1i} - c_2^*(c_1) - \underline{c}_2\right)$$

▸ Details

- **"Fundamental:"** equate all margins and use up budget:

$$u_1'(c_1) = u_2'(c_2 - \underline{c}_2) = u_2'(a - c_1 - c_2 - \underline{c}_2)$$

- **EVS:** trade off c_2, a' **residually** **for each** c_1:

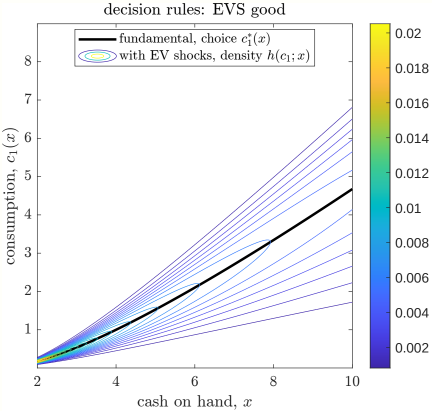
$$v_i(a) \equiv u_1(c_{1i}) + \max_{\underline{c}_2 \leq c_2 \leq a - c_{1i} - \underline{c}_2} u_2(c_2) + u_2(a - c_{1i} - c_2)$$

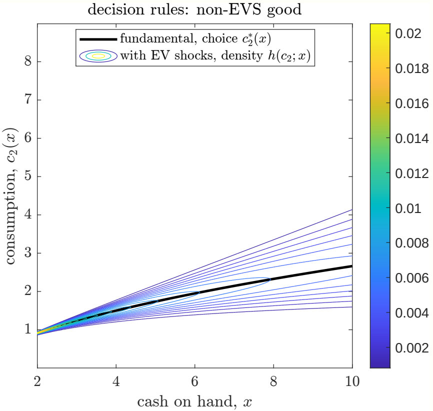
$$\implies u_2'\left(c_2^*(c_1)\right) = u_2'\left(a - c_{1i} - c_2^*(c_1) - \underline{c}_2\right)$$

- Then ex-ante value, decision rules defined as in the baseline.

▸ Details

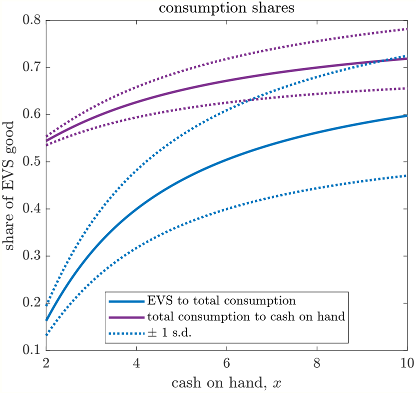
Decision contours: 2 goods, 2 periods

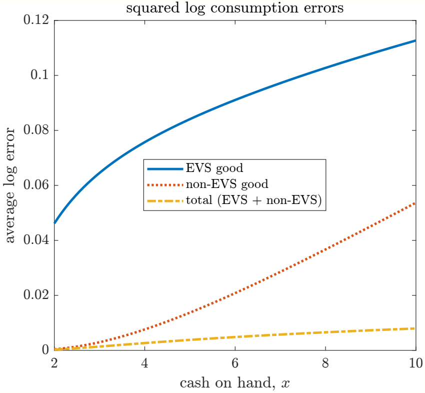




▸ What if preferences are the same?

Shares and errors: 2 goods, 2 periods





Conclusion and Future Directions

Conclusion

We have developed a theory of structural, extreme value preference shocks that imply precautionary savings. This is a new tool.

- very different from shocks to marginal utility

Lots of ways to embed in existing frameworks – complementary to existing types of shocks in precautionary savings models.

Conclusion

We have developed a theory of structural, extreme value preference shocks that imply precautionary savings. This is a new tool.

- very different from shocks to marginal utility
- well-behaved and implies comprehensible formulas

Lots of ways to embed in existing frameworks – complementary to existing types of shocks in precautionary savings models.

Conclusion

We have developed a theory of structural, extreme value preference shocks that imply precautionary savings. This is a new tool.

- very different from shocks to marginal utility
- well-behaved and implies comprehensible formulas
- strong predictions about Euler equation errors as a function of wealth

Lots of ways to embed in existing frameworks – complementary to existing types of shocks in precautionary savings models.

Conclusion

We have developed a theory of structural, extreme value preference shocks that imply precautionary savings. This is a new tool.

- very different from shocks to marginal utility
- well-behaved and implies comprehensible formulas
- strong predictions about Euler equation errors as a function of wealth
    - these predictions are confirmed by data

Lots of ways to embed in existing frameworks – complementary to existing types of shocks in precautionary savings models.

We have developed a theory of structural, extreme value preference shocks that imply precautionary savings. This is a new tool.

- very different from shocks to marginal utility
- well-behaved and implies comprehensible formulas
- strong predictions about Euler equation errors as a function of wealth
    - these predictions are confirmed by data
    - can be used to estimate the key parameter of the EV process

Lots of ways to embed in existing frameworks – complementary to existing types of shocks in precautionary savings models.

We have developed a theory of structural, extreme value preference shocks that imply precautionary savings. This is a new tool.

- very different from shocks to marginal utility
- well-behaved and implies comprehensible formulas
- strong predictions about Euler equation errors as a function of wealth
  - these predictions are confirmed by data
  - can be used to estimate the key parameter of the EV process
- strong precautionary motive: the variance of earnings risk needs to increase by more than 25% to match

Lots of ways to embed in existing frameworks – complementary to existing types of shocks in precautionary savings models.

Thank you Very Much

Log case: derivation

Guess and verify $V(a) = A \ln a + B$, which implies

$$V(a) = \alpha \ln \int_0^a c^{\frac{1}{\alpha}} (a-c)^{\frac{\beta A}{\alpha}} dc + \beta A \ln(1+r) + \beta B + \alpha \ln a$$

Then the change of variables $y = c/a$ implies

$$V(a) = (1 + \beta A + 2\alpha) \ln a + \alpha \underbrace{\ln \int_0^1 y^{\frac{1}{\alpha}} (1-y)^{\frac{\beta A}{\alpha}} dy}_{=\mathcal{B}(1/\alpha+1, \beta A/\alpha+1)} + \beta A \ln(1+r) + \beta B$$

where $\mathcal{B}$ is the beta function. Proceeding, we obtain

$$\begin{aligned} A &= \frac{1+2\alpha}{1-\beta} \\ B &= \frac{\alpha}{1-\beta} \ln \mathcal{B}\left(\frac{1}{\alpha}+1, \frac{\beta(1+2\alpha)}{\alpha(1-\beta)}+1\right) + \frac{\beta}{1-\beta}\frac{1+2\alpha}{1-\beta}\ln(1+r) \end{aligned}$$

▸ Back to log case main ▸ Decision rule

Log case: decision rule

By plugging in the form of the value function from the log case, we obtain

$$h(c;a) = \frac{1}{a}\frac{\left(\frac{c}{a}\right)^{p-1}(\left(1-\frac{c}{a}\right)^{q-1}}{B} \sim \mathcal{B}(p,q;[0,a])$$

- $p = \frac{1}{\alpha} + 1$ and $q = \frac{\beta(1+2\alpha)}{\alpha(1-\beta)} + 1$ are the shape parameters
- B is the constant from the previous slide
- $\mathcal{B}(p,q;[0,a])$ is the (generalized) beta distribution with shape parameters p and q defined over the extended interval $[0,a]$

▸ Back to log case main ▸ Back to log case derivation

MU failure details (I): form of the value function

If we guess that $V(x,\theta) = A(\theta)\frac{x^{1-\gamma}}{1-\gamma}$ for a set of constants $A(\theta)$ with mean $\overline{A} = \sum_\theta \pi(\theta)A(\theta)$. Then, solving the Euler equation yields

$$\frac{c}{(1+r)(x-c)} = \underbrace{\left[\frac{\beta(1+r)\overline{A}}{\theta}\right]^{-\frac{1}{\gamma}}}_{\equiv\Gamma(\theta;\overline{A})} \implies c^*(x,\theta) = \underbrace{\frac{(1+r)\Gamma(\overline{A},\theta)}{1+(1+r)\Gamma(\overline{A},\theta)}}_{\equiv\Lambda(\theta;\overline{A})} x$$

Tomorrow's cash on hand will be

$$x'^*(x,\theta) = (1+r)(x - c^*(x,\theta)) = \underbrace{(1+r)(1-\Lambda(\theta;\overline{A}))}_{\equiv\Delta(\theta;\overline{A})} x$$

and so under the guess of $V(x,\theta)$ (which implies $\overline{V}(x) = \sum_\theta \pi(\theta)V(x,\theta) = \overline{A}\frac{x^{1-\gamma}}{1-\gamma}$),

$$\begin{aligned} \max_c \theta u(c) + \beta\overline{V}((1+r)(x-c)) &= \theta\frac{(c^*)^{1-\gamma}}{1-\gamma} + \beta\overline{A}\frac{(x'^*)^{1-\gamma}}{1-\gamma} \\ \implies A(\theta)\frac{x^{1-\gamma}}{1-\gamma} &= \left[\theta\Lambda(\theta;\overline{A})^{1-\gamma} + \beta\Delta(\theta;\overline{A})^{1-\gamma}\right]\frac{x^{1-\gamma}}{1-\gamma} \end{aligned}$$

Given N levels of θ and existing expressions for $\overline{A}$, Λ, and Δ, this is a system of N equations in N unknowns (the $A(\theta)$), and so it must have a unique solution.

MU failure details (II): figure

- MU shocks affect consumption share of wealth along wealth distribution in a homogenous fashion

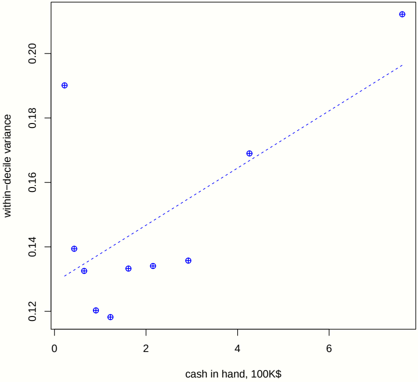
- make the log consumption figure streamlined, include analog for EV case.

Parameters

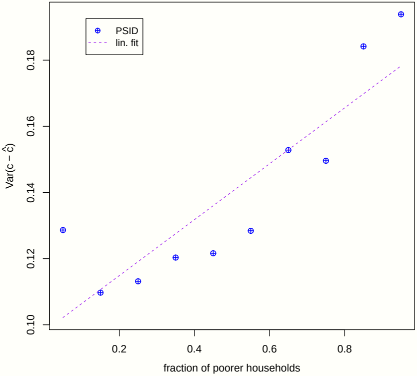
▸ Back: alternative economies

parameter	model		value	notes
CRRA		γ	2.0	standard
subjective discount factor		β	0.96	standard for annual model
capital share		λ	0.30	"
depreciation rate		δ	0.072	"
STY (2004) earnings process				
standard deviation, perm comp.	STY	$\sigma(\epsilon_1)$		log-normal, 5-point discret
persistence, persi comp.	STY	$\rho(\epsilon_2)$		AR(1), 10-point discret
st dev, pers comp.	STY	$\sigma(\epsilon_2)$		normally distributed innovation
st dev, transitory comp.	STY	$\sigma(\epsilon_3)$		log-normal, 5-point discret
specific to certain model variant				
coef. of variation, labor productivity	ER	$\sigma(\zeta)$	0.2	2/3 or 1% precautionary savings
coef. of variation, marginal utility	MUR	$\sigma(\theta)$	0.328	match r from ER economy
scale parameter, simple model	EVS	α	0.048	"
scale parameter, full model	EVS+STY	$\tilde{\alpha}$	0.114	calibration to PSID data
augmented transt earnings risk	STY aug	$\sigma(\tilde{\epsilon}_3)$	0.456	match r from EVS+STY Ec
augmented marg ut risk	MUR+STY	$\sigma(\tilde{\theta})$	0.465	match r from EVS+STY Ec

Figure: empirical results



(a) By decile mean of cash on hand



(b) By decile mean of cash on hand

Back

More on simple 2-good case (I)

Assume the following functional forms:

- **EVS good:** $u_1(c_1) = \frac{c_1^{1-\gamma_1}}{1-\gamma_1}$, γ_1 low
- **non-EVS good:** $u_2(c_2) = \frac{(c_2-\underline{c}_2)^{1-\gamma_2}}{1-\gamma_2}$, γ_2 high
  - $\underline{c}_2 \geq 0$: floor to capture the "necessity" nature of this good
  - $\implies c_1 \leq a - \underline{c}_2$, since an Inada condition holds at $\underline{c}_2$ rather than 0
- **tomorrow:** $u_3(c') = \frac{(c')^{1-\gamma'}}{1-\gamma'}$, $\gamma' \in [\gamma_1, \gamma_2]$ (or just non-EVS)

Fundamental solution: equalize marginal utilities and use up budget

$$c_1^{-\gamma_1} = (c_2 - \underline{c}_2)^{-\gamma_2} = (a - c_1 - c_2)^{-\gamma'}$$

$$\implies c_2 = \underline{c}_2 + c_1^{\frac{\gamma_1}{\gamma_2}} \implies c_1 + c_1^{\frac{\gamma_1}{\gamma_2}} + c_1^{\frac{\gamma_1}{\gamma'}} = a - \underline{c}_2$$

Can solve for c_1 via bisection, then plug into c_2 expression.

More on simple 2-good case (II)

EVS solution: equalize marginal utilities only for non-EVS good and future consumption, use up budget

$$(c_2 - \underline{c}_2)^{-\gamma_2} = (a - c_{1i} - c_2)^{-\gamma'}$$

Can solve for $c_2^*(c_1)$ via bisection, then plug back into budget to get $a'^*(c_1)$

Formulas: 2 goods, 2 periods

The ex-ante value function and decision rules can then be defined as in the baseline:

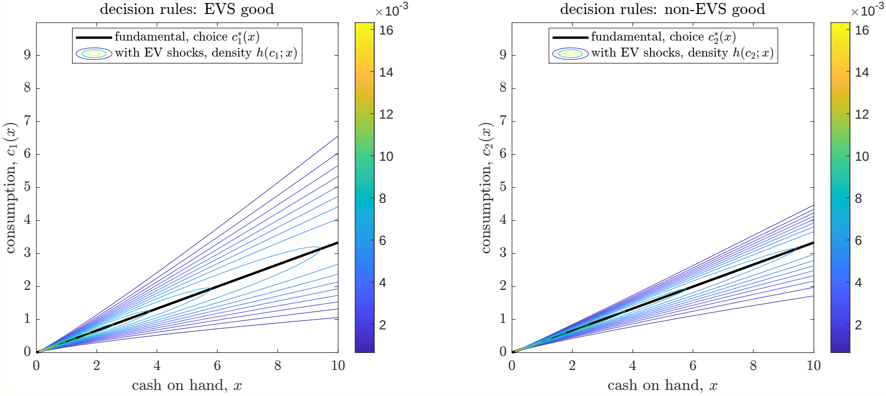
$$V(a) = \alpha \ln \int_0^a \exp\left(\frac{v_{c_1}(a)}{\alpha}\right) dc_1 + \alpha \ln a$$

$$h(c_1; a) = \frac{\exp\left(\frac{v_{c_1}(a)}{\alpha}\right)}{\int_0^a \exp\left(\frac{v_{c_1}(a)}{\alpha}\right) dc_1}$$

Note that the density over c_1 induces a density over c_2 via $c_2^*(c_1)$.

▸ Back

Decision contours: 2 goods, 2 periods, same $u(\cdot)$ function



Back