Econ 702, Spring 2005
Problem Set 5

Problem 1 *Formulate the Gini index and kurtiosis of the distribution of our agents over wealth levels given by $X(a)$. Gini index is given by the area between the 45 degree line and Lorenz curve when the area of the triangle between the axes and the 45 degree line is normalized to one.*

Problem 2 *Consider the following transition matrix for employment and unemployment states,*

$$\begin{pmatrix} \Gamma_{ee} & \Gamma_{ue} \\ \Gamma_{eu} & \Gamma_{uu} \end{pmatrix}$$

noting that the consistency implies $\Gamma_{ee} + \Gamma_{eu} = 1$ .Assuming that it satisfies the requiriements for the existence of an unique stationary distribution associated with it, solve for this stationary distribution.

Problem 3 *Let $(E, \mathcal{E})$ and $(A, \mathcal{A})$ be measurable spaces; let $(F, \mathcal{F})$ be the product space and let Γ be the transition matrix defined on E. Let $g : F \to A$ be a measurable function. Then show that,*

$$Q(e, a, B) = \sum_{e_j \in B} \Gamma_{ee'} 1_{[\{e_j, g(e,a)\} \in B]}$$

defines a transition function on $(F, \mathcal{F})$.

Problem 4 *Imagine a Archipelago that has a continuum of islands. There is a fisherman on each island. The fishermen get an endowment e each period and e follows a first degree Markov chain with transition $\Gamma_{ee'}$ and,*

$$e \in \{e^1, ..., e^{n_e}\}$$

The fishermen cannot swim. There is a storage technology such that if the fishermen store q units of fish today, they get 1 unit of fish tomorrow. The problem of the fisherman is:

$$V(a, e) \underset{c \geq 0, \overline{a} \geq a' \geq 0}{= \max} U(c) + \beta \sum_{e'} \Gamma_{ee'} V(a', e') \tag{1}$$

$$s.t. \quad c + qa\prime = e + a$$

Show that the problem above has a solution if $q > \beta$, using Blackwell's sufficient conditions.