Wealth, Wages, and Employment

Preliminary

Per Krusell IIES
Jinfeng Luo Bonn
José-Víctor Ríos-Rull Penn, UCL, CAERP

February 6, 2023

Econ 8200, Penn 2023

Motivation

- Search models are proved successful in explaining in many features of the labor market:
  - co-existence of unemployed workers and unfilled vacancies
  - the lengths of employment and unemployment spells
  - the job ladder
- The canonical search model, at its core, features
  - a (random) search & match process that takes time
  - a bargaining process that pins down wage
  - a free entry condition that pins down market tightness

Motivation

- However, the canonical search model has fundamental difficulties over some important dimensions.
- Shimer puzzle
    - in the data, employment moves a lot during the business cycle while wages move little
    - search models, however, tend to predict the opposite
    - wage rigidity
- Wealth effect
    - bargaining implies wages increasing in worker wealth
    - rich workers earn more, work less, and stay longer while unemployed
    - at odds with data and hard to solve (not time consistent)

Motivation

- We propose a alternative with directed search and wage commitment.
- Very easy to nest wealth into the analysis
  - Firms only need to know the expected worker duration, which is simply a function of wage.
  - Rep agent version (this time): only level of wealth
  - Heterogeneous agent version: wealth distribution
- Business cycle analysis
  - Firms can commit to a wage plan, i.e., depend on aggregate economic conditions.
  - If wages are fully flexible, employed workers would not change job-to-job moving behavior along the business cycle.
  - The readily available data on j2j moves can thus discipline the extent of wage rigidity.

Model: Precautionary Savings, Competitive Search

- Jobs are created by firms (plants). A plant with capital plus a worker produce one unit of the good
  - Firms pay flow cost $\bar{c}(v)$ to post v vacancies in market $\{w, \theta\}$.
  - Firms cannot change the wage afterwards (like a machine programmed to pay w)
  - Plants (and their capital) are destroyed at rate δ^f.
  - Workers quit exogenously at rate δ^h leaving firms idle.
- The Rep Household with many family members differing in wages.
- The choice of wages/market tightness is made by household members.
- Small open economy: the Rep agent can be poorer than the aggregate because some wealth is held by others.
- General equilibrium: Workers own firms.

Baseline Model: Theory

Order of Events

1. The Rep Agent enters period with a measure of members $x = \{\{x^e(w)\}, x^u\}$. w

2. Production & Consumption: Household collects $x^u b + \int w dx^e(w)$ and chooses how much to save.

3. Firm Destruction and Quits: Some Firms are destroyed at rate δ^f. They cannot search this period. Some workers quit their jobs for exogenous reasons δ^h. Total job destruction $\delta = 1 - (1 - \delta^h)(1 - \delta^f)$.

4. Search: Firms and the unemployed choose wage w and tightness θ.

5. Job Matching: $M(V, U)$: Some vacancies meet some unemployed job searchers. A match becomes operational the following period.
   Job finding and job filling rates $\psi^h(\theta) = \frac{M(V,U)}{U}$, $\psi^f(\theta) = \frac{M(V,U)}{V}$.

1: Centralization: Household Head chooses savings and wages

- Substitute the budget constraint in the Utility function

$$V(a, x^e, x^u) = \max_{a', w'} U\left[(1+r)\, a + b\, x^u + \int_w w\, x^e(dw) - a'\right] + \beta\, V(a', x^{e\prime}, x^{u\prime})$$

s.t.

$$x^{e\prime}(\widehat{w}) = (1-\delta)\, x^e(\widehat{w}) + x^u\, \psi^h\left[\theta\left(\widehat{w}\right)\right]\, \mathbb{1}_{\widehat{w}=w'}$$

$$x^{u\prime} = \delta \int_w dx^e(w) + x^u \left\{1 - \psi^h[\theta(w')]\right\}$$

- First Order Conditions

$$U_c = \beta\, V_{a'}(a', x^{e\prime}, x^{u\prime})$$

$$0 = x^u \int \left\{V_{x^{e\prime}(\hat{w})}(a', x^{e\prime}, x^{u\prime}) - V_{x^{u\prime}}(a', x^{e\prime}, x^{u\prime})\right\} \left\{\frac{\partial\left(\psi^h[\theta(\widehat{w})]\mathbb{1}_{\widehat{w}=w'}\right)}{\partial w'}\right\} d\hat{w}$$

- Envelope Conditions

$$V_a(a, x^e, x^u) = (1+r)\, U_c$$

$$V_{x^e(w)}(a, x^e, x^u) = w\, U_c + \beta\, (1-\delta)\, V_{x^{e\prime}(w)}(a', x^{e\prime}, x^{u\prime}) + \beta\, \delta\, V_{x^{u\prime}}(a', x^{e\prime}, x^{u\prime})$$

$$V_{x^u}(a, x^e, x^u) = b\, U_c + \beta \max_{w'} \left\{\psi^h\left[\theta\left(w'\right)\right] V_{x^{e\prime}(w')}(a', x') + (1 - \psi^h\left[\theta\left(w'\right)\right]) V_{x^{u\prime}}(a', x')\right\}$$

Characterization of saving and wage applying decision

Proposition
In the centralized economy, the HH head's saving decision is characterized by

$$U_c(c) = \beta(1+r)U_c(c')$$

And the wage choice is characterized by

$$\psi^h_{w'}(w')\Phi^c(a', x', w') + \psi^h(w')\Phi^c_3(a', x', w') = 0$$

where

$$\Phi^c(a', x', w') = V'_{x^{e'}(w')} - V'_{x^{u'}}$$

- Saving decision is straightforward.
- Wage choice says the household head is weighing the probability versus the value of putting one unemployed to work.

Characterization of saving and wage applying decision

- The saving decision is characterized by the standard Euler equation

$$U_c(c) = \beta(1+r)\ U_c(c')$$

- Wage applying decision

$$0 = \int \left\{ V_{x^{e\prime}(\hat{w})}(a', x') - V_{x^{u\prime}}(a', x') \right\} \left\{ \frac{\partial \left(\psi^h[\theta(\widehat{w})] \mathbb{1}_{\widehat{w}=w'} \right)}{\partial w'} \right\} \mathrm{d}\hat{w}$$

$$= \frac{\partial \left(\psi^h[\theta(w')][V_{x^{e\prime}(w')}(a', x') - V_{x^{u\prime}}(a', x')] \right)}{\partial w'}$$

$$= \left\{ \psi^h_{w'}(w') \left(V_{x^{e\prime}(w')(a', x')} - V_{x^{u\prime}}(a', x') \right) + \psi^h(w') \left(\frac{\partial V_{x^{e\prime}(w')}}{\partial w'} - \frac{\partial V_{x^{u\prime}}}{\partial w'} \right) \right\}$$

Characterization of saving and wage applying decision

- Wage applying decision (continue)

Use the envelops we have

$$
\begin{aligned}
V'_{x^{e'}(w')} - V'_{x^{u'}} &= (w' - b)U_c(c') + \beta(1-\delta)[V''_{x^{e''}(w')} - V''_{x^{u''}}] \\
&\quad - \beta \max_{w''}\{\psi^h(w'')[V''_{x^{e''}(w'')} - V''_{x^{u''}}]\} \\
&\equiv \Phi^c(a', x', w')
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial(V'_{x^{e'}(w')} - V_{x^{u'}})}{\partial w'} &= U_c(c') + \beta(1-\delta)\frac{\partial(V''_{x^{e''}(w')} - V''_{x^{u''}})}{\partial w'} + \beta \underbrace{0}_{\text{by FOC}} \\
&\equiv \Phi^c_3(a', x', w')
\end{aligned}
$$

So the FOC can be written as

$$
\psi^h_{w'}(w')\Phi^c(a', x', w') + \psi^h(w')\Phi^c_3(a', x', w') = 0
$$

2: Perfect Insurance: Members Choose where to Apply

- Individual state: wealth, measure of workers $\{a, x\} \equiv \{a, x^e(w), x^u\}$, and wages w
- Employed take $a' = h(a, x)$, $x' = \chi(a, x)$ as given
- Unemployed choose w', employed make no choices

$$v^e(a, x^e, x^u, w) = U(c^e) + (w - c^e)U_{c^f} + \beta(1-\delta)\ \ v^e(a', x', w) + \beta\ \delta\ \ v^u(a', x')$$

$$v^u(a, x^e, x^u) = U(c^u) + (b - c^u)U_{c^f} + \beta \max_{w'} \left\{\psi^h[\theta(w')]\ v^e(a', x', w') + \left[1 - \psi^h[\theta(w')]\right]\ v^u(a', x')\right\}$$

- With perfect insurance, $c^u = c^e = c^f = c$, We thus have the net value of working at wage w: $\Phi(\cdot, w)$.

$$\begin{aligned}
\Phi(a, x^e, x^u, w) \equiv & v^e(a, x, w) - v^u(a, x) \\
= & (w - b)U_c(a, x) + \beta(1-\delta)[v^e(a', x', w) - v^u(a', x')] \\
& - \beta \max_{w'} \left\{\psi^h(w')[v^e(a', x', w') - v^u(a', x')]\right\} \\
= & (w - b)U_c(a, x) + \beta\ (1-\delta)\ \Phi(a', x', w) - \beta\ \max_{w'} \left\{\psi^h(w')\ \Phi(a', x', w')\right\}
\end{aligned}$$

Job Applying Problem

- Wage applying problem:

$$\max_{w'} \quad \psi^h[\theta(w')] \; \Phi(a', x^{e\prime}, x^{u\prime}, w')$$

- FOC yields

$$\psi^h_\theta[\theta(w')] \; \theta_w(w') \; \Phi(a', x', w') + \psi^h[\theta(w')] \; \Phi_w(a', x', w') = 0$$

with solution $w' = \omega(a', x')$

- Solve $\Phi_w(a', x^{e\prime}, x^{u\prime}, w')$ forward

$$\Phi_w(a', x^{e\prime}, x^{u\prime}, w') = \sum_{s=0}^{\infty} [\beta \; (1-\delta)]^s \; U_c(a^{1+s}, x^{1+s}, x^{0,1+s})$$

where a^s and x^s are wealth and distribution s periods forward

Job Applying Problem: at Steady state

- Steady state implies

$$\Phi(a, x^e, x^u, w) = v^e(a, x, w) - v^u(a, x)$$
$$= \frac{w - b}{1 - \beta(1 - \delta - \psi^h[\theta(w)])} U_c(a, x)$$
$$\Phi_w(a, x^e, x^u, w) = \frac{1}{1 - \beta(1 - \delta)} U_c(a, x)$$

- At s-s, the solution to the job applying problem boils down to

$$\frac{\theta_w(w)}{\theta(w)} = -\frac{1}{(1 - \eta)(w - b)} \left(1 + \frac{\beta\chi\theta(w)^{1-\eta}}{1 - \beta(1 - \delta)}\right)$$

- With perfect insurance, the member's wage choice does not involve the level of consumption.

Household Head chooses saving only

- Substitute the budget constraint in the Utility function.

$$
\begin{aligned}
V(a, x^e, x^u) \quad &= \quad \max_{c,a'} \; U\left[(1+r)\; a + b\; x^u + \int_w w\; x^e(dw) - a'\right] + \beta\; V(a', x') \\
\text{s.t.} \quad & \\
& x^{e\prime}(\widehat{w}) = (1-\delta)\; x^e(\widehat{w}) + x^u\; \psi^h\, [\theta\,(\widehat{w})]\, \mathbb{1}_{\widehat{w}=\omega(a',x')} \quad \forall \hat{w} \\
& x^{u\prime} = \delta \int_w dx^e(w) + x^u \left\{1 - \psi^h[\theta(\omega(a', x'))]\right\}
\end{aligned}
$$

- Writing the FOCs

$$
U_c = \beta\; V_{a'}(a', x')
$$

- The Envelope

$$
\begin{aligned}
V_a(a, x^e, x^u) &= (1+r)U_c(c) - U_c(c)\frac{\partial a'}{\partial a} + \beta\{V'_{a'}\frac{\partial a'}{\partial a} + \int V'_{x^{e\prime}(w)}\frac{\partial x^{e\prime}(w)}{\partial a}\mathrm{d}w + V'_{x^{u\prime}}\frac{\partial x^{u\prime}}{\partial a}\} \\
&= (1+r)U_c(c) + \beta x^u \int \left\{(V'_{x^{e\prime}(w)} - V'_{x^{u\prime}})\frac{\partial\left(\psi^h[\theta(w)]\mathbb{1}_{w=\omega(a',x')}\right)}{\partial a}\right\}\mathrm{d}w
\end{aligned}
$$

Characterization of Savings

Proposition
Saving decision is characterized by the Euler equation

$$U_c(c) = \beta(1+r)U_c(c')$$

- **Proof.** All is to show is that the second term of the envelop is zero. First note that

$$\int \left\{ V_{x^{e'}(w')}(a',x') - V_{x^{u'}}(a',x') \right\} \left\{ \frac{\partial \left(\psi^h[\theta(w')]\mathbb{1}_{w'=\omega(a',x')} \right)}{\partial a} \right\} \mathrm{d}w'$$

$$= \left\{ \int V_{x^{e'}(w')}(a',x') - V_{x^{u'}}(a',x') \right\} \left\{ \underbrace{\frac{\partial \left(\psi^h[\theta(w')]\mathbb{1}_{w'=\omega(a',x')} \right)}{\partial \omega}}_{\text{total effect of wage on distribution}} \quad \underbrace{\frac{\partial \omega(a',x')}{\partial a}}_{\text{total effect of } a \text{ on wage choice}} \right\} \mathrm{d}w'$$

applying the Lemma

$$= \frac{\partial \left(\psi^h[\theta(w')][V_{x^{e'}(w')}(a',x') - V_{x^{u'}}(a',x')] \right)}{\partial w'} \frac{\partial \omega(a',x')}{\partial a}$$

$$= \left\{ \psi^h_{w'}(w')(V'_{x^{e'}(w')} - V'_{x^{u'}}) + \psi^h(w') \left(\frac{\partial V'_{x^{e'}(w')}}{\partial w'} - \frac{\partial V'_{x^{u'}}}{\partial w'} \right) \right\} \frac{\partial \omega(a',x')}{\partial a}$$

It is left to show the first term of the above is zero.

Proposition

Saving decision is characterized by the Euler equation

$$U_c(c) = \beta(1+r)U_c(c')$$

- **Proof. (continue)** We have

$$\begin{aligned}
V'_{x e'(w')} - V'_{x u'} =& (w' - b)U_c(c') + \beta(1-\delta)[V''_{x e''(w')} - V''_{x u''}] \\
& - \beta\{\psi^h(w'')[V''_{x e''(w'')} - V''_{x u''}]\}\mathbb{1}(w'' = \omega(a'', x'')) \\
=& \Phi(a', x', w') \\
\frac{\partial(V'_{x e'(w')} - V_{x u'})}{\partial w'} =& U_c(c') + \beta(1-\delta)\frac{\partial(V''_{x e''(w')} - V''_{x u''})}{\partial w'} + \beta \underbrace{0}_{\text{by FOC}} \\
=& \Phi_3(a', x', w')
\end{aligned}$$

Plug into the firm term of the previous page yields the member's FOC. Things follow that the second term of the envelop is zero and $U_c(c) = \beta(1+r)U_c(c')$.

Equivalence of centralization and decentralization

Theorem

The centralized economy and the decentralized economy are equivalent.

- **Proof.** To establish the equivalence, it is sufficient to show both the saving decisions and the wage applying decisions in these two economies are the same.
  Note first that, as we have shown in page 8 and 14, the saving decisions of the household in both economies are characterized by

$$U_c(c) = \beta(1+r)U_c(c')$$

  Second, simply comparing the definitions yields the observation that $\Phi^c = \Phi$ and $\Phi^c_3 = \Phi_3$, i.e., the values of moving a worker from unemployed to a job with wage w are the same given fixed (a, x). It then follows that the wage applying decisions in both economies are characterized by

$$\psi^h_{w'}(w')\Phi(a', x', w') + \psi^h(w')\Phi_3(a', x', w') = 0$$

  We thus can conclude the centralized economy and the decentralized economy are equivalent.

Firms Post vacancies: Choose wages & filling probabilities

- Firms of type j need k^j and produce y^j

- Value of an idle firm j: capital depreciates at rate δ^k

$$\Omega^j = -\delta^k k^j + \frac{1-\delta^f}{1+r}\left(-\bar{c} + \psi^f(w)\Omega^j(w) + [1-\psi^f(w)]\Omega^j\right)$$

- Value of wage-w job

$$\Omega^j(w) = y^j - \delta^k\ k^j - w + \frac{1-\delta^f}{1+r}\left[(1-\delta^h)\,\Omega^j(w) + \delta^h\Omega^j\right]$$

- Value of creating firm j:

$$\psi^f[\theta(w)]\Omega^j(w) + [1-\psi^f[\theta(w)]]\Omega^j$$

Firms Post vacancies: Choose wages & filling probabilities

- Free entry implies that (newly created firm can not immediate search or immediately be destroyed)

$$k^j = \frac{1}{1+r}\left(-\bar{c} + \psi^f[\theta(w)]\Omega^j(w) + [1-\psi^f[\theta(w)]]\Omega^j\right)$$

- Free entry condition implies that the idle value of the firm is simply

$$\Omega^j = (1-\delta^k-\delta^f)k^j$$

- It follows that the operating value of the firm is affine in w:

$$\Omega^j(w) = \left[y^j + k^j\left(\frac{(1-\delta^f)(1-\delta^f-\delta^k)}{1+r}\delta^h - \delta^k\right) - w\right]\frac{1+r}{r+\delta} = A^j - \frac{1+r}{r+\delta}w$$

- Firms are identical: only j^* firms exist where $j^* = \arg\max A^j$

Firms Post vacancies: Choose wages & filling probabilities

- Free entry condition then yields

$$\psi^f[\theta(w)] = \chi\theta(w)^{-\eta} = \frac{(r-\delta^f-\delta^k)k^{j*}+\bar{c}}{\Omega^{j*}(w)-(1-\delta^f-\delta^k)k^{j*}}$$

- Express market tightness $\theta(w)$ and its derivative as a function of $\Omega^j(w)$ (and $j = j*$)

$$\theta(w) = \left[\frac{\chi\left(\Omega^j(w)-(1-\delta^f-\delta^k)k^j\right)}{\bar{c}+(r-\delta^f-\delta^k)k^j}\right]^{\frac{1}{\eta}}$$

$$\theta_w(w) = -\frac{1}{\eta}\frac{1+r}{r+\delta}\left[\frac{\chi}{\bar{c}+(r-\delta^f-\delta^k)k^j}\right]^{\frac{1}{\eta}}\left[\Omega^j(w)-(1-\delta^f-\delta^k)k^j\right]^{\frac{1-\eta}{\eta}}$$

Combining Firms and Households: at Steady State

- Equating $\frac{\theta_w(w)}{\theta(w)}$ from the firm problem and worker problem yields

$$\theta(w) = \frac{1-\beta(1-\delta)}{\beta\chi}\left[\frac{1-\eta}{\eta}\frac{1+r}{r+\delta}\frac{w-b}{\Omega^j(w)-\Omega^j} - 1\right]^{\frac{1}{1-\eta}}$$

  which links $\theta(w)$ to the surplus ratio between workers and firms $\frac{w-b}{\Omega^j(w)-\Omega^j}$. Note also this is a function increasing in w.

- We also have the free entry condition of firms stating $\theta(w)$ is decreasing in w

$$\theta(w) = \left[\frac{\chi\left(\Omega^j(w) - \Omega^j\right)}{\bar{c} + (r - \delta^f - \delta^k)k^j}\right]^{\frac{1}{\eta}}$$

- The above two conditions pin down the unique labor market equilibrium (w^*, θ^*), independent of c.

Combining Firms and Households: at Steady State

- Solving w^* and θ^* yields

$$w^* = b + \frac{r+\delta}{1+r}\frac{\eta}{1-\eta}\left[1 + \left(\frac{\beta\chi}{1-\beta(1-\delta)}\right)^{1-\eta}\left(\frac{\chi}{\bar{c} + (r-\delta^f-\delta^k)k^j}\right)^{\frac{1-\eta}{\eta}}\right]\left(\Omega^j(w^*) - \Omega^j\right)^{\frac{1}{\eta}}$$

$$\theta^* = \left[\frac{\chi\left(\Omega^j(w^*) - \Omega^j\right)}{\bar{c} + (r-\delta^f-\delta^k)k^j}\right]^{\frac{1}{\eta}}$$

where $\Omega^j(w) = A^j - \frac{1+r}{r+\delta}w,\ \Omega^j = (1-\delta^f-\delta^k)k^j$

- Steady-state e and u

$$e^* = \frac{\chi\ \theta^{*1-\eta}}{\delta + \chi\ \theta^{*1-\eta}}$$

$$u^* = \frac{\delta}{\delta + \chi\ \theta^{*1-\eta}}$$

Steady State Equilibrium

- Wealth, employment, wage, market tightness, and measure of idle firms $\{a, e, w, \theta, \mu\}$ values and decisions $\{V, \Omega^{j*}(\cdot), h, \Phi, \omega\}$, an interest rate r, and a stationary distribution x^e over w, s.t.

1. $\{V, h\}$ solve the household problem, $\{\Phi, \omega\}$ solve members' problems, $\{\Omega^{j*}(\cdot)\}$ solves the firm's problem.

2. Free entry condition holds

$$k^{j*} = \frac{1}{1+r}\left(-\bar{c} + \psi^f[\theta(w)]\Omega^{j*}(w) + [1-\psi^f[\theta(w)]]\Omega^{j*}\right)$$
$$\Omega^{j*} = (1-\delta^f-\delta^k)k^{j*}$$

3. Wealth aggregates (closed economy)

$$a = \int \Omega^{j*}(w)\mathrm{d}x^e + x^u\Omega^{j*}$$

4. The measure $\{x^e, x^u\}$ is stationary

3. Imperfect Insurance: Members Choose Jobs

- Individual state: wealth and measure of wages $\{a, x^e(w), x^u\}$
- Employed members consume what told $c(a, x)$ give the rest
- Unemployed members consume b and choose where to apply:

$$\begin{aligned} v^e(a, x^e, x^u, w) &= U(c^e) + (w - c^e)U_{c^f} + \beta \left\{(1-\delta)\, v^e(a', x', w) + \delta\, v^u(a', x')\right\} \\ v^u(a, x^e, x^u) &= U(c^u) + (b - c^u)U_{c^f} + \beta \max_{w'} \left\{\psi^h[\theta(w')]\, v^e(a', x', w') + [1 - \psi^h[\theta(w')]]\, v^u(a', x')\right\} \\ &= U(c^u) + (b - c^u)U_{c^f} + \beta\, v^u(a', x') + \beta \max_{w'} \left\{\psi^h[\theta(w')]\, \left[v^e(a', x', w') - v^u(a', x')\right]\right\} \end{aligned}$$

With imperfect insurance, we assume $c^e = c^f = c$, while $c^u = b$. So we have

$$\begin{aligned} \Phi(a, x^e, x^u, w) = v^e(a, x, w) - v^u(a, x) &= U(c) - U(b) + (w - c)U_c + \beta(1-\delta)\, [v^e(a', x', w') - v^u(a', x')] \\ &\quad - \beta \max_{w'} \left\{\psi^h(w')[v^e(a', x', w') - v^u(a', x')]\right\} \\ &= U(c) - U(b) + (w - c)U_c + \beta(1-\delta)\, \Phi(a', x', w) - \beta \max_{w'} \left\{\psi^h(w')\, \Phi(a', x', w')\right\} \end{aligned}$$

FOC yields

$$\psi^h_\theta[\theta(w)]\, \theta_w(w)\, \Phi(a, x, w) + \psi^h[\theta(w)]\, \Phi_w(a, x, w) = 0$$

with solution $w' = \omega(a', x^{e\prime}, x^{u\prime})$

HH head problem

- The household chief chooses consumption of the employed E

$$V(a, x^e, x^u) = \max_{a'} \left(1 - x^u\right) U \left[\frac{(1+r)a + \int w \mathrm{d}x^e - a'}{1 - x^u} \right] + x^u U(b) + \beta V(a', x^{e\prime}, x^{u\prime})$$

$$\text{s.t.} \quad x^{e\prime}(w') = (1-\delta)x^e(w') + x^u \ \psi^h[\theta(w')] \mathbb{1}(w' = \omega(a', x'))$$

$$x^{u\prime} = x^u[1 - \psi^h(\omega(a', x'))] + \delta \int \mathrm{d}x^e(w)$$

- With FOC and envelopes

$$U_c(c) = \beta V_a(a', x')$$

- And the Envelop

$$V_a(a, x^e, x^u) = (1+r)U_c(c) - U_c(c)\frac{\partial a'}{\partial a} + \beta \left\{ V'_{a'} \frac{\partial a'}{\partial a} + \int V'_{x^{e\prime}(w')} \frac{\partial x^{e\prime}(w')}{\partial a} \mathrm{d}w' + V'_{x^{u\prime}} \frac{\partial x^{u\prime}}{\partial a} \right\}$$

$$= (1+r)U_c(c) + \beta x^u \int \left\{ (V'_{x^{e\prime}(w')} - V'_{x^{u\prime}}) \frac{\partial \left(\psi^h[\theta(w')] \mathbb{1}_{w' = \omega(a', x')} \right)}{\partial a} \right\} \mathrm{d}w'$$

$$= (1+r)U_c(c) + \beta x^u \left\{ \psi^h_{w'}(w')(V'_{x^{e\prime}(w')} - V'_{x^{u\prime}}) + \psi^h(w') \left(\frac{\partial V'_{x^{e\prime}(w')}}{\partial w'} - \frac{\partial V'_{x^{u\prime}}}{\partial w'} \right) \right\} \frac{\partial \omega(a', x')}{\partial a}$$

Characterization of Savings

- Denote $c = \frac{(1+r)a + \int w dx - a'}{1-x^0}$, We can write

$$
\begin{aligned}
V_{x^{e'}(w')}(a', x^{e'}, x^{u'}) &= w' U_c(c') + \beta(1-\delta) V''_{x^{e''}(w')} + \beta\delta V''_{x^{u''}} \\
V_{x^{u'}}(a', x^{e'}, x^{u'}) &= -U(c') + (1 - x^{u'}) U_c(c') \frac{c'}{(1 - x^{u'})} + U(b) \\
&\quad + \beta V_{x^{e''}(w'')} \psi^h(w'') \mathbb{1}(w'' = \omega'') + \beta V_{x^{u''}} (1 - \psi^h(w'') \mathbb{1}(w'' = \omega'') \\
V'_{x'(w')} - V'_{x^{u'}} &= \underbrace{U(c') - U(b)}_{\text{mass effect}} + \underbrace{(w' - c') U_c(c')}_{\text{consumption effect}} \\
&\quad + \beta(1-\delta)(V''_{x^{e''}(w')} - V''_{x^{u''}}) + \beta(V_{x^{e''}(w'')} - V_{x^{u''}}) \psi^h(w'') \mathbb{1}(w'' = \omega'') \\
&= \Phi(a', x', w')
\end{aligned}
$$

- It follows that $\left(\frac{\partial V'_{x^{e'}(w')}}{\partial w'} - \frac{\partial V'_{x^{u'}}}{\partial w'} \right) = \Phi_{w'}(a', x', w')$, $V_a(a, x) = (1 + r) U_c(c)$, and $U_c(c) = \beta(1 + r) U_c(c')$

- Under imperfect insurance, wage applying is still independent of decision makers, and saving decision is still characterized by the traditional Euler equation.

- What is changed is the labor market equilibrium per se.

- Job applying FOC

$$\psi^h_\theta[\theta(w)]\ \theta_w(w)\Phi(a,x,w) + \psi^h[\theta(w)]\ \Phi_w(a,x,w) = 0$$

- Steady state implies

$$\Phi(a,x^e,x^u,w) = \frac{U(c) - U(b) + (w-c)U_c}{1-\beta(1-\delta-\psi^h[\theta(w)])}$$

$$\Phi_w(a,x^e,x^u,w) = \frac{U_c(a,x)}{1-\beta(1-\delta)}$$

- At s-s, the solution to the job applying problem boils down to

$$\frac{\theta_w(w)}{\theta(w)} = -\frac{1}{1-\eta}\frac{1}{w - c + \frac{U(c)-U(b)}{U_c(a,x)}}\left(1 + \frac{\beta\chi\theta(w)^{1-\eta}}{1-\beta(1-\delta)}\right)$$

- Now wage choice is dependent on consumption c at the steady-state.

Summary: Properties of Exogenous Quits Model

- It is like a two-agent model (employed, unemployed) of Pissarides with curved utility and savings
- A big family structure makes the economy easily decentralized.
- Only one type of firms, j^* in equilibrium because $\Omega^j(w) = A^j - \frac{1+r}{r+\delta} w$
- In the s-s there is only one wage. Out of steady state there will be multiple wages.

Baseline Model: Implementation

Discrete Wages and Aiming Shocks

- Wages are discrete: a worker can choose to apply to a fixed basket of discrete wages $\mathcal{W} = \{w_1, w_2, \ldots, w_n\}$.
- A Gumbel distributed aiming shock may divert the worker to non-optimal wages.
- The advantage: when computing the transition there is no need to keep track of many new wages, but just the move of the wage distribution over n bins.

Imperfect Insurance with Discrete Wages and Aiming Shocks

- Individual state: wealth and measure of wages $\{a, x^e(w), x^u, w\}$

$$v^e(a, x^e, x^u, w) = U(c) + (w - c)U_c + \beta \left\{(1-\delta)v^e(a', x', w) + \delta v^u(a', x')\right\}$$

$$v^u(a, x^e, x^u) = U(b) + \beta \mathbb{E}\left[\max_{w' \in \mathcal{W}} \left\{\psi^h[\theta(w')]v^e(a', x', w') + [1 - \psi^h[\theta(w')]]v^u(a', x') + \epsilon^w\right\}\right]$$

$$= U(b) + \beta v^u(a', x') + \beta \mathbb{E}\left[\max_{w' \in \mathcal{W}} \left\{\psi^h[\theta(w')]\left[v^e(a', x', w') - v^u(a', x')\right] + \epsilon^w\right\}\right]$$

- Denote Φ the value of putting an unemployed member to work

$$\Phi(a, x^e, x^u, w) = v^e(a, x, w) - v^u(a, x)$$

$$= U(c) - U(b) + (w - c)U_c + \beta(1-\delta)\Phi(a', x', w)$$

$$- \beta \mathbb{E}\left[\max_{w' \in \mathcal{W}} \left\{\psi^h(w')\Phi(a', x', w') + \epsilon^w\right\}\right]$$

- The ex-post wage applying policy $w' = \omega(a', x', \epsilon^w)$, and the ex-ante wage applying profile

$$\pi(w'; a', x') = \frac{\exp \frac{\psi^h(w')\Phi(a', x', w')}{\alpha}}{\sum_{w \in \mathcal{W}} \exp \frac{\psi^h(w)\Phi(a', x', w)}{\alpha}}$$

HH head problem

- The household chief chooses consumption c of the employed to maximize the family's welfare

$$
\begin{aligned}
V(a, x^e, x^u) = \max_{a'} & \left(1 - x^u\right) U\left(c\right) + x^u U(b) + \beta x^u J + \beta V(a', x^{e\prime}, x^{u\prime}) \\
\text{s.t.} \quad & x^{e\prime}(w') = (1-\delta) x^e(w') + x^u \pi(w'; a', x') \psi^h[\theta(w')] \\
& x^{u\prime} = x^u \left(1 - \sum_{w' \in \mathcal{W}} \pi(w'; a', x') \psi^h[\theta(w')]\right) + \delta \sum_{w' \in \mathcal{W}} x^e(w) \\
& (1 - x^u) c + a' = (1+r) a + \sum_{w' \in \mathcal{W}} w x^e(w)
\end{aligned}
$$

- A term J to HH head to ensure the value of moving an unemployed to work is the same for the worker and the HH head.

- Computing the dynamics becomes identical to computing the steady-state.

- All nice properties of the model are maintained.

Parameters

	Description	Value	Note
β	Discount factor	0.985	
σ	Risk aversion	3	
b	Unemp. value	0.0375	$0.3y$
y	Productivity	1/8	annual GDP = 1
k	Firm capital	3	$3\times$ annual GDP
χ	Matching efficiency	0.15	
η	Matching elasticity	0.62	
$\bar{c}$	Job posting cost	0.0037	$0.03y$
δ^h	Worker quitting rate	0.36%	
δ^f	Firm destruction rate	0.3%	
δ^k	Capital depreciation rate	0.45%	

Table 1: Parameter Values: Half Quarter

Steady States

Moments	Model
Interest rate	12.85%
Consumption	0.87
Wealth	3.14
Unemployment rate	3.93%
Avg Wage	0.49
Avg Tightness	4.20
Avg Job Finding Prob	0.16
Avg Vacancy Filling Prob	0.25

Table 2: Closed Economy Steady State: in Annual Terms

Baseline Model: 1% z Shock ($\rho = 0.95$)

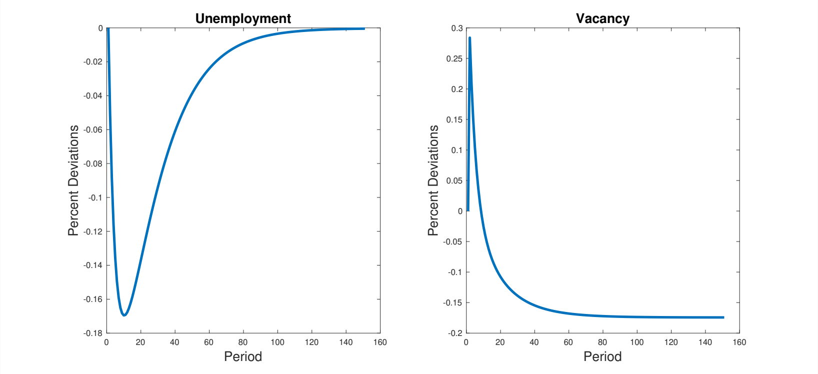

Figure 1: Move of Unemployment and Vacancy

Baseline Model: 1% z Shock ($\rho = 0.95$)

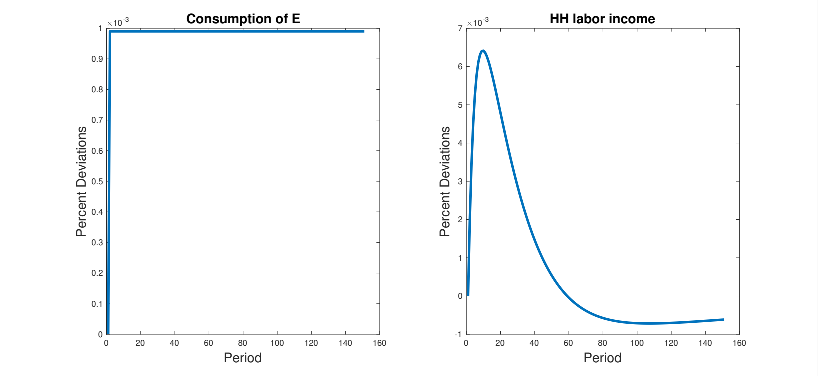

Figure 2: Move of Consumption (E) and Labor Income

Baseline Model: 1% z Shock ($\rho = 0.95$)

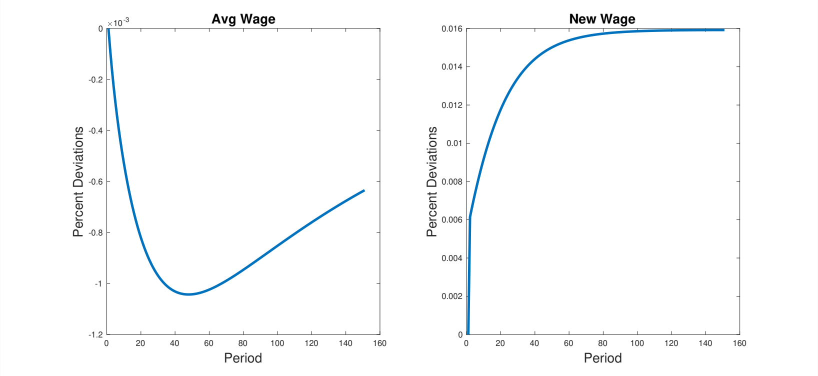

Figure 3: Move of Average Wage and Newly Formed Wage

Baseline Model: 1% z Shock ($\rho = 0.95$)

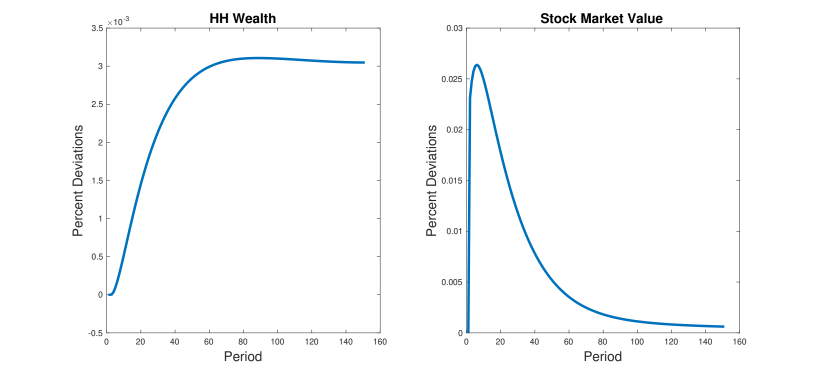

Figure 4: Move of Wealth and Stock Market Value

Baseline Model: 1% z Shock ($\rho = 0.95$)

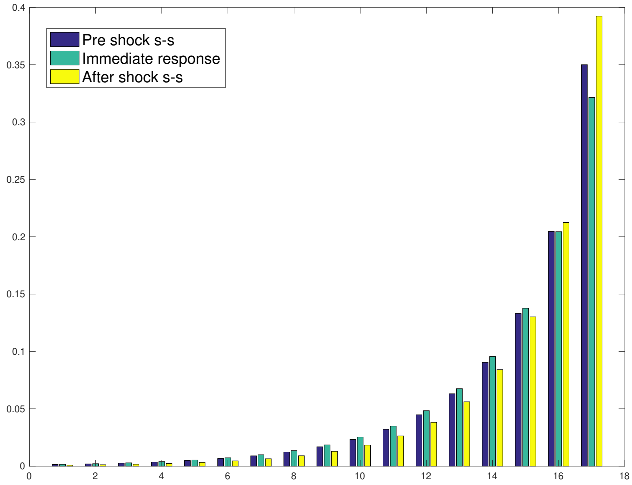

Figure 5: Move of Wage Applying Profile

Baseline Model: 1% z Shock ($\rho = 0.95$)

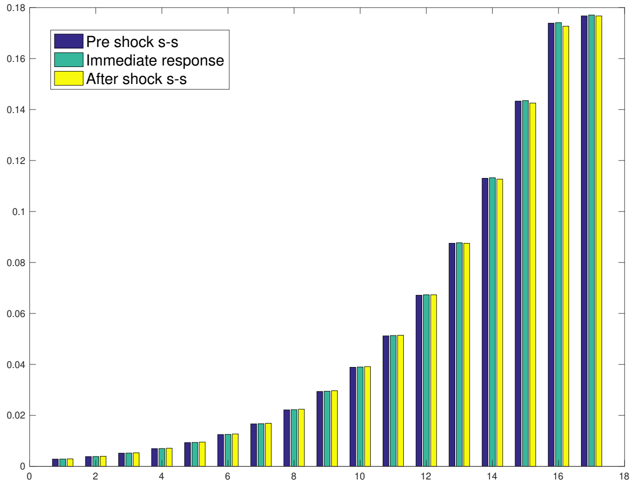

Figure 6: Move of Wage Distribution of the Employed

Summary

- We build a labor search model with directed search and wage commitment.
- Can nest wealth easily and has great potential in business cycle analysis.
- Easy to use and extend.

Endogenous Quitting

4. Endogenous Quits Imperfect Insurance: Members Choose

- HH head state: wealth, measure of wages, and of unemployed $\{a, x^e, x^u\} = \{a, x\}$
- Member state: If unemp same if emp add wage w. Understand $a' = h(a, x)$ and $x' = \chi(a, x)$
- Employed get ut Shocks $\{\epsilon^w, \epsilon^q\}$, $\epsilon^i \sim G(\mu, \alpha)$; after work & cons choose whether to quit
- Unemployed members consume b and choose where to apply.

$$v^u(a, x^e, x^u) = U(b) + \beta\ v^u(a', x') + \beta \max_{w'} \left\{\psi^h[\theta(w')]\ \left[v^e(a', x', w') - v^u(a', x')\right]\right\}$$

$$v^e(a, x^e, x^u, w) = U(c) + (w - c)\ U_c + \beta\left[(1-\delta)\ \widehat{v}^e(a', x', w) + \delta\ v^u(a', x')\right]$$

$$\widehat{\widehat{v}}^e(a', x^{e\prime}, x^{u\prime}, w, \epsilon) = \max\left\{v^e(a', x', w) + \epsilon^w, v^u(a', x') + \epsilon^q\right\}$$

$$\widehat{v}^e(a', x^{e\prime}, x^{u\prime}, w) = \mathbb{E}\left[\widehat{\widehat{v}}^e(a', x^{e\prime}, x^{u\prime}, w, \epsilon)\right] = \mu + \alpha \ln(e^{\frac{v^e(a',x',w)}{\alpha}} + e^{\frac{v^u(a',x')}{\alpha}}) + \alpha\ \gamma$$

$$\pi^q(a, x^e, x^u, w) = \frac{e^{\frac{v^u(a',x')}{\alpha}}}{e^{\frac{v^u(a',x')}{\alpha}} + e^{\frac{v^e(a',x',w)}{\alpha}}} = \frac{e^{\frac{v^u[h(a,x),\chi(a,x)]}{\alpha}}}{e^{\frac{v^u[h(a,x),\chi(a,x)]}{\alpha}} + e^{\frac{v^e[h(a,x),\chi(a,x)],w}{\alpha}}}\text{: Quitting Probb}$$

$$\Phi(a, x^e, x^u, w) \equiv v^e(a, x, w) - v^u(a, x) = U(c) - U(b) + (w-c)U_c + \beta(1-\delta)$$
$$[\widehat{v}^e(a', x', w) - v^u(a', x')] - \beta \max_{w'}\left\{\psi^h(w')[v^e(a', x', w') - v^u(a', x')]\right\}$$
$$= U(c) - U(b) + (w-c)U_c + \beta(1-\delta)\ \hat{\Phi}[h(a,x), \chi(a,x), w] - \beta \max_{w'}\left\{\psi^h(w')\ \Phi[h(a,x), \chi(a,x), w']\right\}$$

$$\text{where } \hat{\Phi}(a', x', w) = \hat{v}^e(a', x', w) - v^u(a', x') = \mu + \alpha\gamma + \alpha \ln(1 + e^{\Phi(a',x',w)/\alpha})$$

FOC: $\psi^h_\theta[\theta(w')]\ \theta_w(w')\ \Phi[h(a,x), \chi(a,x), w'] + \psi^h[\theta(w')]\Phi_3[h(a,x), \chi(a,x), w'] = 0$

& sltn $w' = \omega(a', x')$

$$V(a,x^e,x^u) = \max_{a'} (1-x^u)\, U\left[\frac{(1+r)a + \int w dx - a'}{1-x^u}\right] + x^u U(b) + \beta \widehat{V}(a',x^e,x^u) \qquad \text{s.t.}$$

$$\widehat{V}(a',x^e,x^u) = \int (1-\delta)\; J(\pi^q,\Phi')x^e(\mathrm{d}w) + V(a',x^{e\prime},x^{u\prime})$$

$$x^{e\prime}(w) = [(1-\delta)(1-\pi^q(a,x,w))]\; x^e(w) + \psi^h[\omega(h(a,x),\chi(a,x)]\; x^u$$

$$x^{u\prime} = \int [\delta + (1-\delta)\pi^q(a,x,w)]\; x^e(dw) + \{1-\psi^h[\omega(h(a,x),\chi(a,x)]\}\; x^u$$

$J(\pi^q,\Phi')$ is the joy from the extreme value shocks that the head takes as given. FOC yields

$$U_c(c) = \beta V_a(a',x^{e\prime},x^{u\prime})$$

And Envelopes

$$V_a(a,x^e,x^u) = (1+r)U_c(c) - U_c(c)\frac{\partial a'}{\partial a} + \beta\left\{V'_{a'}\frac{\partial a'}{\partial a} + \int V'_{x^{e\prime}(w)}\frac{\partial x^{e\prime}(w)}{\partial a}dw + V'_{x^{u\prime}}\frac{\partial x^{u\prime}}{\partial a}\right\}$$

$$= (1+r)U_c(c) + \beta x^u \int \underbrace{\left\{(V'_{x^{e\prime}(w)} - V'_{x^{u\prime}})\frac{\partial\left(\psi^h[\theta(w)]\mathbb{1}_{w=\omega(a',x')}\right)}{\partial a}\right\}}\mathrm{d}w$$

$$\left\{\psi^h_{w'}(w')(V_{x^{e\prime}(w')} - V_{x^{u\prime}}) + \psi^h(w')\left(\frac{\partial V_{x^{e\prime}(w')}}{\partial w'} - \frac{\partial V_{x^{u\prime}}}{\partial w'}\right)\right\}\frac{\partial\omega(a',x')}{\partial a}$$

Characterization of Savings

- We again verify that the value of moving a worker from unemployed to employed is the same for the head and the members

$$
\begin{aligned}
V_{x^{e\prime}(w')}(a', x^{e\prime}, x^{u\prime}) &= w' U_c(c') + \beta(1-\delta)J(\pi^{q\prime}, \Phi'') \\
&\quad + \beta(1-\delta)(1-\pi^q(a', x', w'))V''_{x^{e\prime\prime}(w')} + \beta(\delta + (1-\delta)\pi^q(a', x', w'))V''_{x^{u\prime\prime}} \\
V_{x^{u\prime}}(a', x^{e\prime}, x^{u\prime}) &= -U(c') + (1-x^{u\prime})U_c(c')\frac{c'}{(1-x^{u\prime})} + U(b) \\
&\quad + \beta V_{x^{e\prime\prime}(w'')}\psi^h(w'')\mathbb{1}(w'' = \omega'') + \beta V_{x^{u\prime\prime}}(1-\psi^h(w'')\mathbb{1}(w'' = \omega'')) \\
V'_{x^{e\prime}(w')} - V'_{x^{u\prime}} &= U(c') - u(b) + (w' - c')U_c(c') + \beta(1-\delta)J(\pi^{q\prime}, \Phi'') \\
&\quad + \beta(1-\delta)(1-\pi^q(a', x', w'))(V''_{x^{e\prime\prime}(w')} - V''_{x^{u\prime\prime}}) \\
&\quad - \beta(V_{x^{e\prime\prime}(w'')} - V_{x^{u\prime\prime}})\psi^h(w'')\mathbb{1}(w'' = \omega'')
\end{aligned}
$$

- Now define $\Phi^c(a', x', w') = V_{x^{e\prime}(w')}(a', x') - V_{x^{u\prime}}(a', x')$, we have

$$
\begin{aligned}
\Phi^c(a', x', w') &= U(c') - u(b) + (w' - c')U_c(c') + \beta(1-\delta)J(\pi^{q\prime}, \Phi'') \\
&\quad + \beta(1-\delta)(1-\pi^q(a', x', w'))\Phi^c(a'', x'', w') - \beta\psi^h(w'')\mathbb{1}(w'' = \omega'')\Phi^c(a'', x'', w'')
\end{aligned}
$$

- It is now evident to align the interest, J has to satisfy

$$
\begin{aligned}
J + (1-\pi^{q\prime})\Phi^{c\prime\prime} &= \mu + \alpha\gamma + \alpha \ln\left(1 + e^{\Phi''/\alpha}\right) \\
\implies J(\pi^q; \Phi') &= \mu + \alpha\gamma + \alpha \ln\left(1 + e^{\Phi'/\alpha}\right) - (1-\pi^q)\Phi^{c\prime} \\
&= \mu + \alpha\gamma + \alpha \ln\left(1 + e^{\Phi'/\alpha}\right) - (1-\pi^q)\Phi' \text{ , given that } \Phi' = \Phi^{c\prime}
\end{aligned}
$$

Job-to-Job Movements with no Preference Shocks

5. Job to Job Movements, Imperf Insur, No Shocks

- Employed workers can always search on the job.
- No preference shocks. No quits. No search costs.

$$
\begin{aligned}
v^u(a, x^e, x^u) &= U(b) + \beta v^u(a', x') + \beta \max_{w'} \left\{ \psi^h[\theta(w')] \left[v^e(a', x', w') - v^u(a', x') \right] \right\} \\
v^e(a, x^e, x^u, w) &= U(c) + (w - c)U_c + \beta\delta V^u(a', x') + \beta(1-\delta)v^e(a', x', w) \\
&\quad + \beta(1-\delta) \max_{w'} \left\{ \psi^h[\theta(w')] \left[v^e(a', x', w') - v^e(a', x', w) \right] \right\} \\
\Phi(a, x, w) &\equiv v^e(a, x, w) - v^u(a, x) \\
&= U(c) - U(b) + (w - c)U_c + \beta(1-\delta)\Phi(a', x', w) \\
&\quad + \beta(1-\delta) \max_{w'} \left\{ \psi^h[\theta(w')] \left[\Phi(a', x', w') - \Phi(a', x', w) \right] \right\} \\
&\quad - \beta \max_{w'} \left\{ \psi^h[\theta(w')] \Phi(a', x', w') \right\} \\
\text{Envelop: } \Phi_w(a, x, w) &= U_c + \beta(1-\delta)\Phi_w(a', x', w) + 0 \\
\text{FOC of E: } \psi^h_{w'}(w')[\Phi(a', x', w') - \Phi(a', x', w)] &+ \psi^h(w')\Phi_{w'}(a', x', w') = 0 \\
\text{FOC of U: } \psi^h_{w'}(w')\Phi(a', x', w') &+ \psi^h(w')\Phi_{w'}(a', x', w') = 0
\end{aligned}
$$

$$V(a, x^e, x^u) = \max_{a'} \left(1 - x^u\right) U \left[\frac{(1+r)a + \int w x^e(\mathrm{d}w) - a'}{1 - x^u}\right] + x^u U(b) + \beta V(a', x^{e\prime}, x^{u\prime})$$

$$x^{e\prime}(w) = (1-\delta)\left\{1 - \psi^h[\omega^s(a', x', w)]\right\} x^e(w)$$

$$+ \mathbb{1}(\omega^s(a', x', \hat{w}) = w)\psi^h[w](1-\delta)x^e(\hat{w}) + \mathbb{1}(\omega(a', x') = w)\psi^h[w]x^u$$

$$x^{u\prime} = \int \delta x^e(\mathrm{d}w) + \{1 - \psi^h[\omega(a', x')]\}x^u$$

FOC yields

$$U_c(c) = \beta V_{a'}(a', x')$$

And Envelopes

$$V_a(a, x) = (1+r)U_c(c) \underbrace{- U_c(c)\frac{\partial a'}{\partial a} + \beta V'_{a'}\frac{\partial a'}{\partial a}}_{=0 \text{ by HH head FOC}} + \beta \int V'_{x^{e\prime}(w)} \frac{\partial x^{e\prime}(w)}{\partial a} \mathrm{d}w + \beta V'_{x^{u\prime}} \frac{\partial x^{u\prime}}{\partial a}$$

$$= (1+r)U_c(c) + \beta x^u \int \left[V'_{x^{e\prime}(w)} - V'_{x^{u\prime}}\right] \frac{\partial\left[\psi^h(w)\mathbb{1}(w = \omega(a', x'))\right]}{\partial a} \mathrm{d}w$$

$$+ \beta(1-\delta)\int x^e(w) \int \left[V'_{x^{e\prime}(\hat{w})} - V'_{x^{e\prime}(w)}\right] \frac{\partial\left[\psi^h[\hat{w}]\mathbb{1}(\hat{w} = \omega^s(a', x', w))\right]}{\partial a} \mathrm{d}\hat{w}\mathrm{d}w$$

Characterization of Savings

- By definition we have

$$
\begin{aligned}
V_{x^{e\prime}(w')}(a', x^{e\prime}, x^{u\prime}) &= w' U_c(c') + \beta V''_{x^{e\prime\prime}(w')}(1-\delta)(1-\psi^h(\omega^{s\prime\prime})) \\
&+ \beta V''_{x^{e\prime\prime}(\hat{w})}\mathbb{1}(\omega^s(a'', x'', w') = \hat{w})\psi^h[w'](1-\delta) \\
&+ \beta V''_{x^{u\prime\prime}}\delta \\
V_{x^{u\prime}}(a', x^{e\prime}, x^{u\prime}) &= -U(c') + U_c(c')c' + U(b) \\
&+ \beta V_{x^{e\prime\prime}(w'')}\psi^h(w'')\mathbb{1}(w'' = \omega'') + \beta V_{x^{u\prime\prime}}(1-\psi^h(w''))\mathbb{1}(w'' = \omega'') \\
V'_{x^{e\prime}(w')} - V'_{x^{u\prime}} &= U(c') - U(b) + (w' - c')U_c(c') \\
&+ \beta(1-\delta)(V''_{x^{e\prime\prime}(w')} - V''_{x^{u\prime\prime}}) \\
&+ \beta(1-\delta)\psi^h(\hat{w})(V''_{x^{e\prime\prime}(\hat{w})} - V''_{x^{e\prime\prime}(w')})\mathbb{1}(\omega^s(a'', x'', w') = \hat{w}) \\
&- \beta(V_{x^{e\prime\prime}(w'')} - V_{x^{u\prime\prime}})\psi^h(w'')\mathbb{1}(w'' = \omega'')
\end{aligned}
$$

- Now define $\Phi^c(a', x', w') = V_{x^{e\prime}(w')}(a', x') - V_{x^{u\prime}}(a', x')$, we have

$$
\begin{aligned}
\Phi^c(a', x', w') &= U(c') - u(b) + (w' - c')U_c(c') \\
&+ \beta(1-\delta)\Phi^c(a'', x'', w') \\
&+ \beta(1-\delta)\psi^h(w'')(\Phi^c(a'', x'', w'') - \Phi^c(a'', x'', w'))\mathbb{1}(w'' = \omega^{s\prime\prime}) \\
&- \beta\psi^h(w'')\Phi^c(a'', x'', w'')\mathbb{1}(w'' = \omega'')
\end{aligned}
$$

- So $\Phi^c = \Phi$. Interests are aligned.

Characterization of Savings

- It's left to characterize the saving decision, which boils down to the last term of the envelop.

- Note that given $\Phi^c(a', x', w) = V_{x^{e\prime}(w)}(a', x') - V_{x^{u\prime}}(a', x')$, the last term of the envelop

$$
\begin{aligned}
&\int x^e(w) \int \left[V'_{x^{e\prime}(\hat{w})} - V'_{x^{e\prime}(w)}\right] \frac{\partial \left[\psi^h[\hat{w}]\mathbb{1}(\hat{w} = \omega^s(a', x', w))\right]}{\partial a} \mathrm{d}\hat{w}\mathrm{d}w \\
&= \int x^e(w) \left\{ \left[V'_{x^{e\prime}(\omega^s(w))} - V'_{x^{e\prime}(w)}\right] \psi^h_{\omega^s}(\omega^s(w)) + \left[\frac{\partial V'_{x^{e\prime}(\omega^s(w))}}{\partial \omega^s(w)} - \frac{\partial V'_{x^{e\prime}(w)}}{\partial \omega^s(w)}\right] \psi^h(\omega^s(w)) \right\} \frac{\partial \omega^s}{\partial a} \mathrm{d}w \\
&= \int x^e(w) \left\{ \left[\Phi^c(a', x', \omega^s(w)) - \Phi^c(a', x', w)\right] \psi^h_{\omega^s}(\omega^s(w)) + \Phi^c_{\omega^s}(a', x', \omega^s(w))\psi^h(\omega^s(w)) \right\} \frac{\partial \omega^s}{\partial a} \mathrm{d}w \\
&= \int x^e(w)\ 0\ \frac{\partial \omega^s}{\partial a} \mathrm{d}w = 0, \text{ by FOC of the employed searchers for all } w
\end{aligned}
$$

- The head is not incentivized to change the job-to-job moves by adjusting the wealth level. Saving decision is solely to smooth consumption of the employed.

$$
U_c(c) = (1 + r)U_c(c')
$$

6. Job to Job Movements, Imperf Insur, No Shocks, Centralization

$$V(a, x^e, x^u) = \max_{a', \omega, \{\omega^s\}} (1 - x^u)\, U\left[\frac{(1+r)a + \int w x^e(\mathrm{d}w) - a'}{1 - x^u}\right] + x^u U(b) + \beta V(a', x^{e\prime}, x^{u\prime})$$

$$x^{e\prime}(w) = (1-\delta)\left\{1 - \psi^h[\omega^s(w)]\right\} x^e(w)$$
$$+ (1-\delta)\mathbb{1}(w \in \{\omega^s\})\psi^h[w]x^e(\omega^{s-1}(w)) + \mathbb{1}(w = \omega)\psi^h[w]x^u$$
$$x^{u\prime} = \int \delta x^e(\mathrm{d}w) + \{1 - \psi^h[\omega]\}x^u$$

FOC yields

$$U_c(c) = \beta V_{a'}(a', x')$$
$$0 = x^u \int \left[V'_{x^{e\prime}(w)} - V'_{x^{u\prime}}\right] \left\{\frac{\partial\left[\psi^h[w]\mathbb{1}(w = \omega)\right]}{\partial \omega}\right\} \mathrm{d}w$$
$$0 = x^e(w) \int \left[V'_{x^{e\prime}(\hat{w})} - V'_{x^{e\prime}(w)}\right] \left\{\frac{\partial\left[\psi^h[\hat{w}]\mathbb{1}(\hat{w} = \omega^s(w))\right]}{\partial \omega^s(w)}\right\} \mathrm{d}\hat{w}, \forall w$$

6. Job to Job Movements, Imperf Insur, No Shocks, Centralization

And Envelopes

$$V_a(a,x) = (1+r)U_c(c) \underbrace{-U_c(c)\frac{\partial a'}{\partial a} + \beta V'_{a'}\frac{\partial a'}{\partial a}}_{=0 \text{ by saving FOC}} + \beta \underbrace{\left\{ \int V'_{x^{e\prime}(w)} \frac{\partial x^{e\prime}(w)}{\partial a} \mathrm{d}w + V'_{x^{u\prime}} \frac{\partial x^{u\prime}}{\partial a} \right\}}_{=0 \text{ by wage applying FOC}}$$

$$\begin{aligned} V'_{x^{e\prime}(w')} - V'_{x^{u\prime}} &= U(c') - U(b) + (w' - c')U_c(c') \\ &+ \beta(1-\delta)(V''_{x^{e\prime\prime}(w')} - V''_{x^{u\prime\prime}}) \\ &+ \beta(1-\delta)\psi^h(\hat{w})(V''_{x^{e\prime\prime}(\hat{w})} - V''_{x^{e\prime\prime}(w')})\mathbb{1}(\omega^s(a'', x'', w') = \hat{w}) \\ &- \beta(V_{x^{e\prime\prime}(w'')} - V_{x^{u\prime\prime}})\psi^h(w'')\mathbb{1}(w'' = \omega'') \end{aligned}$$

Denote $\Phi^{c\prime} = V'_{x^{e\prime}(w')} - V'_{x^{u\prime}}$ and apply the Lemma, the FOC of ω becomes

$$\psi^h_{w'}(w')\Phi^c(a', x', w') + \psi^h(w')\Phi^c_{w'}(a', x', w') = 0$$

Also for each w, one can apply the Lemma to the FOC of $\omega^s(w)$

$$0 = x^e(w) \int \left[V'_{x^{e\prime}(\hat{w})} - V'_{x^{e\prime}(w)} \right] \left\{ \frac{\partial \left[\psi^h[\hat{w}] \mathbb{1}(\hat{w} = \omega^s(w)) \right]}{\partial \omega^s(w)} \right\} \mathrm{d}\hat{w}$$

$$= x^e(w) \left\{ \left[V'_{x^{e\prime}(\omega^s(w))} - V'_{x^{e\prime}(w)} \right] \psi^h_{\omega^s}(\omega^s(w)) + \left[\frac{\partial V'_{x^{e\prime}(\omega^s(w))}}{\partial \omega^s(w)} - \frac{\partial V'_{x^{e\prime}(w)}}{\partial \omega^s(w)} \right] \psi^h(\omega^s(w)) \right\}$$

Note $V'_{x^{e\prime}(\omega^s(w))} - V'_{x^{e\prime}(w)} = \Phi^c(a', x', \omega^s(w)) - \Phi^c(a', x', w)$, and $\frac{\partial V'_{x^{e\prime}(w)}}{\partial \omega^s(w)} = 0$, the above FOC boils down to

$$0 = \left[\Phi^c(a', x', \omega^s(w)) - \Phi^c(a', x', w) \right] \psi^h_{\omega^s}(\omega^s(w)) + \Phi^c_{\omega^s}(a', x', \omega^s(w)) \psi^h(\omega^s(w)), \forall w$$

Comparing the definitions yields $\Phi^c = \Phi$, and wage applying behavior (of both the employed and unemployed) being independent of decision makers.

6. Job to Job Movements, Imperf Insur, No Shocks, Centralization

Potentially on-the-job searching could encounter the corner solution when the current wage is sufficiently high. In this case there exists a cutoff wage $\bar{w}$. When $w \leq \bar{w}$, FOC applies thus

$$0 = \left[\Phi^c(a', x', \omega^s(w)) - \Phi^c(a', x', w)\right] \psi^h_{\omega^s}(\omega^s(w)) + \Phi^c_{\omega^s}(a', x', \omega^s(w))\psi^h(\omega^s(w)), \forall w \leq \bar{w}$$

When $w > \bar{w}$, FOC does not apply and

$$\omega^s(w) = \tilde{w} \text{ where } \psi^f(\tilde{w}) = 1, \forall w > \bar{w}$$

Put together, $\omega^s(a', x', w)$ is characterized by either the FOC ($w \leq \bar{w}$), or $\psi^f(\omega^s(w)) = 1$ ($w > \bar{w}$).

Job To Job Movements with Preference Shocks

7. Job to Job Movements, Imperf Insur: Members Choose

- In addition to Endog quits:
- Employed get ut Shocks $\{\epsilon^w, \epsilon^q, \epsilon^s\}$, $\epsilon^i \sim G(\mu, \alpha)$; after work & cons choose whether to quit, search or do nothing

$$v^u(a, x^e, x^u) = U(b) + \beta\, v^u(a', x') + \beta \max_{w'} \left\{\psi^h[\theta(w')]\ \left[v^e(a', x', w') - v^u(a', x')\right]\right\}$$

$$v^e(a, x^e, x^u, w) = U(c) + (w - c)U_c + \beta\left[(1-\delta)\, \widehat{v}^e(a', x', w) + \delta\, v^u(a', x')\right]$$

$$v^s(a, x^e, x^u, w) = U(c) + (w - c)U_c - \bar{c}^s + \beta\delta V^u(a', x') + \beta(1-\delta)v^e(a', x', w) + \beta(1-\delta)\max_{w'} \left\{\psi^h[\theta(w')]\ \left[v^e(a', x', w') - v^e(a', x', w)\right]\right\}$$

need one period to search like the unemployed

$$\widehat{\widehat{v}}^e(a', x^{e\prime}, x^{u\prime}, w, \epsilon) = \max\left\{v^e(a', x', w) + \epsilon^w, v^u(a', x') + \epsilon^q, v^s(a', x', w) + \epsilon^s\right\}$$

$$\widehat{v}^e(a', x^{e\prime}, x^{u\prime}, w) = \mathbb{E}\left[\widehat{\widehat{v}}^e(a', x^{e\prime}, x^{u\prime}, w, \epsilon)\right] = \mu + \alpha \ln\left(e^{\frac{v^e(a', x', w)}{\alpha}} + e^{\frac{v^u(a', x')}{\alpha}} + e^{\frac{v^s(a', x', w)}{\alpha}}\right) + \alpha\,\gamma$$

$$\pi^q(a, x^e, x^u, w) = \frac{e^{\frac{v^u[h(a,x),\chi(a,x)]}{\alpha}}}{e^{\frac{v^u[h(a,x),\chi(a,x)]}{\alpha}} + e^{\frac{v^e[h(a,x),\chi(a,x),w]}{\alpha}} + e^{\frac{v^s[h(a,x),\chi(a,x),w]}{\alpha}}} : \text{Quitting Probb}$$

$$\pi^s(a, x^e, x^u, w) = \frac{e^{\frac{v^s[h(a,x),\chi(a,x),w]}{\alpha}}}{e^{\frac{v^u[h(a,x),\chi(a,x)]}{\alpha}} + e^{\frac{v^e[h(a,x),\chi(a,x),w]}{\alpha}} + e^{\frac{v^s[h(a,x),\chi(a,x),w]}{\alpha}}} : \text{Searching Probb}$$

7. Job to Job Movements, Imperf Insur: Members Choose

$$
\begin{aligned}
V^e(a,x,w) = &\ U(c) + (w-c)U_c + \beta\delta V^u(a',x') \\
&+ \beta(1-\delta)\left[\mu + \alpha\gamma + \alpha \ln\left(e^{V^e(a',x',w)/\alpha} + e^{V^u(a',x')/\alpha} + e^{V^s(a',x',w)/\alpha}\right)\right]
\end{aligned}
$$

Define

$$
\begin{aligned}
\Phi^e(a,x,w) \equiv V^e(a,x,w) - V^u(a,x) = &\ U(c) - U(b) + (w-c)U_c - \beta \max_{w'}\left\{\psi^h(w')\Phi^e(a',x',w')\right\} \\
&+ \beta(1-\delta)\left[\mu + \alpha\gamma + \alpha\ln\left(1 + e^{\Phi^e(a',x',w)/\alpha} + e^{\Phi^s(a',x',w)/\alpha}\right)\right] \\
\Phi^s(a,x,w) \equiv V^s(a,x,w) - V^u(a,x) = &\ -\bar{c}^s + U(c) - U(b) + (w-c)U_c \\
&- \beta\max_{w'}\left\{\psi^h(w')\Phi^e(a',x',w')\right\} \\
&+ \beta(1-\delta)\max_{\hat{w}'}\left\{\psi^h(\hat{w}')\left[\Phi^e(a',x',\hat{w}') - \Phi^e(a',x',w)\right]\right\}
\end{aligned}
$$

The members' problem is jointly characterized by Φ^e and Φ^s. The FOCs are

$$
0 = \quad \psi^h_\theta[\theta(w')]\ \theta_w(w')\ \Phi^e[h(a,x),\chi(a,x),w'] + \psi^h[\theta(w')]\ \Phi^e_w[h(a,x),\chi(a,x),w']
$$

$$
0 = \quad \psi^h_w(w')\ [\Phi^e(a',x',w') - \Phi^e(a',x',w)] + \psi^h_w(w')\ \Phi^e_w(a',x',w')
$$

Endog Quits HH head problem

$$
\begin{aligned}
V(a, x^e, x^u) = &\max_{a'} (1-x^u)\, U\left[\frac{(1+r)a + \int w x^e(\mathrm{d}w) - a'}{1-x^u}\right] + x^u U(b) \\
&+ \beta(1-\delta)\int J(a', x', w)x^e(\mathrm{d}w) + \beta V(a', x^{e\prime}, x^{u\prime}) \\
x^{e\prime}(w) = &(1-\delta)\left\{1 - \pi^q(a', x', w) - \pi^s(a', x', w)\psi^h[\omega^s(a', x', w)]\right\} x^e(w) \\
&+ \mathbb{1}(\omega^s(a', x', \hat{w}) = w)\psi^h[w]\pi^s(a', x', \hat{w})(1-\delta)x^e(\hat{w}) + \mathbb{1}(\omega(a', x') = w)\psi^h[w]x^u \\
x^{u\prime} = &\int [\delta + (1-\delta)\pi^q(a', x', w)]x^e(\mathrm{d}w) + \{1 - \psi^h[\omega(a', x')]\}x^u
\end{aligned}
$$

$J(a', x', w)$ is the joy from the extreme value shocks that the head takes as given. FOC yields

$$
U_c(c) = \beta(1-\delta)\int J_{a'}(a', x', w)x^e(\mathrm{d}w) + \beta V_{a'}(a', x')
$$

And Envelopes

$$
\begin{aligned}
V_a(a, x^e, x^u) = &(1+r)U_c(c) \underbrace{- U_c(c)\frac{\partial a'}{\partial a} + \beta(1-\delta)\int J_{a'}(a', x', w)x^e(\mathrm{d}w)\frac{\partial a'}{\partial a} + \beta V'_{a'}\frac{\partial a'}{\partial a}}_{=0 \text{ by HH head FOC}} \\
&+ \beta\int V'_{x^{e\prime}(w)}\frac{\partial x^{e\prime}(w)}{\partial a}\mathrm{d}w + \beta V'_{x^{u\prime}}\frac{\partial x^{u\prime}}{\partial a} \\
= &(1+r)U_c(c) + \beta\int\left\{V'_{x^{e\prime}(w)}\frac{\partial x^{e\prime}(w)}{\partial a} + V'_{x^{u\prime}}\frac{\partial x^{u\prime}}{\partial a}\right\}\mathrm{d}w
\end{aligned}
$$

Characterization of Savings

- The term $\int \left\{ V'_{x^{e\prime}(w)} \frac{\partial x^{e\prime}(w)}{\partial a} + V'_{x^{u\prime}} \frac{\partial x^{u\prime}}{\partial a} \right\} dw$ captures the effect of a on next period V' through changing the measure x'.
- By definition we have

$$
\begin{aligned}
V_{x^{e\prime}(w')}(a', x^{e\prime}, x^{u\prime}) &= w' U_c(c') + \beta(1-\delta)J(a'', x'', w') + \beta V''_{x^{e\prime\prime}(w')}(1-\delta)(1-\pi^{q\prime\prime} - \pi^{s\prime\prime}\psi^h(\omega^{s\prime\prime})) \\
&+ \beta V''_{x^{e\prime\prime}(\hat{w})} \mathbb{1}(\omega^s(a'', x'', w') = \hat{w})\psi^h[w']\pi^s(a'', x'', w')(1-\delta) \\
&+ \beta V''_{x^{u\prime\prime}}(\delta + (1-\delta)\pi^{q\prime\prime}) \\
V_{x^{u\prime}}(a', x^{e\prime}, x^{u\prime}) &= -U(c') + U_c(c')c' + U(b) \\
&+ \beta V_{x^{e\prime\prime}(w'')}\psi^h(w'')\mathbb{1}(w'' = \omega'') + \beta V_{x^{u\prime\prime}}(1-\psi^h(w''))\mathbb{1}(w'' = \omega'') \\
V'_{x^{e\prime}(w')} - V'_{x^{u\prime}} &= U(c') - U(b) + (w' - c')U_c(c') + \beta(1-\delta)J(a'', x'', w') \\
&+ \beta(1-\delta)(1-\pi^{q\prime\prime})(V''_{x^{e\prime\prime}(w')} - V''_{x^{u\prime\prime}}) \\
&+ \beta(1-\delta)\pi^{s\prime\prime}\psi^h(\hat{w})(V''_{x^{e\prime\prime}(\hat{w})} - V''_{x^{e\prime\prime}(w')})\mathbb{1}(\omega^s(a'', x'', w') = \hat{w}) \\
&- \beta(V_{x^{e\prime\prime}(w'')} - V_{x^{u\prime\prime}})\psi^h(w'')\mathbb{1}(w'' = \omega'')
\end{aligned}
$$

- Now define $\Phi^c(a', x', w') = V_{x^{e\prime}(w')}(a', x') - V_{x^{u\prime}}(a', x')$, we have

$$
\begin{aligned}
\Phi^c(a', x', w') &= U(c') - u(b) + (w' - c')U_c(c') + \beta(1-\delta)J(a'', x'', w') \\
&+ \beta(1-\delta)(1-\pi^{q\prime\prime})\Phi^c(a'', x'', w') \\
&+ \beta(1-\delta)\psi^h(w'')\pi^{s\prime\prime}(\Phi^c(a'', x'', w'') - \Phi^c(a'', x'', w'))\mathbb{1}(w'' = \omega^{s\prime\prime}) \\
&- \beta\psi^h(w'')\Phi^c(a'', x'', w'')\mathbb{1}(w'' = \omega'')
\end{aligned}
$$

Job To Job Movs with Preference Shocks & Unemp Insurance

8. J2J Movements, Unemp Insur & capital Income: Members Choose

- In addition to Endog quits:
- Empld get ut Shocks $\{\epsilon^w, \epsilon^q, \epsilon^s\}$, $\epsilon^i \sim G(\mu, \alpha)$; after work & cons choose either quit, search or nothing

$$v^u(a, x^e, x^u) = U\left[b + (1+r)a - a' + \frac{\int w\tau dx^e}{x^u}\right] + \beta \max_{w'} \left\{\psi^h[\theta(w')] \left[v^e(a', x', w') - v^u(a', x')\right]\right\}$$

$$v^e(a, x^e, x^u, w) = U[(1-\tau)w + (1+r)a - a'] + \beta\left[(1-\delta)\,\widehat{v}^e(a', x', w) + \delta\, v^u(a', x')\right]$$

$$v^s(a, x^e, x^u, w) = U[(1-\tau)w + (1+r)a - a'] - \bar{c}^s + \beta\delta V^u(a', x') + \beta(1-\delta)v^e(a', x', w) + \beta(1-\delta)\max_{w'}\left\{\psi^h[\theta(w')] \left[v^e(a', x', w') - v^e(a', x', w)\right]\right\}$$

need one period to search like the unemployed

$$\widehat{\widehat{v}}^e(a', x^{e\prime}, x^{u\prime}, w, \epsilon) = \max\left\{v^e(a', x', w) + \epsilon^w, v^u(a', x') + \epsilon^q, v^s(a', x', w) + \epsilon^s\right\}$$

$$\widehat{v}^e(a', x^{e\prime}, x^{u\prime}, w) = \mathbb{E}\left[\widehat{\widehat{v}}^e(a', x^{e\prime}, x^{u\prime}, w, \epsilon)\right] = \mu + \alpha \ln\left(e^{\frac{v^e(a',x',w)}{\alpha}} + e^{\frac{v^u(a',x')}{\alpha}} + e^{\frac{v^s(a',x',w)}{\alpha}}\right) + \alpha\,\gamma$$

$$\pi^q(a, x^e, x^u, w) = \frac{e^{\frac{v^u[h(a,x),\chi(a,x)]}{\alpha}}}{e^{\frac{v^u[h(a,x),\chi(a,x)]}{\alpha}} + e^{\frac{v^e[h(a,x),\chi(a,x),w]}{\alpha}} + e^{\frac{v^s[h(a,x),\chi(a,x),w]}{\alpha}}} : \text{Quitting Probb}$$

$$\pi^s(a, x^e, x^u, w) = \frac{e^{\frac{v^s[h(a,x),\chi(a,x),w]}{\alpha}}}{e^{\frac{v^u[h(a,x),\chi(a,x)]}{\alpha}} + e^{\frac{v^e[h(a,x),\chi(a,x),w]}{\alpha}} + e^{\frac{v^s[h(a,x),\chi(a,x),w]}{\alpha}}} : \text{Searching Probb}$$

8. J2J Movements, Unemp Insur & capital Income: Members Choose

$$V^e(a,x,w) = U\left[(1-\tau)w + (1+r)a - a'\right] + \beta\delta V^u(a',x')$$
$$+ \beta(1-\delta)\left[\mu + \alpha\gamma + \alpha \ln\left(e^{V^e(a',x',w)/\alpha} + e^{V^u(a',x')/\alpha} + e^{V^s(a',x',w)/\alpha}\right)\right]$$

Define

$$\Phi^e(a,x,w) \equiv V^e(a,x,w) - V^u(a,x) = U[(1-\tau)w + (1+r)a - a'] - U\left[b + (1+r)a - a' + \frac{\int w\tau dx^e}{x^u}\right]$$
$$+ \beta \max_{w'}\left\{\psi^h(w')\Phi^e(a',x',w')\right\}$$
$$+ \beta(1-\delta)\left[\mu + \alpha\gamma + \alpha \ln\left(1 + e^{\Phi^e(a',x',w)/\alpha} + e^{\Phi^s(a',x',w)/\alpha}\right)\right]$$

$$\Phi^s(a,x,w) \equiv V^s(a,x,w) - V^u(a,x) = -\bar{c}^s + U[(1-\tau)w + (1+r)a - a'] - U\left[b + (1+r)a - a' + \frac{\int w\tau dx^e}{x^u}\right]$$
$$- \beta \max_{w'}\left\{\psi^h(w')\Phi^e(a',x',w')\right\}$$
$$+ \beta(1-\delta)\max_{\hat{w}'}\left\{\psi^h(\hat{w}')\left[\Phi^e(a',x',\hat{w}') - \Phi^e(a',x',w)\right]\right\}$$

The members' problem is jointly characterized by Φ^e and Φ^s. The FOCs are still need work as they are not updated yet

$$0 = \quad \psi^h_\theta[\theta(w')]\ \theta_w(w')\ \Phi^e[h(a,x),\chi(a,x),w'] + \psi^h[\theta(w')]\ \Phi^e_w[h(a,x),\chi(a,x),w']$$

$$0 = \quad \psi^h_w(w')\ [\Phi^e(a',x',w') - \Phi^e(a',x',w)] + \psi^h_w(w')\ \Phi^e_w(a',x',w')$$

Endog Quits HH head problem

max weigthed average of unemp utilt & the utility of a worker with average workers' cons not average utilt

$$\widehat{U}(a,x,a') = (1-x^u)\, U\left[(1+r)a - a' + \frac{\int(1-\tau)w\ dx^e(w)}{1-x^u}\right] + x^u\ U\left[b + (1+r)a - a' + \frac{\int w\tau dx^e}{x^u}\right]$$

$$\widehat{U}_a(a,x,a') = (1+r)\left\{x^e U_c\left[(1+r)a - a' + \frac{\int(1-\tau)w dx^e(w)}{1-x^u}\right] + x^u U_c\left[b + (1+r)a - a' + \frac{\int w\tau dx^e}{x^u}\right]\right\}$$

$$-\widehat{U}_{a'}(a,x,a') = (1-x^u)\, U_c\left[(1+r)a - a' + \frac{\int(1-\tau)w\ dx^e(w)}{1-x^u}\right] + x^u\ U_c\left[b + (1+r)a - a' + \frac{\int w\tau dx^e}{x^u}\right]$$

$$V(a,x^e,x^u) = \max_{a'}\ \widehat{U}(a,x,a') + \beta(1-\delta)\int J(a',x',w)x^e(dw) + \beta V(a',x^{e\prime},x^{u\prime})$$

$$x^{e\prime}(w) = (1-\delta)\left\{1 - \pi^q(a',x',w) - \pi^s(a',x',w)\psi^h[\omega^s(a',x',w)]\right\}x^e(w)$$
$$+ \mathbb{1}(\omega^s(a',x',\hat{w}) = w)\psi^h[w]\pi^s(a',x',\hat{w})(1-\delta)x^e(\hat{w}) + \mathbb{1}(\omega(a',x') = w)\psi^h[w]x^u$$

$$x^{u\prime} = \int[\delta + (1-\delta)\pi^q(a',x',w)]x^e(dw) + \{1 - \psi^h[\omega(a',x')]\}x^u$$

$J(a',x',w)$ is the joy from the extreme value shocks that the head takes as given. FOC yields

$$-\widehat{U}_{a'}(a,x,a') = \beta(1-\delta)\int J_{a'}(a',x',w)\ x^e(dw) + \beta V_{a'}(a',x')$$

And Envelope

$$V_a(a,x^e,x^u) = \widehat{U}_a(a,x,a') \underbrace{-\widehat{U}_{a'}(a,x,a')\frac{\partial a'}{\partial a} + \beta(1-\delta)\int J_{a'}(a',x',w)x^e(dw)\frac{\partial a'}{\partial a} + \beta V'_{a'}\frac{\partial a'}{\partial a}}_{=0 \text{ by HH head FOC}}$$

$$+\beta\int V'_{x^{e\prime}(w)}\frac{\partial x^{e\prime}(w)}{\partial a}dw + \beta V'_{x^{u\prime}}\frac{\partial x^{u\prime}}{\partial a} = \widehat{U}_a(a,x,a') + \beta\int\left\{V'_{x^{e\prime}(w)}\frac{\partial x^{e\prime}(w)}{\partial a} + V'_{x^{u\prime}}\frac{\partial x^{u\prime}}{\partial a}\right\}dw$$

Characterization of Savings NEEDS UPDATE

- The term $\int \left\{ V'_{x^{e\prime}(w)} \frac{\partial x^{e\prime}(w)}{\partial a} + V'_{x^{u\prime}} \frac{\partial x^{u\prime}}{\partial a} \right\} dw$ captures the effect of a on next period V' through changing the measure x'.
- By definition we have

$$\begin{aligned}
V_{x^{e\prime}(w')}(a', x^{e\prime}, x^{u\prime}) &= w' U_c(c') + \beta(1-\delta)J(a'', x'', w') + \beta V''_{x^{e\prime\prime}(w')}(1-\delta)(1-\pi^{q\prime\prime} - \pi^{s\prime\prime}\psi^h(\omega^{s\prime\prime})) \\
&\quad + \beta V''_{x^{e\prime\prime}(\hat{w})} \mathbb{1}(\omega^s(a'', x'', w') = \hat{w})\psi^h[w']\pi^s(a'', x'', w')(1-\delta) \\
&\quad + \beta V''_{x^{u\prime\prime}}(\delta + (1-\delta)\pi^{q\prime\prime}) \\
V_{x^{u\prime}}(a', x^{e\prime}, x^{u\prime}) &= -U(c') + U_c(c')c' + U(b) \\
&\quad + \beta V_{x^{e\prime\prime}(w'')}\psi^h(w'')\mathbb{1}(w'' = \omega'') + \beta V_{x^{u\prime\prime}}(1 - \psi^h(w''))\mathbb{1}(w'' = \omega'') \\
V'_{x^{e\prime}(w')} - V'_{x^{u\prime}} &= U(c') - U(b) + (w' - c')U_c(c') + \beta(1-\delta)J(a'', x'', w') \\
&\quad + \beta(1-\delta)(1-\pi^{q\prime\prime})(V''_{x^{e\prime\prime}(w')} - V''_{x^{u\prime\prime}}) \\
&\quad + \beta(1-\delta)\pi^{s\prime\prime}\psi^h(\hat{w})(V''_{x^{e\prime\prime}(\hat{w})} - V''_{x^{e\prime\prime}(w')})\mathbb{1}(\omega^s(a'', x'', w') = \hat{w}) \\
&\quad - \beta(V_{x^{e\prime\prime}(w'')} - V_{x^{u\prime\prime}})\psi^h(w'')\mathbb{1}(w'' = \omega'')
\end{aligned}$$

- Now define $\Phi^c(a', x', w') = V_{x^{e\prime}(w')}(a', x') - V_{x^{u\prime}}(a', x')$, we have

$$\begin{aligned}
\Phi^c(a', x', w') &= U(c') - u(b) + (w' - c')U_c(c') + \beta(1-\delta)J(a'', x'', w') \\
&\quad + \beta(1-\delta)(1-\pi^{q\prime\prime})\Phi^c(a'', x'', w') \\
&\quad + \beta(1-\delta)\psi^h(w'')\pi^{s\prime\prime}(\Phi^c(a'', x'', w'') - \Phi^c(a'', x'', w'))\mathbb{1}(w'' = \omega^{s\prime\prime}) \\
&\quad - \beta\psi^h(w'')\Phi^c(a'', x'', w'')\mathbb{1}(w'' = \omega'')
\end{aligned}$$

Endog Quits HH head problem (Kosher Version)

max average utilty

$$\widetilde{U}(a,x,a') = \int U\left[(1+r)a - a' + (1-\tau)\ w\right]\ dx^e(w) + x^u\ U\left[b + (1+r)a - a' + \frac{\int w\tau dx^e}{x^u}\right]$$

$$\widetilde{U}_a(a,x,a') = (1+r)\left\{\int U_c\left[(1+r)a - a' + (1-\tau)\ w\right]\ dx^e(w) + x^u\ U_c\left[b + (1+r)a - a' + \frac{\int w\tau dx^e}{x^u}\right]\right\}$$

$$-\widetilde{U}_{a'}(a,x,a') = \int U_c\left[(1+r)a - a' + (1-\tau)\ w\right]\ dx^e(w) + x^u\ U_c\left[b + (1+r)a - a' + \frac{\int w\tau dx^e}{x^u}\right]$$

$$V(a,x^e,x^u) = \max_{a'}\ \widetilde{U}(a,x,a') + \beta(1-\delta)\int J(a',x',w)x^e(\mathrm{d}w) + \beta V(a',x^{e\prime},x^{u\prime})$$

$$x^{e\prime}(w) = (1-\delta)\left\{1 - \pi^q(a',x',w) - \pi^s(a',x',w)\psi^h[\omega^s(a',x',w)]\right\}x^e(w)$$
$$+\ \mathbb{1}(\omega^s(a',x',\hat{w}) = w)\psi^h[w]\pi^s(a',x',\hat{w})(1-\delta)x^e(\hat{w}) + \mathbb{1}(\omega(a',x') = w)\psi^h[w]x^u$$

$$x^{u\prime} = \int[\delta + (1-\delta)\pi^q(a',x',w)]x^e(\mathrm{d}w) + \{1 - \psi^h[\omega(a',x')]\}x^u$$

$J(a',x',w)$ is the joy from the extreme value shocks that the head takes as given. FOC yields

$$-\ \widetilde{U}_{a'}(a,x,a') = \beta(1-\delta)\int J_{a'}(a',x',w)\ x^e(\mathrm{d}w) + \beta V_{a'}(a',x')$$

And Envelope

$$V_a(a,x^e,x^u) = \widetilde{U}_a(a,x,a')\underbrace{-\widetilde{U}_{a'}(a,x,a')\frac{\partial a'}{\partial a} + \beta(1-\delta)\int J_{a'}(a',x',w)x^e(\mathrm{d}w)\frac{\partial a'}{\partial a} + \beta V'_{a'}\frac{\partial a'}{\partial a}}_{=0\text{ by HH head FOC}}$$

$$+\beta\int V'_{x^{e\prime}(w)}\frac{\partial x^{e\prime}(w)}{\partial a}\mathrm{d}w + \beta V'_{x^{u\prime}}\frac{\partial x^{u\prime}}{\partial a} = \widetilde{U}_a(a,x,a') + \beta\int\left\{V'_{x^{e\prime}(w)}\frac{\partial x^{e\prime}(w)}{\partial a} + V'_{x^{u\prime}}\frac{\partial x^{u\prime}}{\partial a}\right\}\mathrm{d}w$$

Thanks for listening!

Appendix

Baseline Model Continuous Wage: Steady State

- Solve the steady state
    - Guess the HH consumption level c
    - Given c, The unemployed member's problem yields wage applying function $w = w^u(\theta; c)$.
    - Firm's free entry condition yields wage posting function $w = w^f(\theta)$.
    - The above two functions give the equilibrium (w, θ), along with the member distribution x over w and b.
    - HH head solves the consumption and saving problem given $\{w, x\}$, yields c^f.
    - Adjust c given c^f and loop until converge.
- Note: under perfect insurance the outside loop of c is redundant.

Baseline Model Continuous Wage: Transition

- Solve the transition path given an MIT interest rate (r) shock
  - Take the s-s value of $\{w, x, \theta, c(a, w), h(a, w)\}$ and the interest rate path $\{r_t\}_{t=1}^{T}$. Let the last period be at the s-s.
  - Guess a consumption path $\{c_t\}_{t=1}^{T-1}$.
  - At $T-1$, solve w^u and w^f given r_{T-1} and c_{T-1}, and the corresponding equilibrium w_{T-1} and θ_{T-1}.
  - Solve the distribution x_{T-1} using the worker law of motion $e^T(w) = (1-\delta)e^{T-1}(w) + u^{T-1}\psi^h[\theta(w_{T-1})]$ and $u^T = \delta \sum_w e^{T-1}(w) + (1 - \psi^h[\theta(w_{T-1})])u^{T-1}$.
  - Solve HH problem given $\{w_{T-1}, e_{T-1}(w), u_{T-1}\}$, and V^T.
  - Move backward to $t = 1$. Compute the produced consumption path $\{c_t^f\}_{t=1}^{T-1}$.
  - Adjust $\{c_t\}_{t=1}^{T-1}$ given $\{c_t^f\}_{t=1}^{T-1}$ and loop until converge.

Baseline Model Discrete Wage: Steady State

- Solve the steady state
  - Guess the HH consumption level c.
  - Use the firm's free entry condition to get $\{\theta(w), \psi^h(w), \psi^f(w)\}$ for each w. With exogenous quitting and no on-the-job search this does not involve c.
  - Given c, solve for each w the value of putting one unemployed worker to work $\Phi(w; c)$. This is just solving a system of n equations. Compute wage applying profile $\{\pi(w; c)\}$.
  - Compute J from Φ. And Get stationary $\{x^e, x^u\}$ from $\{\pi(w; c)\}$.
  - Solve the HH head problem given $\{J, x^e, x^u, \psi^h\}$, yields c^f.
  - Adjust c given c^f and loop until converge.

Baseline Model Discrete Wage: Transition

- Solve the transition path given an MIT interest rate (r) shock
  - Guess the HH consumption path $\{c_t\}$.
  - Use the firm's free entry condition to get $\{\theta_t(w), \psi_t^h(w), \psi_t^f(w)\}$ path for each w. With exogenous quitting and no on-the-job search this does not depend on $\{c_t\}$.
  - Given $\{c_t\}$, solve for each w the value of putting one unemployed worker to work $\{\Phi_t(w; c_t)\}$. Compute wage applying profile $\{\pi_t(w; c_t)\}$.
  - Compute J_t from Φ_t. And generate the path of $\{x_t^e, x_t^u\}$ from $\{\pi_t(w; c_t)\}$, given $\{x_0^e, x_0^u\}$.
  - Solve the HH head problem given $\{J_t, x_t^e, x_t^u, \psi_t^h\}$, yields c_t^f.
  - Adjust c_t given c_t^f and loop until converge.