Problem Set 1

Econ 702, Spring 2005

January 15, 2005

Problem 1 *For a representative agent economy prove the following:*

$$x^* \in PO(\varepsilon) \Leftrightarrow x^* \in \arg\max_{x \in X} u(x) \tag{1}$$

Problem 2 *Consider the following social planner's problem:*

$$\max_{\{c_t, l_t, n_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2}$$

$$\begin{aligned} s.t. \qquad & c_t + k_{t+1} = f(k_t, n_t) + (1-\delta)k_t \\ & k_0 \quad \textit{given,} \\ & c_t, k_{t+1}, n_t, \ell_t \geq 0 \\ & \ell_t + n_t = 1 \end{aligned}$$

Show that the set of feasible allocations is convex and compact.

Problem 3 *Consider the social planner's problem (SPP)above with Cobb-Douglas technology, partial depreciation and CRRA preferences. Derive the euler equation.*

Problem 4 *Defining the commodity space as a space of bounded real sequences,*

$$\mathcal{L} = \{ \{\ell_{it}\}_{t=0}^{\infty}, \ \sup_{i,t} |\ell_{it}| < \infty \quad \forall \ell \} \tag{3}$$

Prove that $\mathcal{L}$ endowed with the supnorm is a topological vector space (TVS). Also prove that R^n endowed with the usual Euclidian norm is a TVS.

Problem 5 *Show that the consumption possibility set, X, and the production possibility set, Y are convex and has an interior point (endowed with supnorm).*

Problem 6 *Show that the set of feasible allocations is compact ($X \cap Y$)*

Problem 7 *Let (p^*, x^*, y^*) be an AD equilibrium. Setup the household and firm problem in AD language and derive the prices from the given equilibrium allocations and FOCs. Show that the following mapping constitutes a SME (by verifying the FOCs of SME problem is satisfied)*

$$
\begin{aligned}
c_t^* &= x_{1t}^* - x_{2t+1}^* && \forall t && (4)\\
n_t^* &= x_{3t}^*, \quad \ell_t^* = 0 && \forall t && (5)\\
k_t^* &= x_{2t}^* && \forall t && (6)\\
R_t^* &= \frac{p_{2t}^*}{p_{1t}^*} && \forall t && (7)\\
w_t^* &= \frac{p_{3t}^*}{p_{1t}^*} && \forall t && (8)
\end{aligned}
$$