MONOPOLISTIC COMPETITION ENDOGENOUS GROWTH AND R&D

Romer's monopolistic competition model has three production sectors, the final goods production, intermediate goods production and R&D i.e. variety production. Our usual TFP parameter in production function will represent the 'variety' in production inputs and as we will see the growth of varieties through research and development firms will make sure a balanced growth path is sustainable. The production function in this economy is,

$$Y_t = L_{1t}^{\alpha} \int_0^{A_t} x_t(i)^{1-\alpha} di$$

where $x_t(i)$ is the type i intermediate good and there is a measure A_t of different intermediate goods and L_{1t} is the amount of labor allocated to the final good production. The production function exhibits CRTS. The intermediate goods are produced with the following linear technology,

$$\int_0^{A_t} \eta x_t(i) di = K_t$$

Now suppose the variety of goods grows at rate γ, $A_{t+1} = \gamma A_t$, is long run sustainable growth possible? The answer to this question will depend whether our final goods production technology is linear in growing terms. We do know by the curvature of the technology, optimality implies equal amount of each variety will be used in production, $x_t(i) = x_t$,then we have,

$$A_t \eta x_t = K_t$$

and our output at this equal variety becomes,

$$Y_t = L_{1t}^{\alpha} A_t {x_t}^{1-\alpha}$$

then substituting for x_t we have,

$$Y_t = \frac{L_{1t}^{\alpha}}{\eta^{1-\alpha}} A_t^{\alpha} K_t^{1-\alpha}$$

thus if both A_t and K_t are growing at rate γ, then production function is linear in growing terms and long run balanced growth is feasible. Note that this model becomes very similar to our previous exogenous labor productivity growth under these assumptions. The purpose of this model is to determine γ endogenously. What will be the source of growth, where does γ come from? As we will see, there will be incentives for R&D firms to produce new 'varieties' because there will be a demand for it. These new varieties will be patented to intermediate good production firms, where a patent will mean exclusive rights to produce that intermediate good. So we will have monopolistic competition in the intermediate goods production. Now suppose the law of motion for 'varieties', which is the technology in R&D sector is given by,

$$A_{t+1} = (1 + L_{2t}\zeta)A_t$$

where L_{2t} is the labor employed in R&D sector. Note that this is not a regular law of motion in the sense every new variety produced helps the production of further new varieties.such that there is a positive externality to variety production. Also assume leisure is not valued and we have aggragate feasibility condition for labor as,

$$L_{2t} + L_{1t} = 1$$

As a homework, we have calculated the BG rate of SP version of this economy, now we will decentralize this economy and characterize the equlibrium growth rate.and see that it is sub-optimal. The period t problem of a firm in the competitive final good production sector is,

$$\max_{x_t(i),L_{1t}}\{L_{1t}^{\alpha}\int_0^{A_t} x_t(i)^{1-\alpha}di - w_tL_{1t} - \int_0^{A_t} q_t(i)x_t(i)di\}$$

and since we have CRTS with perfect competition we have zero profit with following FOCs,

$$w_t = \alpha L_{1t}^{\alpha-1}\int_0^{A_t} x_t(i)^{1-\alpha}di$$

$$\text{and } \ q_t(i) = (1-\alpha)L_{1t}^{\alpha}x_t(i)^{-\alpha}$$

notice that the inverse demand function for good of variety i is,

$$\left(\frac{q_t(i)}{(1-\alpha)L_{1t}^{\alpha}}\right)^{\frac{-1}{\alpha}} = x_t(i)$$

The intermediate goods industry will operate under monopolistic competition. There is only one firm, that is one patent holder, producing each variety. Each firm takes the demand of its variety and prices as given, and solves the following problem each period,

$$\Pi_t(i) = \max_{x_t(i),K_t(i)}\{q_t(i)x_t(i) - R_tK_t(i)\}$$

$$s.t. \quad x_t(i) = \frac{K_t(i)}{\eta}$$

plugging in the inverse demand function and the technology constraint, the FOC is,

$$(1-\alpha)^2 x_t(i)^{-\alpha} L_{1t} = R_t\eta$$

and because of the symmetry we mentioned $x_t(i) = x_t = \frac{K_t}{\eta A_t}$ we can write this FOC as,

$$(1-\alpha)^2 (\frac{K_t}{\eta A_t})^{-\alpha} L_{1t} = R_t\eta$$

i.e. the rental price of capital is not equal to it's marginal product and there is opportunuties for positive profit. But also remember there is a fixed cost of entering to this industry, namely the price paid for the patent. Then as we will see the relation between the two will be one of our equilibrium conditions. Now lets look at the problem of R&D firms,

$$\max_{A_{t+1},L_{2t}}\{p_t^P(A_{t+1} - A_t) - w_tL_{2t}\}$$

$$s.t. A_{t+1} = (1 + L_{2t}\zeta)A_t$$

where p_t^P is the patent of the price. Free entry is assumed thus there will be zero profit in equilibrium. Notice also the R&D firm is solving a static problem without realizing the positive externality this period's decision creates on next periods production. As we will see, this and the monopoly power of the patent owners will be the sources of sub-optimality in decentralized solution.The FOC is,

$$p_t^P = \frac{w_t}{\zeta A_t}$$

where wage is determined in the final goods market and given this price equilibrium quantity will come from the deman function. As we mentioned before, one equilibrium condition will be that at any point in time, total profit a patent generates will be equal to price of it such that there will also be zero profit in the intermediate goods market.

$$p_t^P = \sum_{\tau=t}^{\infty} \frac{\Pi_t(i)}{(1+r)^{\tau-t}}$$

These conditions with constant growht equations for the growing variables is sufficient to characterize the equilibrium growth rate of this economy. For more details see Problem Set 9.