1 April 8

1.1 Unemployment Insurance Policy Analysis (Continued)

Given policy parameter (τ, θ), agent's problem is:

$$V(s,a) = \max_{c \geq 0, a' \geq 0} u(c) + \beta \sum_{s'} \Gamma_{ss'} V(s', a') \tag{1}$$

subject to

$$c + qa' = w(1-\tau) 1_{s=e} + \theta 1_{s=u} + a \tag{2}$$

Optimal solution is $g(s,a;\tau,\theta)$. We can find stationary distribution $x(\tau,\theta)$.

We assume that government has to balance its period by period budget constraint.

$$\int b 1_{s=u} dx = \int w\tau 1_{s=e} dx \tag{3}$$

Since the fraction of people who are unemployed/employed are exogenous, we use $x_{e\cdot}$ denote the proportion of employed people and $x_{u\cdot}$ for unemployed people. Then the government budget constraint is

$$\theta x_{u\cdot} = w\tau x_{e\cdot} \tag{4}$$

From (4), the unemployment benefit, or called replacement rate, θ is totally determined given τ since $x_{e\cdot}$, $x_{u\cdot}$ and w are all exogenous. The unemployment insurance policy is trivially computed given τ.

Remark 1 *Markets are incomplete in this economy. People want to trade state contingent claims or borrow but constrained from doing so. But we can assume "chicken government" in the sense that government has power which is beyond people's ability. (People like chicken. People do not know how to make chicken. Government knows how to make chicken. Government makes chicken). In this economy, government provide unemployment insurance. In the next model, we will see that government can also borrow.*

Now, suppose the current policy is $\widehat{\tau}$, will it be a better policy if τ is set to zero? In other words, should we get ride of unemployment insurance? First thing we should know is that the goodness of policy is measured in social welfare. Under current policy $\widehat{\tau}$, social welfare is

$$\int u[c(s,a;\widehat{\tau})]\, dx(\widehat{\tau})$$

where the optimal decision of individual is $c(s,a;\widehat{\tau})$ and $x(\widehat{\tau})$ is wealth distribution in steady state indexed by policy parameter $\widehat{\tau}$. To investigate the effect of changing policies, can we compare the following social welfare, $\int u[c(s,a;\widehat{\tau})]\, dx(\widehat{\tau})$ and $\int u[c(s,a;0)]\, dx(0)$? The two terms are both in steady state, which means that people have managed to adjust their behavior to the prevailing policies.

But it does not make sense to do such comparison in welfare analysis of policies. To compare welfare among policies, we have to put the economy in the same initial conditions (steady state obtained under $\widehat{\tau}$) and then impose different policies (in our example, the choices are to continue with $\widehat{\tau}$ or to have $\tau = 0$). Another illustrative example: suppose there is a full coverage unemployment policy. People will get the same endowment whether they are unemployed or not. In this case, people will have not incentive to save against risk of s. Now if all the benefit is abolished, people will want to dissave when they are hit by bad shock, but they do not have much assets. The whole adjustment to steady state is a long-run thing. Therefore, we get nothing from direct welfare comparison of two steady states. Another example is that suppose Nigeria is now imposing a perfect set of policy for economy. But will you choose to live in Nigeria or in US which has less perfect policy today? Although in 500 years, Nigeria may be a much better place to live than US, but for now, you will not choose to move there. So, we have to compare policies under same initial conditions.

Initial Condition $\left\langle \begin{array}{ll} \pi^A & u\left[\pi^A, IC\right] \\ \pi^B & u\left[\pi^B, IC\right] \end{array} \right.$

In this case, because price does not depend on the whole wealth distribution, policy analysis is easy. We can analyze in the following steps:

1. Solve agent's problem when policy parameter is $\widehat{\tau}$ and 0 respectively. Decision rules are $g(s, a; \widehat{\tau})$ and $g(s, a; 0)$.

2. The current society wealth distribution is $x(\widehat{\tau})$. When the policy continue to be $\widehat{\tau}$, social welfare is

$$\begin{aligned} W(x(\widehat{\tau}), \widehat{\tau}) &= \int u[c(s, c; \widehat{\tau})] \, dx(\widehat{\tau}) + \beta \int u[c(s, c; \widehat{\tau})] \, dx(\widehat{\tau}) \\ &\quad + ... + \beta^t \int u[c(s, c; \widehat{\tau})] \, dx(\widehat{\tau}) + ... \qquad (5) \\ &= \frac{1}{1-\beta} \int u[c(s, c; \widehat{\tau})] \, dx(\widehat{\tau}) \qquad (6) \end{aligned}$$

   where $W(x(\widehat{\tau}), \widehat{\tau})$ denotes the welfare for economy with distribution $x(\widehat{\tau})$ under policy $\tau = \widehat{\tau}$.

3. If the policy parameter change and stay at 0 now, the evolution of wealth distribution can be obtained in the following way. Construct transition function $Q(s, a, B; 0)$ from agent's decision rule $g(s, a; 0)$ and transition matrix Γ. The sequence of wealth distribution is

$$\begin{aligned} x_0 &= x(\widehat{\tau}) \\ x_1(B) &= \int_{S \times A} Q(s, a, B; 0) \, dx_0, \forall B \in \mathcal{A} \\ &\cdots \\ x_t(B) &= \int_{S \times A} Q(s, a, B; 0) \, dx_{t-1}, \forall B \in \mathcal{A} \qquad (7) \end{aligned}$$

$$
\begin{array}{c}
\cdots \\
\downarrow \\
x(0)
\end{array}
$$

since $x(0)$ is the unique stationary distribution associated with $\tau = 0$. Note here, equivalently, we can also define a mapping operator

$$T(Q, x) = \int_{S \times A} Q(s, a, B; 0) \, dx$$

and find the sequence of distribution.

Then, the social welfare under the new policy is

$$
\begin{aligned}
W(x(\widehat{\tau}), 0) \quad = \quad & \int u[c(s, c; \widehat{\tau})] \, dx_0 + \beta \int u[c(s, c; \widehat{\tau})] \, dx_1 \\
& + ... + \beta^t \int u[c(s, c; \widehat{\tau})] \, dx_t + ...
\end{aligned} \tag{8}
$$

where $W(x(\widehat{\tau}), 0)$ denotes the welfare for economy with distribution $x(\widehat{\tau})$ under policy $\tau = 0$.

4. Compare $W(x(\widehat{\tau}), \widehat{\tau})$ and $W(x(\widehat{\tau}), 0)$. The one with higher social welfare is better.

But in most times, price is not exgonenous. And also, the government's budget constraint may depend on the whole wealth distribution.

1.2 Second example with unemployment insurance policy

Assume that under this unemployment insurance policy, people who have jobs have to pay a proportion of their whole income as unemployment premium. The interest rate on storage is r. That is, with policy (τ, θ), the agent solves the following problem:

$$V(s, a) = \max_{c \geq 0, a' \geq 0} u(c) + \beta \sum_{s'} \Gamma_{ss'} V(s', a') \tag{9}$$

subject to

$$c + a' = [w 1_{s=e} + ra](1 - \tau) + \theta 1_{s=u} + a \tag{10}$$

With this economy, government's revenue depends on total value of wealth, including with labor income and interest from storage. Now, We want to find the implication of a new policy in which τ is cut by one half.

Case 1:

Assume the government is facing period by period budget constraint

$$\int (1_{s=e} w + ra) \tau dx = \theta x_u. \tag{11}$$

We rewrite it as

$$w\tau x_{e\cdot} + \int ra\tau dx = \theta x_{u\cdot}$$

Because $\int ra\tau dx$ is capital revenue and depends on the whole wealth distribution $x(\tau, \theta)$, we cannot infer one policy parameter θ from the other one τ. What we can do is that we guess a sequence of θ_t and see whether it satisfies government period by period budget constraint. Steps:

1. Current policy is $\left(\widehat{\tau}, \widehat{\theta}\right)$, $\tau = \frac{\widehat{\tau}}{2}$, guess a sequence $\{\theta_t\}_{t=0}^{\infty}$ which agents take as given in their optimization problem.

2. Solve

$$\max_{\{c_t(h_t), a_{t+1}(h_t)\}} \sum_t \beta^t \sum_{h_t} \Pi(h_t) u(c_t(h_t)) \tag{12}$$

subject to

$$\begin{aligned} c_t(h_t) + a_{t+1}(h_t) &= \left[a_t(h_{t-1}) + w 1_{s(h_t)=e}\right](1-\tau) \\ &\quad + 1_{s(h_t)=u}\theta_t + a(h_{t-1}), \forall t, h_t \\ &\quad a_0, s_0 given \end{aligned} \tag{13}$$

where $h_t = \{s_0, s_1, ...\}$ is a history of an agent. The solution of the problem is:

$$g_t\left(s, a; \tau, \{\theta_t\}\right) \tag{14}$$

Find distribution x_t accordingly.

3. The government budget constraint is satisfied for all t.

$$\begin{aligned} w\frac{\widehat{\tau}}{2}x_{e\cdot} + \int ra\tau dx_0 &= \theta_0 x_{u\cdot} \\ w\frac{\widehat{\tau}}{2}x_{e\cdot} + \int ra\tau dx_1 &= \theta_1 x_{u\cdot} \\ &\ldots \\ w\frac{\widehat{\tau}}{2}x_{e\cdot} + \int ra\tau dx_t &= \theta_t x_{u\cdot} \\ &\ldots \end{aligned} \tag{15}$$

In practise, we can assume after 100 years, say, economy converges to new steady state. Therefore, (15) is a system of 100 equations with 100 unknowns. We can solve for $\{\theta_t\}_{t=0}^{99}$

Case 2:

We can see from last example that the economy does not converge to new steady state immediately. Because if so, government period by period budget constraint does not hold. But if we assume that government can borrow and lending, then there is little constraint on policy parameter and θ can be any constant. We will work on the implication of change policy $\left(\widehat{\tau}, \widehat{\theta}\right)$ to $\left(\overline{\tau}, \overline{\theta}\right)$.

Now, given initial wealth distribution x, a policy (τ, θ) is a feasible policy if

1. Agents solve

$$V(s,a) = \max_{c\geq 0, a'\geq 0} u(c) + \beta \sum_{s'} \Gamma_{ss'} V(s', a') \tag{16}$$

subject to

$$c + a' = [w1_{s=e} + ra](1-\tau) + \theta 1_{s=u} + a \tag{17}$$

Decision rule is

$$g(s, a; \tau, \theta)$$

Note that since government can issue domestic debt, individual wealth can take the form of storage or debt. But both kinds of assets have the same rate of return which is exogenously given by storage technology.

Transition function is $Q(s, a, B; \tau, \theta)$ and the distribution mapping operator on distribution is $T(Q, x)$. And the wealth distribution evolves

$$\begin{aligned} x_0 &= x \\ x_1 &= T(Q, x_0) \\ &... \\ x_t &= T(Q, x_{t-1}) \\ &... \end{aligned}$$

2. Government budget constraint is satisfied

Present value of government expenditure = Present value of government revenue

That is

$$\sum_{t=0}^{\infty} \frac{\theta x_{u\cdot}}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{\int \tau (1_{s=e} w + ra)\, dx_t}{(1+r)^t}$$

3. Government cannot issue ridiculous amount of debt. Let D_t denote the total government debt at period t. Law of motion for D_t is

$$D_{t+1} = (1+r) D_t - \int \tau [w1_{s=e} + ra]\, dx_t + \theta x_{u\cdot}$$

The total government debt and households asset cannot be negative

$$D_{t+1} + \int a dx_t \geq 0 \tag{18}$$

Therefore, we make sure that the society does not store negative amount.

Remark 2 *If we assume the government can borrow from aboard, then there is no (18) constraint.*

Remark 3 *If (18) is violated, storage becomes negative which cannot be true. Therefore, price will not be exogenously given by storage. It is endogenous to clear the asset market demand and supply.*