

# 8200, 2023: basic Life Cycle Model with Intangible Capital

---

José Víctor Ríos Rull (joint with Vincenzo Quadrini)

Penn 2023

- Large Increase in Intangible Capital vs Tangible Capital

- Large Increase in Intangible Capital vs Tangible Capital
- Intangible Capital is more mobile across countries and can evade capital taxation more than tangible Capital

- Large Increase in Intangible Capital vs Tangible Capital
- Intangible Capital is more mobile across countries and can evade capital taxation more than tangible Capital
- What are the implications for the determination of capital income tax rates across countries?

- Large Increase in Intangible Capital vs Tangible Capital
- Intangible Capital is more mobile across countries and can evade capital taxation more than tangible Capital
- What are the implications for the determination of capital income tax rates across countries?
- Is it good to coordinate across countries?

- Large Increase in Intangible Capital vs Tangible Capital
- Intangible Capital is more mobile across countries and can evade capital taxation more than tangible Capital
- What are the implications for the determination of capital income tax rates across countries?
- Is it good to coordinate across countries?
- Answer these questions without assuming commitment on the part of governments.

# MODEL 1

---

- Households stay in country  $j$  like  $u(c, h)$  and discount  $\beta$

# MODEL 1

- Households stay in country  $j$  like  $u(c, h)$  and discount  $\beta$
- Country  $j$  multinationals (owned by its citizens). Observable output and inputs.

$$y^j = Q^j(m^{j,1}, m^{j,2}) \quad q^{j,i} = \frac{\partial Q^j(m^{j,1}, m^{j,2})}{\partial m^{j,i}}.$$

$m^{j,i}$  intermediate input from country  $i \in \{1, 2\}$  for  $Q^j(., .)$  HD1. Only trade is via  $m^{j,i}$



- Households stay in country  $j$  like  $u(c, h)$  and discount  $\beta$
- Country  $j$  multinationals (owned by its citizens). Observable output and inputs.

$$y^j = Q^j(m^{j,1}, m^{j,2}) \quad q^{j,i} = \frac{\partial Q^j(m^{j,1}, m^{j,2})}{\partial m^{j,i}}$$

$m^{j,i}$  intermediate input from country  $i \in \{1, 2\}$  for  $Q^j(., .)$  HD1. Only trade is via  $m^{j,i}$

- 

$$m^{j,i} = F(k_{j,i}, x_{j,i}, \ell_{j,i}) = z \left( k_{j,i}^\alpha x_{j,i}^{1-\alpha} \right)^\theta \ell_{j,i}^{1-\theta},$$

$k$  is tangible,  $x$  intangible capital,  $\ell$  labor of multinational  $j$  in country  $n$ .

- Households stay in country  $j$  like  $u(c, h)$  and discount  $\beta$
- Country  $j$  multinationals (owned by its citizens). Observable output and inputs.

$$y^j = Q^j(m^{j,1}, m^{j,2}) \quad q^{j,i} = \frac{\partial Q^j(m^{j,1}, m^{j,2})}{\partial m^{j,i}}$$

$m^{j,i}$  intermediate input from country  $i \in \{1, 2\}$  for  $Q^j(., .)$  HD1. Only trade is via  $m^{j,i}$

- $$m^{j,i} = F(k_{j,i}, x_{j,i}, \ell_{j,i}) = z \left( k_{j,i}^\alpha x_{j,i}^{1-\alpha} \right)^\theta \ell_{j,i}^{1-\theta},$$
  
 $k$  is tangible,  $x$  intangible capital,  $\ell$  labor of multinational  $j$  in country  $n$ .

- Profits are country specific

$$\pi^{j,i} = q^{j,i} F(k_{j,i}, x_{j,i}, \ell_{j,i}) - w^i \ell_{j,i} - \delta(k_{j,i} + x_{j,i}),$$

- Households stay in country  $j$  like  $u(c, h)$  and discount  $\beta$
- Country  $j$  multinationals (owned by its citizens). Observable output and inputs.

$$y^j = Q^j(m^{j,1}, m^{j,2}) \quad q^{j,i} = \frac{\partial Q^j(m^{j,1}, m^{j,2})}{\partial m^{j,i}}$$

$m^{j,i}$  intermediate input from country  $i \in \{1, 2\}$  for  $Q^j(., .)$  HD1. Only trade is via  $m^{j,i}$

- $$m^{j,i} = F(k_{j,i}, x_{j,i}, \ell_{j,i}) = z \left( k_{j,i}^\alpha x_{j,i}^{1-\alpha} \right)^\theta \ell_{j,i}^{1-\theta}$$
  
 $k$  is tangible,  $x$  intangible capital,  $\ell$  labor of multinational  $j$  in country  $n$ .

- Profits are country specific

$$\pi^{j,i} = q^{j,i} F(k_{j,i}, x_{j,i}, \ell_{j,i}) - w^i \ell_{j,i} - \delta(k_{j,i} + x_{j,i}),$$

- Aggregate state is  $s$ . A  $j$  firm's state is  $\{k_{j,1}, k_{j,2}, x_j\}$ . Both capitals are different

- $\tau^{A,j}$ , Taxes on Corporation's Headquarters residual income  $\pi^{A,j}$ ,

- $\tau^{A,j}$ , Taxes on Corporation's Headquarters residual income  $\pi^{A,j}$ ,
- $\tau^{K,j}$ , Taxes on capital income  $\pi^{K,i,j}$ , (intermediate output net of labor income)

- $\tau^{A,j}$ , Taxes on Corporation's Headquarters residual income  $\pi^{A,j}$ ,
- $\tau^{K,j}$ , Taxes on capital income  $\pi^{K,i,j}$ , (intermediate output net of labor income)
- $\tau^{\ell,j}$ , Taxes on Labor income where it is used  $\pi^{j,i}$ , (Capital income where produced)

# FIRM'S PROBLEM: STATE $\{k_1, k_2, x\}$

- Dynamic Problem

$$\begin{aligned} V^j(\mathbf{s}, k_1, k_2, x; \Psi) &= \max_{i_1, i_2, n} \left\{ d + (R^j)^{-1} V^j(\mathbf{s}', k'_1, k'_2, x'; \Psi) \right\} \\ &\text{s.t.} \\ d &= (1 - \tau_j^A) \left[ (1 - \tau_1^K) \pi^{j,1} + (1 - \tau_2^K) \pi^{j,2} \right] - i_1 - i_2 - n, \\ k'_i &= k_i + i_i, \quad i \in \{1, 2\} \\ x' &= (1 - \delta)x + n, \\ \tau &= \Psi(\mathbf{s}), \\ \mathbf{s}' &= \Phi(\mathbf{s}; \Psi). \end{aligned}$$

- Static part yields  $q^{j,i} F_\ell(k_{j,i}, x_{j,i}, \ell_{j,i}) = w^i$ ,  $i \in \{1, 2\}$  and

$$\left[ q^{j,1} F_x(k_{j,1}, x_{j,1}, \ell_{j,1}) - \delta \right] (1 - \tau^{K,1}) = \left[ q^{j,2} F_x(k_{j,2}, x_{j,2}, \ell_{j,2}) - \delta \right] (1 - \tau^{K,2}).$$



- Static part yields  $q^{j,i} F_\ell(k_{j,i}, x_{j,i}, \ell_{j,i}) = w^i$ ,  $i \in \{1, 2\}$  and

$$\left[ q^{j,1} F_x(k_{j,1}, x_{j,1}, \ell_{j,1}) - \delta \right] (1 - \tau^{K,1}) = \left[ q^{j,2} F_x(k_{j,2}, x_{j,2}, \ell_{j,2}) - \delta \right] (1 - \tau^{K,2}).$$

- Dynamic FOCS

$$\left[ 1 + (1 - \tau_{j,t}^A)(1 - \tau_{i,t}^K) \frac{\partial \pi^{j,i'}}{\partial k'_i} \right] = R^j,$$

$$\left[ 1 + (1 - \tau_{j,t}^A)(1 - \tau_{i,t}^K) \frac{\partial \pi^{j,i'}}{\partial x'} \right] = R^j,$$

with  $i \in \{1, 2\}$ .

$$\begin{aligned}\Omega^j(\mathbf{s}, a; \Psi) &= \max_{c, h, a'} \left\{ u(c, h) + \beta \Omega^j(\mathbf{s}', a'; \Psi) \right\} \quad \text{s.t.} \\ c &= (1 - \tau^{Lj}) w^j h + (d^j + p^j) a + T^j - p^j a', \\ \tau &= \Psi(\mathbf{s}), \\ \mathbf{s}' &= \Phi(\mathbf{s}; \Psi),\end{aligned}$$

with FOCs

$$\begin{aligned}-u_h(c, h) &= w^j (1 - \tau^{Lj}) u_c(c, h) \\ u_c(c, h) p^j &= \beta u_c(c', h') (d^{j'} + p^{j'}).\end{aligned}$$

- State  $\mathbf{s}$  is capital in each country.
- **Equilibrium** is standard and generates  $\mathbf{s}' = G(\mathbf{s}; \Psi)$ , when  $a^j = V^j(\mathbf{s}; \Psi)$

## THE DETERMINATION OF POLICIES: ARBITRARY TAXES TODAY

- Governments play Nash with each other. Take as given future policy **rules**

# THE DETERMINATION OF POLICIES: ARBITRARY TAXES TODAY

- Governments play Nash with each other. Take as given future policy **rules**
- Need to define notion of one-period-only deviation to  $\tau$

$$\begin{aligned}\widehat{V}_j(\mathbf{s}, k_1, k_2, x, \tau; \Psi) &= \max_{i_k, i_x} \left\{ d + \left( R^j(\mathbf{s}, \tau) \right)^{-1} V(\mathbf{s}', k'_1, k'_2, x'; \Psi) \right\} && \text{s.t.} \\ d &= \widehat{\pi}_j(\mathbf{s}, k_1, k_2, x, \tau) - i_1 - i_2 - i_x, \\ k'_i &= (1 - \delta)k_i + i_i, \\ x' &= (1 - \delta)x + i_x, \\ \mathbf{s}' &= \widehat{\Phi}(\mathbf{s}, \tau; \Psi), \\ \tau^L &= \widehat{\varphi}(\mathbf{s}, \tau; \Psi).\end{aligned}$$

# THE DETERMINATION OF POLICIES: ARBITRARY TAXES TODAY

- Governments play Nash with each other. Take as given future policy **rules**
- Need to define notion of one-period-only deviation to  $\tau$

$$\begin{aligned}\widehat{V}_j(\mathbf{s}, k_1, k_2, x, \tau; \Psi) &= \max_{i_k, i_x} \left\{ d + \left( R^j(\mathbf{s}, \tau) \right)^{-1} V(\mathbf{s}', k'_1, k'_2, x'; \Psi) \right\} & \text{s.t.} \\ d &= \widehat{\pi}_j(\mathbf{s}, k_1, k_2, x, \tau) - i_1 - i_2 - i_x, \\ k'_j &= (1 - \delta)k_j + i_j, \\ x' &= (1 - \delta)x + i_x, \\ \mathbf{s}' &= \widehat{\Phi}(\mathbf{s}, \tau; \Psi), \\ \tau^L &= \widehat{\varphi}(\mathbf{s}, \tau; \Psi).\end{aligned}$$

- Households

$$\begin{aligned}\widehat{\Omega}_j(\mathbf{s}, a, \tau; \Psi) &= \max_{c, h, a'} \left\{ u(c, h) + \beta \Omega_j(\mathbf{s}', a'; \Psi) \right\} & \text{s.t.} \\ c &= (1 - \tau_j^L)w_j h + (d_j + p_j)a + T_j - p_j a', \\ \mathbf{s}' &= \widehat{\Phi}(\mathbf{s}, \tau; \Psi), \\ \tau^L &= \widehat{\varphi}(\mathbf{s}, \tau; \Psi),\end{aligned}$$

# THE DETERMINATION OF POLICIES: ARBITRARY TAXES TODAY

- Governments play Nash with each other. Take as given future policy **rules**
- Need to define notion of one-period-only deviation to  $\tau$

$$\begin{aligned}\widehat{V}_j(\mathbf{s}, k_1, k_2, x, \tau; \Psi) &= \max_{i_k, i_x} \left\{ d + \left( R^j(\mathbf{s}, \tau) \right)^{-1} V(\mathbf{s}', k'_1, k'_2, x'; \Psi) \right\} & \text{s.t.} \\ d &= \widehat{\pi}_j(\mathbf{s}, k_1, k_2, x, \tau) - i_1 - i_2 - i_x, \\ k'_i &= (1 - \delta)k_i + i_i, \\ x' &= (1 - \delta)x + i_x, \\ \mathbf{s}' &= \widehat{\Phi}(\mathbf{s}, \tau; \Psi), \\ \tau^L &= \widehat{\varphi}(\mathbf{s}, \tau; \Psi).\end{aligned}$$

- Households

$$\begin{aligned}\widehat{\Omega}_j(\mathbf{s}, a, \tau; \Psi) &= \max_{c, h, a'} \left\{ u(c, h) + \beta \Omega_j(\mathbf{s}', a'; \Psi) \right\} & \text{s.t.} \\ c &= (1 - \tau_j^L)w_j h + (d_j + p_j)a + T_j - p_j a', \\ \mathbf{s}' &= \widehat{\Phi}(\mathbf{s}, \tau; \Psi), \\ \tau^L &= \widehat{\varphi}(\mathbf{s}, \tau; \Psi),\end{aligned}$$

- Yields equilibrium  $\widehat{G}(\mathbf{s}, \tau; \Psi)$



$$\max_{\tau^{Kj}} \widehat{\Omega}^j(\mathbf{s}, 1, \tau_1, \tau_2; \Psi) \quad \text{s.t.}$$

$$\tau^{K,1}(\pi^{1,1} + \pi^{2,1}) + \tau^{A,1} \left[ (1 - \tau^{K,1})\pi^{1,1} + (1 - \tau^{K,2})\pi^{1,2} \right] + \tau^{L,1} w^1 (L_{1,1} + L_{2,1}) = T^1,$$

$$\tau^{K,2}(\pi^{1,2} + \pi^{2,2}) + \tau^{A,2} \left[ (1 - \tau^{K,1})\pi^{2,1} + (1 - \tau^{K,2})\pi^{2,2} \right] + \tau^{L,2} w_2 (L_{1,2} + L_{2,2}) = T^2.$$

- $$\max_{\tau^{Kj}} \widehat{\Omega}^j(\mathbf{s}, 1, \tau_1, \tau_2; \Psi) \quad \text{s.t.}$$
$$\tau^{K,1}(\pi^{1,1} + \pi^{2,1}) + \tau^{A,1} \left[ (1 - \tau^{K,1})\pi^{1,1} + (1 - \tau^{K,2})\pi^{1,2} \right] + \tau^{L,1} w^1 (L_{1,1} + L_{2,1}) = T^1,$$
$$\tau^{K,2}(\pi^{1,2} + \pi^{2,2}) + \tau^{A,2} \left[ (1 - \tau^{K,1})\pi^{2,1} + (1 - \tau^{K,2})\pi^{2,2} \right] + \tau^{L,2} w_2 (L_{1,2} + L_{2,2}) = T^2.$$
- First Step is to get a Nash Equil of both Countries Policies.



- $$\max_{\tau^{Kj}} \widehat{\Omega}^j(\mathbf{s}, 1, \tau_1, \tau_2; \Psi) \quad \text{s.t.}$$
$$\tau^{K,1}(\pi^{1,1} + \pi^{2,1}) + \tau^{A,1} \left[ (1 - \tau^{K,1})\pi^{1,1} + (1 - \tau^{K,2})\pi^{1,2} \right] + \tau^{L,1} w^1 (L_{1,1} + L_{2,1}) = T^1,$$
$$\tau^{K,2}(\pi^{1,2} + \pi^{2,2}) + \tau^{A,2} \left[ (1 - \tau^{K,1})\pi^{2,1} + (1 - \tau^{K,2})\pi^{2,2} \right] + \tau^{L,2} w_2 (L_{1,2} + L_{2,2}) = T^2.$$
- First Step is to get a Nash Equil of both Countries Policies.
- Let them be  $\tau = \psi(\mathbf{s}, \tau; \Psi)$

- $$\max_{\tau^{Kj}} \widehat{\Omega}^j(\mathbf{s}, 1, \tau_1, \tau_2; \Psi) \quad \text{s.t.}$$
$$\tau^{K,1}(\pi^{1,1} + \pi^{2,1}) + \tau^{A,1} \left[ (1 - \tau^{K,1})\pi^{1,1} + (1 - \tau^{K,2})\pi^{1,2} \right] + \tau^{L,1} w^1 (L_{1,1} + L_{2,1}) = T^1,$$
$$\tau^{K,2}(\pi^{1,2} + \pi^{2,2}) + \tau^{A,2} \left[ (1 - \tau^{K,1})\pi^{2,1} + (1 - \tau^{K,2})\pi^{2,2} \right] + \tau^{L,2} w^2 (L_{1,2} + L_{2,2}) = T^2.$$
- First Step is to get a Nash Equil of both Countries Policies.
- Let them be  $\tau = \psi(\mathbf{s}, \tau; \Psi)$
- The equilibrium time-consistent policy rule satisfies  $\Psi(\mathbf{s}) = \psi(\mathbf{s}; \Psi)$ .

- Asset Holdings across Countries (same  $R$ )

- Asset Holdings across Countries (same  $R$ )
- Intangible Capital is not rival

- Asset Holdings across Countries (same  $R$ )
- Intangible Capital is not rival
- Sell Locally.

- Asset Holdings across Countries (same  $R$ )
- Intangible Capital is not rival
- Sell Locally.
- Transfer Pricing not Perfectly Observable

- Asset Holdings across Countries (same  $R$ )
- Intangible Capital is not rival
- Sell Locally.
- Transfer Pricing not Perfectly Observable
- Production of  $x$  Good has a positive Externality

- Asset Holdings across Countries (same  $R$ )
- Intangible Capital is not rival
- Sell Locally.
- Transfer Pricing not Perfectly Observable
- Production of  $x$  Good has a positive Externality
- Legacy Investment makes it easier to post the factor in that country



- Asset Holdings across Countries (same  $R$ )
- Intangible Capital is not rival
- Sell Locally.
- Transfer Pricing not Perfectly Observable
- Production of  $x$  Good has a positive Externality
- Legacy Investment makes it easier to post the factor in that country
- No Notion of  $j$ -country firms

- Country  $j$ , generic firm, note  $x^{j,m}$  is non-rival. Output is

$$y^{j,i} = F^j(k_{j,i}, x_j, \ell_{j,i}) = z \left( k_{j,i}^\alpha x_{j,i}^{1-\alpha} \right)^{\theta_k} \ell_{j,i}^{1-\theta_\ell},$$

- Country  $j$ , generic firm, note  $x^{j,m}$  is non-rival. Output is

$$y^{j,i} = F^j(k_{j,i}, x_j, \ell_{j,i}) = z \left( k_{j,i}^\alpha x_{j,i}^{1-\alpha} \right)^{\theta_k} \ell_{j,i}^{1-\theta_\ell},$$

- Intangible Capital is home produced

$$x^{j'} = (1 - \delta)x^j + f^j(\ell_{j,x})$$

- Country  $j$ , generic firm, note  $x^{j,m}$  is non-rival. Output is

$$y^{j,i} = F^j(k_{j,i}, x_j, \ell_{j,i}) = z \left( k_{j,i}^\alpha x_{j,i}^{1-\alpha} \right)^{\theta_k} \ell_{j,i}^{1-\theta_\ell},$$

- Intangible Capital is home produced

$$x^{j'} = (1 - \delta)x^j + f^j(\ell_{j,x})$$

- There is an Armington (CES) aggregator for final output in

$$Q^j = \left( Y^{j,1}, Y^{j,2} \right)$$

## MODEL 2

- Country  $j$ , generic firm, note  $x^{j,m}$  is non-rival. Output is

$$y^{j,i} = F^j(k_{j,i}, x_j, \ell_{j,i}) = z \left( k_{j,i}^\alpha x_{j,i}^{1-\alpha} \right)^{\theta_k} \ell_{j,i}^{1-\theta_\ell}$$

- Intangible Capital is home produced

$$x^{j'} = (1 - \delta)x^j + f^j(\ell_{j,x})$$

- There is an Armington (CES) aggregator for final output in

$$Q^j = \left( Y^{j,1}, Y^{j,2} \right)$$

- Profits in each country are allocated using transfer pricing. There are quadratic costs from deviating too much from standard accounting prices  $q^*$

$$\pi^{j,j} = p^j F^{j,j}(k_{j,j}, x_j, \ell_{j,j}) - w^j \ell_{j,j} - \delta k_{j,j} + q^j x_j - C(q^j - q^*, x_j)$$

$$\pi^{j,i} = p^j F^{j,i}(k_{j,i}, x_j, \ell_{j,i}) - w^i \ell_{j,i} - \delta k_{j,i} - q^j x_j$$

## MODEL 2

- Country  $j$ , generic firm, note  $x^{j,m}$  is non-rival. Output is

$$y^{j,i} = F^j(k_{j,i}, x_j, \ell_{j,i}) = z \left( k_{j,i}^\alpha x_{j,i}^{1-\alpha} \right)^{\theta_k} \ell_{j,i}^{1-\theta_\ell},$$

- Intangible Capital is home produced

$$x^{j'} = (1 - \delta)x^j + f^j(\ell_{j,x})$$

- There is an Armington (CES) aggregator for final output in

$$Q^j = \left( Y^{j,1}, Y^{j,2} \right)$$

- Profits in each country are allocated using transfer pricing. There are quadratic costs from deviating too much from standard accounting prices  $q^*$

$$\pi^{j,j} = p^j F^{j,j}(k_{j,j}, x_j, \ell_{j,j}) - w^j \ell_{j,j} - \delta k_{j,j} + q^j x_j - C(q^j - q^*, x_j)$$

$$\pi^{j,i} = p^j F^{j,i}(k_{j,i}, x_j, \ell_{j,i}) - w^i \ell_{j,i} - \delta k_{j,i} - q^j x_j$$

- Aggregate state is  $s$ . A  $j$  firm's state is  $\{k_{j,1}, k_{j,2}, x_j\}$ .

- $\tau^{A,j}$ , Taxes on Corporation's Headquarters residual income  $\pi^{A,j}$ ,

- $\tau^{A,j}$ , Taxes on Corporation's Headquarters residual income  $\pi^{A,j}$ ,
- $\tau^{K,j}$ , Taxes on capital income  $\pi^{K,i,j}$ , (intermediate output net of labor income)



- $\tau^{A,j}$ , Taxes on Corporation's Headquarters residual income  $\pi^{A,j}$ ,
- $\tau^{K,j}$ , Taxes on capital income  $\pi^{K,i,j}$ , (intermediate output net of labor income)
- $\tau^{\ell,j}$ , Taxes on Labor income where it is used  $\pi^{j,i}$ , (Capital income where produced)

- $\tau^{A,j}$ , Taxes on Corporation's Headquarters residual income  $\pi^{A,j}$ ,
- $\tau^{K,j}$ , Taxes on capital income  $\pi^{K,i,j}$ , (intermediate output net of labor income)
- $\tau^{\ell,j}$ , Taxes on Labor income where it is used  $\pi^{j,i}$ , (Capital income where produced)
- Added issue of how difficult is to set  $\varphi^{j,i}$ . We will get to this later.

## FIRM'S PROBLEM: STATE $\{k_1, k_2, x\}$

- Static Problem involves choosing  $q^j$  and it is interconnected across countries. Let  $\pi^{i,i}(s, \ell_{j,i}, q^j)$  yield the static profits conditional on choices

# FIRM'S PROBLEM: STATE $\{k_1, k_2, x\}$

- Static Problem involves choosing  $q^j$  and it is interconnected across countries. Let  $\pi^{j,i}(\mathbf{s}, \ell_{j,i}, q^j)$  yield the static profits conditional on choices
- Dynamic Problem

$$V^j(\mathbf{s}, k_1, k_2, x; \Psi) = \max_{\substack{\ell_1, \ell_2, q \\ i_1, i_2, \ell_x}} \left\{ d + (R^j)^{-1} V^j(\mathbf{s}', k'_1, k'_2, x'; \Psi) \right\} \quad \text{s.t.}$$

$$d = (1 - \tau_j^A) \left[ (1 - \tau_1^K) \pi^{j,1}(\mathbf{s}, \ell_1, q^j) + (1 - \tau_2^K) \pi^{j,2}(\mathbf{s}, \ell_2, q^j) \right] - i_1 - i_2 - w^j \ell_x,$$

$$k'_i = k_i + i_i, \quad i \in \{1, 2\}$$

$$x' = (1 - \delta)x + f(\ell_x),$$

$$\tau = \Psi(\mathbf{s}),$$

$$\mathbf{s}' = \Phi(\mathbf{s}; \Psi).$$

- Static

$$w^i = (1 - \tau_j^A) (1 - \tau_i^K) p^j F_\ell^{j,i}$$

$$w^j = (1 - \tau_j^A) (1 - \tau_j^K) p^j F_\ell^{j,j}$$

$$(1 - \tau_j^A) (1 - \tau_i^K) x_j = (1 - \tau_j^A) (1 - \tau_j^K) [x_j - C_q]$$

- Static

$$\begin{aligned}
 w^i &= (1 - \tau_j^A) (1 - \tau_i^K) p^j F_\ell^{j,i} \\
 w^j &= (1 - \tau_j^A) (1 - \tau_j^K) p^j F_\ell^{j,j} \\
 (1 - \tau_j^A) (1 - \tau_i^K) x_j &= (1 - \tau_j^A) (1 - \tau_j^K) [x_j - C_q]
 \end{aligned}$$

- Dynamic FOCS

$$\begin{aligned}
 R^i &= \left[ 1 + (1 - \tau_j^{A'}) (1 - \tau_i^{K'}) p^{j'} F_{k_i'}^{j,i} \right], \\
 R^j &= \left[ 1 + (1 - \tau_j^{A'}) (1 - \tau_j^{K'}) p^{j'} F_{k_j'}^{j,j} \right], \\
 R^q &= 1 + (1 - \tau_j^{A'}) p^{j'} \left[ (1 - \tau_i^{K'}) \left( F_{x'}^{j,i} - q^{j'} \right) F_{x'}^{j,i} + (1 - \tau_j^{K'}) \left( F_{x'}^{j,j} + q^{j'} \right) \right],
 \end{aligned}$$

