Macro 702, Sp 2005, First Midterm.

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In the following there are **??** questions for **??** points. Be as BRIEF as you can and good luck. You have 80 minutes.

Growth Models (lagged vacation jealousy)

There is an economy with many identical consumers and infinite time. Consumers have preferences

$$E \left\{ \sum_{t=0}^{\infty} \beta^t \, u(c_t, n_t, N_{t-1}) \right\}$$

where c_t is own consumption at time t, n_t is the fraction of time worked by the agent at time t and N_{t-1} is the economy wide average time worked in period $t-1$. The first partial derivative of u is positive while the others are negative.

Output can be produced with labor and capital according to a standard neoclassical production function

$$z_t \, F(K_t, N_t)$$

where K_t, is capital. Shocks to productivity z have finite support and follow a Markov chain with transition matrix Γ. Capital depreciates at rate δ. Output can be used either for consumption or for investment purposes.

1. (10 points) Define an Arrow-Debreu competitive equilibrium. Carefully define the commodity space, and the consumption and production possibility sets.

2. (5 points) State the two welfare theorems.

3. (5 points) Briefly describe what may go wrong for the first welfare theorem to hold.

Suppose now that the household owns capital and rents it to firms.

4. (10 points) Define a recursive competitive equilibrium. Make sure that you list the state variables. Briefly describe what is NOT standard in this problem.

Now assume that the government taxes labor at rate τ and returns the proceeds as lump sum subsidies.

5. (8 points) Write a formula that links the equilibrium subsidy as a function of the τ and the state variables.

6. (7 points) Imagine that the utility function is separable in all its arguments. Does this imply any simplification to your answer to the definition of recursive competitive equilibrium? Explain.

Lucas trees

Assume there is a representative agent economy. Each agent owns a tree that produces fruit d_t which follows a Markov chain with transition matrix Γ. In addition, each agent has one (divisible) unit of time that can be transformed into fruit on a one to one basis. The agent has preferences given by

$$E\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{c_t^{1-\sigma}}{1-\sigma}+\alpha(1-n_t)\right]\right\}$$

where n_t is the amount of time spent producing additional fruit.

7. (5 points) Define equilibria recursively assuming that the tree prices only depend on the current yield and that there is no market for labor (each agent produces with their own time fruit for themselves).

8. (5 points) Write a formula for an option to buy land tomorrow at price p_1 and then reselling it at price p_2 the period after.

9. (5 points) Make any assumptions that you want to ensure that in equilibrium the amount of consumption is constant.

10. (5 points) Under the assumptions of the previous question, characterize tree prices.