Econ 702, Spring 2005
Problem Set 3
Due Thursday February 11th

Problem 1 Consider the Cobb-Douglas production function $Y = F(K, N) = AK^a N^{1-a}$. Assume that the prices of factors are determined competitively.

Show that the payments to factors exhaust the product (this is just Euler's Theorem for homogeneous functions).

What is the capital's share?

What is the labor's share?

Show that the Cobb-Douglas function is compatible with the stylized facts characterizing modern economies:

output per capita is increasing, capital per capita is increasing, the real wage is increasing and the interest rate is considerably constant.

Problem 2 Consider an infinite horizon, representative agent economy with an externality. In particular the agent enjoys leisure, but she'd rather not spend her free hours alone. So her utility function is given by

$$u(c, n, N), with \ u_1 > 0 \ , \ u_2 < 0 \ \ and \ \ u_3 < 0$$

where n is the number of hours worked by the agent and N is the total amount of hours worked in this economy (by

her friends).

Set up the dynamic problem. Which are the state varibles?

Define the Recursive Competitive Equilibrium and characterize it as much as you can.

Show in which dimension the equilibrium allocation is different than the Social Planner's allocation.

Finally, prove that the total amount of hours worked in the competitive economy is greater than the amount of hours that the Social Planner's solution would imply.

Problem 3 Consider an infinite horizon, representative agent economy with goverment, as the one studied in class.

There is a constant income tax (both on wage and capital income) and the tax revenues are returned to the consumers through a lump sum transfer. Repeat the analysis that we did in class (derive the First Order Conditions, find the Envelope Conditions and compare your solution to the Social Planner's Problem), assuming now that leisure (or hours worked) appears in the utility function of the representative agent.

Problem 4 (A combination of problems 2 and 3). In problem 2 you showed that the total amount of hours worked in the competitive economy is greater

than the amount of hours that the Social Planner's solution would imply. Working in the same setting as in Problem 2, introduce goverment into the economy. Find a suitable tax policy, so that the interaction of the tax and the externality leads to a Pareto Optimal allocation (in the sense that your new allocation coincides with the allocation implied by the solution to the Social Planner's Problem). Remember to keep a balanced Goverment Budget Constraint.

Problem 5 Consider an economy in which the goverment raises taxes and issues debt in order to pay previous debt. Set up the problem in a recursive form. Argue that a sequence of debt $\{B_t\}_{t=0}^{\infty}$ that satisfies $\lim_{t \to \infty} B_t = \infty$ cannot be part of an equilibrium.