Sources of Lifetime Inequality

Mark Huggett, Gustavo Ventura and Amir Yaron (AER 2011)

February 2022

Research Question

- To what degree is lifetime inequality due to differences across people established early in life as opposed to differences in luck experienced over the working life time?
- Among the individual initial differences, which ones are the most important?
  - Initial human capital
  - Initial wealth
  - Learning ability

Motivation

Why this is relevant?

- To contrast the potential importance of policies related to initial conditions (e.g. public education) against those directed at shocks over the working lifetime (e.g unemployment insurance).
- A discussion of lifetime inequality cannot go too far before discussing which specific type of initial condition is the most critical for determining how one fares in life.
- A useful framework for answering these questions should also be central in the analysis of a wide range of policies.

Motivation

How is the question answered

- They view lifetime inequality through the lens of a risky human capital model:
    - Human capital and labor earnings are risky, as human capital is subject to idiosyncratic shocks each period.
- Agents differ in terms of three initial conditions:
    - Initial human capital
    - Learning ability
    - Financial wealth

Motivation: How

- They ask the model to account for key features of the dynamics of the earnings distribution for cohorts of US males (how mean earnings and measures of earnings dispersion and skewness evolve for cohorts).
- Two forces within the model account for the increase in earnings dispersion:
    - Agents differ in learning ability
    - Agents differ in idiosyncratic human capital shocks received over the lifetime
- They build a benchmark model to identify the contribution of each of these forces.

Motivation: How

Benchmark Model

- Shocks: The model implies that late in life little or no new human capital is produced. Therefore, moments of the change in wage rates for these agents are almost entirely determined by shocks.
- They estimate the shock process using precisely those moments for older males in US data. Given an estimate of the shock process and other model parameters, they choose the initial distribution of financial wealth, human capital, and learning ability across agents to best match the earnings facts seen on the data.
- Caviat: The results for variation in human capital at age 23 need to be understood as applying at that age.

Model

An agent maximizes expected lifetime utility, taking initial financial wealth k_1, initial human capital h_1, and learning ability a as given.
The decision problem for an agent born at time t is

$$\max_{\{c_j, l_j, s_j, h_j, k_j\}_{j=1}^{J}} E\left[\sum_{j=1}^{J} \beta^{j-1} u(c_j)\right] \text{ subject to}$$

$$\text{(i)}\quad c_j + k_{j+1} = k_j(1 + r_{t+j-1}) + e_j - T_{j,t+j-1}(e_j, k_j), \forall j \text{ and } k_{J+1} = 0.$$

$$\text{(ii)}\quad e_j = R_{t+j-1} h_j l_j \text{ if } j < J_R, \text{ and } e_j = 0 \text{ otherwise.}$$

$$\text{(iii)}\quad h_{j+1} = \exp(z_{j+1}) H(h_j, s_j, a) \text{ and } l_j + s_j = 1, \forall j.$$

Model

- The only source of risk to an agent over the working lifetime comes from idiosyncratic shocks to an agent's human capital. Let $\mathbf{z}^j = (z_1, ..., z_j)$ denote the j-period history of these shocks. Thus, the optimal consumption choice $c_{j,t+j-1}(\mathbf{x}_1, z^j)$ for an age j agent at time $t+j-1$ is risky as it depends on shocks $\mathbf{z}^j$ as well as initial conditions $\mathbf{x}_1 = (h_1, k_1, a)$.
- Although the model has a single source of shocks, which are independently and identically distributed over time, this structure is sufficient to endogenously produce many of the statistical properties of earnings that researchers have previously estimated.
- They embed the agent's decision problem within a general equilibrium framework and focus on balanced-growth equilibria.

Balanced Growth Equilibria

DEFINITION: *A balanced-growth equilibrium is a collection of decisions* $\{\{c_{j,t}, l_{j,t}, s_{j,t}, h_{j,t}, k_{j,t}\}_{j=1}^{J}\}_{t=-\infty}^{\infty}$*, factor prices, government spending and taxes* $\{R_t, r_t, G_t, T_t\}_{t=-\infty}^{\infty}$*, and a distribution* ψ *over initial conditions such that*

(1) *Agent decisions are optimal, given factor prices.*

(2) *Competitive Factor Prices:* $R_t = A_t F_2(K_t, L_t A_t)$ *and* $r_t = F_1(K_t, L_t A_t) - \delta$.

(3) *Resource Feasibility:* $C_t + K_{t+1}(1+n) + G_t = F(K_t, L_t A_t) + K_t(1-\delta)$.

(4) *Government Budget:* $G_t = T_t$.

(5) *Balanced Growth: (i)* $\{c_{j,t}, k_{j,t}\}_{j=1}^{J}$ *grow at rate g as a function of time, whereas* $\{l_{j,t}, s_{j,t}, h_{j,t}\}_{j=1}^{J}$ *are time invariant. (ii)* (G_t, T_t, R_t) *grow at rate g, whereas* r_t *is time invariant.*

Empirical Analysis

- They use data to address two issues
  - To characterize how mean earnings and measures of earnings dispersion and skewness evolve with age for a cohort.
  - To estimate a human capital shock process from wage rate data.
- They estimate age profiles for mean earnings and measures of earnings dispersion and skewness for ages 23 to 60 using earnings data for males who are the head of the household from the Panel Study of Income Dynamics (PSID) 1969–2004 family files.
- Two measures of dispersion: the variance of log earnings and the Gini coefficient of earnings.
- They measure skewness by the ratio of mean earnings to median earnings.

Empirical Analysis:

Age profiles for mean earnings and measures of earnings dispersion and skewness

- Age effects: They calculate the statistic of interest $stat_{j,t}$ (for example $stat_{j,t} = ln(e_{jt})$) and propose the following statistical model for the evolution of it:

$$stat_{j,t} = \alpha_c^{stat} + \beta_j^{stat} + \gamma_t^{stat} + \epsilon_{j,t}^{stat}$$

  The earnings statistic is generated by several factors: cohort (c), age (j), and time (t) effects.

- They wish to estimate the age effects β_j^{stat}.
- There's a colinearity problem ($c = t - j$), so they provide two alternative measures for age effects: **cohort effects view** where they set $\gamma_t^{stat} = 0$, $\forall t$ and **time effects view** where they set $\alpha_c^{stat} = 0$, $\forall c$.
- Use ordinary least squares to estimate the coefficients.

Empirical Analysis

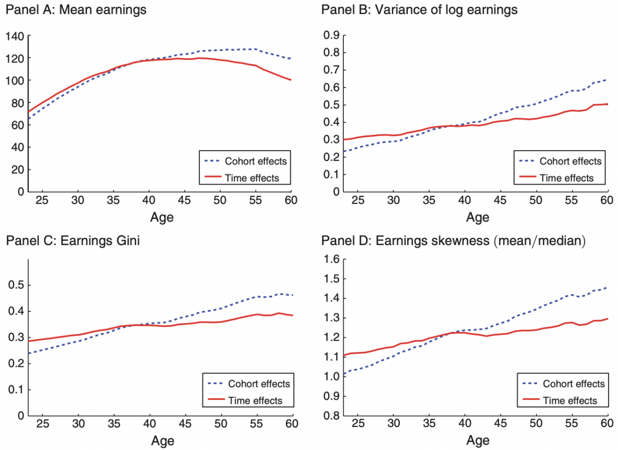

FIGURE 1. MEAN, DISPERSION, AND SKEWNESS OF EARNINGS BY AGE

They will ask the economic model to match both views of the evolution of the earnings distribution.

Human Capital Shocks

- Agent's wage rate (earnings per unit of work time), equals the product of the rental rate and an agent's human capital.
- Work time and learning time are distinct activities.
- The model also implies that late in the working lifetime human capital investments are approximately zero (the number of working periods over which an agent can reap the returns to these investments falls as the agent approaches retirement).
- The upshot is that when there is no human capital investment, then the change in an agent's wage rate is in theory entirely determined by rental rates and the human capital shock process and not by any other model parameters.

Human Capital shocks

- Assume that in periods t through $t+n$ an individual devotes zero time to learning.
- wage w_{t+n} is determined by the rental rate R_{t+n}, shocks $(z_{t+1},...,z_{t+n})$, and human capital h_t.
  $h_{t+1} = exp(z_{t+1})H(h_t, s_t, a) = exp(z_{t+1})[h_t + f(h_t, s_t, a)]$ and $f(h, s, a) = 0$ when $s = 0$.

$$
\begin{aligned}
\omega_{t+n} =& R_{t+n}h_{t+n} \\
=& R_{t+n}exp(z_{t+n})H(h_{t+n-1}, 0, a) \\
=& R_{t+n}\Pi_{i=1}^{n} exp(z_{t+i})h_t
\end{aligned}
$$

- Taking logs

$$
\hat{\omega}_{t+n} = ln(w_{t+n}) = \hat{R}_{t+n} + \sum_{i=1}^{n} z_{t+i} + \hat{h}_t
$$

Human Capital shocks

- Measured n-period log wage differences (denoted $y_{t,n}$) are true log wage differences plus measurement error differences $\epsilon_{t+n} - \epsilon_t$

$$y_{t,n} = \hat{\omega}_{t+n} - \hat{\omega}_t + \epsilon_{t+n} - \epsilon_t = \hat{R}_{t+n} - \hat{R}_t + \sum_{i=1}^{n} z_{t+i} + \epsilon_{t+n} - \epsilon_t$$

  thus log wage differences are due solely to rental rate differences and shocks.

- They assume that both human capital shocks z_t and measurement errors ϵ_t are independent and identically distributed over time and people. Furthermore, we assume that $z_t \sim N(\mu, \sigma^2)$ and $var(\epsilon_t) = \sigma_\epsilon^2$.

- The assumptions made imply the following cross-sectional moment conditions:

$$E[y_{t,n}] = \hat{R}_{t+n} - \hat{R}_t + n\mu$$
$$var(y_{t,n}) = n\sigma^2 + 2\sigma_\epsilon^2$$
$$cov(y_{t,n}, y_{t,m}) = m\sigma^2 + \sigma_\epsilon^2 \quad \text{for} \quad m < n.$$

- They follow males for four years and thus calculate three log wage differences $(n = 1, 2, 3)$.
- They use all cross-sectional variances and all cross-sectional covariances aggregated across panel years. For each year they generate the sample analog to the moments:

$$\mu_{t,n} \equiv \frac{1}{N_t} \sum_{i=1}^{N_t} y_{t,n}^i \quad \sum_{i=1}^{N_t} (y_{t,n}^i - \mu_{t,n})^2 \quad \sum_{i=1}^{N_t} (y_{t,n}^i - \mu_{t,n})(y_{t,m}^i - \mu_{t,m})$$

They stack the moments across the panel years and use a two-step General Method of Moments estimation with an identity matrix as the initial weighting matrix.

Estimation of Human Capital Shocks

TABLE 1—ESTIMATION OF HUMAN CAPITAL SHOCKS

Age range	Period	$e_{\min}$	$e_{\max}$	N	σ	SE(σ)	σ_ϵ	SE(σ_ϵ)
55–65	1969–2004	2,000	1.8 M	103	0.111	0.007	0.137	0.005
50–60	1969–2004	2,000	1.8 M	199	0.117	0.006	0.142	0.004

- The point estimate for the age 55–65 sample is $\sigma = 0.111$ so that a one standard deviation shock moves wages by about 11%.
- This is the shock estimate that they employ in the analysis of lifetime inequality.

Setting parameters

- The first collection of model parameters is set without solving the model.
- The remaining model parameters are set so that the equilibrium properties of the model best match the earnings distribution facts while matching some steady-state quantities.
- The parameters of the shock process are (μ, σ). The standard deviation of human capital shocks is set to $\sigma = 0.111$ based on the estimate from Table 1. The mean is set to be $\mu = 0.029$, so that the model matches the average rate of decline of mean earnings for the cohorts of older workers in US data.

Human Capital Profile

Figure 2. Mean, Dispersion, and Skewness of Earnings by Age: Time Effects

Frame Title

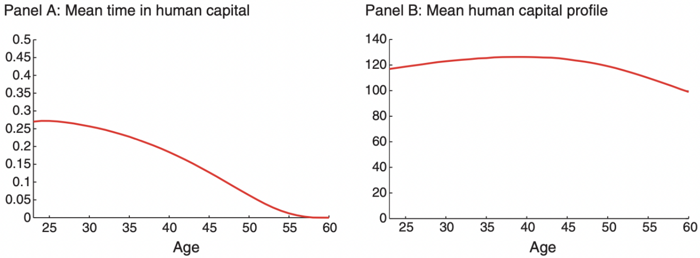

FIGURE 3. PROPERTIES OF HUMAN CAPITAL BY AGE

- That the mean human capital profile is flatter than the earnings profile means that average human capital as of age 23 is quite high. This is a key reason of why they find next that human capital differences are such an important (at age 23) compared to ability differences.

Benchmark Model

- What's the quantitative importance of risk and ability differences for producing the increase in earnings dispersion in the benchmark model?
- They answer it by either eliminating ability differences (everyone gets the median ability) or eliminating shocks.
- Eliminating ability differences flattens the rise in the variance of log earnings with age. Earnings dispersion actually falls over part of the working lifetime.
- Without risk and without ability differences, all agents within an age group produce the same amount of new human capital regardless of the current level of human capital.

Benchmark Model: Decomposed

- They eliminate idiosyncratic risk by setting $\sigma = 0$. They adjust the mean log shock μ to keep the mean shock level constant.
- Removing idiosyncratic risk leads to a counterclockwise rotation of the mean earnings profile and an L-shaped earnings dispersion profile.
- When idiosyncratic risk is eliminated, human capital accumulation becomes more attractive for risk averse agents. Thus, all else equal, agents spend a greater fraction of time accumulating human capital early in life.
- Eliminating risk results in substantial changes in the time allocation decisions of agents with relatively high learning ability. They allocate an even larger fraction of time into human capital accumulation. This leads to very high earnings dispersion early in life.

Benchmark Model: Decomposed

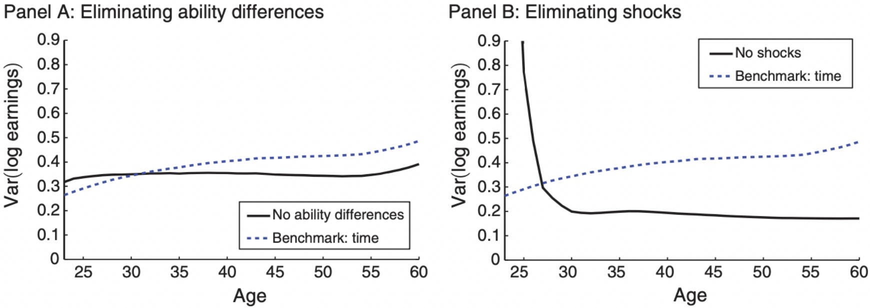

FIGURE 4. DISPERSION OF EARNINGS BY AGE

Statistical Model of earnings

- They generate data from the model and ask what an empirical researcher would find (using the common way of viewing earning).
- An empirical researcher would conclude that their human capital model produces coefficients governing persistent shocks that are similar to results found using US data.
- One should be cautious in making statements about the true nature of shocks based on information derived from such statistical models.
    - Shocks in the human capital model are independent and identically distributed over time, but produce what an empirical researcher might describe as persistent earnings shocks.
- In sum, their benchmark model, with shocks inferred from the wage rates of older workers, is broadly consistent with the dynamics of earnings and earnings growth rates documented in the literature.

Results

TABLE 5—SOURCES OF LIFETIME INEQUALITY

Statistic	Benchmark model	Benchmark model with initial wealth differences
Fraction of variance in lifetime utility due to initial conditions	0.640	0.661
Fraction of variance in lifetime earnings due to initial conditions	0.615	0.613
Fraction of variance in lifetime wealth due to initial conditions	0.615	0.626

Variable	Change in variable	Equivalent variation (%)
Human capital	+1 st. deviation	39.3
	−1 st. deviation	−28.3
Learning ability	+1 st. deviation	5.7
	−1 st. deviation	−2.6
Initial wealth	+1 st. deviation	7.1
	−1 st. deviation	−1.6

- Analyze the importance of different initial conditions by asking an agent how much compensation is equivalent to starting at age 23 with a one standard deviation change in any initial condition, other things equal.
- One standard deviation movement in log human capital is substantially more important than a one in either log learning ability or log initial wealth.

Summary

Contribution

- Quick summary: They determine age affect, estimate shocks, select parameters in order to build a benchmark model. With that they can decompose the variance. finally they generate data from the model and study what an empirical researcher would find with that data.
- This paper offers a quantitative model that works as a bridge between the macroeconomic literature with incomplete markets and the human capital literature. Provides an alternative workhorse model for quantitative work and policy analysis.
- Gives awareness on how to interpret data.

Concerns

- 23 is to old to make the analysis.
- iid shocks every period and same for everyone