Optimal unemployment insurance (with observable effort)

Consider an economy where the probability of finding a job $p(a)$ is a function of effort $a \in [0,1]$. And we assume that once the agent gets a job, she will have wage w for ever. Thus, the individual problem is

$$\max_{a_t} E \sum_t \beta^t \left[u(c_t) - a_t\right]$$

There are two cases: when the agent has got a job, she will pay no effort and enjoy w for ever. The life long utility is

$$V^E = \sum_t \beta^t u(w) = \frac{u(w)}{1-\beta} \tag{1}$$

When the agent is still unemployed, she will have nothing to consumer. Her problem is

$$V^u = \max_a \left\{ u(0) - a + \beta \left[p(a) V^E + (1 - p(a) V^{u\prime}) \right] \right\} \tag{2}$$

If the optimal solution of a is interior, $a \in (0,1)$, then the first order condition gives

$$-1 + \beta p'(a) \left(V^E - V^u\right) = 0 \tag{3}$$

And since the V^u is stationary,

$$V^u = \max_a \left\{ u(0) - a + \beta \left[p(a) V^E + (1 - p(a) V^u) \right] \right\} \tag{4}$$

Solving (3)(4) gives the optimal a and V^u. Another way is to successively substitute a and obtain solution because (**??**) defines a contraction mapping operator. We can fix V_0^u, then solve (4) to get $a(V_0^u)$ and obtain V_1^u. Keeping going until $V_n^u = V_{n+1}^u$. In a word, optimal effort level a^* solves (4) with $V^u = V^{u\prime}$.

The probability of finding a job $p(a)$ is called hazard rate. If agents did not find a job with effort level a^*, next period, she will still execute the same effort level a^*. Why? Because the duration of unemployment is not state variable in agent's problem. (If agents do not have enough realization about the difficulty of getting a job. With learning, their effort a will increase as they revise their assessment of the difficulty. But such revision of belief is not in this model.)

Now suppose resource is given to people who is unemployed to relive her suffering by a benevolent planner. This planner has to decide the minimal cost of warranting agent a utility level V: $c(V)$. To warrant utility level V, the planner tells the agent how much to consume, how much effort to exert and how much utility she will get if she stay unemployed next period. Obviously, the cost function $c(V)$ is increasing in V.

The cost minimization problem of the planner can be written in the following recursive problem:

$$c(V) = \min_{c,a,V^u} c + [1 - p(a)] \frac{1}{1+r} c(V^u) \tag{5}$$

subject to

$$V = u(c) - a + \beta \left[p(a) V^E + (1 - p(a)) V^u \right] \tag{6}$$

To solve the problem, construct Lagragian function

$$\mathcal{L} = c + [1 - p(a)] \beta c(V^u) + \theta \left[V - u(c) + a - \beta \left[p(a) V^E + (1 - p(a)) V^u \right] \right]$$

FOC: (c)

$$\theta = \frac{1}{u_c} \tag{7}$$

(a)

$$c(V^u) = \theta \left[\frac{1}{\beta p'(a)} - \left(V^E - V^u \right) \right] \tag{8}$$

(V^u)

$$c'(V^u) = \theta \tag{9}$$

Envelope condition

$$c'(V) = \theta \tag{10}$$

We will work on some implication of these conditions:

1. Compare (8) and (3), we can see that the substitution between consumption and effort is different from the one in agent's problem without unemployment insurance. This is because the cost of effort is higher for work that it is from the viewpoint of planner.

2. (9) tells us that the marginal cost of warranting an extra unit of utility tomorrow is θ.,provided that tomorrow V^u is optimally chosen when today's promise is V. And (10) tells us that the marginal cost of warranting an extra unit of V today is θ.

3. Given that c is strictly concave, $V = V^u$.

4. Regardless of unemployment duration, $V = V^u$. So, effort required the the planner is the same over time. Hazard rate is still constant.

Next , we will study the case when effort is not observable. Planner can only choose consumption and V^u. Effort level is chosen optimally by worker and it is unobservable.

Now suppose there is a social planner who will warrant utility level V for unemployed agent, where V summarize all the past information. The cost minimization problem is

$$c(V) = \min_{c,a,V^u} c + [1 - p(a)] \beta c(V^u) \tag{11}$$

subject to

$$V = u(c) - a + \beta \left[p(a) V^E + (1 - p(a)) V^u \right] \tag{12}$$

FOC: (c)

$$\theta = \frac{1}{u_c} \tag{13}$$

(a)

$$c\left(V^u\right) = \theta\left[\frac{1}{\beta p'\left(a\right)} - \left(V^E - V^u\right)\right] \tag{14}$$

(V^u)

$$c'\left(V^u\right) = \theta \tag{15}$$

Envelope condition

$$c'\left(V\right) = \theta \tag{16}$$

The optimal promise for tomorrow is $V^u\left(V\right)$. Now, let's work out the property of $V^u\left(.\right)$.

Lemma 1 *If* $V > V^A$, *then* $c(V) > 0$, *where* V^A *is the utility for unemployed agent when they are in autarky..*

The intuition for the lemma is that if the planner promises the agent something more than what agent can achieve by herself, it will cost the planner something because the planner cannot do anything more than what people can do on their own.

Lemma 2 *Lagrangian multiplier* $\theta > 0$.

The second lemma tells us that if the planner promise more, she has to pay more.

In the problem without unemployment insurance, (**??**) implies that

$$\frac{1}{\beta p'\left(a\right)} = V^E - V^u$$

In the planner's problem, since $c\left(V\right) > 0$, (14) implies that the effort level chosen by the planner are different from agent's choice in autarky. The reason is that effort does not cost that much in planner's thought.

Unobservable effort

When a is not observable, planner can only choose c and V^u. And households choose a optimally. Now it becomes a principle-agent problem. We will solve the problem backward.

If given c and V^u, the agent will solve

$$\max_a u(c) - a + \beta \left[p(a) V^E + (1 - p(a)) V^u \right] \tag{17}$$

FOC is

$$[p'(a)\beta]^{-1} = V^E - V^u \tag{18}$$

This FOC gives an implicit function of a as a function of V^u: $a = g(V^u)$. (Because c and a are separate in the utility function, a is not a function of c).

Then, the planner solve her cost minimization problem, in which the optimality condition is also one constraint.

$$c(V) = \min_{c,a,V^u} c + [1 - p(a)] \beta c(V^u)$$

subject to

$$\begin{aligned} V &= u(c) - a + \beta \left[p(a) V^E + (1 - p(a)) V^u \right] && (19) \\ 1 &= [p'(a)\beta] \left[V^E - V^u \right] && (20) \end{aligned}$$

Lagragian is

$$\begin{aligned} & c + [1 - p(a)] \beta c(V^u) + \theta \left[V - u(c) + a - \beta \left[p(a) V^E + (1 - p(a)) V^u \right] \right] \\ & + \eta \left[1 - [p'(a)\beta] \left[V^E - V^u \right] \right] \end{aligned}$$

FOC: (c)

$$\theta^{-1} = u_c$$

(a)

$$c(V^u) = \theta \left[\frac{1}{\beta p'(a)} - \left(V^E - V^u \right) \right] - \eta \frac{p''(a)}{p'(a)} \left(V^E - V^u \right) \tag{21}$$

(V^u)

$$c'(V^u) = \theta - \eta \frac{p'(a)}{1 - p(a)} \tag{22}$$

Envelope condition

$$c'(V) = \theta \tag{23}$$

Again, (22) tells the marginal cost to warrant additional amount of delayed promise. (23) gives the marginal cost to increase today's utility. The Lagrangian multiplier associated with constraint (20) is positive, $\eta > 0$, which means that the constraint is binding. So,

$$\eta \frac{p'(a)}{1 - p(a)} > 0$$

Therefore, we have

$$c'(V^u) < c'(V) \Rightarrow V^u < V$$

from the strict concavity of $c(.)$. The delayed promised utility decreases over time.

Let $\theta^u = \theta - \eta \frac{p'(a)}{1-p(a)}$, then $\theta^u < \theta$,which tells us about the consumption path. Consumption decreases over time because $\theta^{-1} = u_c$.

Overall, we get the following model implications: optimal unemployment insurance says that longer unemployment period the agent stays, the less insurance she will be insured for. In this way, the planner induces the higher effort level. Although you cannot let people do what is optimal, such behavior can be achieved by giving out less consumption and promised utility over time. This model implies that time-varying unemployment insurance plan is optimal, under which the replacement rate θ goes down over time.