1 FEBRUARY 11

1.1 Review

- Last class, our purpose was to construct a new market arrangement, sequence of markets, because it is much closer to what we think markets are like in the real world. The fact that all trade takes place at time 0 in the Arrow-Debreu world is not very realistic so we wanted to allow the agents to trade at each period.
- We used certain properties of equilibrium to write a shorter version of SME that did not bother to distinguish between the choice of the firm and the household (for convenience).
- Now we will show that the allocations of the Arrow-Debreu equilibrium and the sequence of markets equilibrium are the same. Namely we will outline the proof of the following theorem:

Theorem

(i) If (x^*, y^*, p^*) is an Arrow-Debreu equilibrium, we can construct the sequence of markets equilibrium with (x^*, y^*).

(ii) If $(\widetilde{x}, \widetilde{y}, \widetilde{r}, \widetilde{w})$ is an sequence of markets equilibrium, we can construct the Arrow-Debreu equilibrium with $(\widetilde{x}, \widetilde{y})$.

Proof (Outline)

Remark 1 *Refer to the solution key of Hw 3 for the complete proof.*

First showing ADE $\Rightarrow$ SME

$(\mathrm{q}^*, x^*, y^*) \Rightarrow \exists \left\{c_t^*, k_{t+1}^*, r_t^*, w_t^*\right\}_{t=0}^{\infty} \in SME$

The first thing we need to do is construct the sequence of markets equilibrium prices from q^*. Remember that $\mathrm{q}^*(x)$ is a function that assigns a value to each commodity bundle in terms of consumption goods AT TIME 0. The prices in the AD world DO NOT correspond to the usual price for consumption goods, wage and rent. In order to get r_t^*, w_t^* we need to transform these prices in terms of units of consumption at time 0, to prices in terms of units of consumption goods at time t. The question is: How much does one unit of time 0 consumption exchange for unit of consumption at time t?

1 unit of time 0 consumption $\rightarrow \frac{1}{q^*(\{0,0,0\},\{1,0,0\},\ldots\ldots\ldots.)}$

i.e. 1 unit of time 0 consumption can get you $\frac{1}{q^*(\{0,0,0\},\{1,0,0\},\ldots\ldots\ldots.)}$ units of time 1 consumption.

Thus, we can write the following,
$w_0^* = q^*(\{0,1,0\}, \{0,0,0\}, ...)$
$r_0^* = q^*(\{0,0,1\}, \{0,0,0\}, ...)$
$w_1^* = \frac{q^*(\{0,0,0\},\{0,1,0\},..........)}{q^*(\{0,0,0\},\{1,0,0\},..........)}$
....

Homework *In the same way, write down the expressions for* r_t^*, w_t^*.

Now the following will be our strategy to show that from ADE we can get to SME:

First construct a candidate $\{\widetilde{r}_t, \widetilde{w}_t\}_{t=0}^{\infty}$ and $\left\{\widetilde{c}_t, \widetilde{k}_{t+1}, \widetilde{n}_t\right\}_{t=0}^{\infty}$ from (q^*, x^*, y^*).

- For $\widetilde{c}_t, \widetilde{k}_{t+1}, \widetilde{n}_t$, pick c_t^*, k_{t+1}^*, n_t^* so that

$$\begin{aligned}
\widetilde{c}_t &= x_{1t}^* + (1-\delta)x_{3t}^* - x_{3t+1}^* \qquad \forall t \\
\widetilde{n}_t &= x_{2t}^* \qquad \forall t \\
\widetilde{k}_t &= x_{3t}^* \qquad \forall t
\end{aligned}$$

- For $\widetilde{r}_t, \widetilde{w}_t$, pick

$$\begin{aligned}
\widetilde{r}_t &= \frac{p_{3t}^*}{p_{1t}^*} = F_k(k_t^*, n_t^*) \qquad \forall t \\
\widetilde{w}_t &= \frac{p_{2t}^*}{p_{1t}^*} = F_n(k_t^*, n_t^*) \qquad \forall t
\end{aligned}$$

Now verify that these candidates solve the firm's and the consumer's maximization problem. For firms, this is obvious from the condition that marginal productivities equal to the prices of factors of production.

But for the consumer, we need to show that,

$$\left\{\widetilde{c}_t, \widetilde{k}_{t+1}, \widetilde{n}_t\right\}_{t=0}^{\infty} \in \underset{\{c_t, k_{t+1}, n_t\}_{t=0}^{\infty}}{\text{argmax}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1-n_t)$$

$$\text{s.t.} \quad c_t + k_{t+1} = \widetilde{w}_t n_t + (1 + \widetilde{r}_t - \delta)k_t$$

We know that the objective function is strictly concave. The next thing we need is that the constraint set is convex.

Homework *Define F as,*

$F{=}\{c_t, k_{t+1}, n_t\}_{t=0}^{\infty} \mid c_t + k_{t+1} = \widetilde{w}_t + (\widetilde{r}_t + 1 - \delta)k_t \quad \forall t$

Show that F is convex

Once we know the above (i.e. the strict concavity of the objective function and convexity of the constraint set), we can say that the solution to the consumer's problem exists, is unique and the First Order Conditions characterize it (together with the Transversality Condition).

Then showing that if c_t^*, k_{t+1}^*, n_t^* satisfies the FOC in the AD world given q^* , it also satisfies the FOC from the consumer's problem above will be enough to complete the proof.

Question: Can we prove it another way, for example through contradiction? Yes, but that will not make our life any easier. Because even when you suppose that there is another allocations other than c_t^*, k_{t+1}^*, n_t^* that solves the consumer's problem in the sequence of markets, you will still need the properties that the solution satisfies as we derived to get the contradiction.

Now showing SME $\Rightarrow$ ADE

We need to build the AD objects (x's and y's) from the SME allocation.

$$\begin{aligned}
x_{1t}^* &= \widetilde{c}_t + \widetilde{k}_{t+1} - (1-\delta)\widetilde{k}_t \quad & \forall t \\
x_{2t}^* &= \widetilde{n}_t & \forall t \\
x_{3t}^* &= \widetilde{k}_t & \forall t
\end{aligned}$$

And the candidate for q^* will be,

$$q^*(x) = \frac{\sum_{t=0}^{\infty} \text{-}x_{1t} + x_{2t}\widetilde{w}_t + x_{3t}\widetilde{r}_t}{\prod_{s=0}^{t} (1{+}r_s^*)}$$

Note that this is a function on a whole sequence. We have to define q not just one a point but everywhere. The other way (ADE to SME) was easy because the wage and the rental prices were just numbers.

Homework *Show that this candidate for $q^*(x)$ is indeed a price (Hint: Show that it is continous and linear).*

Remark 2 *What does continous mean in infinite dimensional space? Bounded. In this context, it implies that the value of the bundle of commodities has to be finite and for that we need prices to go to zero sufficiently fast.*

$\frac{1}{\prod_{s=0}^{t}(1+r_s^*)} \to 0$

A sufficient condition for the prices that we contructed to be bounded and thus continous is that the interest rates are not negative too often.

Now from market clearing we know that the following has to hold,

$$x^* = y^*$$

Homework *Show that x^* and y^* solves the problem of the consumer and the firm.*

And that's the end of the second part of the proof.

1.2 ROAD MAP

What have we done so far?

- We know that the social planner's problem can be solved recursively (you learned this in Randy's class). So with dynamic programming methods, we get a good approximation of the optimal policy (g(k)) and get $\{c_t^*, k_{t+1}^*\}_{t=0}^{\infty}$ .Then we learned that this allocation is Pareto Optimal and that it can be supported as a quasi-equilibrium with transfers.
- We also learned that this allocation $\{c_t^*, k_{t+1}^*\}_{t=0}^{\infty}$ is also the sequence of markets equilibrium allocation and it is the ONLY one.
- So now we know that the dynamic programming problem gives us not only what is good but also what wiil happen in the sequence of markets.

What next?

- The question that we now want to address is: What happens if there are heterogenous agents in the economy (versus the representative agent model that we have been dealing with so far) and if the solution is not Pareto Optimal?
- What can we do when we do not have the luxury of having an economy that does not satisfy the Welfare Theorems or when there are different agents?

- Can we still use dynamic programming to deal with problems like this?
- We will define equilibria recursively so that we can write the problem of the households as a dynamic programming problem and we will use the same methods Randy used to find the optimal policy rule g(k). But now the objects that the agents are choosing over are not sequences. They choose what they will do for today and tomorrow and prices are not a sequence anymore but a function of the states.
- We will do the construction of such equilibria after a short digression on shocks.

1.3 SHOCK AND HISTORY

We will now look at the stochastic RA-NGM.

What is a shock? Unanticipated change? Not really:

In a stochastic environment, we don't know exactly what will happen but we know where it's coming from (we know something about the stochastic process, i.e. the process that the shocks are following)

1.3.1 Markov Chains

In this course, we will concentrate on Markov productivity shock. Markov shock is a stochastic process with the following properties.

1. There are finite number of possible states for each time. More intuitively, no matter what happened before, tomorrow will be represented by one of a finite set.
2. The only thing that matters for the realization tomorrow is today's state. More intuitively, no matter what kind of history we have, the only thing you need to predict realization of shock tomorrow is today's realization.

   More formally, for each period, suppose either z^1 or z^2 happens. Denote z_t is the state of today and Z_t is a set of possible state today, i.e. $z_t \in Z_t = \{z^1, z^2\}$ for all t. Since the shock follow Markov process, the state of tomorrow will only depend on today's state. So let's write the probability that z^j will happen tomorrow, conditional on today's state being z^i as $\Gamma_{ij} = prob[z_{t+1} = z^j | z_t = z^i]$. Since Γ_{ij} is a probability, we know that

   $$\sum_j \Gamma_{ij} = 1 \qquad for\ \forall i$$

   Notice that 2-state Markov process is summarized by 6 numbers: z^1, z^2, Γ_{11}, Γ_{12}, Γ_{21}, Γ_{22}.

The great beauty of using Markov process is we can use the explicit expression of probability of future events, instead of using weird operator called expectation, which very often people don't know what it means when they use.

1.3.2 Representation of History

- Let's concentrate on 2-state Markov process. In each period, state of the economy is $z_t \in Z_t = \{z^1, z^2\}$.
- Denote the history of events up to t (which of $\{z^1, z^2\}$ happened from period 0 to t, respectively) by

  $h_t = \{z_0, z_1, z_2, ..., z_t\} \in H_t = Z_0 \times Z_1 \times ... \times Z_t$.
- In particular, $H_0 = \emptyset$, $H_1 = \{z^1, z^2\}$, $H_2 = \{(z^1, z^1), (z^1, z^2), (z^2, z^1), (z^2, z^2)\}$.
- Note that even if the state today is the same, past history might be different. By recording history of event, we can distinguish the two histories with the same realization today but different realizations in the past (think that the current situation might be "you do not have a girl friend", but we will distinguish the history where "you had a girl friend 10 years ago" and the one where you didn't (tell me if it is not an appropriate example...).)
- Let $\Pi(h_t)$ be the unconditional probability that the particular history h_t does occur. By using the Markov transition probability defined in the previous subsection, it's easy to show that (i) $\Pi(h_0) = 1$, (ii) for $h_t = (z^1, z^1)$, $\Pi(h_t) = \Gamma_{11}$ (iii) for $h_t = (z^1, z^2, z^1, z^2)$, $\Pi(h_t) = \Gamma_{12}\Gamma_{21}\Gamma_{12}$.
- $\Pr\left\{z_{t+1} = z^i \mid z_t = z^j, z_{t-1}, z_{t-2,..........}\right\} = \Gamma_{ji}$
- Having finite support of the distribution is very convenient.

  Homework *Show that a Markov chain of memory 2 can be represented as a Markov chain of memory 1.*

1.3.3 Social Planner's Problem with Shocks

- Social Planner's Problem (the benevolent God's choice) in this world is a state-contingent plan, i.e. optimal consumption and saving (let's forget about labor-leisure choice in this section for simplicity) choice for all possible nodes (imagine the nodes of a game tree. we need to solve optimal consumption and saving for each node in the tree).
- Notice that the number of nodes for which we have to solve for optimal consumption and saving is countable. This feature allows us to use the same argument as the deterministic case to deal with the problem. The only difference is that for deterministic case, the number of nodes is equal to number of periods (which is infinite but countable), but here the number

of nodes is equal to the number of date-events (which is also infinite but countable).

- More mathematically, the solution of the problem is the mapping from the set of date-events (which is specified by history) to the set of feasible consumption and saving.

$$\max_{\{c_t(h_t),k_{t+1}(h_t)\}} \sum_{t=0}^{\infty} \beta^t \sum_{h_t \in H_t} \Pi(h_t) u(c_t(h_t))$$

subject to

$$c_t(h_t) + k_{t+1}(h_t) = (1-\delta)k_t(h_{t-1}) + z_t F[k_t(h_{t-1}), 1] \quad \forall t \ \forall h_t$$

$$k_0, z_0 \text{ given}$$

What is the dynamic programming version of this problem?

When we are writing the dynamic programming version, we need to carefully specify what the states are. States should be things that matter and change and that are predetermined. We will have more on this later.

$$V(z,k) = \max_{c,k\prime} u(c) + \beta \sum_{z\prime \in Z} \Gamma_{zz\prime} V(z\prime, k\prime)$$

subject to

$$c + k\prime = (1-\delta)k + zF(k,1)$$

2 FEB 13: ADE and SME in a stochastic RA-NGM

2.1 Review

- Recall $\Gamma_{ij} = \Pr\left\{z_{t+1} = z^j \mid z_t = z^i\right\}$

- $\Gamma_{i.} = 1$ i.e. the probability of going SOMEWHERE given today's state is z^i is 1.
- $\Pi_t(h_t; \Gamma, z_0)$ is a function from the set of histories up to t.
- A Markov matrix Γ is a square matrix such that
  1. $\Gamma_{i.} = 12.$
  2. $\Gamma_{ij} \geq 0$
- $\Pi_t(h_t; \Gamma, z_0)$: Here Γ denotes possible Markov matrices and z_0 denotes possible initial shocks. Why do we have ; ? This is because Γ and z_0 are given in the problem. They will not be changing while we do the analysis, they are like the parameters of the problem.

2.2 ADE

We will now go over Arrow-Debreu with uncertainty with the inner product representation of prices (rather than using a general continous linear function)

We first need to define the commodity space, the consumption possibility set and the production possibility set.

As in the deterministic environment, define commodity space as space of bounded real sequences with sup-norm $L = l^3_\infty$.

But before in the deterministic case, we only had 3 commodities for each period. Now we have 3 commodities for each date-event (h_t).

Define the consumption possibility set X as:

$$X = \{x \in L = l^3_\infty : \exists \{c_t(h_t), k_{t+1}(h_t)\}_{t=0}^\infty \geq 0 \text{ such that}$$

$$\begin{aligned} k_{t+1}(h_t) + c_t(h_t) &= x_{1t}(h_t) + (1-\delta)k_t(h_t) \qquad \forall t \; \forall h_t \\ x_{2t}(h_t) &\in [0,1] \qquad \forall t \; \forall h_t \\ x_{3t}(h_t) &\leq k_t(h_t) \qquad \forall t \; \forall h_t \\ & k_0, z_0 \text{ given}\} \end{aligned}$$

- Notice that the only difference from before is that now all the constraints has to hold for all periods AND all histories.

  Define the production possibility set Y as:

  $$Y = \{y \in L : y_{1t}(h_t) \leq F(y_{3t}(h_t), y_{2t}(h_t)) \quad \forall t \; \forall h_t\}$$

  The consumer's problem in ADE is:

  $$\max_{x \in X} \sum_{t=0}^{\infty} \beta^t \sum_{h_t \in H_t} \Pi(h_t) u(c_t(h_t))$$

subject to

$$\sum_{t=0}^{\infty}\sum_{h_t\in H_t}\sum_{i=1}^{3}\hat{p}_{it}(h_t)x_{it}(h_t)\leq 0$$

We know that the solution to this problem is Pareto Optimal.

Recall the dynamic programming version of the social planner's problem:

$$V(z,k)=\max_{c,k\prime}u(c)+\beta\sum_{z\prime\in Z}\Gamma_{zz\prime}V(z\prime,k\prime)$$

subject to

$$c+k\prime=(1-\delta)k+zF(k,1)$$

Remember that the state needs to be changing and predetermined. For example, Γ is not a state.

Solution to the above problem is a policy rule k'=g(k) and from this policy rule we can draw the whole path for capital. Also Second Welfare Theorem tells us that the solution can be supported as a quasi-equilibrium with transfers.

2.3 SME

$p_1(h_{17})$: Price of one unit of the consumption good in period 17 at history h.

$p_1(\widetilde{h}_{17})$:Price of one unit of the consumption good in period 17 at history $\widetilde{h}$.

We want to have sequence of markets that are complete. We want the agents to be able to transfer resources not just across time but also across different states of the world.

For this, we need state contingent claims.

The budget constraint for the representative agent in SME world is:

$$c_t(h_t)+k_{t+1}(h_t)+\sum_{z_{t+1}\in Z}q_t(h_t,z_{t+1})l_{t+1}(h_{t+1})=k_t(h_{t-1})[1+r_t(h_t)]+w(h_t)+l_t(h_t)$$

Here $l_{t+1}(h_{t+1})$ is the state contingent claim. By deciding how much $l_{t+1}(h_{t+1})$ to get for each possible h_{t+1}, the agent decides how much of the good he is buying for each possible realization of tomorrow.

Homework *What should the expression below be equal to?*

$$\sum_{z\prime \in Z} q_t(h_t, z\prime) = ?$$

Note that this is the price of an asset that pays one unit of the good to the agent the next period at each state of the world.

A sequence of markets equilibrium is $\{c_t(h_t), k_{t+1}(h_t), l_{t+1}(h_t, z_{t+1})\}$, $\{w(h_t), r(h_t), q_t(h_t, z_{t+1})\}$ such that,

1. Given $\{w(h_t), r(h_t), q_t(h_t, z_{t+1})\}$, $\{c_t(h_t), k_{t+1}(h_t), l_{t+1}(h_t, z_{t+1})\}$ solves the consumer's problem.
2. $w(h_t) = z_t F_2(k_t(h_{t-1}), 1)$
   r(h_t) $= z_t F_1(k_t(h_{t-1}), 1)$
3. $l_{t+1}(h_t, z_{t+1}) = 0 \qquad \forall h_t, z_{t+1}$

3 Feb 18:

What is it that people buy and sell in the sequence of markets?

Consider an economy with two periods. At t=0, the agent's endowment of the good is 2 units. At t=1, two things can happen: The good state or the bad state. The bad state happens with probability π, and the bad state with probability (1-π). In the good state, the agent's endowment is 3 units of the good and in the bad state the agent's endowment is 1 unit of the good.

How many date events are there? 3 date events. Because in addition to the first period, we also have the two possible "events" that can take place at t=1.

The consumer's problem in this economy is:

$$\max \; u(c_0) + \pi u(c_b) + (1-\pi)u(c_g)$$

$$s.t. \;\; c_0 + p_g c_g + p_b c_b = 2 + 3p_g + p_b$$

Suppose the solution to the consumer's problem is $\{2, 2, 4\}$. What does this mean?

He signs a contract in period 0, then he consumes c_0 (regardless of anything). After period 0, nature determines whether the good state or the bad state happens. NO TRADE happens in period 1. All trade already took place at t=0. All that takes place at t=1, is the fullfilment of whatever promises for deliveries were made at t=0. For example, the given allocation above tells us the following: The guy signs a contract at t=0 promising that he will give up his endowment of 3 units of the good in the good state for delivery of 2 units AND he will give up his endowment of 1 unit of the good in the bad state for the delivery of 4 units. And it also tells us that he will consume 2 units of the good at period 0, no matter what happens.

Remember not to think of this concept as just insurance. Because insurance is only a subset of possible state contingent claims. We are talking about any kind of state contingent claims here, not just the ones which are only geared towards insuring you against the bad state.

Now let's extend this to three periods. We will now have 7 commodities.

The agent's objective function is:

$$u(c_0) + \pi u(c_b) + (1-\pi)u(c_g) + \pi^2 u(c_{bb}) + \pi(1-\pi)\left[u(c_{bg}) + u(c_{gb})\right] + (1-\pi)^2 u(c_{gg})$$

In the Arrow-Debreu world, in complete markets, how many commodities are traded? 7 commodities. It is 7 commodities because the agent need to decide what he wants for each date-event. For t=2, we have four date events, for t=1 we have two date-events, and for t=0 we have one. These date-events are the nodes.

Recall that in the AD world, after period 0, all people do is honour their commitment and deliver promises. No trade takes place after period 0.

How about in the sequence of markets? Trades can occur at more than one node. We want to implement the same type of allocation as in AD with a market arrangement that is simpler and recurrent. Think of the same world that we described above, with two periods and two states. And take note of the fact that at each one of those nodes, trade CAN take place now, unlike in the AD arrangement.

In the sequence of markets, how many things are traded at period 0? Only 2. This is because in the sequence of markets, the agent does not trade for two periods ahead. Also once we go on to t=1, at one of the nodes, say the good state, the agent again only trades for two commodities, he does not do anything about the other states that are following the other node (the bad state) anymore, because the bad state has not happened.

We will characterize what happens in this world through backwards induction. We will first go to the last period (t=1 in this case) and work backwards.

So at t=1, the agent is either at the good state or the bad state. Let's first consider the node associated with the good state.At this node, the agent consumes c_g and he chooses what he will consume if tomorrow's period is good again (c_{gg}) and he chooses what he will consumer if tomorrow's period is bad (c_{gb}). His objective function consists of the utility that he gets from consuming c_g and the expected value of his utility in the next period.

$$V_g(x_g;p) = \max \ u(c_g) + \pi u(c_{gb}) + (1-\pi)u(c_{gg})$$

$$s.t. \ \ c_g + \frac{p_{gb}}{p_g}c_{gb} + \frac{p_{gg}}{p_g}c_{gg} = x_g + 1\frac{p_{gb}}{p_g} + 3\frac{p_{gg}}{p_g}$$

x_g : The agent's past choice on what to get at the node associated with the good state at t=1.

Now consider the node associated with the bad state:

$$V_b(x_b;p) = \max \ u(c_b) + \pi u(c_{bb}) + (1-\pi)u(c_{bg})$$

$$s.t. \ \ c_b + \frac{p_{bb}}{p_b}c_{bb} + \frac{p_{bg}}{p_b}c_{bg} = x_b + 1\frac{p_{bb}}{p_b} + 3\frac{p_{bg}}{p_b}$$

We have basically collapsed what the agent cares for after period 1 to the V functions.

Now go to time 0. The consumer's problem is:

$$\max \ \ \mathrm{u}(\mathrm{c}_0) + \pi V_b(x_b;p) + (1-\pi)V_g(x_g;p)$$

$$s.t. \ \ \mathrm{c}_0 + x_g p_g + x_b p_b = 2 + 3p_g + p_b$$

Constructing ADE from SME and vice versa in this environment:

This is trivial because this time we don't even need to bother with constructing the prices from one world to the other. Notice that in the formulations of the consumer's problem in the sequence of markets, we already have been implicitly using the AD prices given that we know the allocations will be the same. The p's are the AD prices and the SM prices are, for example, $\frac{p_{bg}}{p_b}$, etc.

However, one thing you should be aware of is that ADE gives us certain prices and allocations; whereas in SME we need to determine the prices, allocations AND x_g and x_b.

From ADE to SME:

1. Construct the SM prices:

   $\mathrm{p}_b = \mathrm{p}_b$

   $\mathrm{p}_g = p_g$

2. Use the same allocation:

   $\mathrm{c}_0, c_g, c_b, c_{gg}$

3. Construct the missing items (x_g and x_b)

   Using the budget constraint, get x_g from the prices and allocations in state g; and get x_b from the prices and allocations in state b.

4. Verify that the following SME conditions are satisfied:

   -Markets clear at last period (This is trivial from ADE)

   -x's add up to 0 across consumers.

   -c and x solve the consumer's problem.

From SME to ADE:

1. Get rid of the x's.

2. Verify conditions of ADE.

With two periods:

In the sequence of markets how many things are traded? 9 because we have 3 commodities at each of the 3 nodes.

In the Arrow-Debreu it was 7.

Now suppose we have 100 goods instead. In Arrow-Debreu, we will have 700 things to trade. On the other hand, in SM, we will have 102 goods to trade per node and thus we will have only 306 things to trade.

In the sequence of markets, we have minimal number of trades to get the best allocation. Arrow-Debreu has nice properties but it's messy to deal with.

3.1 Back to the Growth Model

$$\max_{\{c_t(h_t),k_{t+1}(h_t)\}} \sum_{t=0}^{\infty} \beta^t \sum_{h_t \in H_t} \Pi(h_t) u(c_t(h_t))$$

s.t. $$c_t(h_t) + k_{t+1}(h_t) + \sum_{z_{t+1}} q_t(h_t, z_{t+1}) x(h_t, z_{t+1}) = k_t(h_{t-1})[1 + r_t(h_t) - \delta] + w(h_t) + x(h_{t-1}, z_t(h_t))$$

Note: The notation $z_t(h_t)$ just refers to the z that is consistent with history h_t.

In the representative agent model, market clearing requires that $x(h_t, z_{t+1}) = 0 \quad \forall h_t \quad \forall z_{t+1}$

Homework *Consider an economy with 2 periods. There are two states of nature: The good state and the bad state.Both states have equal probabilities. There is only one agent in the economy and he has an endowment of 1 coconut and 2 scallops. In the good state, he will have an endowment of 3 units of the goods and in the bad state, he will have an endowment of 1 unit of each good. The agent's utility function takes the following form:*

$$u(s, c) = \log s + \log c$$

Compute the equilibrium for this economy.

As before when we write down the equilibrium, we do a shortcut and we ignore the x and q. This does not mean that markets are not complete. If all agents are identical then state contingent claims have to be 0 for all nodes.

3.2 General Overview

So far we have shown the following:

SPP $\Leftrightarrow$ AD (From the Welfare Theorems)
SPP$\Leftrightarrow$ Dynamic Programming Problem(What Randy did)

AD$\Longleftrightarrow$ SME

SME $\leftarrow$ sc $\rightarrow$ RCE
RCE $\leftarrow$ sc $\rightarrow$ Dynamic Programming Problem

-sc denotes "something in common"

Notice that RCE and DP are not necessarily equivalent.
Also, SME and RCE are not necessarily equivalent.

Why would it be that SPP $\not\Leftrightarrow$ AD
-Markets may not be complete.
-Externalities

-Heterogenous Agents

So for most equilibria, we need to compute the equilibria directly. We don't have the luxury of solving the social planner's problem to get the equilibrium allocation. Solving the problem from AD and SME, it's very messy. So we will use the RCE notion to characterize what happens in the economy.

4 Feb 20: Defining RCE

4.1 Review

Consider the following two period economy:

The goods A and B at time 0 are denoted by x_0^A ,x_0^B and the goods at time 1 are denoted by x_1^A ,x_1^B.

In Arrow-Debreu, the consumer's problem is:

$$\max \quad \mathrm{u}(\mathrm{x}_0, x_1)$$

$$s.t. \sum_{i=0}^{1} \sum_{l=A,B} p_i^l x_i^l \quad \leq 0$$

In SME,

$$\Omega(b_1^A; q) = \max_{\mathrm{x}_1^A, x_1^B} \quad \mathrm{u}(\mathrm{x}_1^A, x_1^B)$$

$$s.t.\ x_1^A + q_{1B} x_1^B = b_1^A$$

where b_1^A is what the consumer chose to bring from the past. Assume that loans are in the form of good A (don't need to transfer resources in the form of all goods. Saving in the form of only one good is enough.)

Now go to period 0,

$$\max_{\mathrm{x}_0^A, x_0^B, b_1^A} \quad \mathrm{u}(\mathrm{x}_0^A, x_0^B) + \beta \Omega(b_1^A; q)$$

$$s.t.\ x_0^A + q_0^{bond} b_1^A + q_{0B} x_0^B = endowment$$

Homework *Take this simple economy and show the equivalence between SME and ADE*

Homework *Given an ADE, write two sequence of markets equilibria. In one of them, take good A as the good used to transfer resources into the future.In the other, take it as good B. Show that the two allocations are equivalent.*