Econ 702, Spring 2005
Problem Set 7
Due Thursday March 24

Problem 1 Hugget Economy
Read the 03/15/05 lecture notes. Consider a different version of the model. In particular, consider an economy which allows loans and storage, but in which there is a negative externality on storage. For example, if too much of the commodity is stored, then rats eat it.

Define equilibrium following the three different ways, exactly as we did in class.

Growth Models

Problem 2 Use a Lucas Tree economy to price the following "asset": Having a growth rate of 2% instead of 1% per period. In other words, answer the question " how much would you be willing to pay in order to obtain this higher growth rate? ".

Problem 3 Prove or disprove the follwing claim: If the production function is CES, and under different K/L ratios, the capital and labor shares (over income) remain the same.

Problem 4 Consider the following problem of the Social Planner (in an economy with constant population growth γ)

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \ N_t \ u\left(\frac{c_t}{N_t}\right)$$

$$s.t: \quad c_t + k_{t+1} = \ F(k_t, N_t) + (1-\delta)\, k_t,$$

$$\text{where } N_t = \gamma^t N_0.$$

Transform this problem into the equivalent "hat" problem, i.e, the problem in which all variables have been transformed according to the rule $\hat{x}_t = x_t/\gamma^t$, $\gamma > 1$.

Define a Steady State for this economy and characterize it.

What happens in the original economy when the transformed economy is at the Steady State?

Problem 5 Consider an identical enviroment as the one described in the previous problem. Write down a "consistent" problem of the representative agent (consistent in the usual sense that all variables are either states, or controls, or well defined functions of the above). Define a competitive equilibrium for this economy.

Problem 6 Compute the intertemporal rate of substitution for the following preferences

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

in a model with labor augmenting technological progress, i.e, an economy in which output is given by $Y_t = F(k_t, \gamma_n^t N_t)$.

Use the transformation $\hat{c_t} = c_t/\gamma_n^t$, $\gamma_n > 1$.

After applying the transformation, what is the "new" discount rate? What role does does the value of σ play here?

Problem 7 Consider the following specifications of exogenous technological progress:

$$Y_t = F(\gamma_k^t\, k_t, N_t), \text{ capital augmenting technological progress}$$

$$Y_t = \gamma^t F(\gamma_k^t\, k_t, \gamma_n^t\, N_t), \text{ total productivity technological progress, and}$$

$$k_{t+1} = \gamma_i^t\, i_t + (1-\delta)\, k_t, \text{ investment specific technological progress,}$$

$$\text{Where } \gamma_k, \gamma_n, \gamma_i > 1.$$

For each of the above specifications write down the Social Planner's problem, and apply a suitable "hat" transformation so that you can obtain a Steady State for the transformed economy.