Homework 2

Spring2005, Econ702

Problem 1 Consider the sequence of markets setting in a stochastic economy that we discused in class. Find an arbitrage condition that links the price of state contigent claims, $q_{t+1}(h_t, z_{t+1})$ and the interest rate $R_{t+1}(h_{t+1})$.

Problem 2 On Tuesday's lecture we wrote down the problem of the representative agent in the sequence of markets setting, and we obtained a (general) formula for the price of the state "n" contigent claim, $q_{t+1}(h_t, z^n)$. Repeat the analysis using particular functional forms for preferences and technology. More precisely, use a constant relative risk aversion (CRRA) utility function and a Cobb-Douglas (CD) production function. Assume full depreciation ($\delta = 1$). Also assume that the productivity shock evolves according to a three state Markov Chain, so that for every $t,\ z_t \in \{z^1, z^2, z^3\}$. Give formulas for the prices of the state contigent claims. Finally, define a Suquence of Markets Equilibrium (SME) for this specific example.

Problem 3 Consider a stochastic economy in which the production function $f_t = f(k_t, 1)$ is multiplied by a random shock,

$$z_t,\ z_t \in \{z^1, z^2, ..., z^M\}$$

Using the same functional forms as in Problem 2 (but now without full depreciation)

i) Write down the Social Planner's problem,
ii) Find the Euler Equation, and
iii) Characterize the steady state solution as much as you can.

Problem 4 Consider the following problems:

Problem (1)

$$\max_{\{c_t, k_{t+1}\} \geq 0} \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$
$$s.t : c_t + k_{t+1} = A k_t^a\ ,\ A > 0, a \in (0, 1)$$

Problem (2)

$$\max_{\{c_t, A_{t+1}\} \geq 0} \sum_{t=0}^{\infty} \beta^t \frac{1}{1-a} c_t^{1-a}$$
$$s.t :\ A_{t+1} = R(A_{t-c_t}),\ R > 0$$

For both problems find closed form solutions for the value function and the decision rule (Hint: for the second problem "guess" a value function of the form $V(A) = BA^{1-a}$, where B is a constant to be determined).

Problem 5 We have seen in class that the infinite horizon household problem can be formulated in a recursive manner as a functional equation:

$$v(K, a; G^e) = \max_{c, a' \in [\underline{a}, \overline{a}]} u(c) + \beta v(K', a'; G^e) \tag{1}$$

subject to

$$\begin{aligned} c + a' &= w + aR \\ w &= w(K) \\ R &= R(K) \\ K' &= G^e(K) \end{aligned}$$

where $G^e(.)$ is the household's expectation about aggregate capital next period given current value K.

a. Prove that given $G^e(.)$ as a part of the environment and under suitable assumptions on primitives, there exists a unique function $V(K, a; G)$ that solves the Bellman equation above and it is the fixed point of the operator T, where T is defined as;

$$v^{n+1} = Tv^n = \max_{c, a' \in [\underline{a}, \overline{a}]} \{u(c) + \beta v^n(K', a'; G^e)\} \tag{2}$$

b. Show that under appropriate assumptions on primitives, the resulting value function is strictly increasing, strictly concave, continuous and bounded.