NOTES ON INDUSTRY EQUILIBRIUM

Consider a firm who uses labor to produce the output good and has productivity $s \in S$. The production technology of the firm is given by $y = sf(n)$. The firm chooses how much labor to employ given the wage, w, and given its productivity,s. The firm's problem is,

$$\max_n psf(n) - wn$$

The solution n^* solves,

$$psf'(n) = w$$

Then the two period profit of the firm is,

$$\pi_2 = [p^* sf(n^*) - wn^*] \left[1 + \frac{1}{1+r}\right]$$

Now suppose that the firm will only operate next period with probability $(1-\delta)$. With probability δ it will die. In that case, the two period profit of the firm is,

$$\pi_2 = [p^* sf(n^*) - wn^*] \left[1 + \frac{1-\delta}{1+r}\right]$$

Now consider the infinite periods profit of the firm,

$$\begin{aligned} \pi_\infty &= [p^* sf(n^*) - wn^*] \sum_{t=0}^{\infty} \left(\frac{1-\delta}{1+r}\right)^t \\ &= [sf(n^*) - wn^*] \left(\frac{1+r}{r+\delta}\right) \end{aligned}$$

The zero profit condition is that the profit from entry is equal to the cost of entry, denoted by c_e. This condition says that there are no further incentives to enter the industry:

$$c_e = \pi_\infty$$

Define $x : \mathcal{S} \to R$ as the measure of firms, where $\mathcal{S}$ is the $\sigma - algebra$ defined on the set S

DEFINITION

An industry equilibrium is a set $\{p^*, y^*, n^*, x^*(s)\}$, such that

1) $p^* = p(y^*)$ (demand is satisfied)

2) $y^* = x^*(s, p^*)\ \ s\ f(n^*)$ (feasibility)

3) Firms optimize: $n^* \in \arg\max_n \ \ p^* s f(n) - w^* n$

4) Zero profit condition: $c_e = \pi_\infty$

TWEAK No1

Suppose that each firm has to pay a cost of entry c_e, and the productivity shock is drawn from the distribution $\gamma(s)$. Once the firm draws s it keeps it forever.

DEFINITION

An industry equilibrium is a set $\{p^*, y^*, n^*, x^*(s)\}$, such that

1) $p^* = p(y^*)$ (demand is satisfied)

2) $y^* = \int\limits_S \ s\ f(n^*(s))\, dx^*$ (feasibility)

3) Firms optimize: $n^* \in \arg\max_n \ \ p^* s f(n) - w^* n$

4) Zero profit condition: $c_e = \int\limits_S \ \pi(s)\, d\gamma(s)$

Note that here, the distribution of firms completely reflects the distribution from which they draw their productivity shocks, $\gamma(s)$. This is because what types of firms remain or what types of firms exit is not an issue since there is exogenous entry and exit. For example, if exit was endogenous we would expect the 'bad' firms to exit and the better ones to stay, and therefore the type distribution of incumbent firms would be different than the initial distribution $\gamma(s)$. But in our case, the distribution of incumbents and the initial type distribution are identical.

So this model is not interesting because it has no economics. The next version of the model that we study is:

TWEAK No 2 (Changing productivity)

Here s is drawn from $\gamma(s)$ as before, but after the initial shock is obtained, $s' \sim \Gamma_{ss'}$. We will assume that Γ satisfies First Order Stochastic Dominance. This means that

$$For\ s_1, s_2 \in S,\ \ s_1 < s_2 \Rightarrow \int_{\tilde{s}}^{\bar{s}} \Gamma(s_1, s)\ ds \leq \int_{\tilde{s}}^{\bar{s}} \Gamma(s_2, s)\ ds$$

The entry and exit decisions are still exogenous. Incumbent firms die at rate δ. The following condition is needed for stationarity:

$$x^*(B) = \int_S \int_S (1-\delta)\Gamma_{ss'} 1_{s' \in B}\ dx^*(s)\ ds' + \int_S 1_{s' \in B} d\gamma(s'),$$

where B is a Borel set over the state space.

TWEAK No 3 (Endogenous Exit Decision of Firms)

Now at each period firms make entry and exit decisions. What is a sufficient mechanism to get firms to quit? Having fixed costs. A fixed cost c_f must be paid every period by incumbent firms. Each period, incumbent firms decide to stay or exit.

The value of a firm with current productivity shock s is givne by:

$$\pi(s) = \max \ \left[\max_n \ p^* s f(n) - wn - c^f + \tfrac{1}{1+r} \int_S \Gamma_{ss'} \pi(s') ds', \ \ 0 \right]$$

As we saw in class the decision of the firm (under the crucial assumption of FOSD of the transition matrix Γ) will be characterized by a threshold. There exists a $s^* \in S$, such that if $s < s^*$ the firm quits, abd if $s \geq s^*$ the firm stays in the industry.

The formula for a stationary distribution (see also Problem Set 6) is the folloing:

$$x^*(B) = \int_S \left(\int_{s^*}^{\bar{s}} \Gamma_{ss'}\ dx^*(s) \right) 1_{s' \in B}\ ds' + \left(\int_{\underline{s}}^{s^*} dx^* \right) \int_S 1_{s' \in B} d\gamma(s)$$

Note that $\left(\int_{\underline{s}}^{s^*} dx^* \right)$ is the number of firms that exit the market, and which will be equal to the number of firms that enter (in equilibrium). Of course, these firms will draw their productivity shock from the distribution γ, and this is why the integration in the second term is with respect to that distribution instead of x^*.

The market clearing condition is given by

$$y^D(p^*) = \int_S s f(n^*) dx^*(s) + \left(\int_{\underline{s}}^{s^*} dx^* \right) \int_S s f(n^*(s)) d\gamma(s)$$

The zero profit condition:

$$c_f = \int \pi(s) d\gamma(s)$$

The interpretation is that the firm that is out of the market should have expected profits equal to the fixed cost, because if it enters the firm has to pay that cost at least for one period (if it enters and the shock it obtains is really bad, the firm can walk out in the second period).

Employment protection with Firing Costs

Note that in this problem the labor force of the last period is a state variable for the firm. This means that the state space will also be different. The new state space is given by $X = S \times N$. N is the set of the possible values of labor force. For convenience assume that it is bounded, i.e, $N = \left[0, \bar{N}\right]$, where $\bar{N} < \infty$.

Assuming that there is a cost of firing equal to a per worker, the profit function is given by

$$\Pi\left(s, n^{-1}\right) =$$

$$\max \left\{ -a\ n^{-1},\ \max_n \left[p\ s\ f(n) - w\ n -\ a\left(n^{-1} - n\right)\ \{n^{-1} > n\}\ + \tfrac{1}{1+\ r} \sum_{s'} \Gamma_{s\ s'} \Pi\left(s', n\right) \right] \right\}$$

In Problem Set 6 you had to find a formula for the stationary distribution in this model. Maintaining the assumption of FOSD, we saw that this formula will be given by:

$$x^*(B) = \int_S \int_N \left[\int_{s^*(n^{-1})}^{\bar{s}} \int_N \Gamma_{s\ s'} dx^*\left(s, n^{-1}\right) \right] \left\{\left(s', n\left(s, n^{-1}\right)\right) \in B\right\}\ ds' dn +$$

$$+ \left(\int_{\underline{s}}^{s^*(n^{-1})} \int_N dx^*\left(s, n^{-1}\right) \right) \int_S \{(s', n(s, 0)) \in B\}\ d\gamma(s'),$$

where

$B \in \mathcal{X}$, the set of subsets of the (new) state space,

$\int_{\underline{s}}^{s^*(n^{-1})} \int_N dx^*\left(s, n^{-1}\right)$ is the number of firms that quit (or- in equilibrium- enter the market),

and $\int_S \{(s', n(s,0)) \in B\}\, d\gamma(s')$ is the probability measure that a new firm will end up in B (that's why $n^{-1} = 0$)