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1.1 Road map

- From now on, we will look at Recursive Competitive Equilibrium (RCE).
  - In Randy's class, we learned that a Sequential Problem of SPP can be solved using Dynamic Programming. Now we will see that we can use the same Dynamic Programming technique to solve an equilibrium, RCE.
  - First, we know the equivalence between an allocation of SPP and an allocation of ADE, using Welfare Theorems. And we showed that ADE can be represented as SME, where the market arrangements are more palatable. From today, we will see that SME is equivalent to RCE.
  - When Welfare Theorems holds, we do not need to directly solve the equilibrium, because we know that allocation of SPP can be supported as an equilibrium and it is unique, meaning the SPP allocation is the only equilibrium. But if (i) assumptions of Welfare Theorems do not hold or (ii) we have more than one agent, thus we have many equilibrium depending on the choice of the Pareto weight in the Social Planner's Problem, we can solve the equilibrium directly, both in theory and empirically using computer. Since (i) solving ADE is "almost impossible", (ii) solving SME is "very hard", but (iii) solving RCE is "possible", RCE is important for analyzing this class of economies, where Welfare Theorems fail to hold.
- In ADE and SME, sequences of allocations and prices characterize the equilibrium, but in RCE, what characterize the equilibrium are functions from state space to space of controls and values.

1.2 Recursive representation in equilibrium

Remember that the consumer's problem in SME is as follows:

$$\max_{\{k_{t+1},c_t\}_{t=0}^{\infty}} \sum_t \beta^t u(c_t) \tag{1}$$

$$c_t + k_{t+1} = w_t + [1 + r_t]k_t \tag{2}$$

How to translate the problem using recursive formulation? First we need to define the state variables. state variables need to satisfy the following criteria:

1. PREDETERMINED: when decisions are made, the state variables are taken as given.

2. It must MATTER for decisions of agents: there is no sense of adding irrelevant variables as state variable.

3. It VARIES across time and state: otherwise, we can just take it as a parameter.

Be careful about the difference between aggregate state and individual state. Aggregate state is not affected by individual choice. But aggregate state should be consistent with the individual choice (we will consider the meaning of "consistency" more formally later), because aggregate state represents the aggregated state of individuals. In particular, in our RA-NGM, as we have only one agent, aggregate capital turns out to be the same as individual state in equilibrium, but this does not mean that the agent decide the aggregate state or the agent is forced to follow the average behavior, but rather the behavior of the agent turns out to be the aggregate behavior, in equilibrium.

Also note that prices (wages, and rental rates of capital) is determined by aggregate capital, rather than individual capital, and since individual takes aggregate state as given, she also takes prices as given (because they are determined by aggregate state). Again, the aggregate capital turns out to coincide with the individual choice, but it is not because of the agent's choice, rather it is the result of consistency requirement.

One notational note. Victor is going to use a for individual capital and K for aggregate capital, in order to avoid the confusion between K and k. But the problem with aggregate and individual capital is often called as "big-K, small-k" problem, because the difference of aggregate capital and individual capital is crucial. So for our case, the counterpart is "big-K, small-a" problem.

Having said that we guess that candidates for state variables are $\{K, a, w, r\}$. But we do not need $\{r, w\}$. Why? Because they are redundant: K is the sufficient statistics to calculate $\{r, w\}$ and K is a state variable, we do not need $\{r, w\}$ as state variables.

Now let's write the representative consumer's problem in the recursive way. At this point, the time subscript has not be got rid of. People care about today's period utility and the continuation utility from tomorrow t+1:

$$V_t(K, a; G) = \max_{c,a'}[u(c) + \beta V_{t+1}(K', a'; G)] \tag{3}$$

subject to

$$c + a' = w + [1 + r - \delta]a \tag{4}$$

$$w = w(K) \tag{5}$$

$$r = r(K) \tag{6}$$

$$K' = G(K) \tag{7}$$

Fundamental rules to write a well-defined problem:

- All the variables in the problem above: $([u(c)+\beta V(K',a';G^e)])$ have to be either (i) a parameter or an argument of the value function V(.) (state variable), (ii) a choice variable (so appear below max operator, c and a' here), (iii) or defined by a constraint, in order for the problem to be well defined. In the case above, note (i) c and a' is a choice variable, (ii) K' is defined by (7) (which we will discuss below), (iii) the variables in (4) (especially r and w) are also defined by constraints, which only contains state variables (K), thus we know that the problem is well defined.

- Agents need to make expectations about tomorrow's price to make consumption - saving choice. Because prices $\{r,w\}$ are given by marginal product of production functions. Agents have to make "forecast" or "expectations" about the future aggregate state of the world.

- We index the value function with G because the solution of the problem above depends on the choice of G. But what is "appropriate" G? This is revealed when we see the definition of an equilibrium below.

Homework 1.1 *Show the mapping defined by (3) is a contraction mapping. And prove the existence of FP and give the solution's properties.*

1.3 Recursive Competitive Equilibrium:

Now, let's define the Recursive Competitive Equilibrium:

Definition 1.2 *A Representative Agent Recursive Competitive Equilibrium with arbitrary expectation G^E is $\{V(.),g(.),G(.)\}$ such that*

1. $\{V(.),g(.)\}$ *solves consumer's problem:*

$$V(K,a,G^E)=\max_{c,a'}[u(c)+\beta V(K',a';G^E)]$$

subject to

$$c+a'=w\,(K)+[1+r\,(K)-\delta]a \tag{8}$$

$$K'=G^E(K) \tag{9}$$

Solution is $g\left(K,a;G^E\right)$.

2. *Aggregation of individual choice:*

$$K'=G(K;G^E)=g(K,K;G^E) \tag{10}$$

Some comments on the second condition. The second condition means that if a consumer turns out to be average this period (her individual capital stock is K, which is aggregate capital stock), the consumer will choose to be average in the next period (she chooses $G(K)$, which is a belief on the aggregate capital stock in the next period if today's aggregate capital stock is K). You can interpret this condition as "consistency" condition, because this condition guarantees that in an equilibrium, individual choice turns out to be consistent with the aggregate law of motion.

Agents have rational expectation when $G = G^E$. To compute this equilibrium, we can define G^E first, then get g and $G(.)$. The whole sequence of equilibrium choice is obtained by iteration.

Now, let's define A Representative Agent Recursive Competitive Equilibrium with rational expectation.

Definition 1.3 *A Representative Agent Recursive Competitive Equilibrium with rational expectation is* $\{V(.), g(.), G(.)\}$ *such that*

1. $\{V(.), g(.)\}$ *solves consumer's problem:*

$$V(K, a, G) = \max_{c,a'}[u(c) + \beta V(K', a'; G^E)]$$

subject to

$$c + a' = w(K) + [1 + r(K) - \delta]a \tag{11}$$

$$K' = G(K) \tag{12}$$

Solution is $g(K, a; G)$ *.*

2. *Aggregation of individual choice:*

$$K' = G(K) = g(K, K; G) \tag{13}$$

In other words, a RA RCE with rational expectation is a RA RCE with expectation G^E while with additional condition imposed:

$$G\left(K, G^E\right) = G^E(K)$$

1.4 Solve SPP and RCE

When we look at SPP in recursive form, we find a contraction mapping. The SPP is solved as the fixed point of contraction mapping. In math, we define

$$T(V_0)(K) = \max_{K' \in X} R(K, K') + \beta V_1(K')$$

where T maps a continuos, concave function to a continuos and concave function. And we can show T is a contraction mapping. To find the fixed point of this contraction V^*, we can use iteration: for any continuos and concave function V_0,

$$V^* = \lim_{n \to \infty} T^n(V_0)$$

such that

$$V^* = \max_{K' \in X} R(K, K') + \beta V^*(K')$$

But to solve a RE RA RCE, we cannot use such fixed point theorem because we need find (V, G, g) jointly. Similarly, we can define the following mapping $\widehat{T}$ which has three parts corresponding to (V, G) .

$$\begin{aligned} V_1(K, a) &= \widehat{T}_1(V_0, G_0) = \max_{c, a'} u(c) + \beta V_0(G_0(K), a') \\ s.j. \quad c + a' &= w(K) + [1 - \delta + r(K)] a \end{aligned}$$

and the decision rule is

$$a' = g(K, a; G_0)$$

$$G_1(K) = \widehat{T}_2(V_0, G_0) = g(K, K; G_0)$$

We can see the first component of $\widehat{T}$ mapping gives V, and the second part gives G. Fixed point of this mapping $\widehat{T}$ is RE RA RCE.

But, $\widehat{T}$ is not a contraction. It is more difficult to find RE RA RCE in theory, but we will see how we can solve the problem on computer later.

- Another comment about RCE: If there are multiple equilibria in the economy, it is problematic to define RCE. The reason is that RCE solution is functions. Given today's state variable, tomorrow's state is unique. When we construct SME out of RCE $\{..., K_i, K_j, ...\}$, given K_i, there is only one unique K_j.

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2.1 From RCE to SME

Homework 2.1 *Prove that a RCE with RE is a SME.*

Hint 2.2 *You can show by construction. Suppose we have a RCE. Using a_0 (given) and $G(K)$, we can derive a whole sequence of $\{k_t,\ c_t\}_{t=0}^{\infty}$. Using the constructed sequences of allocation, we can construct sequence of prices $\{r_t,\ w_t\}_{t=0}^{\infty}$. Remember that we have necessary and sufficient conditions for SME. we just need to show that the necessary and sufficient conditions are satisfied by the constructed sequences.*

2.2 RCE for the Economy with Endogenous Labor-Leisure Choice

Let's try to write down the problem of consumer. The first try:

$$V(K, a; G) = \max_{c,n,a'}\{u(c, n) + \beta V(K', a'; G)\} \tag{14}$$

subject to

$$c + a' = [1 - \delta + r(K)]a + w(K)n \tag{15}$$

$$K' = G(K) \tag{16}$$

This is an ill-defined problem. Why? Something is missing! $r(K)$ and $w(K)$ are wrong function of price because now K is not sufficient determinant of w and r.From firm's problem, we know

$$w = f_2(K, N)$$

Now we have two options to add the missing piece.

Option 1: write $V(K, a; G, w(.), r(.))$. And the equilibrium condition would be

$$\begin{aligned} w(K). &= f_2(K, N) \\ r(K) &= f_1(K, N) \end{aligned}$$

Option 2: write $V(K, a; G, H)$ where H function is agent's expectation about aggregate labor as function of aggregate capital.

$$N = H(K)$$

then, the price function is

$$\begin{aligned} w(K) &= f_2(K, H(K)) \\ r(K) &= f_1(K, H(K)) \end{aligned}$$

We will use option 2 to write RCE with RE.

Homework 2.3 *Define RCE using option 1*

From now on, we will only look at RCE with rational expectation. Now the consumer's problem is

$$V(K, a; G, H) = \max_{c,n,a'}\{u(c, n) + \beta V(K', a'; G, H)\} \tag{17}$$

subject to

$$c + a' = [1 - \delta + f_1(K, H(K))]a + f_2(K, H(K))n \tag{18}$$

$$K' = G(K) \tag{19}$$

And the solutions are:

$$a' = g(K, a; G, H) \tag{20}$$

$$n = h(K, a; G, H) \tag{21}$$

Definition 2.4 *A RCE is a set of functions* $\{V(.),\ G(.), H(.),\ g(.),\ h(.)\}$ *such that*

1. *Given* $\{G(.), H(.)\}$, $\{V(.),\ g(.),\ h(.)\}$ *solves the consumer's problem.*
2. $$G(K) = g(K, K; G, H) \tag{22}$$

   $$H(K) = h(K, K; G, H) \tag{23}$$

2.3 More on solving RCE

We have known that we can define mapping for

$$V^0(K,a) = T\left(V^1(.)\right) = \max_{a' \in X} u(a,a') + \beta V^1(K',a')$$

Note that we cannot solve a mapping since that's a mechanic thing. Mapping we have here is from a functional space to a functional space. We can only solve equation. For example, the Bellman equation is a functional equation which we can solve.

$$V(K,a) = \max_{a' \in X} u(a,a') + \beta V(K',a')$$

When the mapping we defined above is a contraction mapping (sufficient condition is monotonicity and discounting), then there is a unique fixed point. This fixed point can be obtained by iteration. For RCE, if we fix G and H, we can construct the contraction mapping and get fixed point by iteration. The reason why the value function is fixed point is that in infinite horizon economy, today's view of future is the same as that of tomorrow. For finite horizon economy, we have to solve problem backward, starting from $V_{T-1}(.) = \max u(.) + \beta V_T(.)$

To solve RCE, there are two steps. First, given G and H, we can solve the problem by some approximation methods (you will see this in late May). Second, we have to verify that G and H are consistent in equilibrium. That is agent's expectation is actually correct as what happens in life. Since there is no contraction mapping for

$$\begin{aligned} G'(K) &= g\left(K,K;G^0,H^0\right) \\ H'(K) &= h\left(K,K;G^0,H^0\right) \end{aligned}$$

it is hard to prove existence directly. But we can construct one and verify the equilibrium condition. This is the way to solve RCE.

Although compared with SPP, RCE is hard to solve, it can be used to characterize more kinds of economies, including those environments when welfare theorem does not hold.

2.4 RCE for non-PO economies

What we did with RCE so far can be claimed to be irrelevant. Why? Because, since the Welfare Theorems hold for these economies, equilibrium allocation, which we would like to investigate, can be solved by just solving SPP allocation. But RCE can be useful for analyzing much broader class of economies, many of them is not PO (where Welfare Theorems do not hold). That's what we are going to do from now. Let's define economies whose equilibria are not PO, because of distortions to prices, heterogeneity of agents, etc.

2.5 Economy with Externality

Suppose agents in this economy care about other's leisure. We would like to have beer with friends and share time with them. So other people's leisure enters my utility function. That is, the preference is given by

$$u(c, n, N)$$

where L=1-N is the aggregate leisure.

One example may be

$$\log c + \log(1-n) + (1-n)(1-N)$$[17]

With externality in the economy, competitive equilibrium cannot be solved from SPP.

The problem of consumer is as follows:

$$V(K, a) = \max_{c,n,a'}\{u(c, n, N) + \beta V(K', a')\} \tag{24}$$

subject to

$$\begin{aligned}
c + a' &= [1 - \delta + r]a + wn && (25)\\
r &= F_k(K, N) && (26)\\
w &= F_N(K, N) && (27)\\
K' &= G(K) && (28)\\
N &= H(K) && (29)
\end{aligned}$$

And the solutions are:

$$a' = g(K, a)$$

$$n = h(K, a)$$

We can define RCE in this economy.

Homework 2.5 *Please define a RCE for this economy. Compare the equilibrium with social planner's solution and explain the difference.*

Comments:

1. We will not write G and H in value function since this is the way we see in literature. But you should feel it.

2. What if you only wanna hang out with some friends? Write $\frac{N}{12}$ in the utility function. This is the way we can work with RA framework. We can see how far we can get from RA model. To think how to write a problem with unemployment in a RA model, for example. But if you only wanna hang out with rich guys, RA is not enough. We will see how to model economy with certain wealth distribution later.

2.6 Economy with tax (1)

What is the government? It is an economic entity which takes away part of our income and uses it. The traditional (or right-wing) way of thinking of the role of the government is to assume that the government is taking away part of our disposable income and throw away into ocean. If you are left-wing person, you might think that the government return tax income to household as transfer or they do something we value.

Let's first look at the first version where income tax is thrown into ocean. For now, we assume that the government is restricted by period-by-period budget constraint (so the government cannot run deficit nor surplus).

The consumer's problem is as follows:

$$V(K,a) = \max_{c,n,a'}[u(c,n) + \beta V(G(K), a\prime)] \tag{30}$$

subject to

$$c + a' = a + \{nf_2(K, H(K)) + [f_1(K) - \delta]a\}(1 - \tau) \tag{31}$$

Income tax is proportional tax and only levied on income not on wealth. Depreciation is exempt from tax too.

The government period by period constraint is trivial in this case:

$$\text{government expenditure} = \tau[f(K, H(K)) - \delta K]$$

Remark 2.6 *Notice that the economy does not achieve Pareto Optimality, thus solved by SPP. Because in SPP, marginal rate of substitution equals to marginal rate of transformation. But in this economy, income tax affected equilibrium allocation in the following way: (i) the distortion is in favor of leisure against consumption. Why? Tax is only on income which is needed to get consumption not on leisure, but agent can simply work less to get higher utility. (ii) the distortion is in favor of today against tomorrow. The reason is the return of saving is less due to tax.*

2.7 Economy with tax (2)

Now let's look at an economy where the tax income is returned to household in the form of lump sum transfers.

Consumer's problem is

$$V(K,a) = \max_{c,n,a'}[u(c,n) + \beta V(G(K), a\prime)] \tag{32}$$

subject to

$$c + a' = a + \{nf_2(K, H(K)) + [f_1(K) - \delta]a\}(1-\tau) + T \tag{33}$$

Where T is lump sum transfer. From government period by period constraint, we know

$$T = \tau[f(K, H(K)) - \delta K]$$

The equilibrium in this economy is not Pareto optimal. The reason is that agents tend to work less in order to pay less tax. And they do not realize the lump sum tax they will get from government is affected by their action. But we can not blame them because agent only have power to control what she does, not other's action. Only in a RA world, her action happens to be the aggregate state. We have to separate agent's problem from equilibrium condition.

2.8 Economy with shocks to production

When there is shocks to production, should it be included in state variables?Yes, because shocks matters in two ways: (1) it changes rate of return. (2) it affects the way that economy evolves. Therefore, the state variables are: z, K, a. Consumer's problem is

$$V(z,K,a;G,q_z) = \max_{c,a'(z')}\{u(c) + \beta\sum_{z'}\Gamma_{zz'}V(z',K',a'(z');G,q_{z'})\} \tag{34}$$

subject to

$$\begin{aligned} c + \sum_{z'} q_{z'}(z,K)\,a'_{z'} &= [1-\delta + r(z,K)]a + w(z,K) && (35)\\ r(z,K) &= zf_1(K, H(K)) && (36)\\ w(z,K) &= zf_2(K, H(K)) && (37)\\ K' &= G(z,K) && (38) \end{aligned}$$

- There is a complete set of markets for all possible contingences. So people can sign contract to trade state-contingent goods. What we have in the question above is state-contingent asset. $q(z, z')$ has a fancy name of pricing kernel and it has to induce equilibrium in this economy. Since there is only one RA, in equilibrium, there is no trade.

The decision rule is:

$$a'_{z'} = g_{z'}(z, K, a; G, q_z) \tag{39}$$

Agent is free to choose any asset holding conditional on any z'. That's why there are n_z decision rules. But in equilibrium, there is only one K' get realized which cannot depend on z'.

First, we can get n_z market clearing condition for equilibrium:

$$G(z, K) = g_{z'}(z, K, K) \tag{40}$$

But there are $n_z + 1$ functions to solve in equilibrium: $g_{z'}$ and G. So there is one missing condition. We will see in next class that the missing condition is No Arbitrage condition: If one is free to store capital rather than trade state-contingent claim, the result is the same.

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We have talked about stochastic RCE from last class. The consumer's problem is

$$V(z, K, a) = \max_{c, a'(z')} \{u(c) + \beta \sum_{z'} \Gamma_{zz'} V(z', K', a'(z'))\} \tag{41}$$

subject to

$$\begin{aligned} c + \sum_{z'} q_{z'}(z, K) a'_{z'} &= [1 - \delta + r(z, K)]a + w(z, K) && (42) \\ r(z, K) &= z f_1(K, H(K)) && (43) \\ w(z, K) &= z f_2(K, H(K)) && (44) \\ K' &= G(z, K) && (45) \end{aligned}$$

The decision rule is:

$$a'_{z'} = g_{z'}(z, K, a; G, q_z) \tag{46}$$

In RCE,

$$G(z, K) = g_{z'}(z, K, K) \tag{47}$$

which gives us n_z conditions. But we need $n_z + 1$ conditions. The missing condition is NA.

3.1 No Arbitrage condition in stochastic RCE

If the agent wanna have one unit of capital good for tomorrow, there are two ways to achieve this. One is the give up one unit of consumption today and store it for tomorrow's one unit of capital good. The cost is 1. The other way is to purchase state-contingent asset to get one unit of capital good for tomorrow. How to do this? Buy one unit of state-contingent asset for all the possible z'. That is $a'(z') = 1, \forall z'$.The total cost is

$$\sum_{z'} q_{z'}(z, K)$$

No Arbitrage condition is

$$\sum_{z'} q_{z'}(z, K) = 1 \tag{48}$$

3.2 Steady State Equilibrium

In a sequential market environment, steady state equilibrium is an equilibrium where $k_t = k$, $\forall t$. In a deterministic economy without leisure nor distortion, we can first look at the steady state of SPP. To find steady state, we use Euler equation and equate all the $k's$. Note: Euler equation is a second order difference equation, so there are k's at three different time involved, $k_{t,}k_{t+1}, k_{t+2}$.

In a RCE, steady state equilibrium is when

$$K = G(K)$$

When there is shock in economy, strictly speaking steady state does not exist in the sense of $K = G(K)$. Because now z is evolving stochastically and $K' = G(z, K)$. But we will see the probability measure of (K, z) can be found as a stationary once the capital is set at right range. And of course, the shock has to be stationary somehow itself.

Comments:

1. RCE is stationary automatically in the sense that there is no time subscript in value function and decision rule.
2. For some growing economy, we can always transform it into a non-growing economy, as you may see with Randy.
3. The way that econometricians and macroeconomists look at data are different. Econometricians believe there is a true data generating process underlying the data. Macroeconomists think that real data are generated by people's choice. They test models by

comparing the properties of data generated by model to the real data. We will see how to use model to look at data later in the class.

3.3 FOC in stochastic RCE

$$u'\left(c\left(z,K,K\right)\right)=\beta\sum_{z'}\Gamma_{zz'}u'\left[c\left(z',K',K'\right)\right]\left[1-\delta+r\left(z',K'\right)\right] \tag{49}$$

where from budget constraint,

$$c\left(z,K,a\right)=\left[1-\delta+r\left(z,K\right)\right]a+w\left(z,K\right)-\sum_{z'}q_{z'}\left(z,K\right)a'_{z'}$$

Comment: in (49), the RA condition $a=K$ is used. It is allowed because the substitution is done after we derive first order condition. Agent only optimizes with respect to a', not K'. So, we get correct FOC first. Then, we can apply equilibrium condition that $a'=K'$.

To derive FOC, envelope condition is used.

FOC (a'):

$$-u'\left(c\left(z,K,K\right)\right)q_{z'}\left(z,K\right)+\beta\Gamma'_{zz'}V_3\left(z',K',a'_{z'}\right)=0$$

By envelop condition

$$V_3\left(z',K',a'_{z'}\right)=\left[1-\delta+r\left(z',K'\right)\right]u'\left[c\left(z',K',K'\right)\right] \tag{50}$$

Homework 3.1 *Derive Envelope condition for this problem.*

Therefore,

$$u'\left(c\left(z,K,K\right)\right)=\beta\frac{\Gamma'_{zz'}}{q_{z'}\left(z,K\right)}\left[1-\delta+r\left(z',K'\right)\right]u'\left[c\left(z',K',K'\right)\right] \tag{51}$$

If we can get $c\left(z,K,K\right)$ from SPP, (51) is an equation of $q_{z'}\left(z,K\right)$.

$$q_{z'}\left(z,K\right)=\beta\frac{\Gamma'_{zz'}u'\left[c\left(z',K',K'\right)\right]}{u'\left(c\left(z,K,K\right)\right)}\left[1-\delta+r\left(z',K'\right)\right] \tag{52}$$

(52) gives the price that induce household to choose the same allocation c and a' as from SPP. And such price ensure that agent's decision $g_{z'}(z,K,a)$ does not depend on z' in equilibrium:

$$G\left(z,K\right)=g_{z'}(z,K,K)$$

Remark 3.2 *Price q are related to but not the same as probability $\Gamma'_{zz'}$. It is also weighted by intertemporal rate of substitution to measure people's evaluation on consumption at some event. One simple example: in two period economy where good state and bad state happen with equal probability, to induce people to choose endowment of 2 and 1 at time 1, price for bad state must be higher since consumption at bad state is more valuable to people.*

Remark 3.3 *In this version of stochastic RCE, agent chooses state-contingent asset a' for next period before shocks are realized. When next period comes, z' realizes and production takes place using the saving a'. There are other different timings. Say, consumer chooses consumption and saving after shocks for next period get revealed.*

(49) and (52) are equilibrium condition for RCE. We can also get (49) in the following way:

(52) holds for all z'. If we sum (52) over z' and use the No Arbitrage condition (48), we can get

$$\sum_{z'} \beta \frac{\Gamma'_{zz'} u'\left[c\left(z', K', K'\right)\right]}{u'\left(c\left(z, K, K\right)\right)} \left[1-\delta+r\left(z', K'\right)\right] = \sum_{z'} q_{z'}\left(z, K\right) = 1$$

Therefore,

$$u'\left(c\left(z, K, K\right)\right) = \beta \sum_{z'} \Gamma_{zz'} u'\left[c\left(z', K', K'\right)\right]\left[1-\delta+r\left(z', K'\right)\right]$$

Up to this point, we know that people will same the same amount regardless of tomorrow's state, because the price of state-contingent asset will induce them to do so. Therefore, an equivalent way to write RCE is to let agent choose tomorrow's capital without trade of state-contingent asset. And we can define RCE without $q's$.

Homework 3.4 *Show that if there is a law saying that people have no right to buy state-contingent commodities. Then in equilibrium, the law is not binding. In other word, consumer's problem is equivalent to*

$$V\left(z, K, a\right) = \max_{c, a'} u\left(c\right) + \sum_{z'} \Gamma_{zz'} V(z', K', a')$$

subject to

$$\begin{aligned} c + a' &= [1-\delta+r\left(z, K\right)]a + w\left(z, K\right) \\ K' &= G(z, K) \end{aligned}$$

3.4 Economy with Two Types of Agents

Assume that in the economy there are two types of agents, called type A and type B. Measure of the agents of type A and type B are the same. Without loss of generality, we can think of the economy as the one with two agents, both of whom are price takers.

Agents can be different in many ways, including in terms of wealth, preference, ability, etc. We will first look at an economy where agents are different in wealth. There are 1/2 population of rich people and 1/2 population of poor people. For simplicity, we assume there are no shocks and agents do not value leisure.

The state variables are aggregate wealth of both types, K^A and K^B. Why? We know wage and rental only depends on total capital stock $K = K^A + K^B$. But K is not sufficient as aggregate state variables because agents need know tomorrow's price which depends on tomorrow's aggregate capital.

Agents' preference is the same, so the problem for both types are:

$$V(K_A, K_B, a) = \max_{c,a'} \{u(c) + \beta V(G_A(K_A, K_B), G_B(K_A, K_B))\} \tag{53}$$

subject to

$$c + a' = [r(K) + 1 - \delta]\, a + w(K) \tag{54}$$

Solutions are:

$$a' = g(K_A, K_B, a)$$

In RCE, the equilibrium condition is:

$$\begin{aligned} G_A(K_A, K_B) &= g(K_A, K_B, K_A) \\ G_B(K_A, K_B) &= g(K_A, K_B, K_B) \end{aligned}$$

Homework 3.5 *Show that necessary condition for K to be sufficient state variable is that agents' decision rules are linear.*

Homework 3.6 *Show*

$$G_A(K_A, K_B) = G_B(K_B, K_A)$$

Homework 3.7 *What does the theory say about the wealth distribution in steady state equilibrium for 2-type-agent economy above? Compare it with steady state wealth distribution in island economy where markets do not exist.*